Bipartite Rendezvous for Heterogeneous Agents in Uncertain Cooperation-Competition Networks

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***Abstract*—The article investigated a kind of bipartite rendezvous problem of heterogeneous multi-agent systems in a uncertain cooperation-competition network assuming that it is structurally balanced and undirected. Under this background, the whole network topology can be separated into two cooperation sub-networks. The first-order system considered is to model the heterogeneous agents in this paper. As the problem goes deeper, a distributed adaptive Nussbaum-type-function controller is designed to estimate the nonzero unknown parameters reflecting the model uncertainty is designed, so did the distributed protocol with leader information by using the energy inequality of the multi-agent systems with heterogeneous agents. Through this strategy, the heterogeneous multi-agent system can achieve bipartite rendezvous. Finally, simulation results are shown to demonstrate the validity of the theoretical analysis.**

***Index Terms*—heterogeneous agents, cooperation-competition network, Nussbaum function, bipartite rendezvous**

I. Introduction

Nowadays, the research on multi-agent systems widely exists in control science, which has already received increasing attention. The collective behaviors of Multi-Agent system [1] own their various types on which it is of great significance for researchers to put efforts exploring. The collective behavior in multi-agent system refers to a phenomenon that the agents combine their own properties and the information interaction among agents to complete a series of relevant actions, so that the whole system can realize a certain regular phenomenon [2]-[7]. It is rather remarkable that the types of collective behaviors among agents are determined by their forms of information interaction, accordingly the essence of studying collective behavior is to analyze and design corresponding distributed protocols. Generally, collective behavior includes consensus [6]-[8], distributed tracking [9, 10], formation control [11, 12], and so on.

When studying the collective behavior of multi-agent systems, the issue of consensus tracking caught the eyes of researchers. In the consensus problem, that is to say, the network is separated into manifold sub-networks, where agents exchange message with their neighbors in different ways under the different sub-networks. According to the information of agents' neighbors, the research focus on designing a distributed protocol, and to drive all agents and their leader to reach consensus. In recent years, there have come the great deals of excellent results in the research field of consensus tracking problems. In [13], Shao et al, inspired by the Pigeon Hierarchies problem, based on the assumption that the network do not contain self-loops, analyzed the consensus tracking problem under the switching topology, and further found that the hierarchy has a great influence on the consensus of agents. In [14], Xu et al. studied a consensus problem, where a general class of linear MASs gather three event triggering schemes, and proposed that this kind of event trigger mechanism can obviously decrease the updated frequency of the controllers. In [15], considering the influence of communication delay and switching topology, Lu et al. designed a novel adaptive distributed controller for Euler-Lagrange closed system. In [16], Li et al. studied the distributed tracking problem of linear multi-agent systems with bounded unknown input leaders.According to the relative state of agent's neighbor, Li designed two distributed discontinuous controllers to ensure the state of followers is consistent with the leader.

Regarding the consensus tracking problem, the works in the above literature are all based on a common assumption that there is only one cooperative relationship between agents. From the standpoint of graph theory, weights of edge connection between each agent are nonnegative. However, in real-world scenarios, it is inevitable that agents will have not only cooperative relationships but also competitive relationships. The performance of the network is that the connection weight is negative. We call this network structure of cooperation and competition coexisting as cooperation-competition network, which is way more challenging and practical to study the competitive relationship in swarming behavior. Therefore, the problem of the consensus tracking in cooperation-competition networks has become more necessary to investigate. In [17], Wen et al. studied a leader-following problem linear systems with s single dynamic leader and designed a new distributed non-smoothing protocol rested on the relative state information of the agent's neighbor, and utilized Lyapunov stability theorem to ensure the bipartite consensus tracking works on this system in this framework. Furthermore the bipartite

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consensus tracking problem with different functions based on a preset-time approach and controller considering neighbor states proposed in [18]. Based on the above framework, under the cooperation-competition network Hu et al. [19] studied and analysed the reverse group consensus problem without in-degree equilibrium conditions and found that the competitive relationship between agents plays an active role when realizing the corresponding collective behavior.

Due to the complexity of practical engineering applications, the research on the uncertainty of systems or networks has more practical significance. In [20], Haris et al. used the distributed nonlinear PI function to deal with the problem under switching topology, which the integral agents with different and unknown directions. Under the conditions of unbalanced and non-strongly connected switching topologies, Wang et al. [21] proved that the systems with different and unknown directions can achieve consensus.

Mainly with the above inspirations, we discuss the bipartite rendezvous problem of under the cooperation-competition network, which is a special class of consensus tracking problem. Among this problem, the entire network topology is separated into two sub-networks in particular. Specifically, the agents in the same subnetwork share the positive connection weights, while the agents between different subnetworks own their negative weights. Moreover, for the uncertainty of the system model, a distributed protocol rested on relative error information and a Nussbaum-type function is designed. The boundedness of auxiliary variables and some relevant lemmas are used to prove the controller designed can finally achieve the bipartite rendezvous.

The structure under the article is as follows. Section II gives several basic concepts related to the cooperation-competition network topology, and provide the definition of system model. In Section III, the distributed protocol is designed, and the chief theoretical outcomes are presented. In Section IV, a numerical simulation is given to illustrate our analytic results. Finally, we give a conclusion to sum up this paper in Section V.

II. Preliminaries

A. Cooperation-Competition Network

The information interaction between agents in a MAS is usually described by a topology [22, 23]. Normally, a cooperation-competition network $\mathcal{R} = (\mathbf{V}, \zeta, \boldsymbol{A})$ is proposed to describe the topology of agents. the sign of the weights between agents have two possibilities, that is positive or negative, and $a_{ij} < 0$ refers to the competitive relationship within agent i and agent j. In addition, the agents in the network considered in this paper do not contain self-loops. Symbol π_i is assigned to represent agent i, and a path connecting π_i to π_j in $\mathcal{R}$ can be described as a sequence of edges $(\pi_i, \pi_{i1}), (\pi_{i1}, \pi_{i2}), \ldots, (\pi_{il}, \pi_j)$, where $\pi_{ik}, k = 1, 2, \ldots, l$ are different agents. The cooperation-competition network is said connected while there exists a connection between any two different agents.

Next, some necessary definitions and the following lemmas are given.

Definition 1([24]). A network $\mathcal{R} = (\mathbf{V}, \zeta, \boldsymbol{A})$ with N agents is said to be a structurally balanced network provided that there exists a bipartition $\{V_1, V_2\}$ of the vertex set V, such that $V_1 \cup V_2 = V$, $V_1 \cap V_2 = \varnothing$, and $a_{ij} \geq 0$ for any $\pi_i, \pi_j \in V_k, k \in \{1, 2\}$, while $a_{ij} \leq 0$ for any $\pi_i \in V_{k1}, \pi_j \in V_{k2}, k_1 \neq k_2\,(k_1, k_2 \in \{1, 2\})$. Otherwise, it is said structurally unbalanced network.

Lemma 1([24]). $\mathcal{R}$ is structurally balanced if and only if there exists a matrix $D = \operatorname{diag}\{d_1, d_2, \ldots, d_n\}$ with $d_i \in \{1, -1\}$ such that DAD has all nonnegative entries.

Lemma 2([25]). Let $f : [0, +\infty) \to R$ be a uniformly continuous function, if $\lim_{t\to\infty} \int_0^t f(\tau)d\tau$ exists and is finite, then $f(t) \to 0$ as $t \to 0$.

Definition 2([26]). A differentiable even function $N(\cdot)$ is said to be Nussbaum function provided that the following properties are satisfied:

$$\begin{aligned} &\lim_{k\to\infty} \sup \left(\tfrac{1}{k}\int_0^k N(\tau)d\tau\right) = \infty, \\ &\lim_{k\to\infty} \inf \left(\tfrac{1}{k}\int_0^k N(\tau)d\tau\right) = -\infty. \end{aligned} \tag{1}$$

B. System Model

Consider a MAS composed of N heterogeneous agents. The mathematical model is as follows:

$$\dot{x}_i = c_i u_i + \varphi_i (x_i)^T \theta_i, \forall i = 1, 2, \ldots, N, \tag{2}$$

where x_i, u_i represent the state and the control input acting on agent i, c_i is the unknown system parameter and θ_i is the corresponding unknown constant vector of agent i. Meanwhile, $\varphi_i(\cdot)$ is a known continuous vector function of agent i. It is noteworthy that c_i, θ_i and $\varphi_i(\cdot)$ reflect the individual heterogeneity simultaneously, among which θ_i also gives expression to the model uncertainty.

Furthermore, the following assumption on the system is put forward.

Assumption 1. For system (2), all the unknown nonzero parameters $c_i, \forall i = 1, 2, \ldots, N$ have the same sign.

To summarise the topological relationship between the leader and its followers, a matrix $H = \operatorname{diag}\{a_{10}, a_{20}, \ldots, a_{N0}\}$ is defined with the property that once agent i can receive the leader's information directly, then set $a_{i0} = 1$, otherwise $a_{i0} = 0$.

Definition 3. For a structurally balanced network $\mathcal{R}$ with adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$, (2) is viewed as asymptotically achieving bipartite rendezvous assuming that the solutions of (2) is able to satisfy

$$\lim_{t\to\infty} \|x_i(t) - d_i x_0\| = 0, \forall i = 1, 2, \ldots, N, \tag{3}$$

where $x_0 \in R$ is an arbitrarily given constant.

Lemma 3([27]). Consider a structurally balanced connected network $\mathcal{R}$, suppose that there is at least one agent can connect with the leader and receive its information, then $L + H$ is positive definite matrix.

Lemma 4([28]). Let functions $V(t)$ and $k_i(t), \forall i = 1, 2, \ldots, N$ be smooth defined in $[0, t_f)$, where $V(t) \geq 0$,

$k_i(0) = 0$. At same time, let $N_0(k_i) = \cosh(\lambda k_i)\sin(k_i)$ where $\lambda > \max\left\{\frac{1}{\pi}\ln\frac{\eta_{\max}(N-1)}{\eta_{\min}}, 0\right\}$ with $\eta_{\max} > \eta_{\min} > 0$. If the following inequality holds:

$$\begin{aligned} V(t) \leq & \sum_{i=1}^{N}\eta_i\int_0^t N_0(k_i(\tau))\dot{k}_i(\tau)d\tau \\ & + \sum_{i=1}^{N} a_i\int_0^t \dot{k}_i(\tau)d\tau + c, \forall t \in [0, t_f), \end{aligned} \tag{4}$$

where both a_i and η_i are constants, among which $a_i > 0$, η_i are all positive or negative, and $|\eta_i| \in [\eta_{\min}, \eta_{\max}]$, $i = 1, 2, \ldots, N$. Then $V(t)$, $k_i(t)$ and $\sum_{i=1}^{N}\eta_i\int_0^t N_0(k_i(\tau))\dot{k}_i(\tau)d\tau$ on the interval $[0, t_f)$ are bounded.

III. MAIN RESULTS

Theorem 1. Consider a structurally balanced and connected cooperation-competition network $\mathcal{R}$, as well as the mathematical model is shown by (2). Under assumption 1, (2) can achieve the bipartite rendezvous by the following distributed controller

$$u_i = -N_0(k_i)\left(\xi_i + \varphi_i(x_i)^T\hat{\theta}_i\right), \tag{5}$$

where

$$N_0(k_i) = \cosh(\lambda k_i)\sin(k_i), \tag{6}$$

$$\begin{aligned} \xi_i(t) = & \textstyle\sum_{j=1}^{n}|a_{ij}|\left(x_i(t) - \operatorname{sgn}(a_{ij})x_j(t)\right) \\ & + a_{i0}(x_i(t) - d_i x_0), \end{aligned} \tag{7}$$

and the evolution laws of k_i and $\hat{\theta}_i$ are designed as below:

$$\dot{k}_i = \gamma_i\xi_i\left(\xi_i + \varphi_i(x_i)^T\hat{\theta}_i\right), \tag{8}$$

$$\dot{\hat{\theta}}_i = \zeta_i\varphi_i(x_i)\xi_i. \tag{9}$$

Proof. Denote the state vector $X = [x_1, x_2, \ldots, x_n]^T$. Due to the fact that $\mathcal{R}$ is structurally balanced, the error vector could be denoted as $\bar{X} = [x_1 - d_1x_0, x_2 - d_2x_0, \ldots, x_n - d_nx_0]^T$, $\bar{X}$ could be rewritten as $\bar{X} = X - D1x_0$, where 1 is a column vector that all elements are 1. Let $\bar{L} = L + H$, and from Lemma 3, we conclude $\bar{L}$ is a positive definite matrix, and

$$\begin{aligned} \bar{L}\bar{X} &= (L + H)(X - D1x_0) \\ &= LX - LD1x_0 + HX - HD1x_0 \\ &= LX + H(X - D1x_0) - LD1x_0 \\ &= LX + H(X - D1x_0). \end{aligned} \tag{10}$$

Hence, one can get

$$\bar{L}\bar{X} = \begin{pmatrix} \sum_{j=1}^{n}|a_{1j}|\left(x_1(t) - \operatorname{sgn}(a_{1j})x_j(t)\right) \\ +a_{10}(x_1(t) - d_1x_0) \\ \vdots \\ \sum_{j=1}^{n}|a_{ij}|\left(x_i(t) - \operatorname{sgn}(a_{ij})x_j(t)\right) \\ +a_{i0}(x_i(t) - d_ix_0) \\ \vdots \\ \sum_{j=1}^{n}|a_{nj}|\left(x_n(t) - \operatorname{sgn}(a_{nj})x_j(t)\right) \\ +a_{n0}(x_n(t) - d_nx_0) \end{pmatrix}. \tag{11}$$

Denote $\xi_i(t) = \sum_{i=1}^{n}|a_{ij}|\left(x_i(t) - \operatorname{sgn}(a_{ij})x_j(t)\right) + a_{i0}(x_i(t) - d_ix_0)$ and $\tilde{\theta} = \hat{\theta}_i - \theta_i$, $\tilde{\theta} = \left[\tilde{\theta}_1^T, \tilde{\theta}_2^T, \ldots, \tilde{\theta}_n^T\right]^T$. We can construct the Lyapunov function as below:

$$V(t) = \frac{1}{2}\bar{X}^T\bar{L}\bar{X} + \frac{1}{2}\tilde{\theta}^T\left(\operatorname{diag}\{\zeta\}^{-1}\otimes I\right)\tilde{\theta}, \tag{12}$$

where $\zeta = [\zeta_1, \zeta_2, \ldots, \zeta_n]^T$. When calculating the derivative of Eq. (12), one has Eq. (13), which implies Eq. (14).

According to lemma 4, one can obtain that $V(t)$, $-\sum_{i=1}^{N}\int_0^t\frac{c_i}{\gamma_i}N_0(k_i(\tau))\dot{k}_i(\tau)d\tau$, k_i are bounded. Thus, $\int_0^\infty\xi_i^2(t)dt$ exists. Furthermore, by Lemma 2, one can draw a conclusion that $\xi_i^2(t) \to 0$ as $t \to 0$, indicating that when $t \to 0$, $\xi_i(t) \to 0$. Combine Eq. (11) and Eq. (12), there are $\bar{L}\bar{X} = 0$ as $t \to \infty$. According to the positive definite of $\bar{L}$, it can be found that $\bar{X} \to 0$ as $t \to \infty$, namely,

$$\lim_{t\to\infty}\|x_i(t) - d_ix_0\| = 0, \tag{15}$$

which means that (2) can reach bipartite rendezvous. This completes the proof. ■

IV. NUMERICAL SIMULATION

In the part, we give a numerical sample to verify the validity of the derived analytical consequences.

Consider 6 heterogeneous agents in network $\mathcal{R}$ the same as the above theoretical analysis, agents 1 to 6 are viewed as the followers and the agent 0 represents as the leader in this article, which is depicted in Figure. 1. Seen from Figure. 1, the solid lines mean that there is a cooperative relationship between agents and the edges with positive weights, at the same time the dotted lines indicate there is a competitive relationship between agents and the edges with negative weights. Moreover, the positive weights are taken 1 , while the negative weights are taken -1. Agent 1 can obtain the information from leader 0.

A is the weighted matrix and L is the Laplacian matrix of network topology $\mathcal{R}$, which are given below:

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}, \tag{16}$$

$$L = \begin{bmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 3 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 2 \end{bmatrix}. \tag{17}$$

Obviously, when the agents are in the same sub-network, it means they are cooperative and when the ones are in different sub-networks meaning that tey are competitive, thus the whole network is structurally balanced. The dynamic model of the agents is shown as Eq. (2). Since it is a

$$
\begin{aligned}
\dot{V}(t) &= \bar{X}^T \bar{L} \dot{X} + \tilde{\theta}^T \left(\operatorname{diag}\{\zeta\}^{-1} \otimes I\right) \dot{\hat{\theta}} \\
&= \sum_{i=1}^{N} \dot{x}_i \xi_i + \tilde{\theta}^T \left(\operatorname{diag}\{\zeta\}^{-1} \otimes I\right) \dot{\hat{\theta}} \\
&= \sum_{i=1}^{N} \dot{x}_i \xi_i + \sum_{i=1}^{N} \frac{1}{\zeta_i} \tilde{\theta}_i^T \dot{\hat{\theta}}_i \\
&= \sum_{i=1}^{N} \left[-c_i N_0(k_i) \xi_i - c_i N_0(k_i) \varphi_i(x_i)^T \hat{\theta}_i + \varphi_i(x_i)^T \theta_i\right] \xi_i + \sum_{i=1}^{N} \frac{1}{\zeta_i} \tilde{\theta}_i^T \left(\zeta_i \varphi_i(x_i) \xi_i\right) \\
&= -\sum_{i=1}^{N} \xi_i^2 - \sum_{i=1}^{N} \left[c_i N_0(k_i) \xi_i^2 + c_i N_0(k_i) \varphi_i(x_i)^T \hat{\theta}_i \xi_i - \varphi_i(x_i)^T \hat{\theta}_i \xi_i - \xi_i^2\right] \\
&= -\sum_{i=1}^{N} \xi_i^2 - \sum_{i=1}^{N} \frac{c_i}{\gamma_i} N_0(k_i) \dot{k}_i + \sum_{i=1}^{N} \frac{\dot{k}_i}{\gamma_i}
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
V(t) &= -\sum_{i=1}^{N} \int_0^t \xi_i^2(\tau) d\tau - \sum_{i=1}^{N} \int_0^t \frac{c_i}{\gamma_i} N_0(k_i(\tau)) \dot{k}_i(\tau) d\tau + \sum_{i=1}^{N} \int_0^t \frac{1}{\gamma_i} \dot{k}_i(\tau) d\tau + c \\
&\leq -\sum_{i=1}^{N} \int_0^t \frac{c_i}{\gamma_i} N_0(k_i(\tau)) \dot{k}_i(\tau) d\tau + \sum_{i=1}^{N} \int_0^t \frac{1}{\gamma_i} \dot{k}_i(\tau) d\tau + c
\end{aligned}
\tag{14}
$$

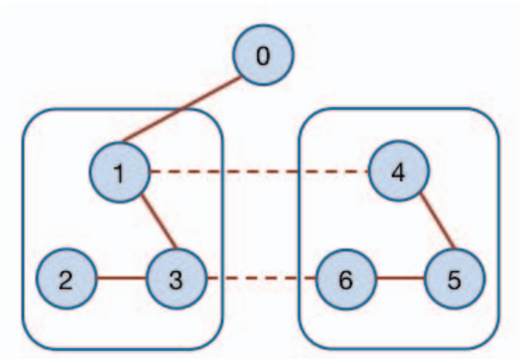

Fig. 1. Network topology $\mathcal{R}$ of 6 agents and 1 leader.

cooperation-competition network, the corresponding canonical transformation matrix D can be obtained:

$$
D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{18}
$$

The unknown parameters c_i, θ_i, ζ_i, γ_i, and the initial value $x_i(0)$ are shown in Table 1. Meanwhile, the continuous function are chosen as $\varphi_1(x_1) = 2\sin(x_1)$, $\varphi_2(x_2) = 2\cos(x_2)$, $\varphi_3(x_3) = x_3$, $\varphi_4(x_4) = x_4$, $\varphi_5(x_5) = 2\sin(x_5)$, $\varphi_6(x_6) = x_6^2$. Let x_0=1.6, furthermore, the initial value of the estimated vector $\hat{\theta}_i$ of θ_i is chosen as 0.

TABLE I
AGENTS' PARAMETERS.

Agents' parameters	Agents					
	1	2	3	4	5	6
c_i	1.25	2.87	2.98	3.18	3.67	3.94
θ_i	-1.3	-2.5	1.6	1.3	-0.8	0.6
ζ_i	0.8	1.8	2.5	1.3	1.5	2.3
γ_i	0.3	1.5	2.7	1.0	2.1	1.4
$x_i(0)$	1.7	0.8	-1.6	1.6	-0.8	-0.7
a_{i0}	1	0	0	1	0	0

Under the Nussbaum distributed controller, the evolution of x_i is shown in Figure. 2. It can be observed that x_i in the two sub-networks eventually asymptotically reach 1.6 and -1.6, respectively, and finally the MAS achieve the bipartite rendezvous, verifying the correctness of the designed distributed protocol. Also, the evolution of k_i is shown in Figure. 3.

V. CONCLUSION

The article has researched the bipartite rendezvous problem for the uncertain cooperation-competition network. In our study, each heterogeneous agent is modeled by the first-order systems integrator. Meanwhile, the unknown parameters of the agents induce the uncertainty of the system model. Given the assumption that the unknown parameters of each agent has the same sign. To deal with this problem, the Nussbaum gain method and the adaptive scheme based upon the neighbor's information interaction is proposed. Moreover, a distributed controller with leader information is designed. Through the boundedness of auxiliary variables and Barbalat's lemma, the

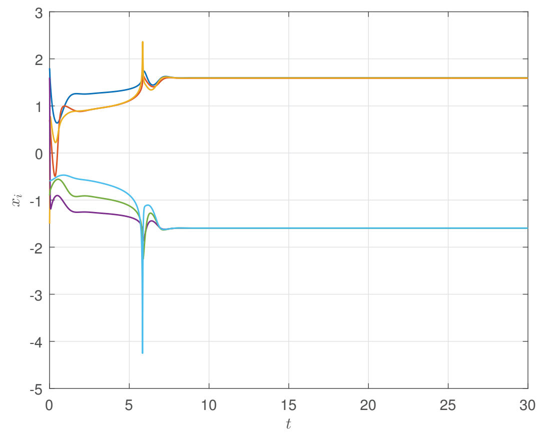

Fig. 2. Evolution of x_i.

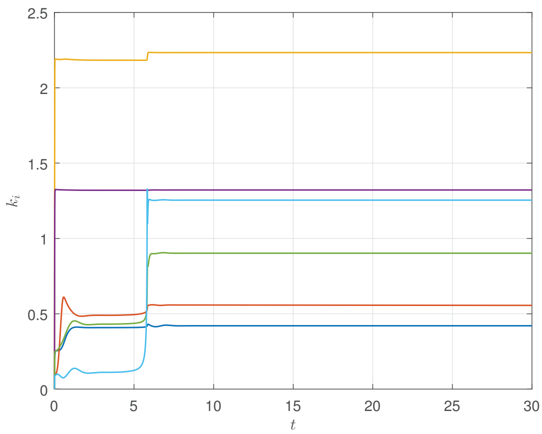

Fig. 3. Evolution of k_i.

outcome has shown the error vector of the heterogeneous system is able to reach zero asymptotically, and thus achieve the bipartite rendezvous. At last, a numerical simulation is used to verify that the designed controller can help the system achieve the desired behaviors.

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