# Driven Brownian transport through arrays of symmetric obstacles

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## Motivation

We study the dynamics of a suspended point-size Brownian particle driven through a two-dimensional rectangular array of circular obstacles with finite radius[1, 2]. Prominent transport features, like negative differential mobilities, excess diffusion peaks, and unconventional asymptotic behaviors, are explained in terms of two distinct lengths, the size of single obstacles (trapping length) and the lattice constant of the array (local correlation length)[3].

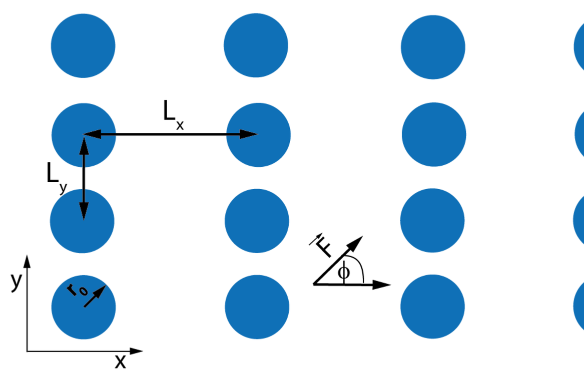


FIGURE 1: Sketch of a 2D rectangular array composed of circular obstacles.

- lattice constants $L_x \times L_y$ and obstacle radius $r_0$
- set of units $L_y = \eta = k_B T = 1$
- overdamped Langevin dynamics (in dimensionless units)

$$\dot{\mathbf{q}} = \mathbf{f} + \sqrt{2}\,\xi(t) \qquad (1)$$

- $\mathbf{f} = f\,(\cos(\phi), \sin(\phi))^T$ with $f = F\,L_y/k_B T$
- Gaussian white noise with $\langle\, \xi(t)\,\rangle = 0$ and $\left\langle\, \xi_i(t), \xi_j(s)\,\right\rangle = \delta_{i,j}\,\delta(t-s)$ for $i, j = x, y$

## 2D rectangular array

- transport quantities

$$\langle\, \dot{q}_i\,\rangle = \lim_{t\to\infty} \frac{q_i(t)}{t} \quad \text{and} \quad D^i_{\text{eff}}/D_0 = \lim_{t\to\infty} \frac{\left\langle\, q_i(t)^2\,\right\rangle - \langle\, q_i(t)\,\rangle^2}{2\,t} \quad \text{for} \quad i = x, y$$

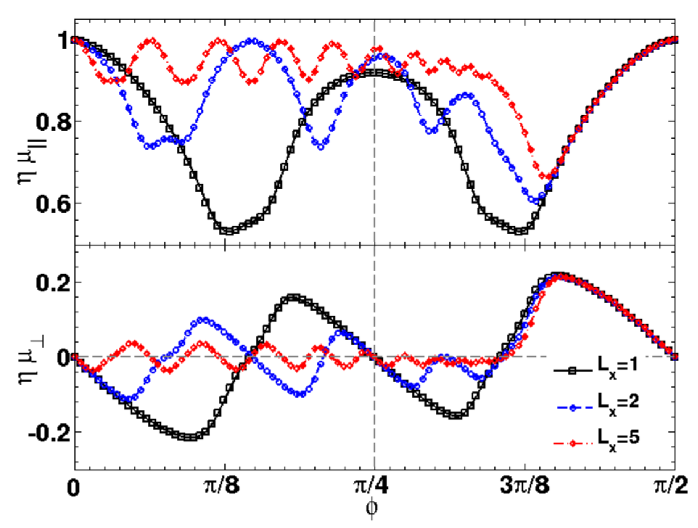


FIGURE 2a: Longitudinal and lateral mobility as a function of forcing angle $\phi$. $r_0 = 0.4$ and $f = 1000$.

- longitudinal and lateral particle mobility

$$\eta\mu_\parallel = \frac{\langle\, \dot{x}\,\rangle\,\cos(\phi) + \langle\, \dot{y}\,\rangle\,\sin(\phi)}{f} \qquad (2a)$$

$$\eta\mu_\perp = \frac{-\langle\, \dot{x}\,\rangle\,\sin(\phi) + \langle\, \dot{y}\,\rangle\,\cos(\phi)}{f} \qquad (2b)$$

- migration angle

$$\alpha = \phi + \arctan\left(\mu_\perp(\phi)/\mu_\parallel(\phi)\right) \qquad (3)$$

- longitudinal and lateral effective diffusivity

$$D_\parallel/D_0 = D^x_{\text{eff}}\,\cos(\phi) + D^y_{\text{eff}}\,\sin(\phi)\,, \qquad (4a)$$

$$D_\perp/D_0 = -\,D^x_{\text{eff}}\,\sin(\phi) + D^y_{\text{eff}}\,\cos(\phi) \qquad (4b)$$

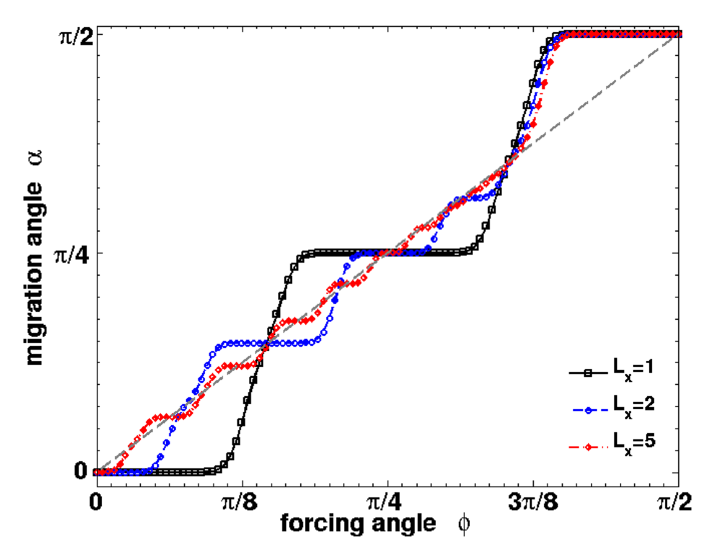


FIGURE 2b: Migration angle $\alpha$ versus forcing angle $\phi$. $r_0 = 0.4$ and $f = 1000$.

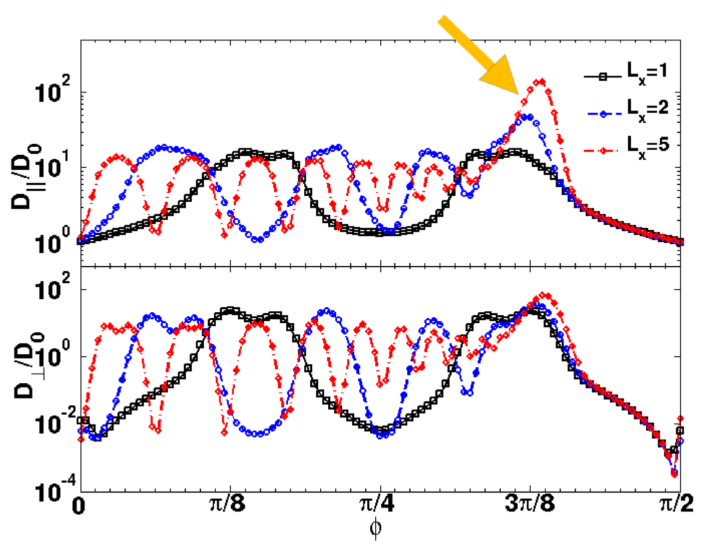
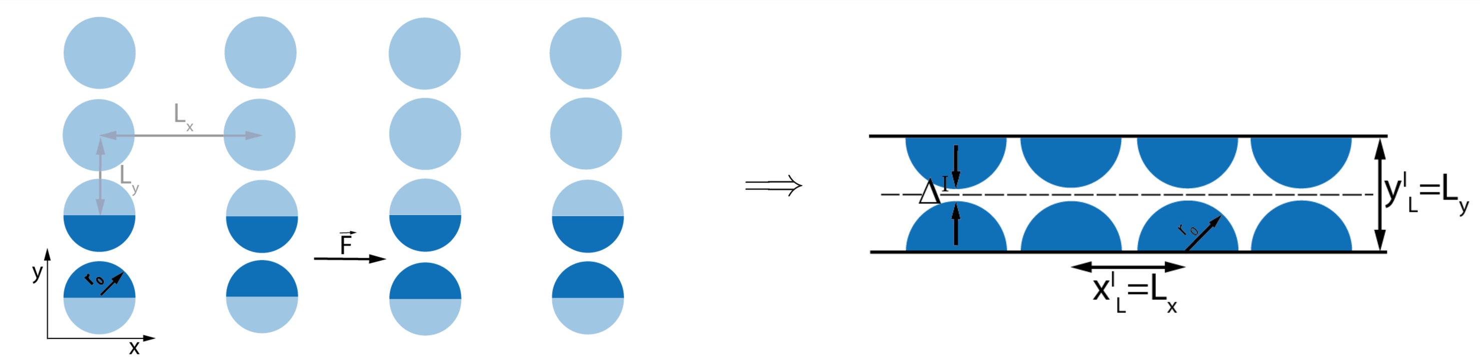


FIGURE 2c: Longitudinal and lateral effective diffusivity versus $\phi$. $r_0 = 0.4$ and $f = 1000$.

- most important results

1. if $L_x/L_y = n, n \in \mathbb{N} \to \alpha$ exhibits $n$ steps in $0 < \phi < \pi/4$ [2]
2. migration angle $\alpha$ equals forcing angle $\phi$ if $\mu_\perp(\phi) = 0$
3. enhancement of long. diffusion $\Longrightarrow L_x \neq L_y \curvearrowright D_\parallel \approx 100 D_0$
4. lateral diffusion $D_\perp \in [10^{-3}, 10^1]\,D_0$

## Quasi 1D channels - Channel I for $\phi = 0$



- longitudinal mobility possesses the following properties:

$$\text{(i)} \quad \mu(-f) = \mu(f)\,, \quad \text{(ii)} \quad \lim_{f\to 0} \eta\mu(f) \approx \sqrt{\frac{1-2\,r_0}{x_L}}\,, \quad \text{(iii)} \quad \lim_{f\to\infty} \eta\mu(f) = 1$$

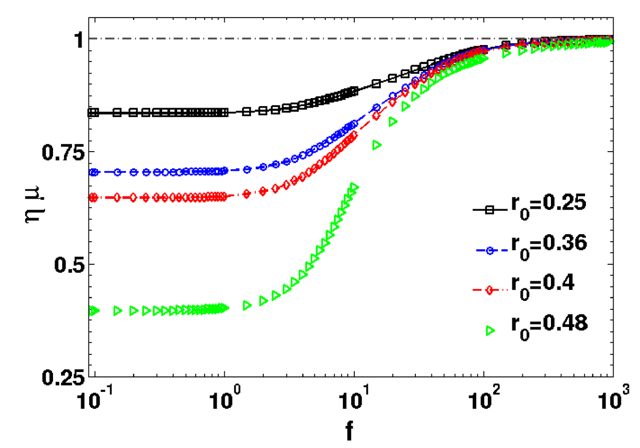


FIGURE 3a: Longitudinal particle mobility $\mu$ as a function of $f$. $x_L = y_L = 1$.

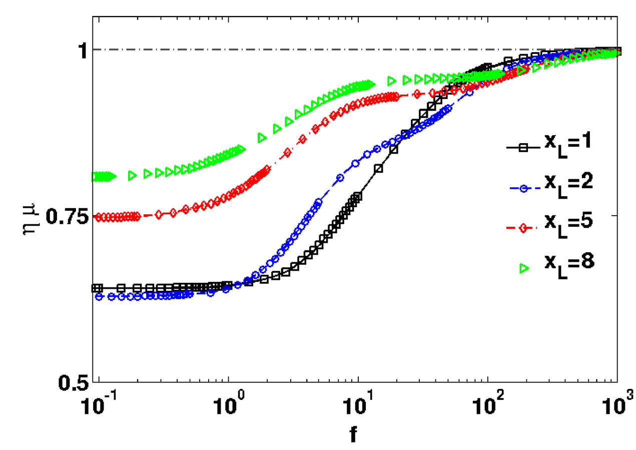


FIGURE 3b: Dependence of longitudinal particle mobility $\mu$ on $f$. $y_L = 1$ and $r_0 = 0.4$.

- for $x_L \gg 1$ mobility exhibits a plateau
- properties of longitudinal effective diffusion coefficient:

$$\text{(i)} \quad D(-f) = D(f)\,, \quad \text{(ii)} \quad \lim_{f\to 0} D(f)/D_0 = \lim_{f\to 0} \mu(f)/\mu_0, \quad \text{(iii)} \quad \lim_{f\to\infty} D(f)/D_0 = 1$$

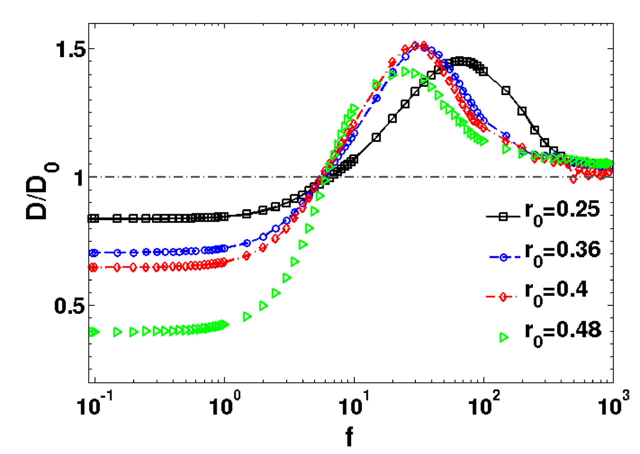


FIGURE 4a: Dependence of longitudinal effective diffusivity $D$ on $f$. $x_L = y_L = 1$.

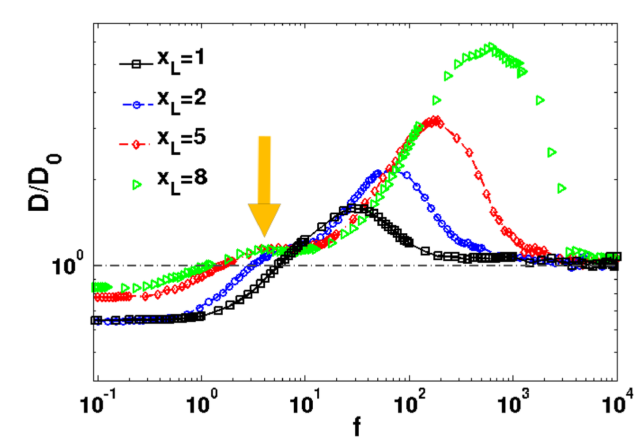


FIGURE 4b: Longitudinal effective diffusivity $D$ versus $f$. $y_L = 1$ and $r_0 = 0.4$.

- for $x_L \gg 1$ occurrence of a distinct shoulder in $D(f)$ independent of the value of $x_L$

### Estimates for diffusion peaks

- strongest separation effect if the longitudinal drift time $\tau_\parallel$ equals the transverse diffusion time $\tau_\perp$

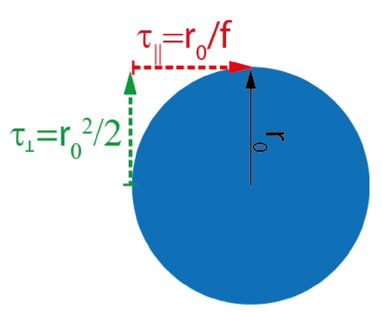


FIGURE 5a: Schematic sketch for 1st depinning mechanism which generates the shoulder in $D(f)$.

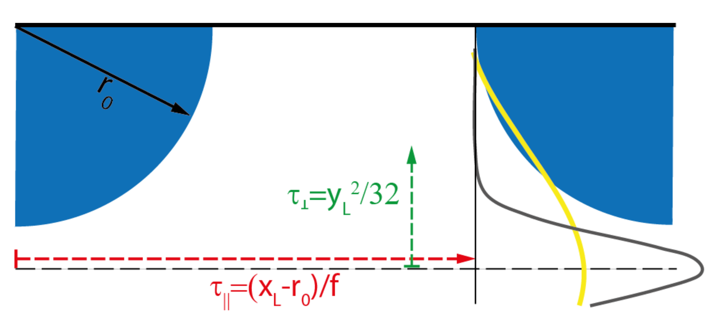
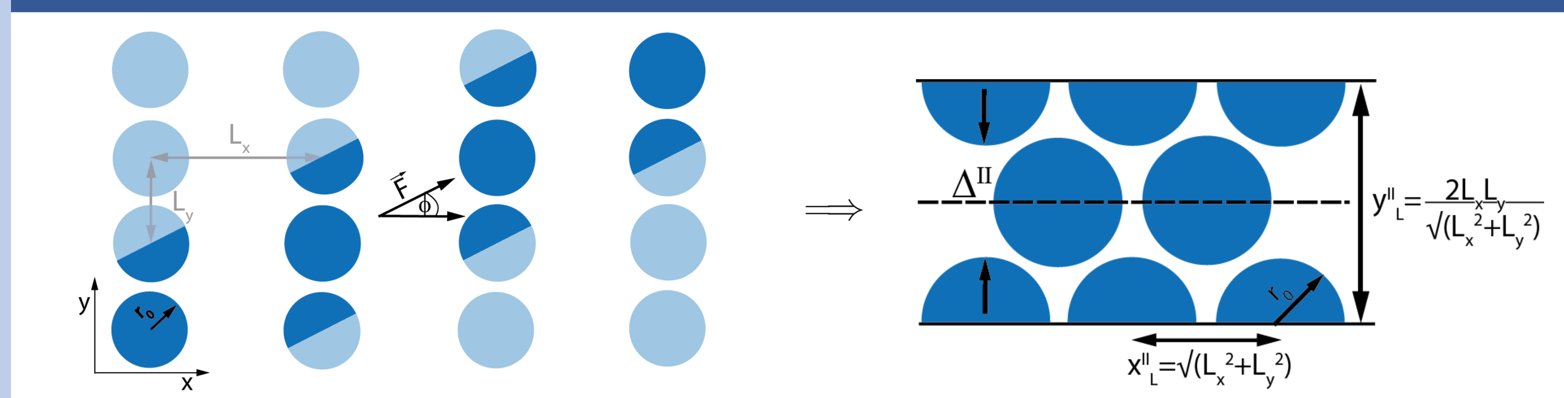


FIGURE 5b: Schematic sketch for 2nd depinning mechanism yielding the dominant diffusion peak.

$$f^{(I/II,1)} \simeq \frac{2}{r_0} \geq 4 \qquad (5)$$

$$f^{(I,2)} \simeq 32\,(x_L - r_0) \qquad (6)$$

## Quasi 1D channels - Channel II for $\phi = \arctan\left(L_y/L_x\right)$



$$\text{(i)} \lim_{f\to 0} \eta\mu(f) \lesssim 1\,, \quad \text{(iia)} \quad \lim_{f\to\infty} \eta\mu(f) = 1 \text{ if } r_0 \leq 1/4\,, \quad \text{(iib)} \quad \lim_{f\to\infty} \eta\mu(f) < 1 \text{ if } r_0 > 1/4$$

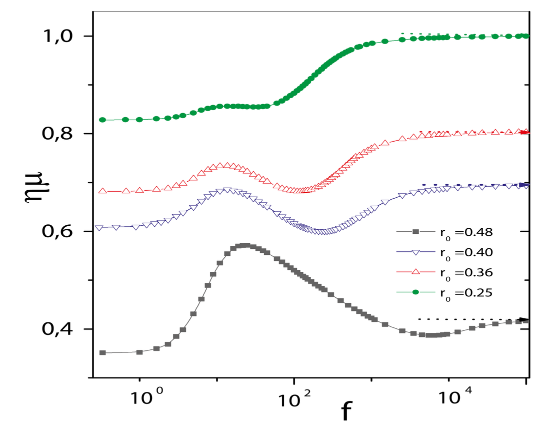


FIGURE 6a: Impact of $f$ and $r_0$ on longitudinal mobility $\mu$. $x_L = 2$ and $y_L = 1$.

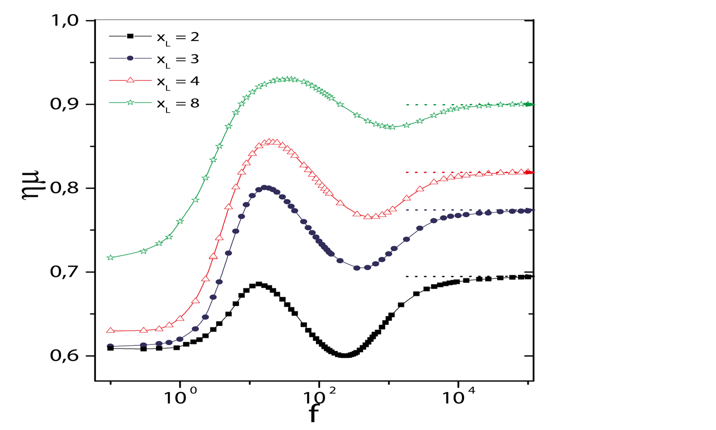


FIGURE 6b: Longitudinal mobility $\mu$ versus $f$ for various $x_L$. $y_L = 1$ and $r_0 = 0.4$.

- occurence of negative differential mobility

$$\text{(i)} \lim_{f\to 0} D/D_0 \lesssim 1\,, \quad \text{(iia)} \quad \lim_{f\to\infty} D/D_0 = 1 \text{ if } r_0 \leq 1/4\,, \quad \text{(iib)} \quad \lim_{f\to\infty} D/D_0 > 1 \text{ if } r_0 > 1/4$$

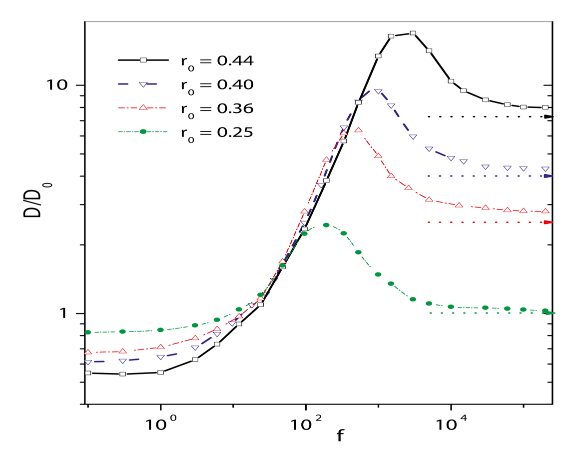


FIGURE 7a: Dependence of $D$ on $f$ and obstacle radius. $x_L = 2$ and $y_L = 1$.

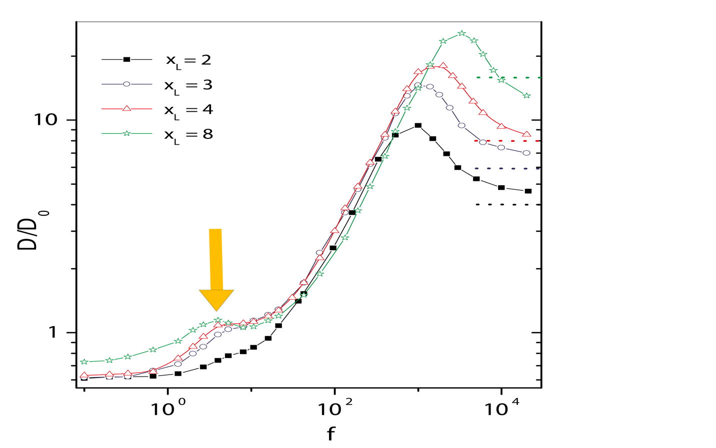
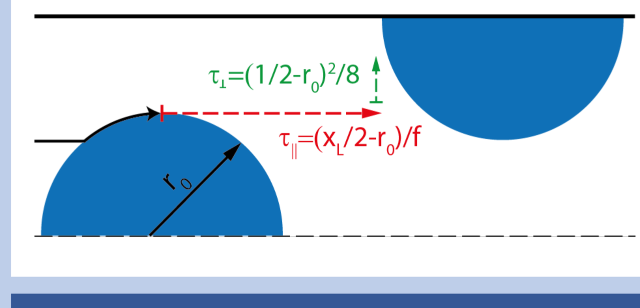


FIGURE 7b: Longitudinal diffusion coefficient $D$ versus $f$ for various $x_L$. $y_L = 1$ and $r_0 = 0.4$.

### Estimates for diffusion peaks



$$f^{(II,2)} \simeq 16\,\frac{(x_L - 2\,r_0)}{(1-2\,r_0)^2} \qquad (7)$$

FIGURE 8: Schematic sketch for 2nd depinning mechanism yielding the dominant diffusion peak.

## Conclusion

- array of circular obstacles
  1. $\alpha(\phi)$ is a "Shapiro-step" like function
  2. $\mu_{\parallel,\perp}$ and $D_{\parallel,\perp}$ show complicated structure of $\phi$
  3. excess diffusion peaks
- quasi 1D geometry - Channel I and II
  1. obstacle size dependent 2nd diffusion peak
  2. negative differential mobility
  3. unconventional asymptotic behaviors

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## References

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[2] J. Hermann et al. PRE **79**, 061404 (2009)
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