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Group consensus for a class of heterogeneous multi-agent networks in the competition systems

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ABSTRACT

In this paper, we investigate the group consensus issues of a continuous-time heterogeneous multi-agent networks. The system consists of two types of agents, the first-order and second-order, respectively. Be distinct from the related works, we propose a new distributed group consensus protocol which modeled by the agents' competition interaction. Based on stability theory and the method of frequency domain, the systems' group consensus including these two cases are discussed: without and with the influence of time delays. Meanwhile, the corresponding criteria are theoretically obtained. Specially, these criteria we obtained do not rely on the conservative conditions which exist in the relevant research works, such as in-degree balance and so on. Furthermore, we also theoretically establish the tolerable upper bounds of the time delays for achieving group consensus. The validity of our findings is verified through some numerical simulation examples.

1. Introduction

As an extended consensus issue, group consensus is also a fundamental problem of coordinate control in multi-agent networks (MANs). It implies that the different consensus can be realized in different subgroups of the whole systems. Namely, agents' consensus can be reached in the same subgroup. Due to their broad applications, lots of interest findings have been emerged recently, such as in the fields of mobile robot [1,2], smart grid [3–5], group decision making [7,8], optimization control of complex systems [9–11], sensor networks [12], delayed complex systems [13–16], hybrid systems [17], pinning control of complex systems [18,19], reinforcement learning and deep learning [20,21] and so on.

It should be pointed out that almost all the works aforementioned are mostly aimed at the homogeneous complex systems. Namely, all the systems' agents own an identical dynamic behaviour. Obviously, it is too absolutely restrictive to be applied in practice. In fact, there may be differences in the dynamics of the agents due to the various limitation, or it may be necessary to accomplish the expected task by the agents with heterogeneous dynamics. Thus, we need to pay more attention to the heterogeneous complex systems. Recently, a lot of relevant research works have been constantly reported. Such as in [22] where the consensus issues including the homogeneous and heterogeneous systems were investigated. In [23], the authors discussed the consensus problem for a class of heterogeneous systems with the undirected topology. In [24], the consensus problems for discrete-time heterogeneous networks with leaderless and leader-following were studied. The global bounded consensus of the heterogeneous systems was discussed and the sufficient condition was addressed as well in [25]. In [26], a novel protocol for heterogeneous complex systems was established and some criteria are proposed by using state transition method. In [27], the output consensus for heterogeneous discrete-time networks was investigated. In [28–30], the authors discussed the synchronization problems for the heterogeneous complex networks. Consider the influence of time delays, in [31,32] where investigated the heterogeneous MANs' consensus problems, respectively. In [33,34], group consensus for heterogeneous systems were studied, respectively. And some criteria were given theoretically. Through the results obtained in [35], it revealed that the eigenvalues of Laplacian matrix of the complex systems, especially the nonzero eigenvalues determined the realization of group consensus. In [36,37], to the heterogeneous systems with fixed and switching topologies, the authors not only discussed the systems' group consensus, but also extended their work to the situation under the influence of input time delays. In [38], the group synchronization for heterogeneous systems including linearly or nonlinearly dynamics were studied, respectively.

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It should be noted that on the one hand, the relevant research work is mainly based on the consensus problems for the heterogeneous systems, the discussion on the group consensus is still relatively rare; on the other hand, there is a need for further discussion on the consensus and group consensus of heterogeneous systems under the influence of time delay. After all, the delay will seriously affect the systems convergence performance [1]. Therefore, group consensus problem of a class of delayed heterogeneous MANs will be mainly investigated in this paper.

The main contributions of this paper are listed as follows: Firstly, a novel group consensus distributed control protocol is designed which modeled by the agents' competition interaction. Almost all of the above works are based on the cooperative interaction of the agents, such as in [22–38]. As known that in a complex systems, there are three kinds of interactions between the agents: cooperative, competitive, competition and cooperation in two aspects. So the case we discussed is a relatively new research perspective. Secondly, we theoretically establish some algebraic criteria based on the frequency domain method, which are different from those in [26,31,34–37]. Their works are mainly based on these two conservative conditions: balance of in-degree and the zero eigenvalue's geometric multiplicity of the Laplacian matrix are at least two. They are so special that limit the topology of the systems and the interaction between the subgroups. In our work, we absolutely relax these preconditions. Thus, our results should be more general. Thirdly, the topology of the systems we investigated is a bipartite digraph, which does not need to be symmetric. As known that the topology of the systems plays an important role in the achievement of group consensus. What has to be mentioned that the more general premise usually means the more challenging job.

Notation: In the paper, R represents the real set and C denotes the complex numbers set. For $\forall x \in C$, $|x|$ denotes its modulus, $\mathrm{Re}(x)$ and $\mathrm{Im}(x)$ respectively denote its real and imaginary parts. To matrix A, $\lambda_i(A)$ is its ith eigenvalue, and $det(A)$ denotes its determinant. I_n represents the n-dimensional identity matrix.

2. Problems statement

For a multi-agent network containing N agents, we can represent the agent and the information interaction between them by a weighted digraph $\eth = (\mathbb{V}, \mathbb{E}, \mathbb{A})$ via the graph theory, where $\mathbb{V}$, $\mathbb{E}$ and $\mathbb{A}$ is the node set, edge set and weighted adjacency matrix, respectively. In this paper, we assume that the adjacency weight between the agent i and j is nonnegative, i.e., $a_{ij} \geq 0, a_{i,j} \in \mathbb{A}$. The ith agent's in-degree and neighbor set are respectively denoted as $d_i = \deg_{in}(i) = \sum_{j=1}^{N} a_{ij}$ and $N_i = \left\{ j \in \mathbb{V} : e_{ij} \in \mathbb{E} \right\}$, so the degree matrix can be represented as $\mathbb{D}=diag\{d_1, d_2, \dots, d_N\}$.

Consider a class of heterogeneous networks $\eth$, which contains $N_s + N_f$ agents. Without loss of generality, assume the first N_s agents are second-order agents, and the rest N_f agents' dynamics is first-order. Meanwhile, the agents' dynamics are described as:

$$\begin{cases} \dot{\zeta}_i(t) = \varsigma_i(t), \\ \dot{\varsigma}_i(t) = u_i(t), \quad i \in \hat{s}_1, \end{cases} \tag{1}$$

$$\dot{\zeta}_i(t) = u_i(t), \quad i \in \hat{s}_2, \tag{2}$$

where $\hat{s}_1 = 1, 2, \dots, N_s, \hat{s}_2 = N_s + 1, N_s + 2, \dots, N_s + N_f, s = \hat{s}_1 \cup \hat{s}_2$. $u_i(t)$, $\varsigma_i(t)$, $\zeta_i(t)$ can represent agent i' state, such as input control, velocity, and position. For the convenience, note that in this paper we only consider the case of two subgroups.

Next, for the convenience of further analysis, we will firstly introduce some preliminary.

Definition 1. To the heterogeneous MASs (1) and (2), it is said that the group consensus can be achieved asymptotically if the following conditions hold:

$$\lim_{t\to+\infty} \left\| \zeta_i(t) - \zeta_j(t) \right\| = 0, \quad \text{if } i, j \in \hat{s}_k, \ k = 1, 2;$$

$$\lim_{t\to+\infty} \left\| \varsigma_i(t) - \varsigma_j(t) \right\| = 0, \quad \text{if } i, j \in \hat{s}_k, \ k = 1, 2.$$

Remark 1. In Definition 1, it not only applies to the whole heterogeneous systems, but also applies to the situations where the agent in the same subgroup is heterogeneous. It is different from the cases in [26,33,35,37]. They only consider the special case that is the agent in the same subgroup is homogeneous. Therefore, the case we discussed is more general.

Lemma 1 [11]**.** *Consider a complex systems $\eth$ including K agents, the following conditions hold if its topology is a bipartite digraph which has a directed spanning tree: if $\lambda_i(\mathbb{D} + \mathbb{A}) \neq 0$, $\mathrm{Re}(\lambda_i(\mathbb{D} + \mathbb{A})) > 0$; $rank(\mathbb{D} + \mathbb{A}) = K - 1$.*

Lemma 2 [31]**.** *For $\forall p > 0$, inequation $p/(1 + p^2) < \arctan(p)$ can be held.*

3. Main results

In this section, we will discuss the group consensus issues of the heterogeneous systems (1) and (2).

Between agents i and j, as known that $x_j - x_i$ and $x_j + x_i$ can represent the cooperation and competition interaction [11]. So far, almost all related references on consensus and group consensus are geared toward the cooperative networks. Motivated by those works but different from them, we design a distinct distributed group consensus control protocol for the heterogeneous MANs modeled by the agents' competitive interaction, which listed as:

$$\begin{cases} \dot{\zeta}_i(t) = \varsigma_i(t), \\ \dot{\varsigma}_i(t) = -\alpha \left[\sum_{j\in N_i} a_{ij}[\zeta_i(t) + \zeta_j(t)] \right] - \beta \left[\sum_{j\in N_i} a_{ij}[\varsigma_i(t) + \varsigma_j(t)] \right], \\ \qquad \forall i \in \hat{s}_1, \end{cases} \tag{3}$$

and

$$\begin{cases} \dot{\zeta}_i(t) = -\beta \left[\sum_{j\in N_i} a_{ij}[\zeta_i(t) + \zeta_j(t)] \right] + \omega_i(t), \\ \dot{\omega}_i(t) = -\alpha \left[\sum_{j\in N_i} a_{ij}[\zeta_i(t) + \zeta_j(t)] \right], \quad \forall i \in \hat{s}_2. \end{cases} \tag{4}$$

where α, β are the positive control parameters. Same with the works in [31,34–37], $\omega_i(t)$ is the ith agent's velocity estimate with first-order dynamics.

Suppose τ is the input time delay of each agent, the delayed systems (3) and (4) can be listed as:

$$\begin{cases} \dot{\zeta}_i(t) = \varsigma_i(t), \\ \dot{\varsigma}_i(t) = -\alpha \left[\sum_{j\in N_i} a_{ij}[\zeta_i(t-\tau) + \zeta_j(t-\tau)] \right] \\ \qquad - \beta \left[\sum_{j\in N_i} a_{ij}[\varsigma_i(t-\tau) + \varsigma_j(t-\tau)] \right], \quad \forall i \in \hat{s}_1, \end{cases} \tag{5}$$

and

$$\begin{cases} \dot{\zeta}_i(t) = -\beta \left[\sum_{j\in N_i} a_{ij}[\zeta_i(t-\tau) + \zeta_j(t-\tau)] \right] + \omega_i(t), \\ \dot{\omega}_i(t) = -\alpha \left[\sum_{j\in N_i} a_{ij}[\zeta_i(t-\tau) + \zeta_j(t-\tau)] \right], \quad \forall i \in \hat{s}_2. \end{cases} \tag{6}$$

Theorem 1. *For the heterogeneous MANs (5) and (6), if system's topology is a bipartite digraph which contains a directed spanning tree, group consensus can be reached asymptotically when* $\sqrt{\alpha^2+\beta^2\omega_{i0}^2} < \frac{\omega_{i0}^2}{\sqrt{\mathrm{Re}^2(\lambda_i)+\mathrm{Im}^2(\lambda_i)}}$, $i \in s$ *holds. Where* ω_{i0} *satisfies* $\alpha \tan\omega_{i0}\tau = \beta\omega_{i0}$.

Proof. First, doing Laplace transforms to (5) and (6), it has

$$\begin{cases} s\zeta_i(\hat{s}) = \varsigma_i(\hat{s}), \\ s\varsigma_i(\hat{s}) = -\alpha\left[\sum_{j\in N_i} a_{ij}[\zeta_i(\hat{s})+\zeta_j(\hat{s})]e^{-\hat{s}\tau}\right] \\ \qquad\qquad -\beta\left[\sum_{j\in N_i} a_{ij}[\varsigma_i(\hat{s})+\varsigma_j(\hat{s})]e^{-\hat{s}\tau}\right], \quad \forall i \in \hat{s}_1, \end{cases} \tag{7}$$

and

$$\begin{cases} s\zeta_i(\hat{s}) = \omega_i(\hat{s}) - \beta\left[\sum_{j\in N_i} a_{ij}[\zeta_i(\hat{s})+\zeta_j(\hat{s})]e^{-\hat{s}\tau}\right], \\ s\omega_i(\hat{s}) = -\alpha\left[\sum_{j\in N_i} a_{ij}[\zeta_i(\hat{s})+\zeta_j(\hat{s})]e^{-\hat{s}\tau}\right], \quad \forall i \in \hat{s}_2. \end{cases} \tag{8}$$

Where $\zeta_i(\hat{s})$, $\varsigma_i(\hat{s})$, $\omega_i(\hat{s})$ respectively present the Laplace transformation of $\zeta_i(t)$, $\varsigma_i(t)$, $\omega_i(t)$. Set $\zeta(\hat{s}) = [\zeta_1(\hat{s}), \zeta_2(\hat{s}), \ldots, \zeta_{N_s+N_f}(\hat{s})]^T$, from the Eqs. (7) and (8), if follows that

$$\hat{s}^2\zeta(\hat{s}) = -(\beta\hat{s}+\alpha)(\mathbb{D}+\mathbb{A})e^{-\hat{s}\tau}\zeta(\hat{s}). \tag{9}$$

By Eq. (9), we obtain the systems' characteristic equation as

$$\det(\hat{s}^2 I + (\beta\hat{s}+\alpha)(\mathbb{D}+\mathbb{A})e^{-\hat{s}\tau}) = 0. \tag{10}$$

Since systems' topology is a bipartite digraph containing a directed spanning trees, by Lemma 1, we have $rank(\mathbb{D}+\mathbb{A}) = N_s + N_f - 1$. So all other eigenvalues' real part of the matrix $(\mathbb{D}+\mathbb{A})$ is positive except the simple zero eigenvalue. For convenience, we suppose $\lambda_1 = 0$, $Re(\lambda_i) > 0$, $i \geq 2$, where λ_i is the ith eigenvalue of the matrix $(\mathbb{D}+\mathbb{A})$. Hence, from Eq. (10), we have

$$\hat{s}^4 \prod_{i=2,\ldots,N_s+N_f} (\hat{s}^2 + \lambda_i(\beta\hat{s}+\alpha)e^{-\hat{s}\tau}) = 0. \tag{11}$$

From Eq. (11), we find that it has four roots at the point $\hat{s} = 0$. When $\hat{s} \neq 0$, it can be rewritten as follows,

$$1 + \lambda_i(\beta\hat{s}+\alpha)e^{-\hat{s}\tau}/\hat{s}^2 = 0, i = 2, 3, \ldots, N_s + N_f. \tag{12}$$

Define $h_i(\hat{s}) = \lambda_i(\alpha+\beta\hat{s})e^{-\hat{s}\tau}/\hat{s}^2$, so $h_i(\hat{s}) + 1 = 0$. From the general Nyquist criterion, it follows that the zeros point of Eq. (12) have negative real part when point $(-1, j0)$ is not enclose by the Nyquist curve of $h_i(\hat{s})$.

Set $\hat{s} = j\omega, \omega \in R$, it yields that

$$h_i(j\omega) = \frac{\lambda_i}{-\omega^2}[(\beta\omega\sin\omega\tau + \alpha\cos\omega\tau) + j(\beta\omega\cos\omega\tau - \alpha\sin\omega\tau)]. \tag{13}$$

So we have

$$|h_i(j\omega)| = \frac{|\lambda_i|}{\omega^2}\sqrt{\alpha^2+\beta^2\omega^2} = \frac{\sqrt{\mathrm{Re}^2(\lambda_i)+\mathrm{Im}^2(\lambda_i)}}{\omega^2}\sqrt{\alpha^2+\beta^2\omega^2}. \tag{14}$$

Assume the Nyquist curve of $h_i(j\omega)$ first crosses the real axis at the ω_{i0} point, it follows that $\tan\omega_{i0}\tau = \beta\omega_{i0}/\alpha$. As $|g_i(j\omega)|$ is monotonously decline when $\omega > 0$. Thus, when $|h_i(j\omega_{i0})| < 1$, the zero point of the Eq. (12) has negative real part.

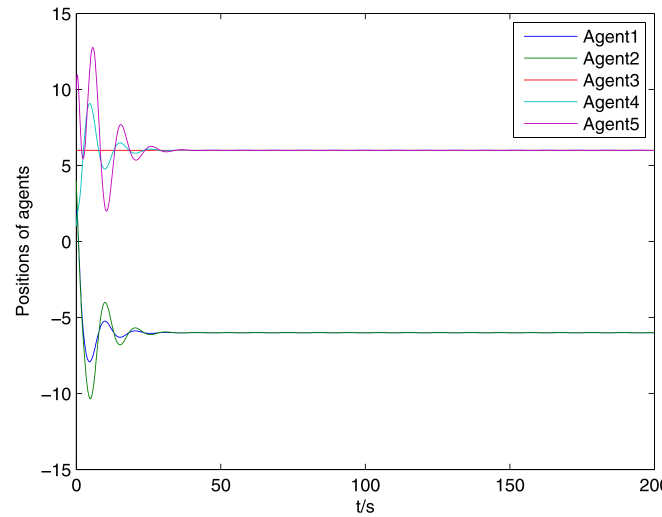


Fig. 1. The state trajectories of the agents when $\tau = 0s$.

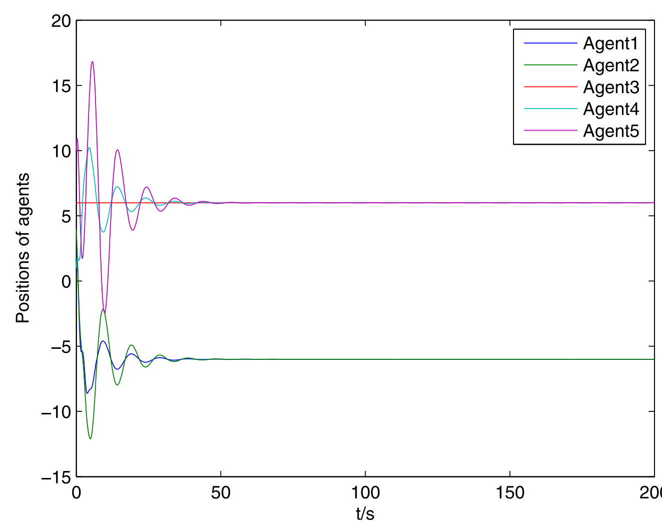


Fig. 2. The state trajectories of the agents when $\tau = 0.35s$.

Combining the above analysis and the stability theory, if $\frac{\sqrt{\mathrm{Re}^2(\lambda_i)+\mathrm{Im}^2(\lambda_i)}}{\omega_{i0}^2}\sqrt{\alpha^2+\beta^2\omega_{i0}^2} < 1$ is satisfied, the systems' group consensus can be reached asymptotically. That completes the proof.

If $\frac{|\lambda_i|}{\omega_{i0}^2}\sqrt{\alpha^2+\beta^2\omega_{i0}^2} < 1$ holds, it follows that

$$\omega_{i0} > \sqrt{\frac{|\lambda_i|^2\beta^2 + \sqrt{|\lambda_i|^4\beta^4 + 4|\lambda_i|^2\alpha^2}}{2}}. \tag{15}$$

From $\tan\omega_{i0}\tau = \beta\omega_{i0}/\alpha$, we can obtain

$$\tau = \frac{\arctan(\frac{\beta\omega_{i0}}{\alpha})}{\omega_{i0}}. \tag{16}$$

Next, from (16), we have

$$\frac{d\tau}{d\omega_{i0}} = \frac{1}{\omega_{i0}^2}\left[\frac{\beta\omega_{i0}/\alpha}{1+(\beta\omega_{i0}/\alpha)^2} - \arctan(\beta\omega_{i0}/\alpha)\right]. \tag{17}$$

When $\omega_{i0} > 0$ holds, by Lemma 2, it has $\frac{d\tau}{d\omega_{i0}} < 0$.

Combining Eqs. (15) and (16), it follows that

$$\tau < \frac{\arctan(\frac{\beta}{\alpha}\tilde{\phi})}{\tilde{\phi}}. \tag{18}$$

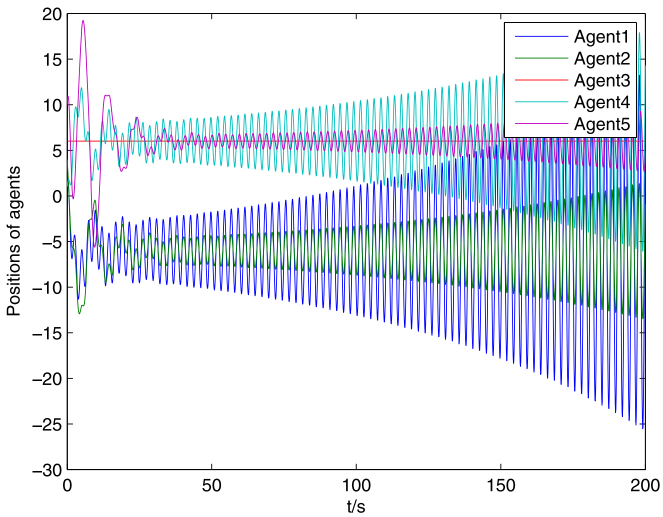


Fig. 3. The state trajectories of the agents when $\tau = 0.45s$.

where $\tilde{\phi} = \sqrt{\frac{|\lambda_i|^2\beta^2+\sqrt{|\lambda_i|^4\beta^4+4|\lambda_i|^2\alpha^2}}{2}}$, $|\lambda_i| = \sqrt{\mathrm{Re}^2(\lambda_i) + \mathrm{Im}^2(\lambda_i)}$.

Corollary 1. *Consider the heterogeneous systems (5) and (6), if system's topology is a bipartite digraph which contains directed spanning trees, group consensus can be reached asymptotically if*

$$\tau < \min_{\lambda_i \neq 0} \left\{ \frac{\arctan(\frac{\beta}{\alpha}\tilde{\phi})}{\tilde{\phi}} \right\}, i \in s. \tag{19}$$

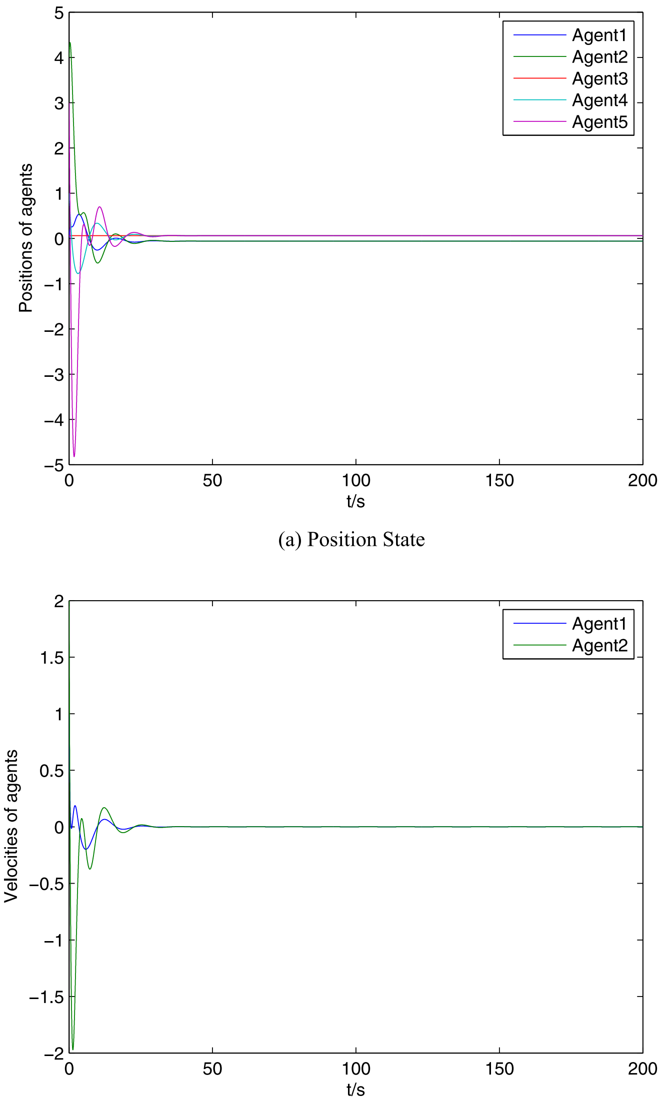
holds, where $\tilde{\phi} = \sqrt{\frac{|\lambda_i|^2\beta^2+\sqrt{|\lambda_i|^4\beta^4+4|\lambda_i|^2\alpha^2}}{2}}$, $|\lambda_i| = \sqrt{\mathrm{Re}^2(\lambda_i) + \mathrm{Im}^2(\lambda_i)}$ *and* λ_i *is the nonzero eigenvalue of matrix* $(\mathbb{D} + \mathbb{A})$.

Remark 2. From Theorem 1 and Corollary 1, it reveals that the system's control parameters and the maximal eigenvalue of matrix $(\mathbb{D} + \mathbb{A})$ associated with systems play key roles on the realization of group consensus. In addition, it is worth noting that the conclusion is similar to those in the references [31] and [37]. But the difference should be listed as follows: in our work, we do not rely on these two assumptions: the geometric multiplicity of Laplacian matrix's zero eigenvalue are not less than two and balance of in-degree. The former condition limits the system topology. In addition, the latter one implies that the interactions between the subgroups of the systems are offset [6] as well. Hence, our results should be more general after relaxing these conservative conditions.

Remark 3. In this paper, the systems' topology we supposed seems to be a special topology. But in fact, to a multi-agent networks, its group consensus can hardly be achieved unless some other conditions are added. For instance, in [26,31,34–37], the systems topology either is an undirected graph or contains directed spanning trees. Further reducing the topological complexity of complex systems is a challenging task. Lots of work need to be explored in the future.

Corollary 2. *When* $\tau = 0$, *consider the heterogeneous MANs (5) and (6), if system's topology is a bipartite digraph which contains a directed spanning tree, group consensus can be asymptotically realized if* $\alpha, \beta > 0$ *hold.*

From the proof of Theorem 1, the results in Corollary 2 can be easily obtained, so we omit it here.



(a) Position State

(b) Velocity State

Fig. 4. The states trajectories of the agents when $\tau = 0s$.

4. Simulation results

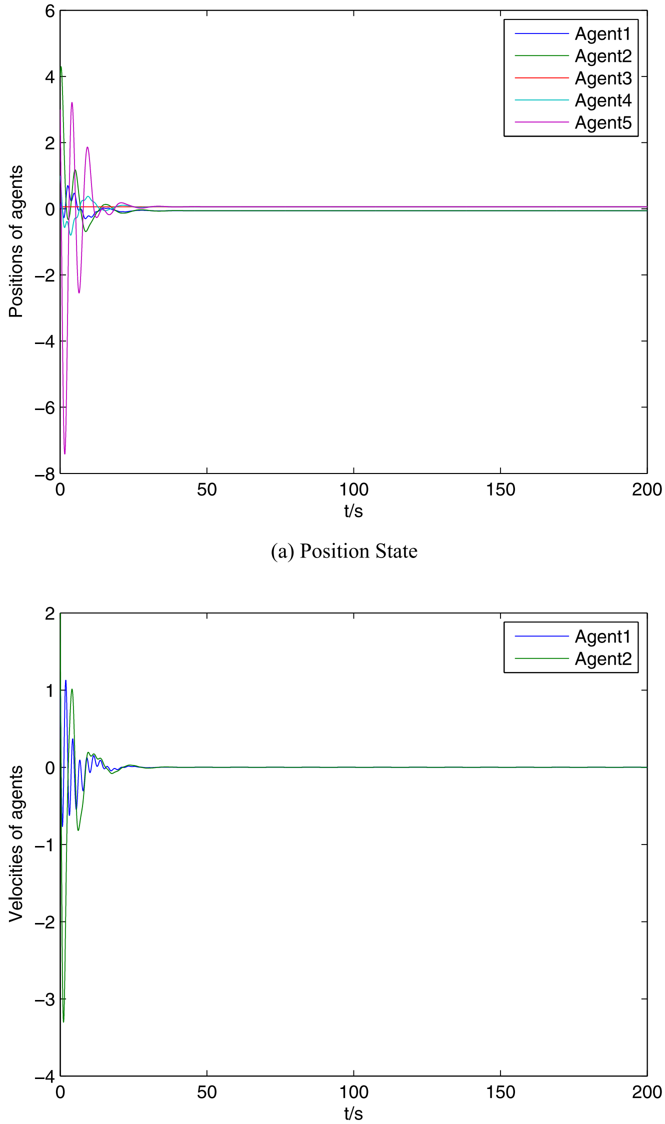
In this section, the correctness of our theoretical results will be illustrated by several numerical simulation examples.

Suppose the systems contains 5 agents and is divided into two subgroups. Thereinto, the first two agents belong to the subgroup $\eth_1$ and the other subgroup $\eth_2$ contains the remaining ones. Without loss of generality, we set the adjacency matrix as

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

It is easy to know that $\lambda_i(\mathbb{D} + \mathbb{A}) = \{0, 0.3820, 1, 1, 2.6180, i = 1, 2, ..., 5\}$.

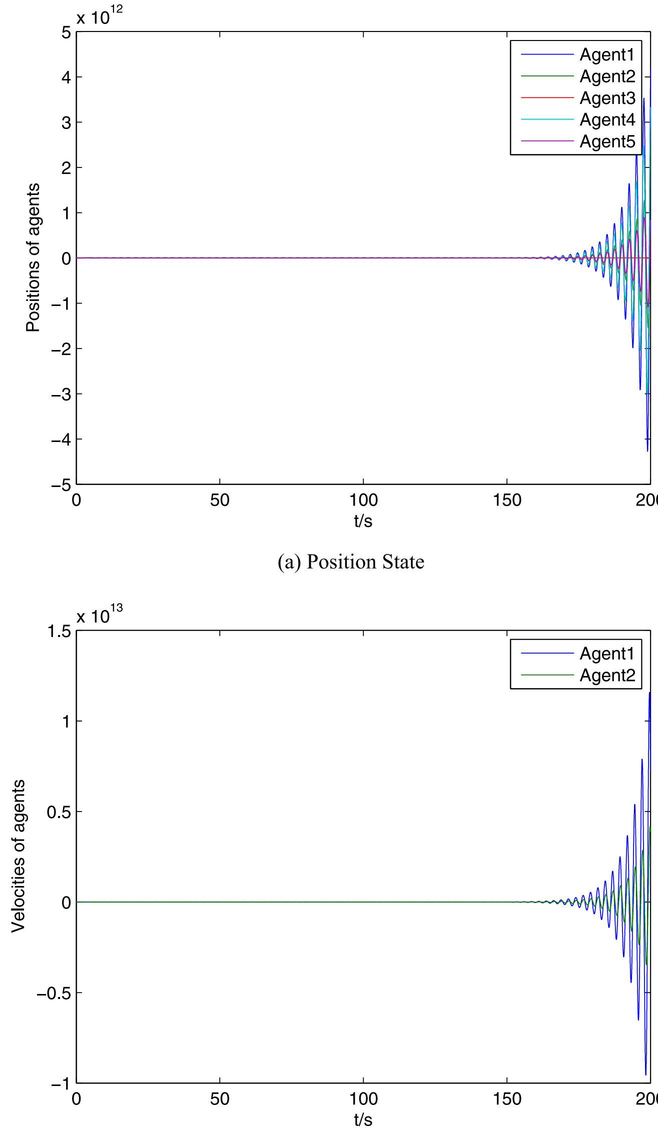
Example 1. In this example, we assume the agents 1, 4, 5 are second-order nodes and the remaining agents are first-order nodes. Hence, the whole systems and these two subgroups are all heterogeneous. Without loss of generality, set $\alpha = 1$ and $\beta = 1$, based on the results in Theorem 1 and Corollary 1, it has $\tau < 0.44s$.



(a) Position State

(b) Velocity State

Fig. 5. The states trajectories of the agents when $\tau = 0.35s$.



(a) Position State

(b) Velocity State

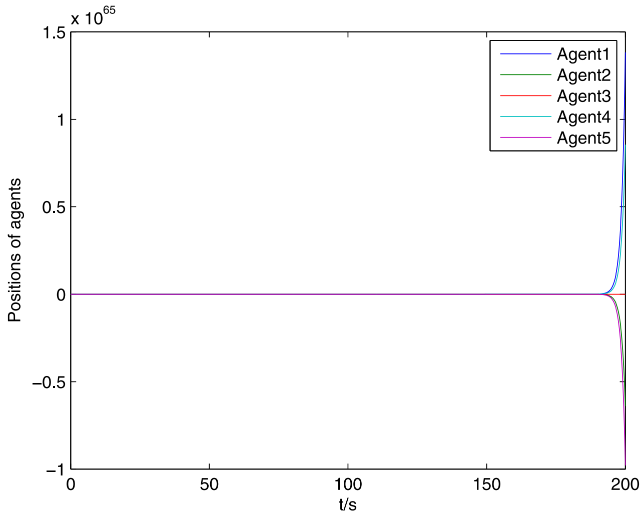
Fig. 6. The states trajectories of the agents when $\tau = 0.45s$.

That's to say, the systems' can achieve group consensus if time delay $\tau < 0.44s$. Next, we will consider the following three cases to verify the correctness of the result: *i)* $\tau = 0s$; *ii)* $\tau = 0.35s$; *iii)* $\tau = 0.45s$. Figs. 1–3 accordingly illustrate the agents' state trajectories. From those results, we find that the group consensus can be realized in these two cases: 1) $\tau = 0s$ and 2) $\tau = 0.35s$. However, the group consensus of the systems does not be achieved when $\tau = 0.45s$. That is to say, the systems' group consensus cannot be achieved when the delay exceeds the upper bound of the time delay that the systems can tolerate. This simulated example can illustrated the validity of the proposed Theorem 1, Corollary 1 and Corollary 2.

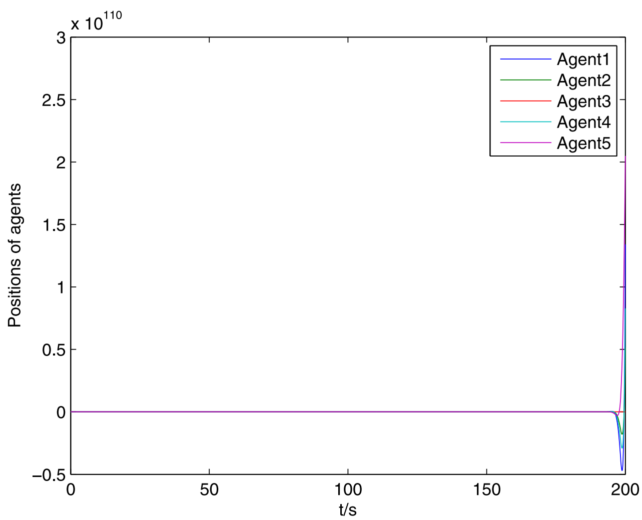
Example 2. Different from the Example 1, in this example, we suppose the agents with second-order dynamics include the agent 1 and the agent 2. And the rest agents are first-order nodes. Under this circumstance, the whole systems is heterogeneous, but the two subgroups, $\eth_1$ and $\eth_2$ are homogeneous. Therefore, if the systems' group consensus can be realized, the velocity state of the agents in $\eth_1$ can also reach consensus asymptotically.

In the example, all the values of the parameters and the considered cases are same with those in Example 1, the agents' trajectories are shown in Figs. 4–6. From the results, it shows that the position state can reach group consensus and the velocity state can reach consensus asymptotically when $\tau = 0s$ and $\tau = 0.35s$. But the system's group consensus cannot be achieved when $\tau = 0.45s$. These results further verifies the correctness of the conclusions we obtained.

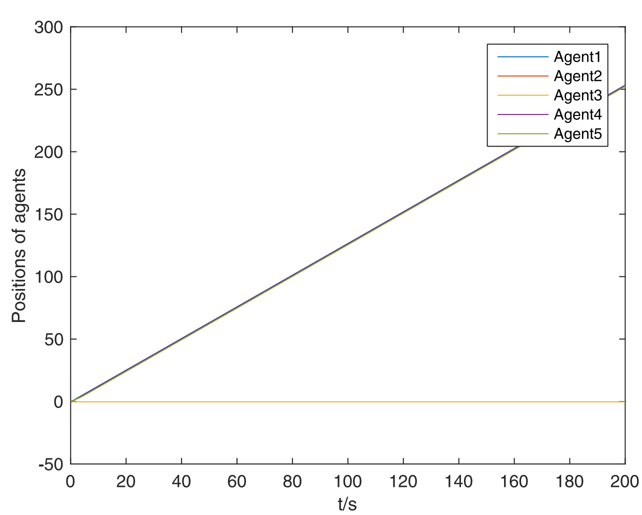
Example 3. The results in Examples 1 and 2 have respectively verify the correctness of Corollary 2. Next, we will continue to consider the following three cases to illustrate the validity of Corollary 2: $\alpha < 0,\ \beta > 0$; $\alpha > 0,\ \beta < 0$ and $\alpha,\ \beta < 0$. Based on Example 1, the agents' trajectory are shown in Fig. 7. Obviously, the group consensus in all these three cases are not achieved. Combining the results of the above examples, it is not difficult to find the correctness of Corollary 2.



(a)



(b)



(c)

Fig. 7. The state trajectories of the agents when $\tau = 0s$. (a) $\alpha = -1, \beta = 1$; (b) $\alpha = 1, \beta = -1$; (c) $\alpha = -1, \beta = -1$.

5. Conclusion

This paper investigates the group consensus issue of a heterogeneous MANs which are constructed by the agents with first-order and second-order dynamics. Be distinct from the most relevant works which based on the agents' cooperation relationship, we propose a new distributed group consensus protocol which modelled by agents' competitive interaction. By graph theory and stability theory, the group consensus issues for this heterogeneous systems with and without time delay are discussed. Meanwhile, the sufficient criteria are proposed and the upper bound of time delay is obtained as well. From the results, it reveals that the systems' control parameters, topology and the coupling strength between agents play an important roles in the achievement of group consensus. Meanwhile, our results do not rely on the related restrictive preconditions existed in the relevant works. Such as, balance of in-degree and the zero eigenvalue's geometric multiplicity of the Laplacian matrix are at least two. These conditions limit the topology of the systems and the interaction between the subgroups. Therefore, our results should be more general in practice. In the future, we will further the work to the cases with multiple time delays and more general topologies.

Conflict of Interest

None.

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