Probabilistic Reasoning Over Time

CS 470 Introduction To Artificial Intelligence

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Outline

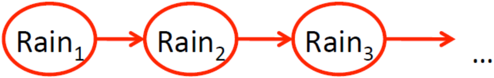
Outline

1. Temporal Inference
   - Dynamic model
   - Applications
2. Markov Process
   - Markov assumption
   - Markov model
3. Bayesian Filter
   - Hidden Markov Model
4. Application : Mapping
   - Grid filter

Dynamic model

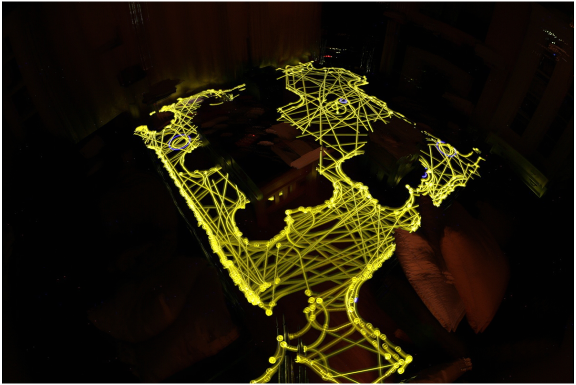
Static model

Dynamic model



Reasoning over time

Application



Robotic Mapping

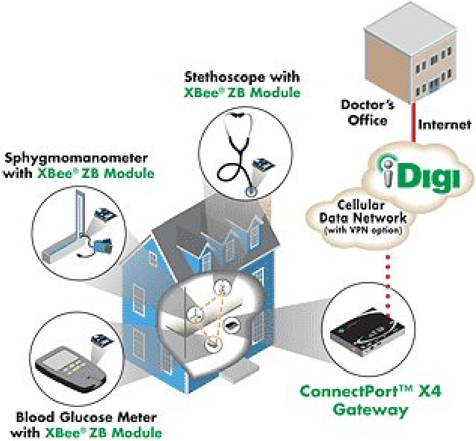
Reasoning over time

Application

Robotic Localization

Reasoning over time

Application



Health Monitoring

Outline

1. Temporal Inference
   - Dynamic model
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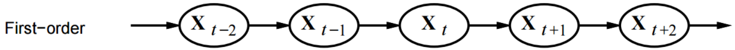
Markov process

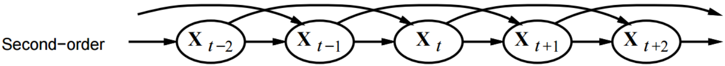
- Dynamic model $P(X_t \mid X_0, \cdots, X_{t-1})$
- Markov assumption : the current state depends on only a finite fixed number of previous states
- Markov process
  $P(X_t \mid X_0, \cdots, X_{t-1}) = P(X_t \mid X_{t-K}, \cdots, X_{t-1})$

Markov process

- First-order Markov Process $P(X_t \mid X_{t-1})$

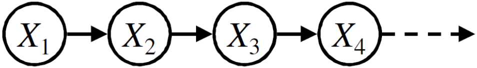


- Second-order Markov Process $P(X_t \mid X_{t-1}, X_{t-2})$



Markov Model

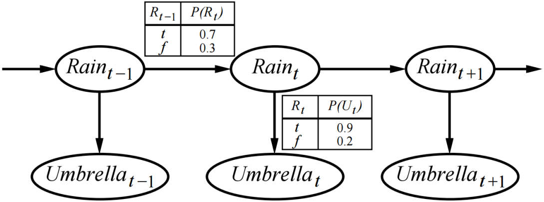
- a chain-structured Bayesian network



- initial state $P(X_1)$
- transition model $P(X_t \mid X_{t-1})$
- conditional independence

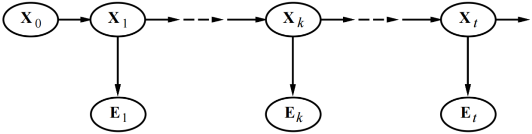
Markov Model

Example: Rain



Markov Model

- Observation model $P(E_t \mid X_0, \cdots, X_t, E_0, \cdots, E_{t-1})$
  What you can observe at time t depends on
  - previous states $X_0, \cdots, X_t$
  - previous observation $E_0, \cdots, E_{t-1}$
- Markov assumption $P(E_t \mid X_t)$



Markov Model

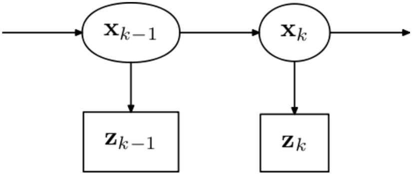
Inference

- Filtering $P(X_t \mid E_1, \cdots, E_t)$
  estimate current state
- Prediction $P(X_{t+k} \mid E_1, \cdots, E_t), k > 0$
  forecasting
- Smoothing $P(X_k \mid E_1, \cdots, E_t), 0 \leq k \leq t$
  smooth data
- Estimation $\arg\max_{\mathbf{x}_1, \cdots, \mathbf{x}_t} P(X_1, \cdots, X_t \mid E_1, \cdots, E_t)$
  Speech recognition

Outline

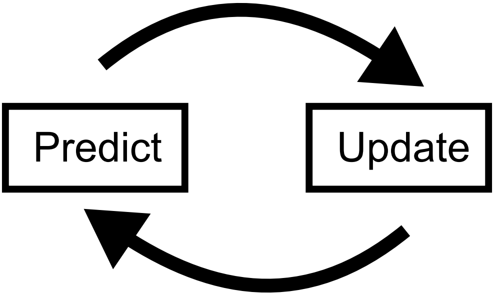
Hidden Markov Model

- observation z
- "hidden" state x
- transition model
  $P(x_k \mid x_0, \cdots, x_{k-1}) = P(x_k \mid x_{k-1})$
- observation model
  $P(z_k \mid x_0, \cdots, x_k) = P(z_k \mid x_k)$



Bayesian filter

Recursive Bayesian Estimation



Bayesian filter

Recursive Bayesian Estimation

- **Prediction** (priori)
  $P(x_k \mid z_1, \cdots, z_{k-1}) \longleftarrow P(x_{k-1} \mid z_1, \cdots, z_{k-1})$
  - Use transition model to predict
- **Update** (posteriori)
  $P(x_k \mid z_1, \cdots, z_k) \longleftarrow P(x_k \mid z_1, \cdots, z_{k-1})$
  - Use new observation to update the prediction

Question?

- $P(x_k \mid z_1, \cdots, z_{k-1}) = P(x_k \mid z_{k-1})$?
- $P(x_k \mid z_1, \cdots, z_k) = P(x_k \mid z_k)$?

Bayesian filter

Prediction

$$P(x_k \mid z_1, \cdots, z_{k-1}) = \int P(x_k \mid x_{k-1}) P(x_{k-1} \mid z_1, \cdots, z_{k-1}) dx_{k-1}$$

- transition model $P(x_k \mid x_{k-1})$
- previous posterior $P(x_{k-1} \mid z_1, \cdots, z_{k-1})$

Bayesian filter

Update

$$
\begin{aligned}
P(x_k \mid z_1, \cdots, z_k) =& \frac{P(z_k \mid x_k) P(x_k \mid z_1, \cdots, z_{k-1})}{P(z_k \mid z_1, \cdots, z_{k-1})} \\
=& \alpha P(z_k \mid x_k) P(x_k \mid z_1, \cdots, z_{k-1})
\end{aligned}
$$

- measurement model $P(z_k \mid x_k)$ (Likelihood)
- current prior $P(x_k \mid z_1, \cdots, z_{k-1})$
- normalization factor α

Bayesian filter

Define

- $bel(x_k) = P(x_k \mid z_1, \cdots, z_k)$
- $\hat{bel}(x_k) = P(x_k \mid z_1, \cdots, z_{k-1})$

Simplification

- Prediction

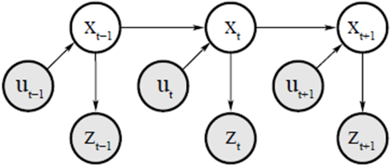
$$\hat{bel}(x_k) = \int P(x_k \mid x_{k-1}) bel(x_{k-1}) dx_{k-1}$$

- Update

$$bel(x_k) = \alpha P(z_k \mid x_k) \hat{bel}(x_k)$$

Bayesian filter

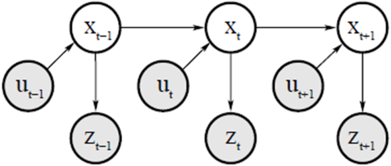
Consider system input u



- transition model $P(x_k \mid x_{k-1}, u_k)$
- $bel(x_k) = P(x_k \mid z_1, \cdots, z_k, u_1, \cdots, u_k)$
- $\hat{bel}(x_k) = P(x_k \mid z_1, \cdots, z_{k-1}, u_1, \cdots, u_{k-1})$

Bayesian filter

Consider system input u



- Prediction

$$\hat{bel}(x_k) = \int P(x_k \mid x_{k-1}, u_k) bel(x_{k-1}) dx_{k-1}$$

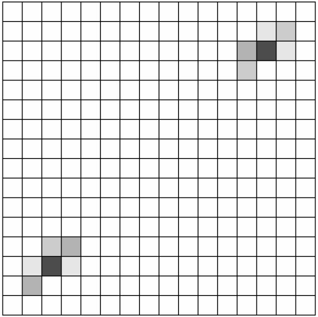
- Update

$$bel(x_k) = \alpha P(z_k \mid x_k) \hat{bel}(x_k)$$

Outline

Grid filter

map discretization



Grid filter

binary Bayes filter with static state

- binary state : x and $\neg x$
- static state : x does not change over time
- belief in state at time k : $bel(x_k) = P(x_k \mid z_1, \cdots, z_k)$

Grid filter

- Prediction

$$\hat{bel}(x_k) = \sum_{x_{k-1}} P(x_k \mid x_{k-1}) bel(x_{k-1})$$

static state $\longrightarrow \hat{bel}(x_k) = bel(x_{k-1})$

- Update

$$bel(x_k) = \alpha_k P(z_k \mid x_k) \hat{bel}(x_k)$$

Simplification

$$bel(x_k) = \alpha_k P(z_k \mid x_k) bel(x_{k-1})$$

Grid filter

- a cell in grid $s_{i,j} \in S$
- the occupancy of the cell
  $\{s_{i,j} = \text{occupied or } s_{i,j} = \neg\text{occupied}\}$
- the observation of the cell
  $\{o_{i,j} = \text{occupied or } o_{i,j} = \neg\text{occupied}\}$

Grid filter

posteriori $P(s_{i,j} \mid O)$

- priori $P(s_{i,j} = \text{occupied})$ and $P(s_{i,j} = \neg\text{occupied})$
- likelihood
  - $P(o_{i,j} = \text{occupied} \mid s_{i,j} = \text{occupied})$
  - $P(o_{i,j} = \neg\text{occupied} \mid s_{i,j} = \text{occupied})$
  - $P(o_{i,j} = \text{occupied} \mid s_{i,j} = \neg\text{occupied})$
  - $P(o_{i,j} = \neg\text{occupied} \mid s_{i,j} = \neg\text{occupied})$