Probabilistic Reasoning

CS 470 Introduction To Artificial Intelligence

Daqing Yi

Department of Computer Science
Brigham Young University

Outline

1. Probabilistic Logic
   - Noisy OR
2. Naive Bayes
   - Bayesian inference
   - Naive Bayes
3. Bayesian Network
   - Bayesian network
   - Interpretation
   - Application

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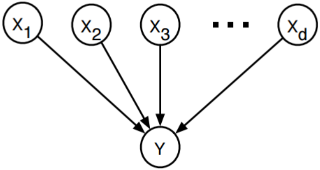
Probabilistic logic

deductive logic + uncertainty

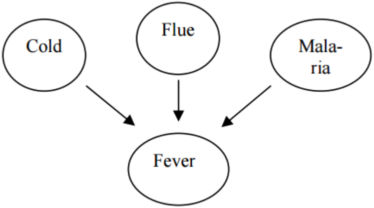
Noisy OR



- Failure probability q_i
- $\neg X_i$ does not have any influence on Y
- Independence of failure probability q_i

$$P(Y \mid X_1 \cdots X_n) = 1 - \prod_{i:X_i=true} q_i$$

Noisy OR



$$
\begin{aligned}
q_{cold} &= P(\neg fever \mid cold, \neg flu, \neg malaria) = 0.6 \\
q_{flu} &= P(\neg fever \mid \neg cold, flu, \neg malaria) = 0.2 \\
q_{malaria} &= P(\neg fever \mid \neg cold, \neg flu, malaria) = 0.1
\end{aligned}
$$

Noisy OR

Cold	*Flu*	*Malaria*	$P(Fever)$	$P(\neg Fever)$
F	F	F		
F	F	T		0.1
F	T	F		0.2
F	T	T		
T	F	F		0.6
T	F	T		
T	T	F		
T	T	T		

Noisy OR

Cold	*Flu*	*Malaria*	$P(Fever)$	$P(\neg Fever)$
F	F	F	0.0	1.0
F	F	T	0.9	0.1
F	T	F	0.8	0.2
F	T	T	0.98	$0.02 = 0.2 \times 0.1$
T	F	F	0.4	0.6
T	F	T	0.94	$0.06 = 0.6 \times 0.1$
T	T	F	0.88	$0.12 = 0.6 \times 0.2$
T	T	T	0.988	$0.012 = 0.6 \times 0.2 \times 0.1$

Outline

Bayesian inference

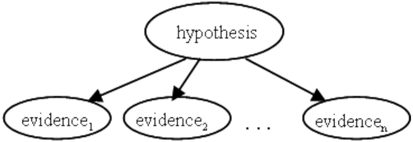
MAP estimation

$$P(x \mid e_1, \cdots, e_n) \propto P(e_1, \cdots, e_n \mid x)P(x)$$

Assumption

Simplification $\longrightarrow$?

Naive Bayes



Different evidences are conditionally independent given the hypothesis

$$P(e_1, \cdots, e_n \mid x) = \prod_{i=1}^{n} P(e_i \mid x)$$

Naive Bayes

Bayesian inference

- Posterior $P(x \mid e_1 \cdots e_n)$
- MAP estimation

$$x^* = \arg\max_x P(x \mid e_1, \cdots, e_n) \propto P(x) \prod_{i=1}^{n} P(e_i \mid x)$$

Naive Bayes

Application: spam filter

- Naive Bayes Classifier
- Label a message as spam if
  $P(spam \mid message) > P(\neg spam \mid message)$
- bag of wods
  - a set of words $\{w_1, \cdots, w_n\}$
  - Conditional independence of words given $(spam \text{ or } \neg spam)$

Naive Bayes

Application: spam filter

$P(spam \mid message)$ vs $P(\neg spam \mid message)$

- $P(spam \mid message) \propto P(spam) \prod_{i=1}^{n} P(w_i \mid spam)$
- $P(\neg spam \mid message) \propto P(\neg spam) \prod_{i=1}^{n} P(w_i \mid \neg spam)$

- likelihood $P(w_i \mid spam), P(w_i \mid \neg spam)$
- priors $P(spam), P(\neg spam)$

How to calculate?

Naive Bayes

Application: spam filter

- The probability of seeing w_i in a spam message

$$P(w_i \mid spam) = \frac{COUNT(w_i \text{ occurrences in spam messages})}{COUNT(\text{all words in spam messages})}$$

- The probability of a message is spam

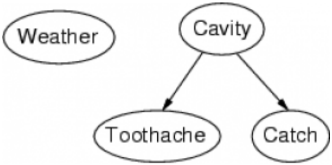
$$P(spam) = \frac{COUNT(\text{spam messages})}{COUNT(\text{all messages})}$$

Outline

Bayesian networks

- a type of graphical model
- a network of random variables
- represent a full joint distribution
- conditional independence between random variables

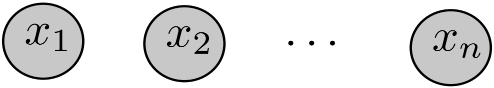
Bayesian networks



- Directed Acyclic Graph
- Node: random variable
  - observed node
  - unobserved node
- Edge: dependence
- Probability:
  - discrete : probability mass function or conditional probability table
  - continuous : probability density function

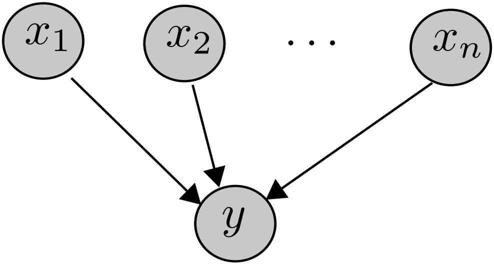
Example

Coin flip



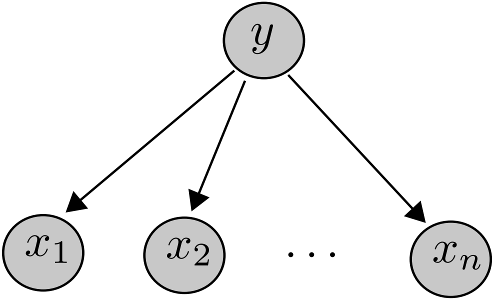
Example

Noisy OR



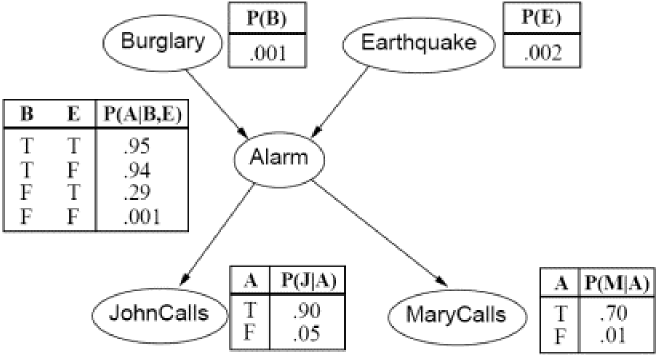
Example

Naive Bayes



Example

Burglar Alarm

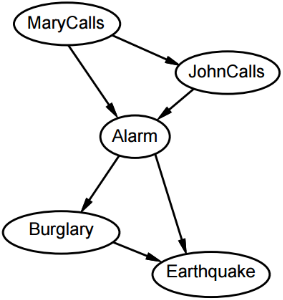


Joint probability

Represent the joint probability distribution

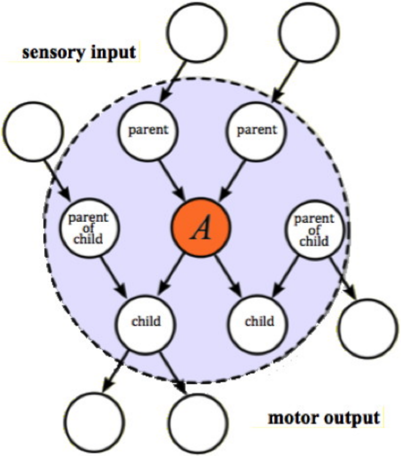
- Chain rule
- Conditional independence
- $$P(x_1, \cdots, x_n) = \prod_{i=1}^{N} P(x_i \mid parents(x_i))$$

Compactness and order



- size of conditional probability table
- locally structured system

Markov blanket



- parent
- child
- parent of child

Find conditional independence $P(A \mid MB(A), B) = P(A \mid MB(A))$

Bayesian inference

problem

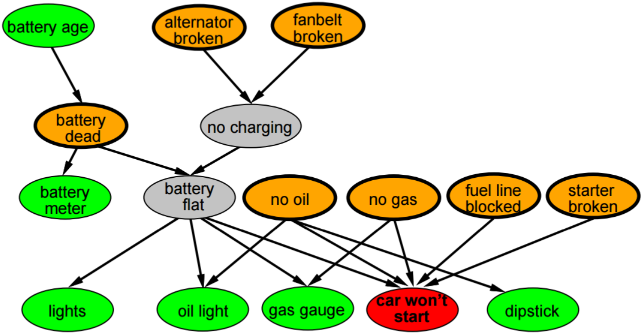
- Query variables X
- Evidence variables E
- Unobserved variables Y

solution

- Margin out Y
- $P(X \mid E)$

Bayesian inference

Car inspection



Bayesian inference

Car insurance