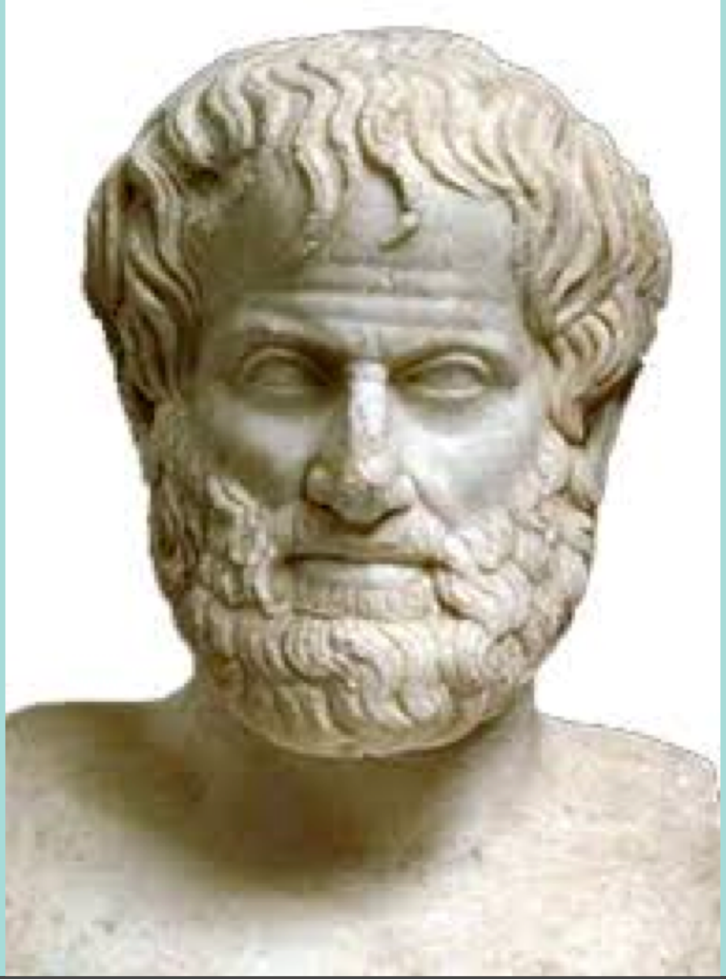
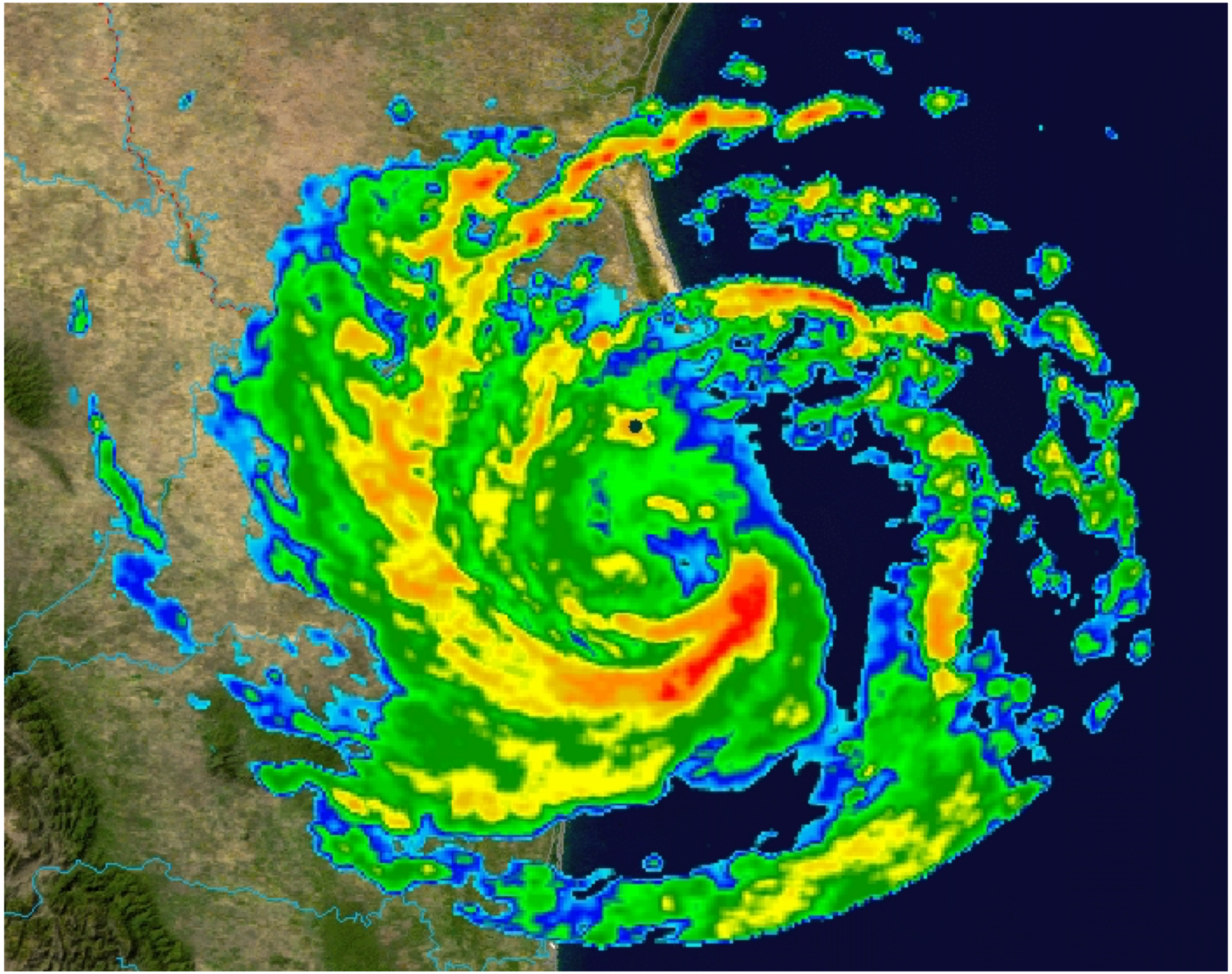


INTRODUCTION TO LOGIC

Josh McCoy

Knowledge Abstraction

Knowledge Abstraction

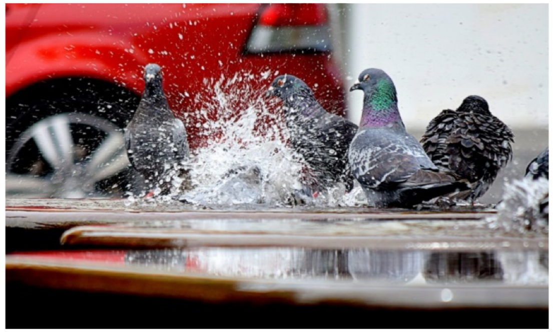


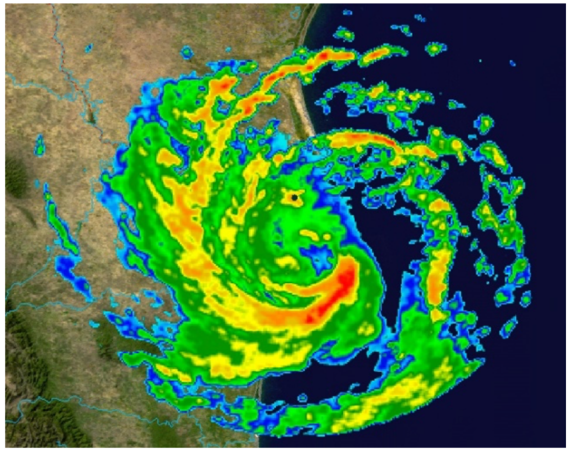
Knowledge Abstraction

Knowledge Abstraction

Set of bindings

Raining =






Logic

A formal language with syntax, semantics, a proof system, percepts, state transitions.

Inference

- RIKLS
- Premises to conclusions.
- Logical or statistical.
  - Certainty or uncertainty.

- How does this compare to learning?

C.F. Peirce

Deduction

- Conclusions concretely supported by premises.
- Top-down reasoning.
- Certain conclusions.

All homo sapiens are mortal.
Arnav is a homo sapien.
Therefore, Arnav is mortal.

All children drink water.
Donald drinks water.
Therefore, Donald is a child.

Induction

- Premises are evidence for the conclusion.
- Inherently uncertain.
- Credible given evidence.

```
All of the swans we have seen are white.
Therefore, we know that all swans are white.
```

- Generalization

```
The proportion Q of the sample has attribute A.
Therefore, the proportion Q of the population has attribute A.
```

Abduction

- Premises and observations used to find explanations.
- Observing, hypothesis, then explanation.
  - Essentially guessing:

```
All the beans from this bag are white.
These beans are white.
Therefore, these beans are from this bag.
```

- Can you think of examples?

Propositional Logic

- Syntax correspond to facts.
- Compositional.
- Context independent.
- Limited expressive power.
  - Cannot say that all cats are cute.
- Sentences
  - Simple: x, cat, w1nt3r_is_c0m1ing
  - Compound: $A \vee B \wedge \neg C$
- Model: A binding of all literals to truth values.

Propositional Logic Operators

- ¬ Not - negation
- ∧ And - conjunction
- ∨ Or - disjunction
- → Single Arrow - implication
- ↔ Double Arrow – iff, equivalence

This is precedence order.

Note on implication:

- antecedent -> consequent
- LHS -> RHS

Truth Table

P	Q	$P \wedge Q$	$P \vee Q$	$P \underline{\vee} Q$	$P \underline{\wedge} Q$	$P \Rightarrow Q$	$P \Leftarrow Q$	$P \Leftrightarrow Q$
T	T	T	T	F	T	T	T	T
T	F	F	T	T	F	F	T	F
F	T	F	T	T	F	T	F	F
F	F	F	F	F	T	T	T	T

Statements

- A
- $\neg A$
- $A \vee B$
- $A \vee B \wedge \neg C$
- $\neg(A \vee B) \vee (\neg C \vee D)$
- $A \rightarrow (B \rightarrow C)$

Satisfiability and Validity, Entailment

- Entailment: is the relationship between sentences whereby one sentence will be true if all the others are also true.
- Consistency: At least one truth assignment makes a set of sentences true.
- Validity: if truth value is true in all interpretations.
  - true, $\neg$ false, $P \lor \neg P$
- Satisfiable sentence: there exists a truth value assignment for the variables that makes the sentence true
  - P, true, $\neg P$
  - Unsatisfiable: $P \land \neg P$, false, $\neg$ true
- What algorithm would implement a check for satisfiability?

Conjunctive Normal Form (CNF)

- Simplification of propositional logic sentences for easier use.
  - Ease of computation.
- Clause with a disjunction of literals.
- Clauses are literals with possible negations.
- Each clause much be satisfiable.
- Every sentence and propositional logic can be written in CNF.
  - Watch out for exponential explosion in size!

Translating to CNF

1. Eliminate arrows using definitions
   - $P \rightarrow Q \equiv \neg P \vee Q$
2. Drive in negations using laws of De Morgan
   - $\neg(\varphi \vee \phi) \equiv \neg\varphi \wedge \neg\phi$
   - $\neg(\varphi \wedge \phi) \equiv \neg\varphi \vee \neg\phi$
3. Distribute disjunction over conjunction
   - $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$

CNF Conversion Example

- $(A \vee B) \rightarrow (C \rightarrow D)$

CNF Conversion Example

- $(A \lor B) \rightarrow (C \rightarrow D)$

1. Eliminate arrows
   - $\neg(A \lor B) \lor (\neg C \lor D)$

CNF Conversion Example

- $(A \lor B) \rightarrow (C \rightarrow D)$

1. Eliminate arrows
   - $\neg(A \lor B) \lor (\neg C \lor D)$
2. Drive in negations
   - $(\neg A \land \neg B) \lor (\neg C \lor D)$

CNF Conversion Example

- $(A \vee B) \rightarrow (C \rightarrow D)$

1. Eliminate arrows
   - $\neg(A \vee B) \vee (\neg C \vee D)$
2. Drive in negations
   - $(\neg A \wedge \neg B) \vee (\neg C \vee D)$
3. Distribute
   - $(\neg A \vee \neg C \vee D) \wedge (\neg B \vee \neg C \vee D)$

Practice

1. $(A \to B) \to C$
2. $A \to (B \to C)$
3. $(A \to B) \vee (B \to A)$
4. $\neg(\neg P \to (P \to Q))$
5. $(P \to (Q \to R)) \to (P \to (R \to Q))$
6. $(P \to Q) \to ((Q \to R) \to (P \to R))$

DPLL

- Satisfaction solving algorithm.
- Davis–Putnam–Logemann–Loveland
- http://www.inf.ufpr.br/dpasqualin/d3-dpll/
- https://www.youtube.com/watch?v=D2o3APE9344

Motivation for First-Order Logic

In first-order logic, variables refer to things in the world and you can quantify over them. That is, you can talk about all or some of them without having to name them explicitly.

- Statements that cannot be made in propositional logic but can be made.
- When you paint a block with green paint, it becomes green.
  - In propositional logic, one would need a statement about every single block, one cannot make the general statement about all blocks.
- When you sterilize a jar, all the bacteria are dead.
  - In FOL, we can talk about all the bacteria without naming them explicitly.
- A person is allowed access to this Web site if they have been formally authorized or they are known to someone who has access.

First Order Logic

- Term
  - Constant symbols: Fred, Japan, Bacterium39
  - Variables: x, y, a
  - Function symbol applied to one or more terms: F(x), F(F(x)), Mother-of(John)
- Sentence
  - A predicate symbol applied to zero or more terms: on(a,b), sister(Jane, Joan), sister(Mother-of(John), Jane), its-raining()
  - t1=t2
  - For v a variable and Φ a sentence, then ∀v.Φ and ∃v.Φ are sentences.
  - Closure under sentential operators: Æ v → ¬ (
    - AKA: same operators with same meaning in propositional logic

First Order Logic Examples

- Everyone loves Tacos
  - $\forall x.\ loves(x, Tacos)$
  - $\neg \exists x.\ \neg\ loves(x, Tacos)$
- Nobody loves OldGlobalFresh
  - $\forall x.\ \neg\ loves(x, OldGlobalFresh)$
  - $\neg \exists x.\ loves(x, OldGlobalFresh)$
- Everybody has a biological parent
  - $\forall x.\ \exists y.\ biologicalParent(y,x)$
- All books have an author and a publisher
  - $\forall x.\ \exists yz.\ author(y,x) \land publisher(z,x)$
- Whoever has a cat, has a furbaby
  - $\forall x.[[\exists y.\ hasCat(y,x)] \rightarrow [\exists y.\ hasFurBaby(y,x)]]$

Someone at UCD is smart

Someone at UCD is smart

$$\exists x\ At(x, UCD) \Rightarrow Smart(x)$$

Someone at UCD is smart

$$\exists x \; At(x, UCD) \Rightarrow Smart(x)$$

$$\equiv$$

$$\exists x \; \neg At(x, UCD) \lor Smart(x)$$

There's someone who either is smart or isn't at UCD.

Someone at UCD is smart

Instead:

$\exists x\ At(x, UCD) \land Smart(x)$

There's someone who is at UCD and is smart.

Comparison of Popular Logic Languages

Language	Ontological commitment	Epistemological commitment
Prop. logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts with degree of truth	known interval value