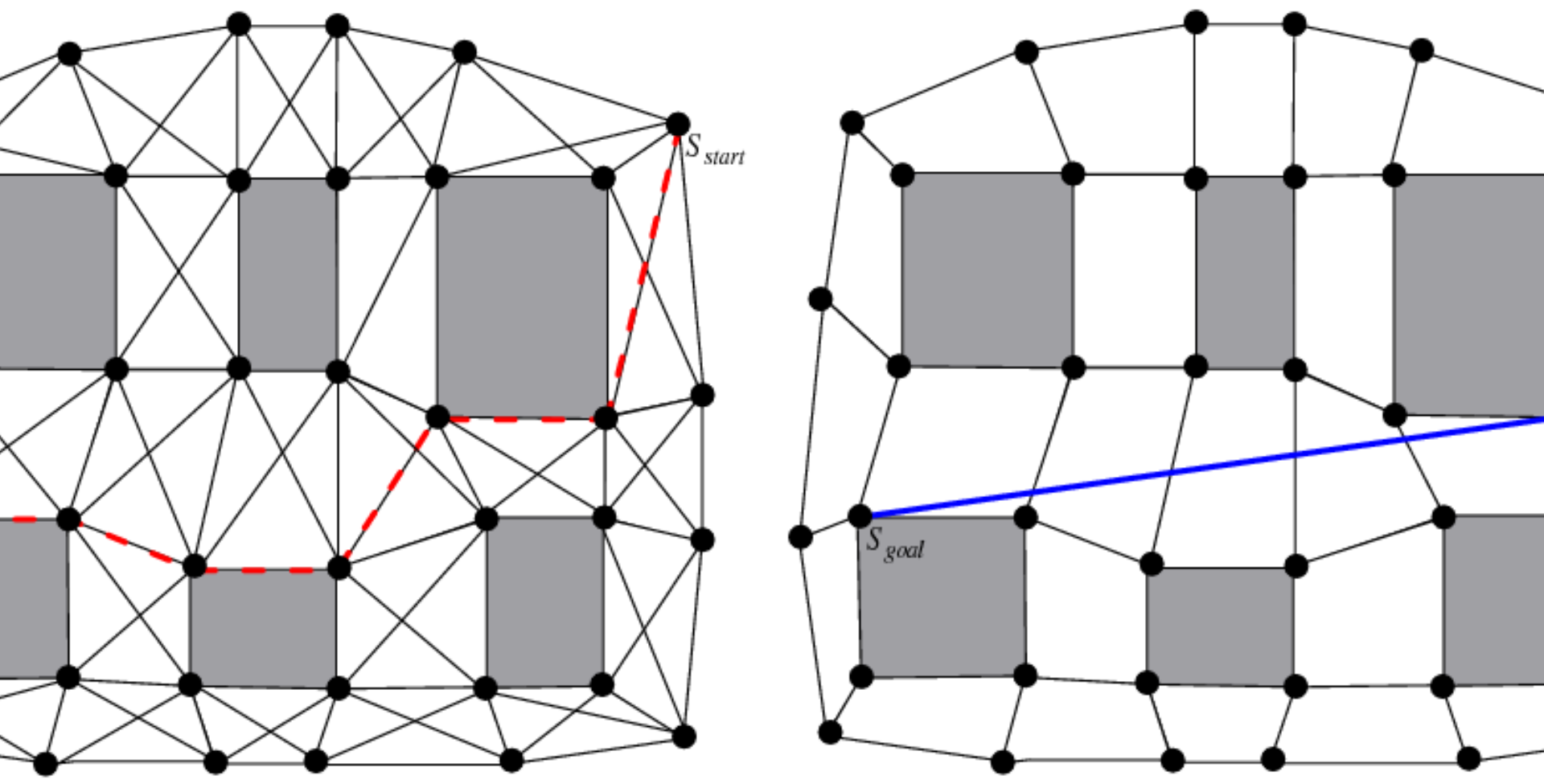


ADVERSARIAL SEARCH

ECS170 Spring 2018
Josh McCoy, @deftjams

Multi-agent Search

- Game Theory
- Competition
- Zero Sum
  - Better described as constant payout
- Perfect Information

Prisoner's dilemma payoff matrix

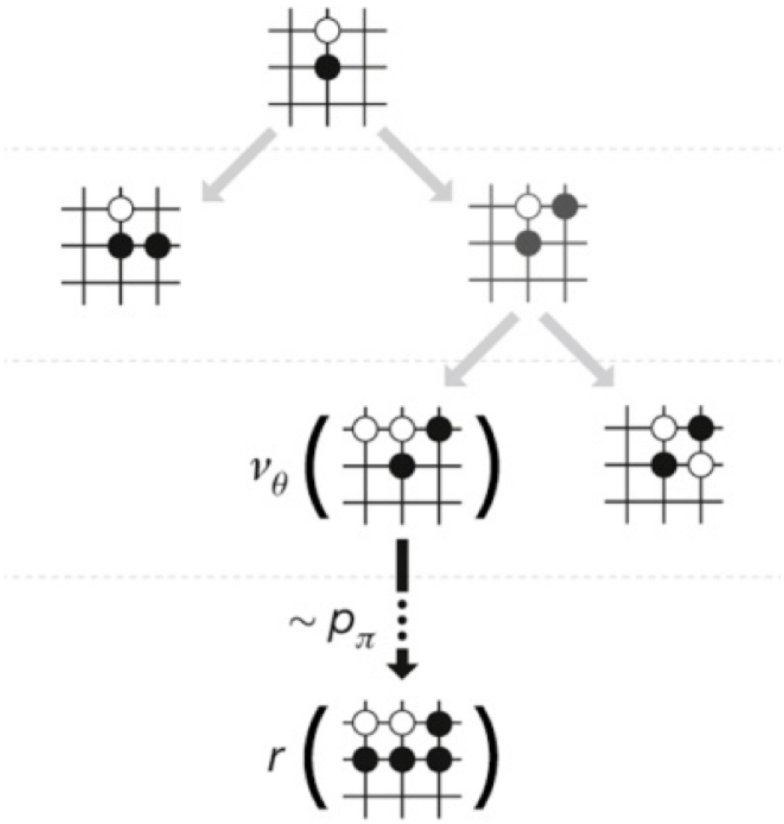
A \ B	B stays silent	B betrays
A stays silent	-1 / -1	0 / -3
A betrays	-3 / 0	-2 / -2

https://en.wikipedia.org/wiki/Prisoner%27s_dilemma

Game Tree

Adversarial search builds a search tree just like previous search algorithms.

Twist: control is passed between players at each level of depth.



https://www.nature.com/articles/nature16961

Game Tree

Adversarial search builds a search tree just like previous search algorithms.

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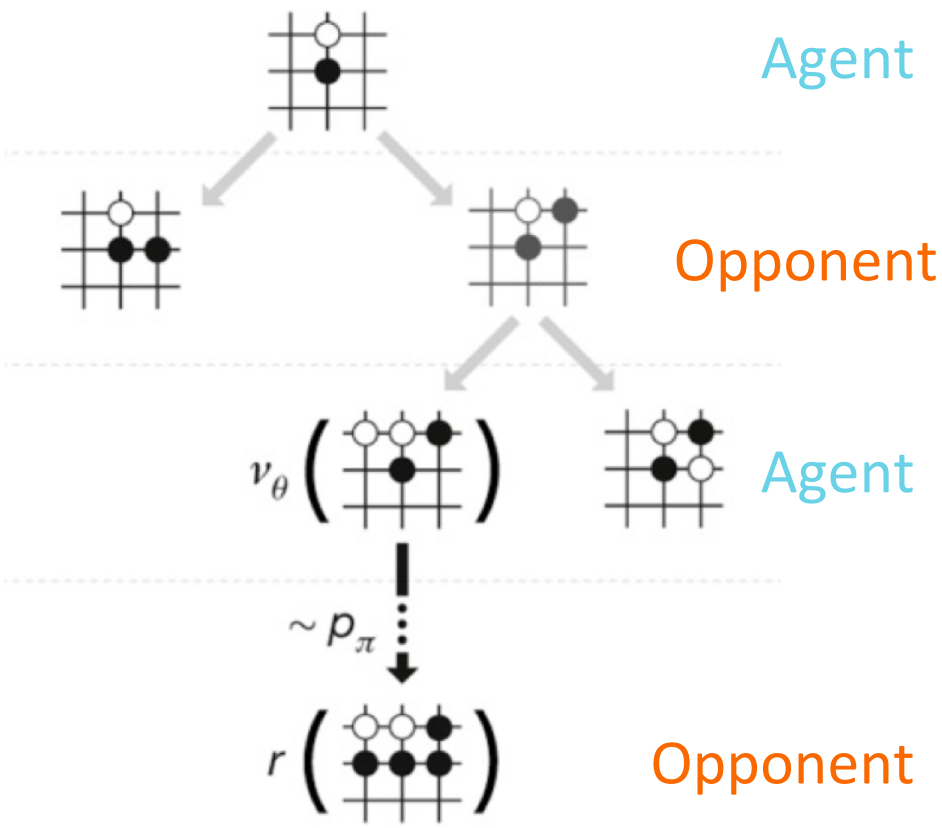
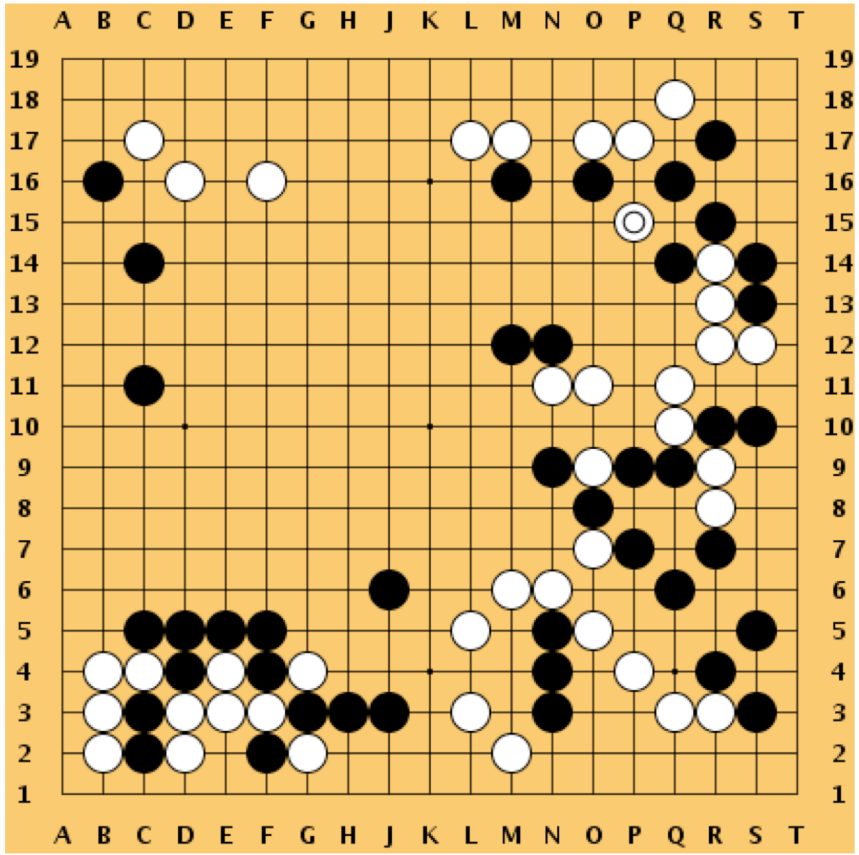


https://www.nature.com/articles/nature16961

Evaluation Functions

$$f\left(\right.$$  $$\left.\right)=v$$

https://commons.wikimedia.org/wiki/File:Go_game_Kobayashi-Kato.png

Human Evaluation (WWLSD?)

https://www.theatlantic.com/technology/archive/2016/03/the-invisible-opponent/475611/

Adversarial Search

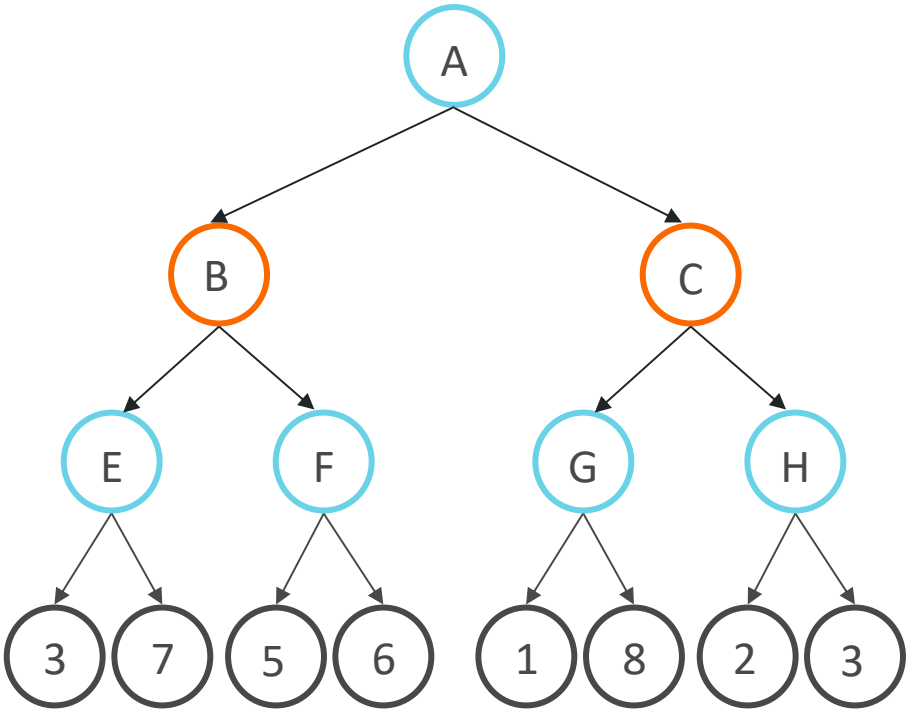
- **Initial State**
- **Player**(state): which player has agency
- **Actions**(state): available moves
- **Result**(state, action): next game state
- **Terminal**(state): true if state is a leaf.
- **Utility**(state, player): score of state for player
- Min(...) - least of args
- Max(...) - most of args

A Tale of Min and Max

- Player wants to maximize their score.
- Opponent wants to minimize the player's score.
- Alternating rounds of decisions.
  - max -> min -> max -> min -> ⋯

Minimax

```
minimax(s=initial_state, p=starting_player):
   if terminal(s) then return utility(s,p)
   if max=player(s) then
      value = -infinity
      for each a in actions(s)
         value = max(value, minimax(result(s,a), min))
      return value
   if min=player(s) then
      value = +infinity
      for each a in actions(s)
         value = min(value, minimax(result(s,a), max))
      return value
```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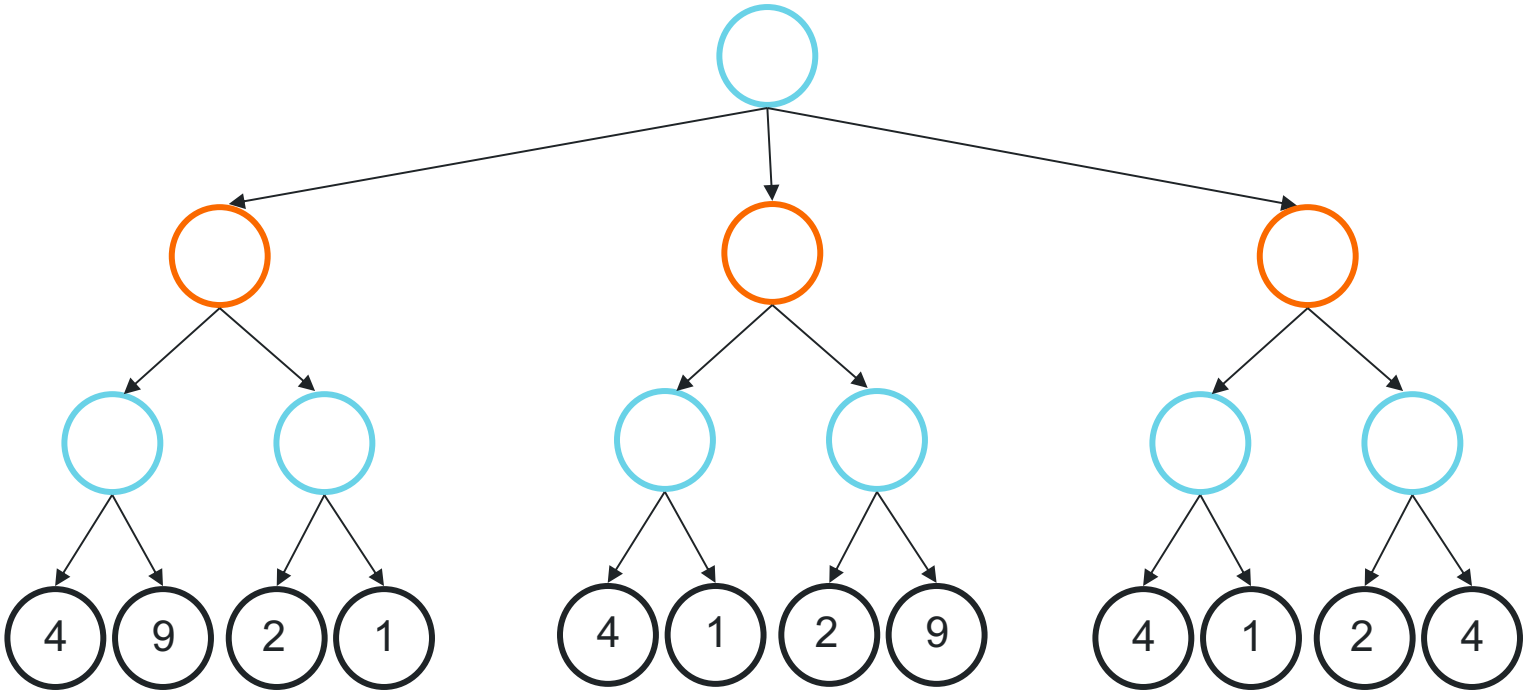
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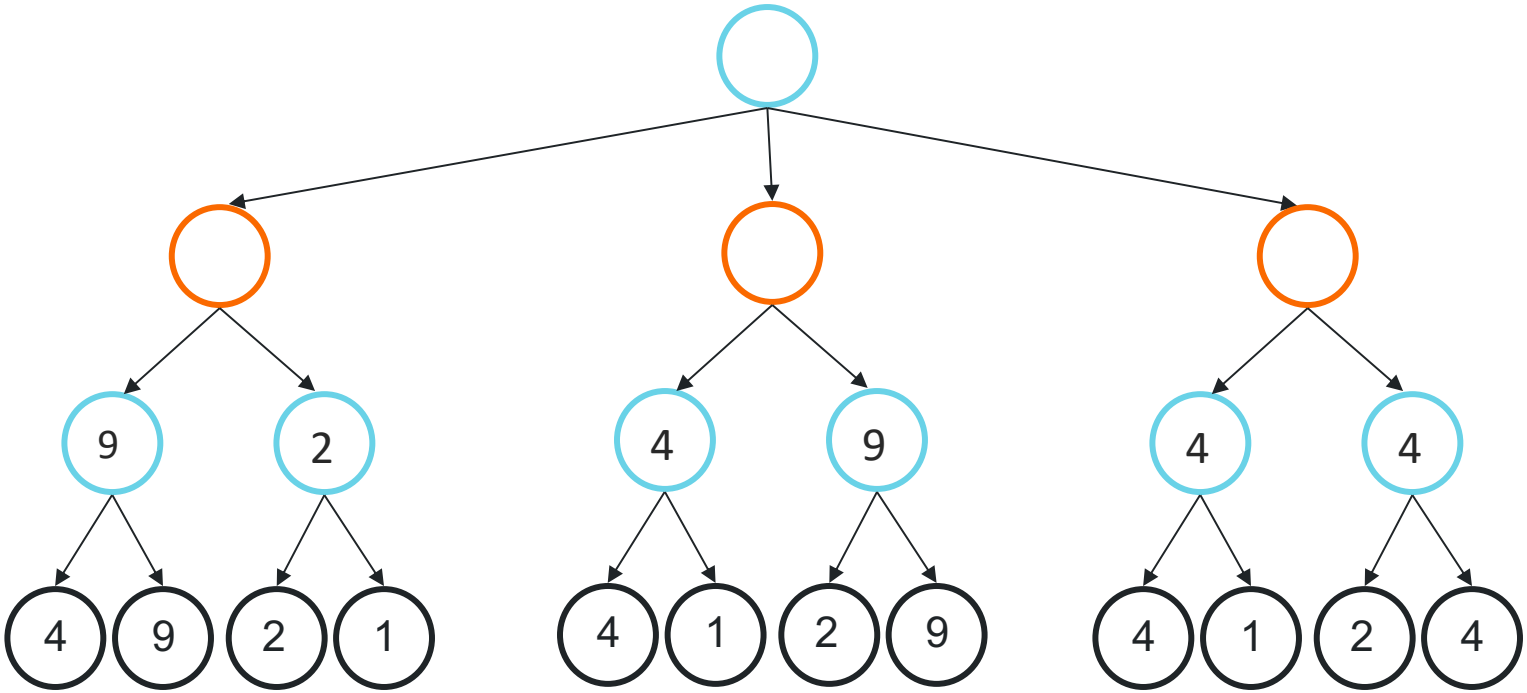
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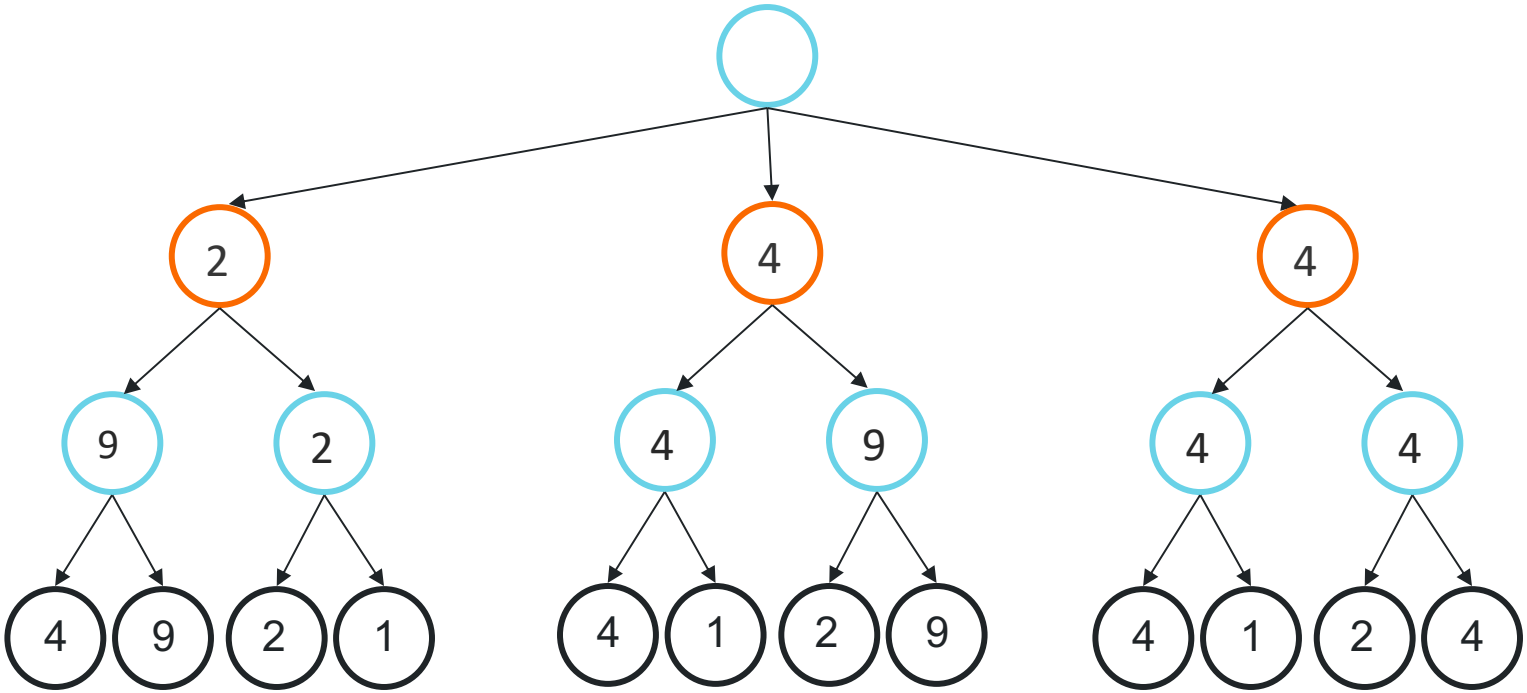
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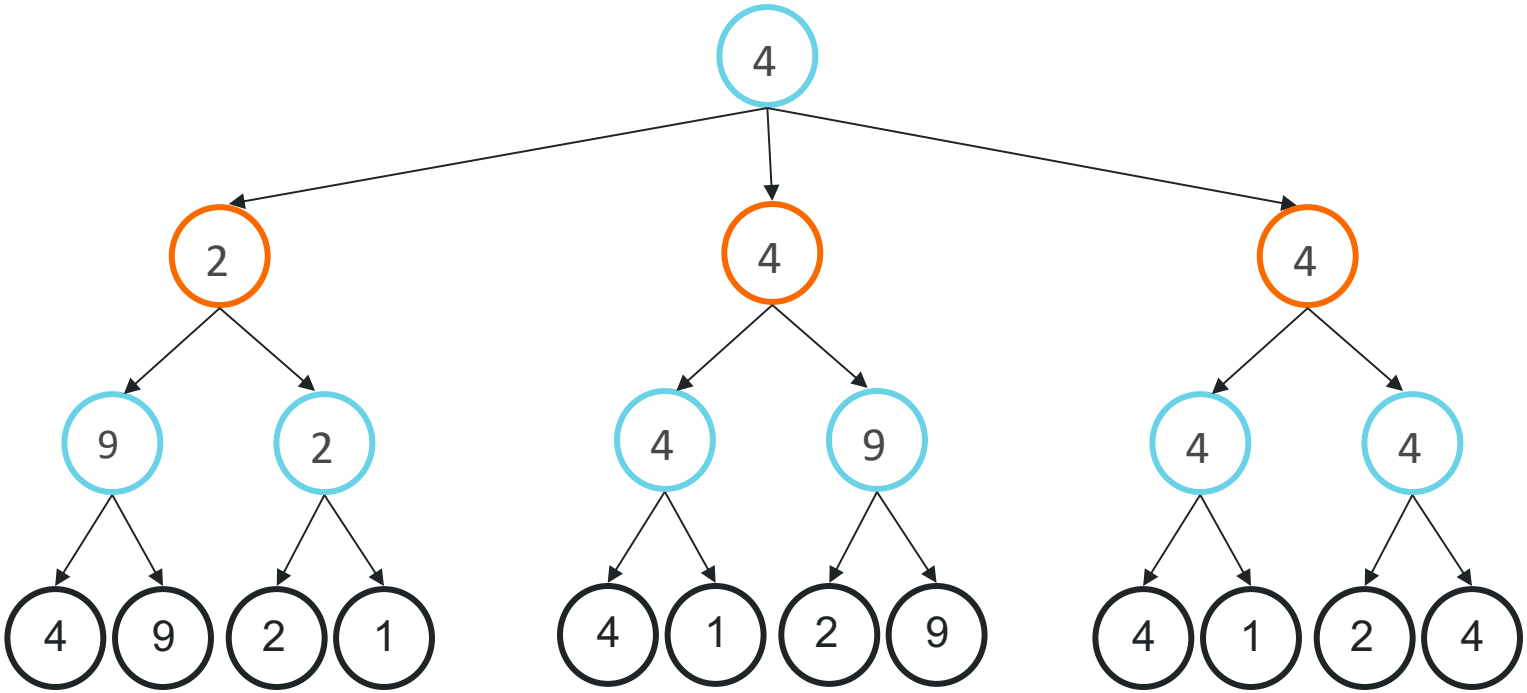
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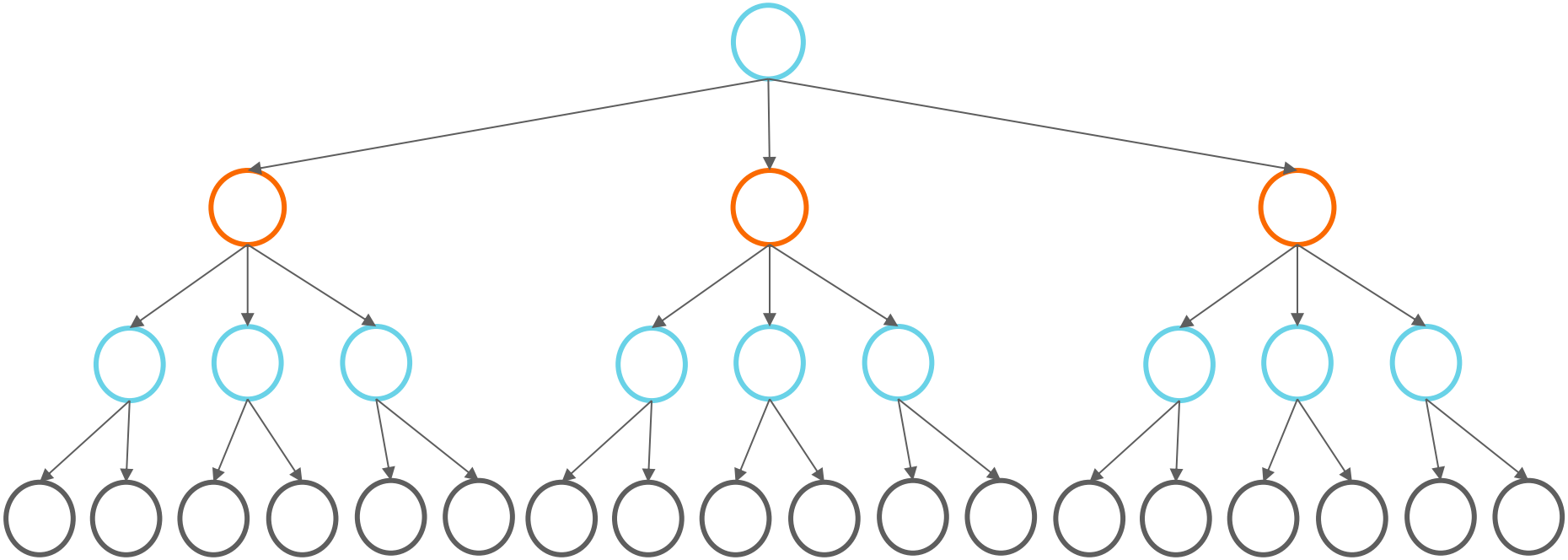
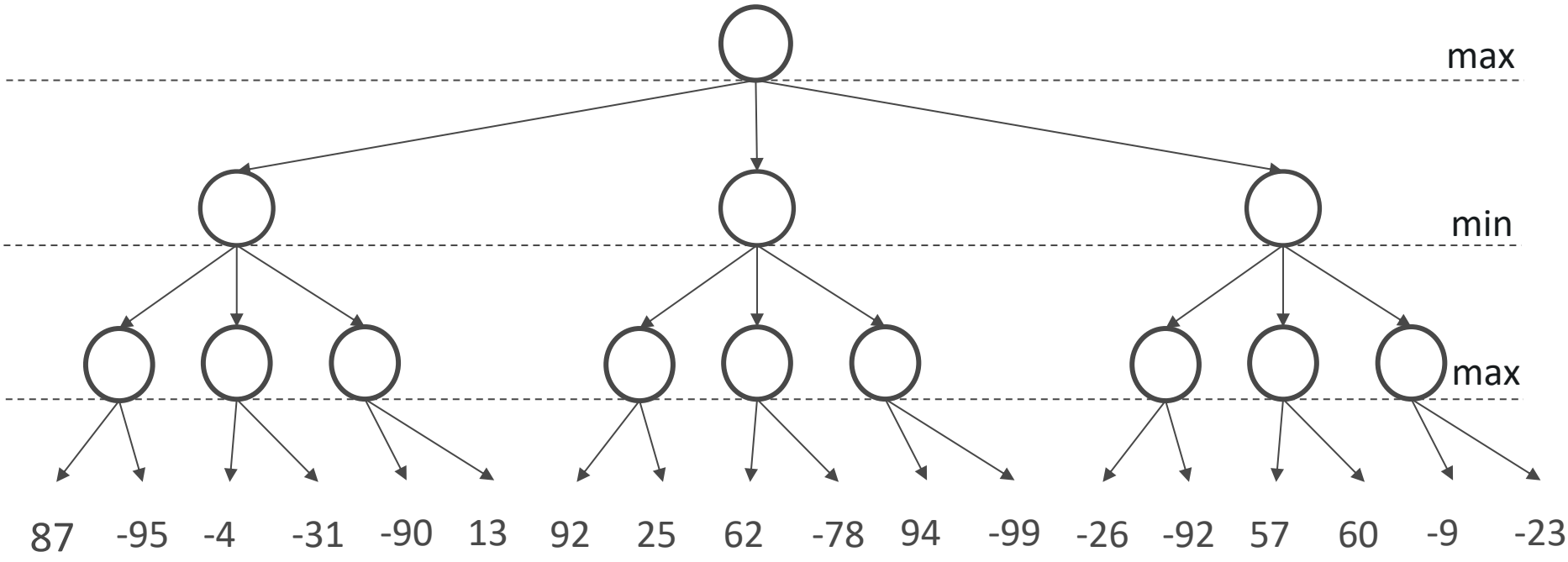
Together!



Together!



Too Many Paths

Time complexity:

$O(moves^{depth})$ or $O(b^m)$

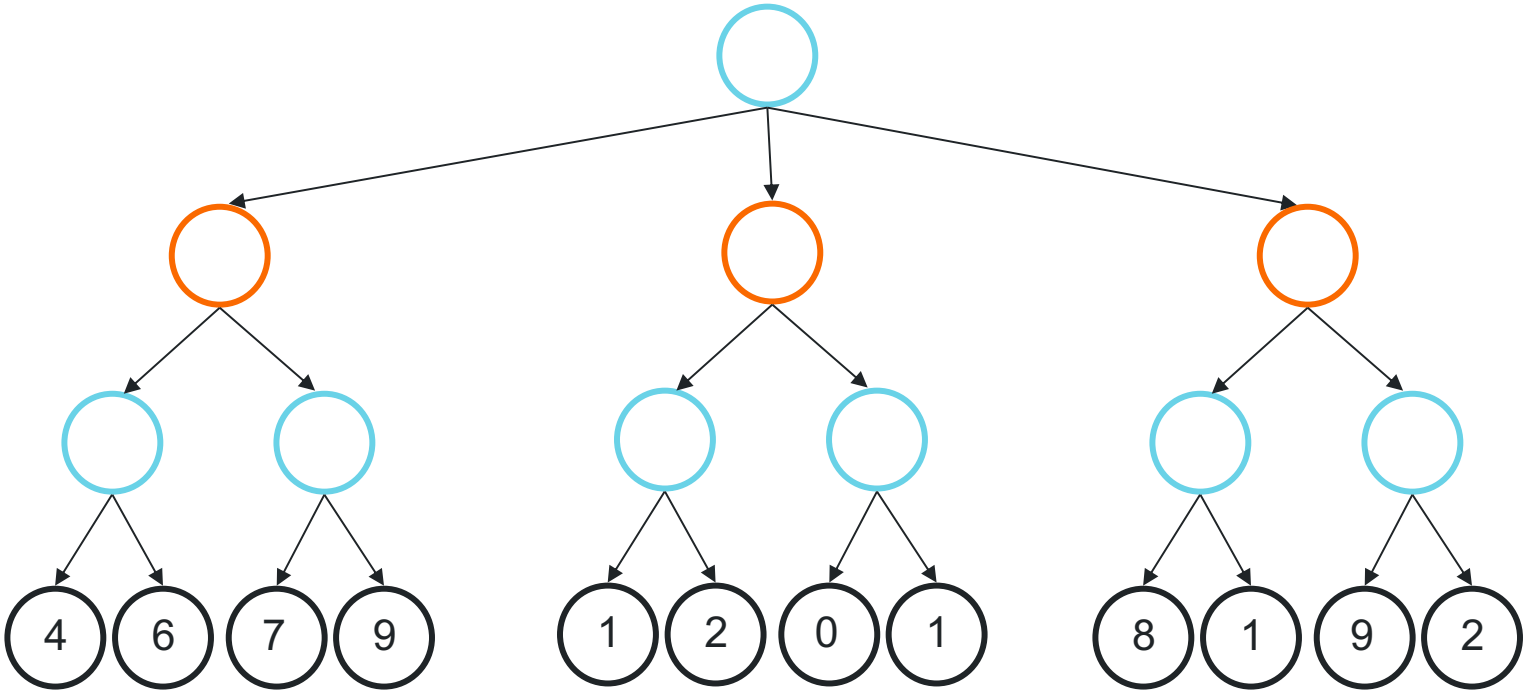
Space complexity:

$O(moves * depth)$ or $O(bm)$

Depth limited minimax?

Alpha-Beta (α-β) Pruning

```
alphabeta(state, α, β):
    if terminal(state) then return utility(state, max)
    else if max=player(state)
        v = -∞
        For each a in actions(state)
            v = max(α, alphabeta(result(state, a), α, β))
            if v >= β return v
            α = max(α, v)
        return v
     else min=player(state)
        v = ∞
        for each a in actions(state)
            v = min(β, alphabeta(result(state, a), α, β))
            if v <= α return v
            β = min(β, v)
        return v
```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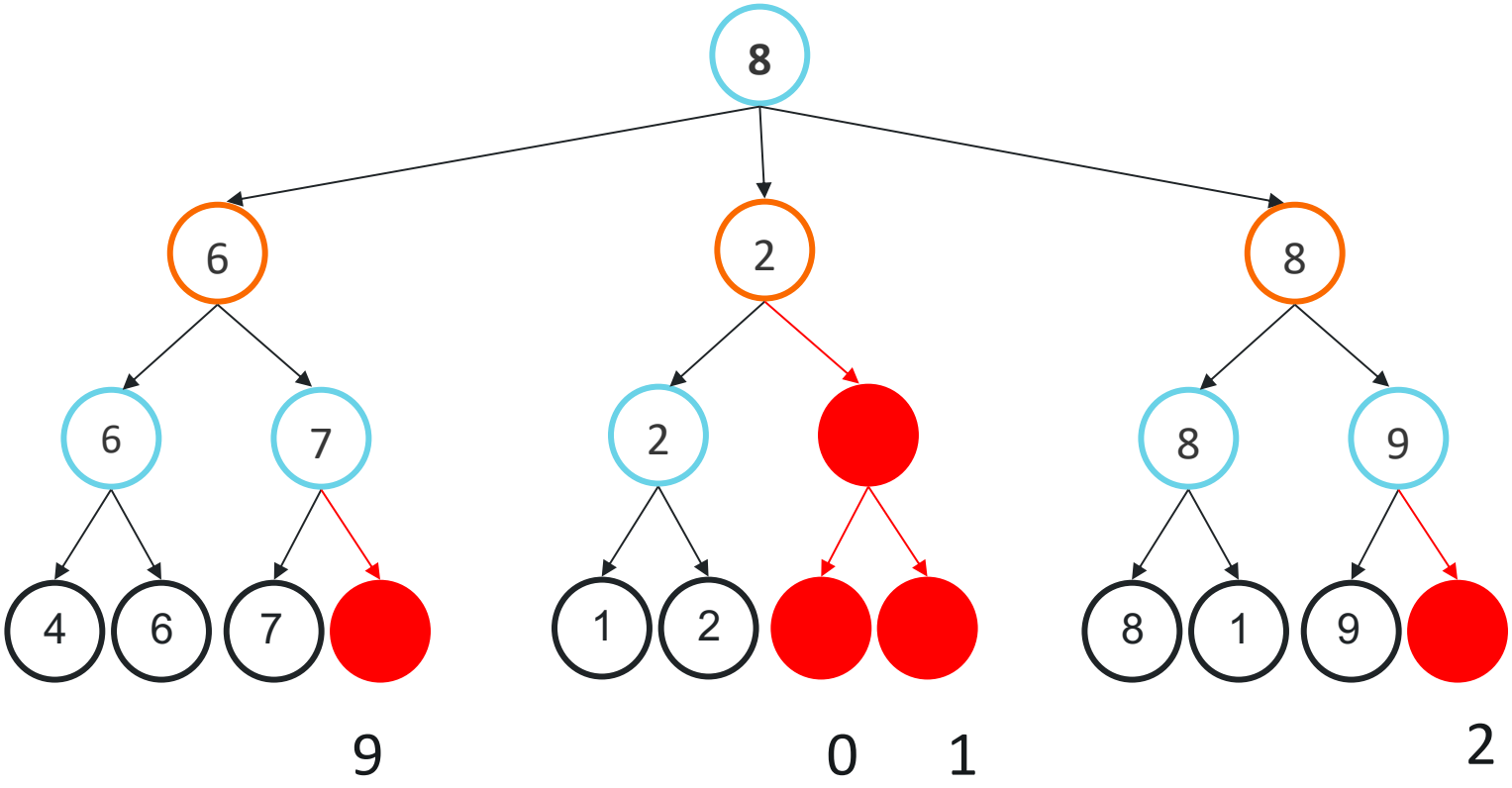
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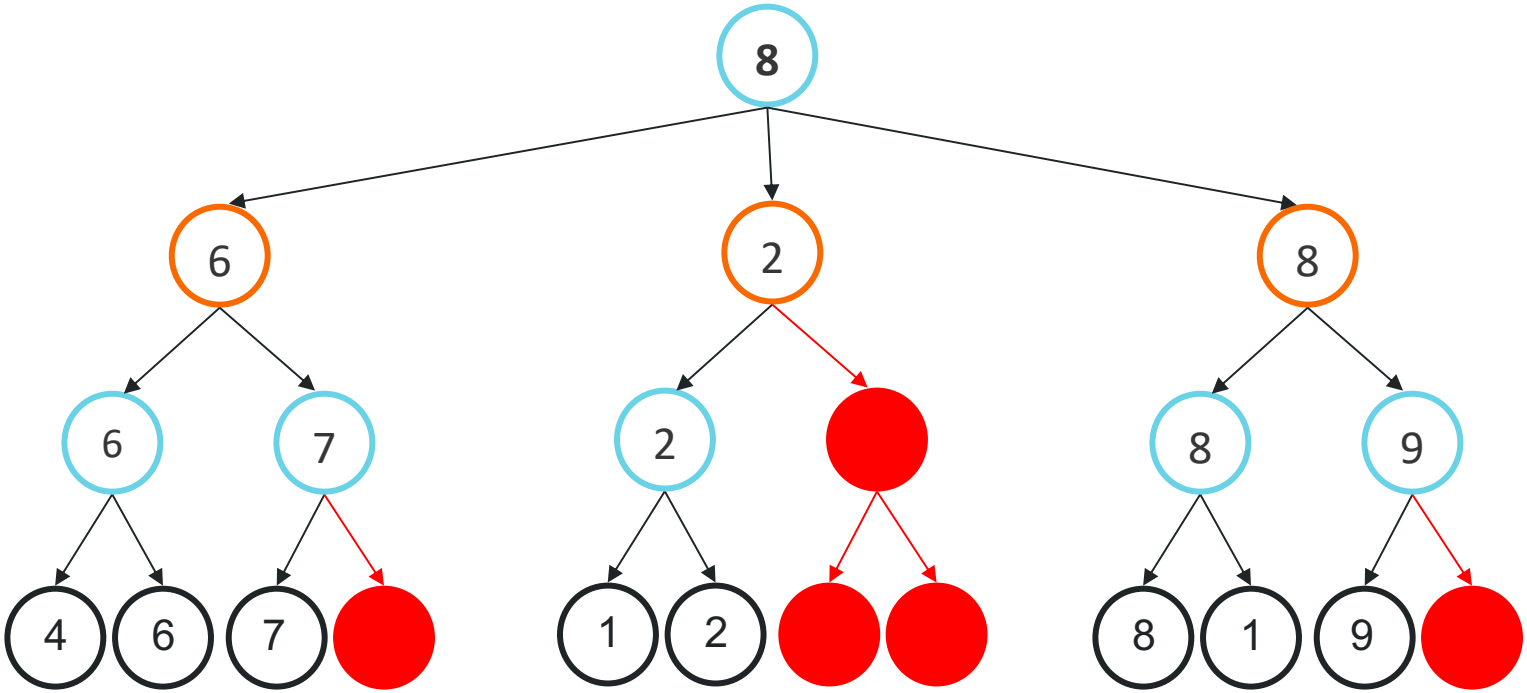
Time complexity:
$O(moves^{depth/2})$ or $O(b^{m/2})$



Too Many Paths

Space complexity:
$O(moves * depth)$ or $O(bm)$