Decision and Utilities

CS 470 Introduction To Artificial Intelligence

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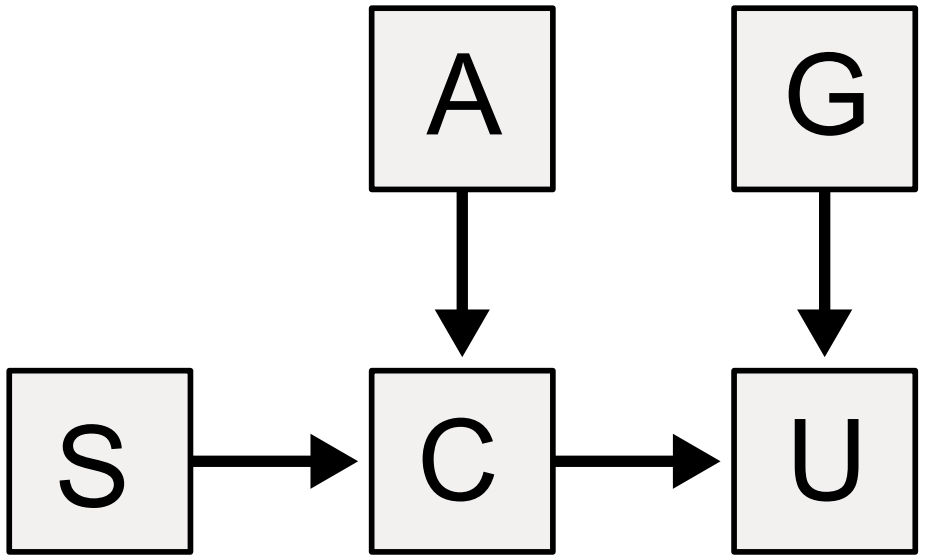
Outline

Outline

Agent

- State
- Action
- Consequence
- Goal
- Utility



Maximize Expected Utility

Application

Algorithmic trading

High frequency trading

Outline

Preference

Model agent's preference

- $A \succ B$ the agent prefer A over B
- $A \sim B$ the agent is indifferent between A and B
- $A \succsim B$ the agent prefers A over B or is indifferent between them

Lottery

$$L = [p_1, S_1; p_2, S_2; \cdots p_n, S_n]$$

- a set of outcomes $\{S_1, S_2, \cdots S_n\}$
- each outcome occurs a probability p_i
  - $\sum_{i=1}^{n} p_i = 1$
- each outcome S_i can either be an atomic state or another lottery

Principles

- **Orderability**
  - Exactly one of $(A \succ B)$, $(A \prec B)$, or $(A \sim B)$ holds.
- **Transitivity**
  - $(A \succ B) \wedge (B \succ C) \Longrightarrow (A \succ C)$.
- **Continuity**
  - $A \succ B \succ C \Longrightarrow \exists p [p, A : 1 - p, C] \sim B$.

Principles

- **Substitutability**
  - $A \sim B \implies [p, A; 1-p, C] \sim [p, B; 1-p, C]$.
- **Monotonicity**
  - $A \succ B \implies (p > q \iff [p, A; 1-p, B] \succ [q, A; 1-q, B])$.
- **Decomposability**
  - $[p, A; 1-p, [q, B; 1-q, C]] \sim [p, A; (1-p)q, B; (1-p)(1-q), C]$.

From Preference to Utility

Existence of Utility Function

- $U(A) > U(B) \iff A \succ B$
- $U(A) = U(B) \iff A \sim B$

From Preference to Utility

Expected Utility of a Lottery

$$U([p_1, S_1; \cdots ; p_n, S_n]) = \sum_i p_i U(S_i)$$

Outline

Decision making

$$a^* = \arg\max_a EU(u \mid e, a)$$

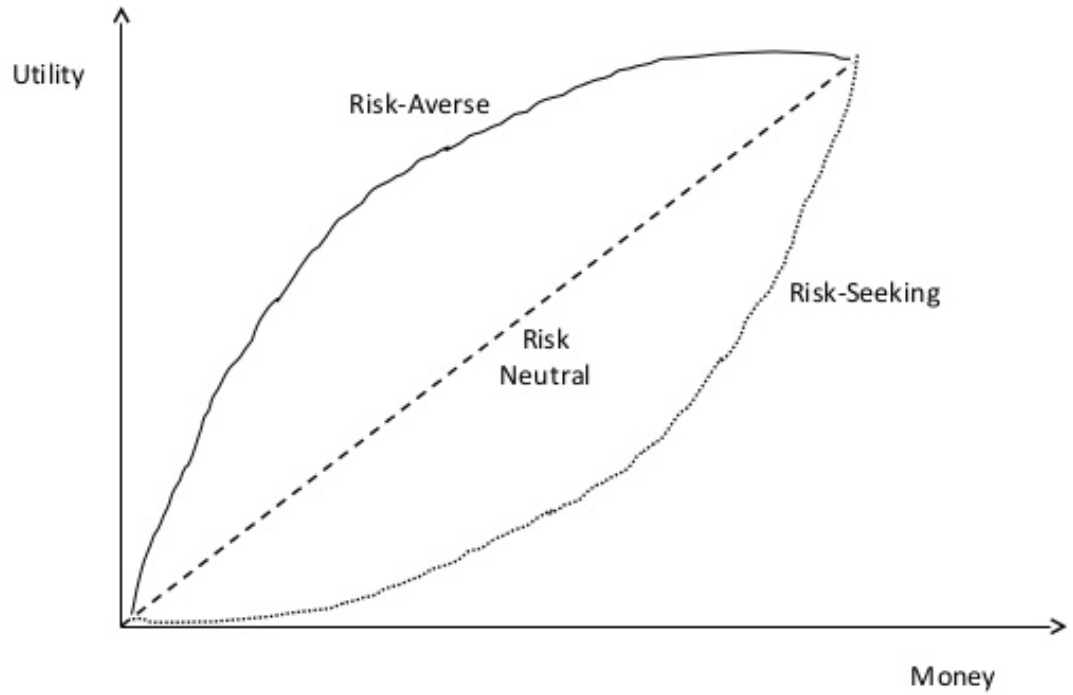
Utility Function

- $U(S)$
- worst $u_\perp$ and best $u_\top$
- standard lottery $[p, u_\perp; (1-p), u_\top]$

- Risk-Seeking
- Risk-Averse
- Risk-Neutral

Utility Function

Bernoulli's Model of Different Risk Perspectives



Source: *Begg, Bratvold and Campbell, Decision-Making Under Uncertainty*

Multi-Attribute Utility

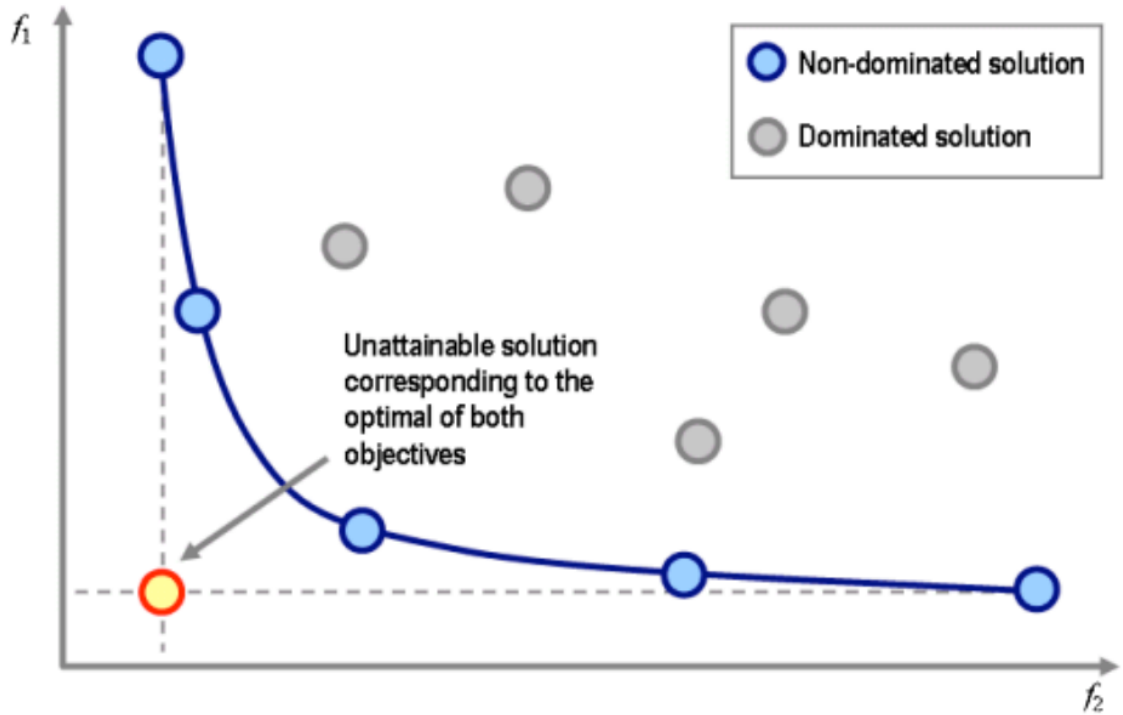
- Usually there are multiple attributes to be considered in decision making.
- **Example : New airport location**
  - the cost of the land
  - the distance from centers of population
  - the noise of flight
  - safety

Multi-Attribute Utility

A vector of attributes $\mathbf{x} =< x_1, \cdots, x_n >$

- **dominance** : better or equivalent in all the attributes $\succsim$
- **strict dominance** : better in all the attributes $\succ$
- **non-dominance** : at least better in one attributes than any other solution
  - Pareto-optimal

Multi-Attribute Utility



Outline

Value of information

- the benefit from uncertainty reduction
- the cost of collecting information
- decision making
  - whether it is worth to collect the information
- **Example - oil drill**