

Homotopy-Aware RRT* : Toward Human-Robot Topological Path-Planning

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Outline

1 Introduction

- Human-Robot Path-Planning
- Homotopy

2 Related Work

3 Homotopy Identification

- Map Decomposition
- Homotopic Equivalence

4 Homotopy-Aware RRT*

5 Experiments and Application

- Experiments
- Single Homotopy Class
- Multiple Homotopy Classes
- Application

6 Conclusion



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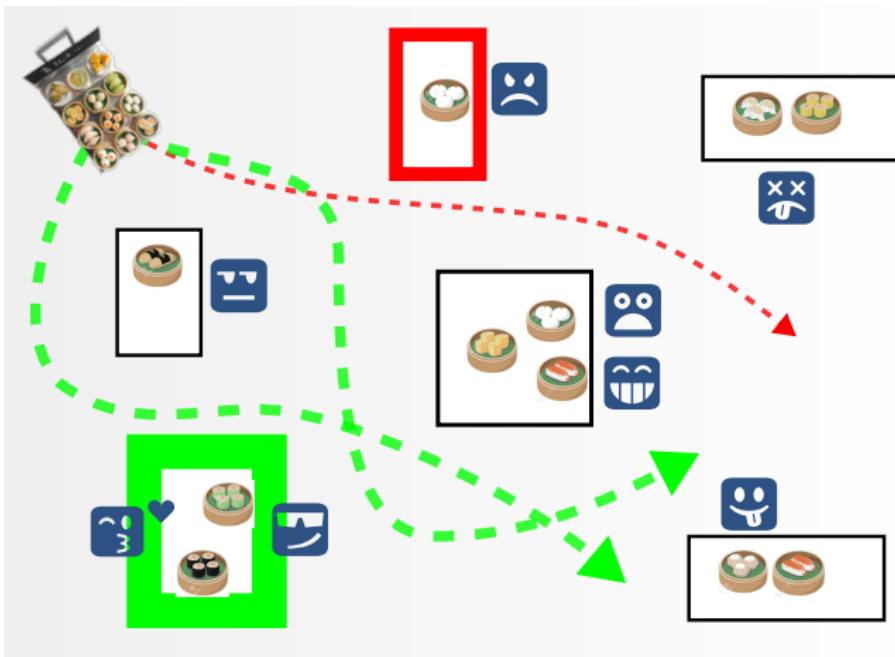
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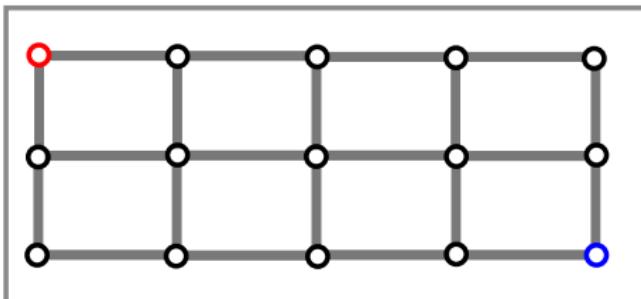
Human-Robot Collaboration

Instruct a robot executing a task

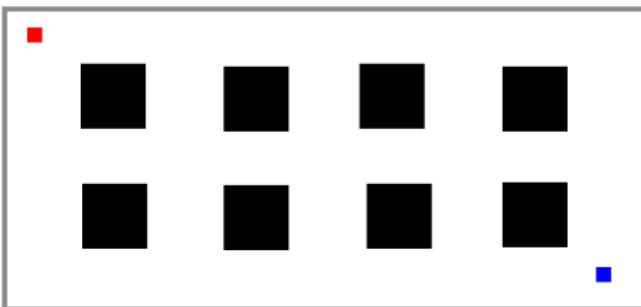
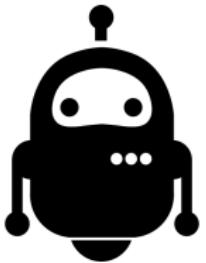




Collaborative Path-Planning



Topology space



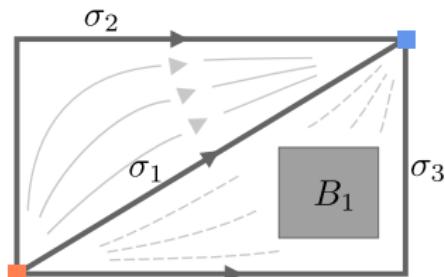
Metric space



Homotopy

Definition

A path can be continuously deformed into another without encroaching any obstacle



- $\sigma_1 \simeq \sigma_2$
- $\sigma_1 \not\cong \sigma_2$

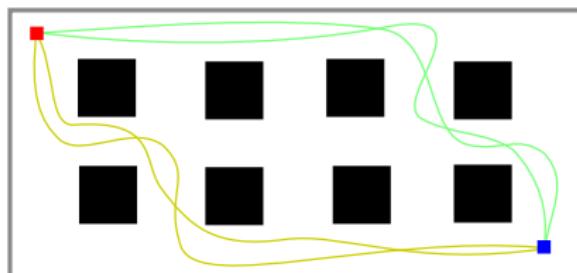
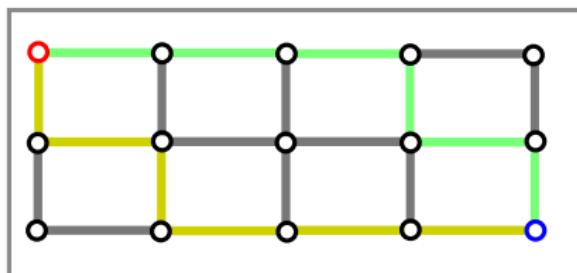
Homotopy Class

A set of paths that are homotopic



Homotopy-based Path Planning

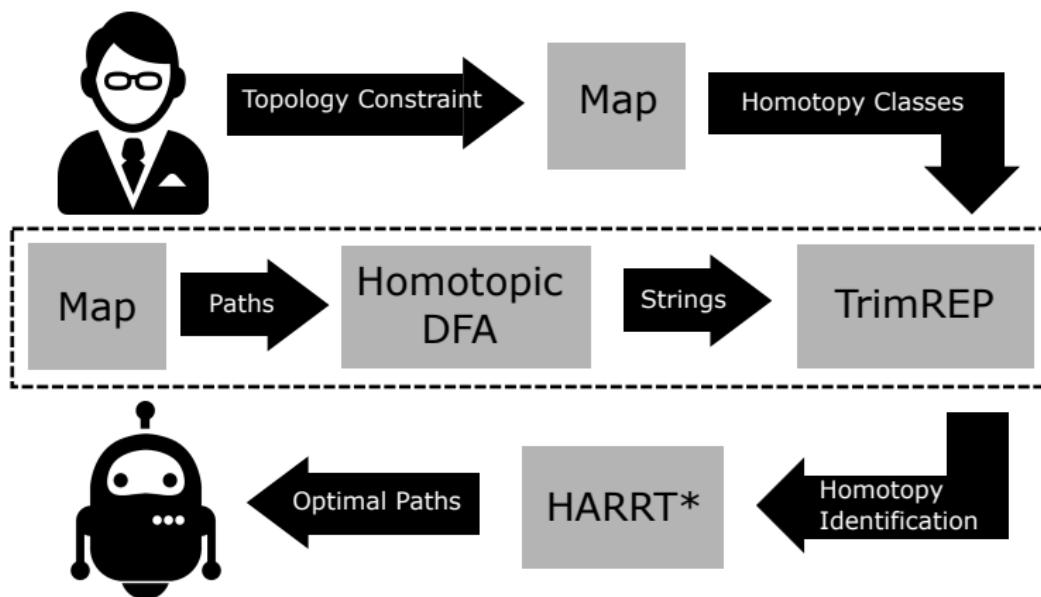
Find the optimal paths of several homotopy classes



- $\sigma(0) = x_{init}$
- $\sigma(1) = x_{goal}$
- $H \subseteq H(x_{init}, x_{goal})$
- $\forall h_i \in H, \sigma_{h_i}^* = \arg \max_{\sigma \in X_{free} \wedge h(\sigma) = h_i} Cost(\sigma)$



Framework





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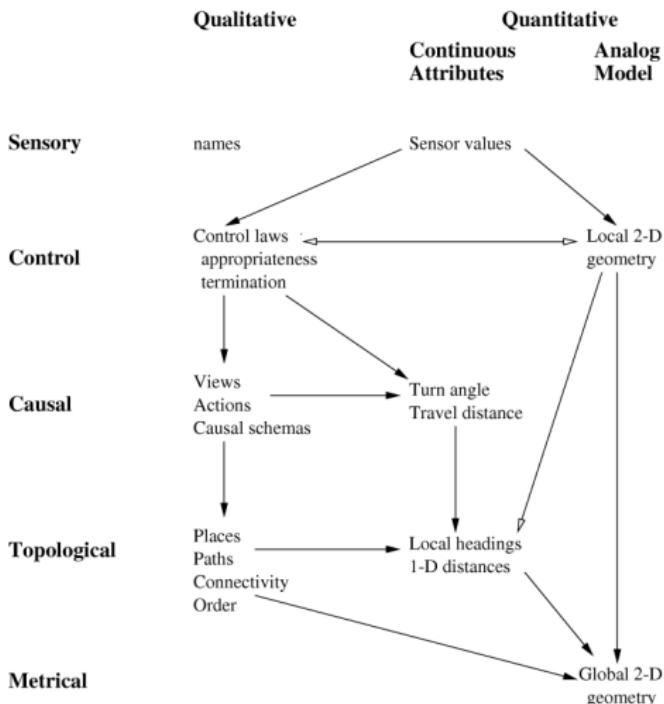
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Spatial Semantic Hierarchy

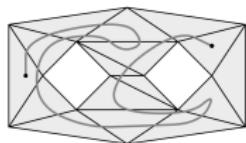


Kuipers, "The spatial semantic hierarchy," Tech. Rep. AI99-281, 29, 1999.

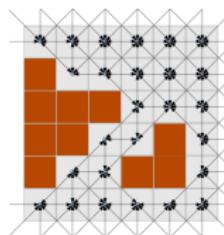
Approaches of Homotopic Path-Planning



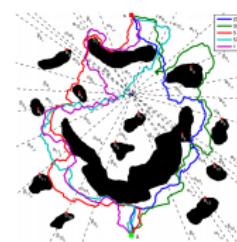
- (a) Space-cut approaches ^{1, 2}
- (b) Homology approaches ^{3, 4}
- (c) Reference-frame approaches ^{5, 6}



(a)



(b)



(c)

¹ Hershberger et al., "Computing minimum length paths of a given homotopy class," Computational Geometry, 1994.

² Grigoriev et al., "Polytime algorithm for the shortest path in a homotopy class amidst semi-algebraic obstacles in the plane," ISSAC 98.

³ Bhattacharya, "Search-based path planning with homotopy class constraints," AAAI 2010.

⁴ Florian et al., "Multiscale topological trajectory classification with persistent homology," RSS 2014.

⁵ Hernandez et al., "A topologically guided path planner for an AUV using homotopy classes," ICRA 2011.

⁶ Kim et al., "Path planning for a tethered mobile robot," ICRA 2014.



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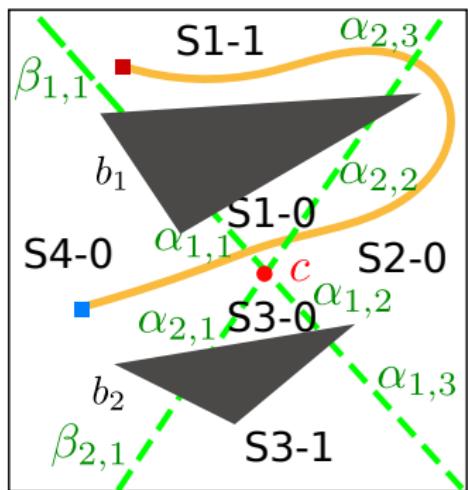
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Map Decomposition

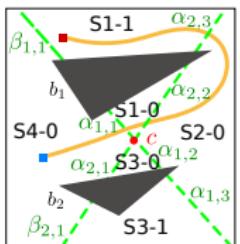


- Sampling representative point in each obstacle
- Sampling the center point
- Generating a radial structure

- Reference frames separate regions
- An ID for a reference frame
- A string for a sequence of reference frames



Topology of DFA



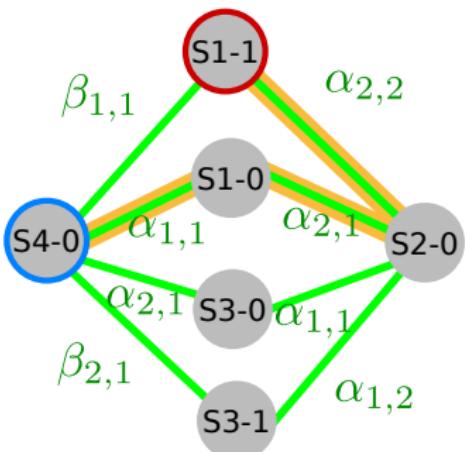
- Region \Rightarrow Node
- Reference frame \Rightarrow Edge

Deterministic Finite Automata

- Path \Rightarrow String

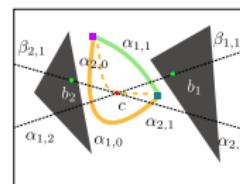
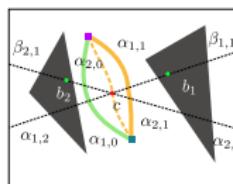
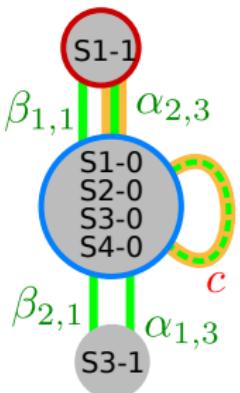
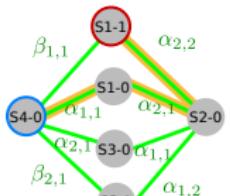
String block

A set of paths that generates the same string by DFA





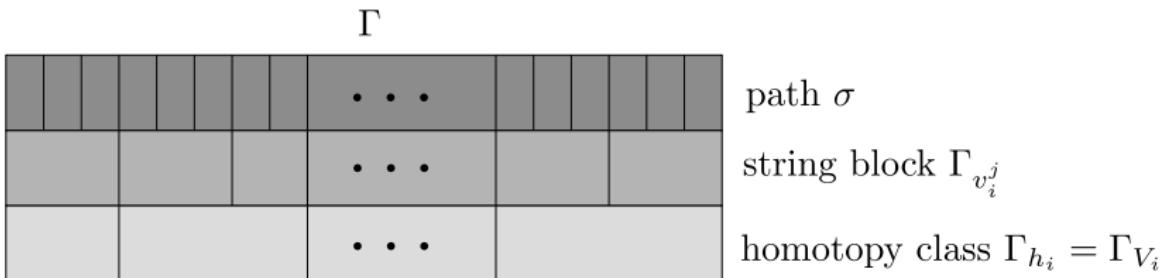
Homotopic DFA



- Homotopic equivalence in crossing the center point
- Merge regions that connect with the center point



Hierarchy of Path Partition



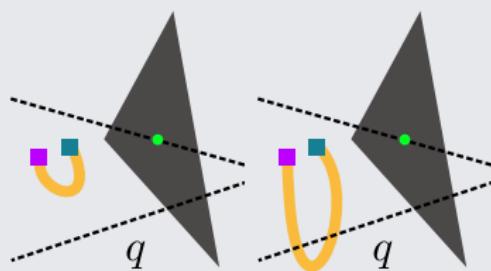


REP String

Definition

A string made up of recursive embedded palindromic substrings

Example



Before deform After deform

- A path crosses several reference frames and backtrack
- Example:
 - ε
 - ww^R
 - $uvv^R ww^R u^R$
- *REPTtrim()*
 - trim REP string



Homotopic Equivalence Identification

Theorem

Removing all the REP substrings of $M^h(\sigma)$ yields the v_i^* for which $\Gamma_{M^h(\sigma)} \simeq \Gamma_{v_i^*}$.

Corollary

$\text{REPTtrim}(M^h(\sigma_i)) = \text{REPTtrim}(M^h(\sigma_j))$ iff $\sigma_i \simeq \sigma_j$.



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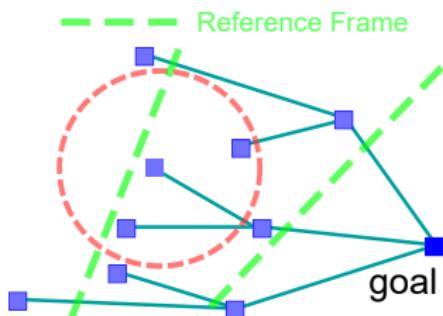
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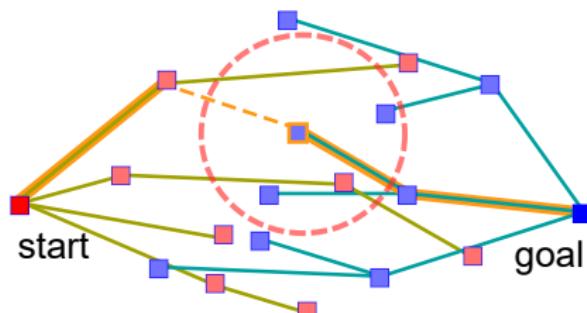


Homotopy-Aware RRT*

- Homotopic identification of each branch
- Bidirectional RRT*

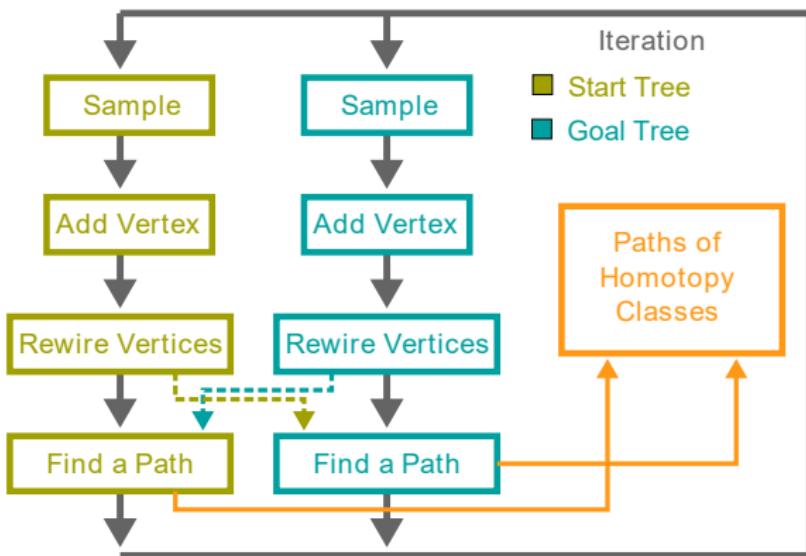


Homotopic identification



Bidirectional RRT*

Flow





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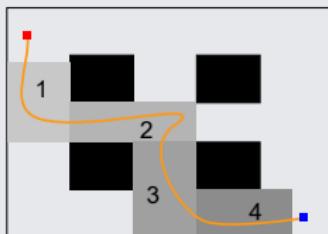
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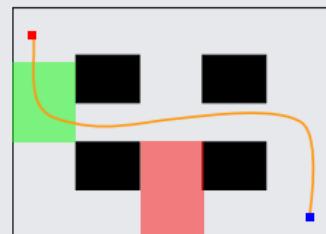
Human-Robot Interactive Path-Planning



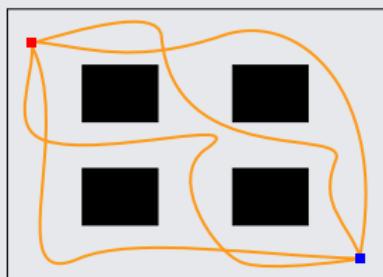
Quickly go from point A to point B through a sequence of specific regions.



Quickly go from point A to point B making sure to visit some regions and avoid other regions.

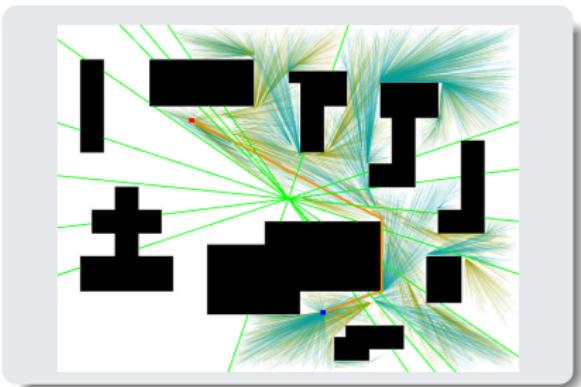
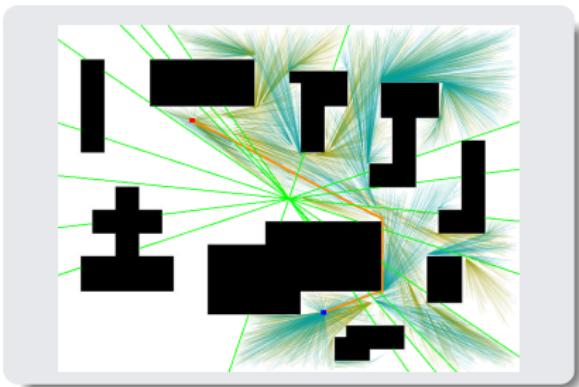


In quickly going from point A to point B, I prefer some types of paths over others, but I recognize that tradeoffs may be needed.





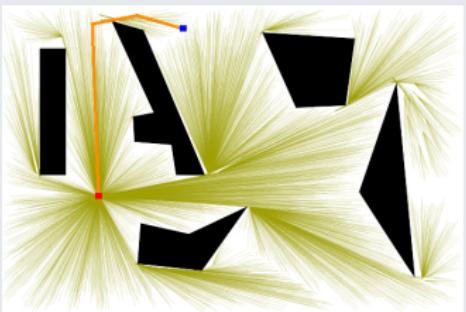
Single Homotopy Class



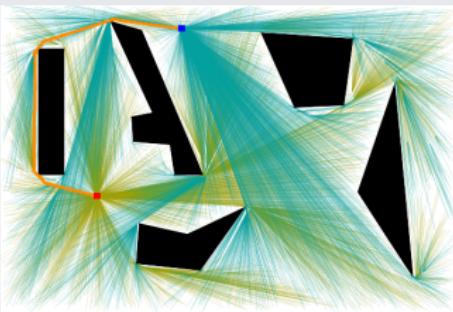


Multiple Homotopy Classes

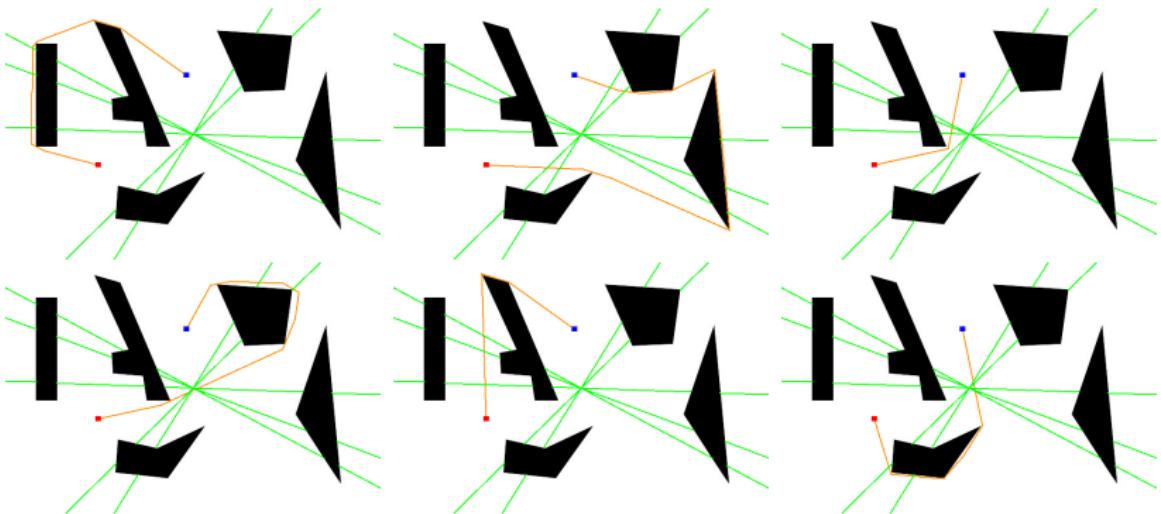
Unidirectional Tree



Bidirectional Tree

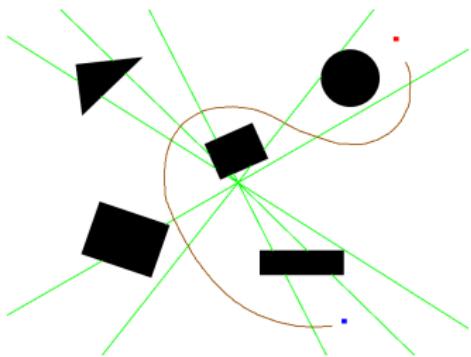


Multiple Homotopy Classes

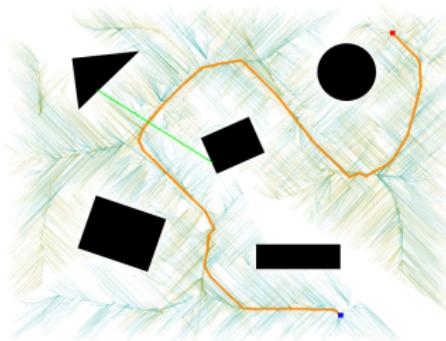




Interactive Graphical Interface



Sketch reference

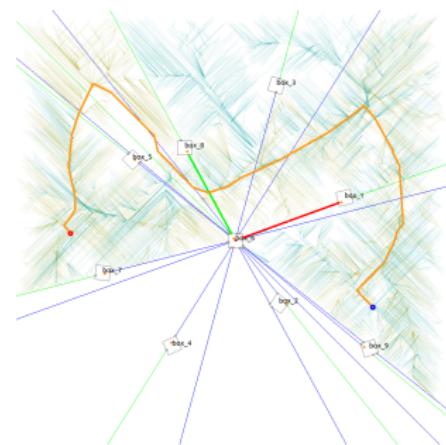
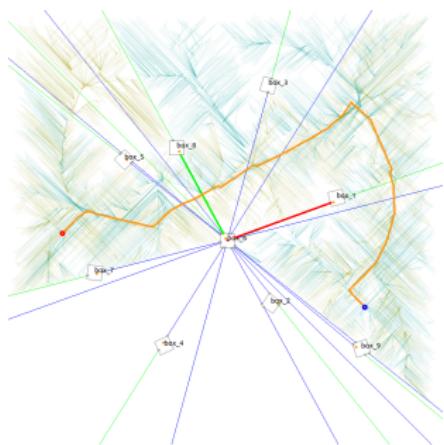


Planned path

Nature Language Interface



"Go between box 8 and box 6, and avoid the left of box 1".





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Conclusion and Futurework



Conclusion

- Encode path into string
- Homotopic identification
- Find optimal paths of homotopy classes

Futurework

- Integrate with multiple performance objectives
- Integrate with moving objects



Thank you