Astro 276: Instrumentation
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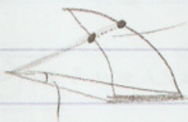
9/27/2007

Spherical Coordinate Systems

pole (degenerate point)
equator
latitude (angle N or S of equator)
pole
meridian lines (east or west of prime meridian)
prime meridian (special arbitrary meridian)
passes through Royal Observatory in Greenwich (currently in Edinburgh)

New York long 73° 58` W
lat 40° 40` N

Westwood 118° 20' W (118.°43)
34° 3' N



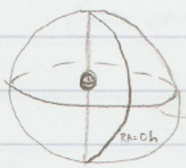
not longitude
equator
this is longitude → measured at equator

Different meridian lines pass under astronomical objects at different times.

24 hrs in a day → 24 ~equal slices of longitude 15° per hour

San Francisco 122.°418 W   SF + Westwood Δlong = 4°   (48."3 └/fares ?)
16 minutes (High Noon) or star transits

RA + Dec



celestial longitude = right ascension
latitude = declination
celestial equator
celestial pole

@ vernal equinox both systems are identical

celestial sphere takes $23^h 56^m$ for rotation   1° on sky or 4m time

Sidereal Time
– Right Ascension of the (local) meridian at a given time
ex RA = $14^h 5^m 13^s$ local sidereal time
object RA = $16^h 15^m 27^s$ → dif: $2^h 10^m 14^s$ hour angle → when object will transit └ highest in sky = its on meridian

same object will transit in SF 16 minutes later

if your lat = 34°, object on celestial equator will transit 34° S of your zenith
angle off zenith is dec – latitude
zenith stars – stars whose declinations match latitude of locations
started by Hawaiians (?)
Hokule`a (Arcturus) dec = 19° south point in Hawaii

Tahiti $-17° = $ dec A'a (Sirius)

Westwood 34° Epsilon Cygnus (2.48) 5th brightest star in Cygnus

San Francisco 38° Vega

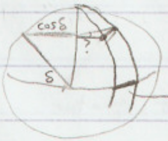
New York 41° Algol

2 stars angular distance between them close so ignoring spherical trig

RA $4^h\ 2^m\ 14^s$ $4^h\ 3^m\ 17^s$

Dec $27°\ 14'\ 4''$ $27°\ 16'\ 6''$

$\Delta RA = 1^m\ 3^s = 63$ sec $\Delta Dec = 2'\ 2'' = 122''$



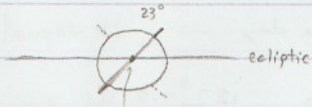
$\therefore\ \Delta RA = 63\ \text{sec}\left(15\frac{''}{\text{sec}}\right)\cos\delta \simeq 400''$

both stars close so chose one or another

ΔRA 15" per sec valid @ equator

$\text{dis} = \sqrt{(\Delta RA)^2 + (\Delta\delta)^2}$

Precession period = 26,000 yrs



current coordinate system 2007.75

equinox — specifies when coordinates were extended

first point of Aries

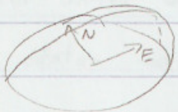
- full circle in 26,000 yrs

$\Rightarrow 50.''4$ per year (not small)

1950
2000 Coordinates → for tables (so dont have to change tables every year)

Other spherical coordinates

Altitude + Azimuth



horizon ~ equator

zenith ~ pole

altitude — angle between horizon and object (~lat)

azimuth — ~longitude 0=North

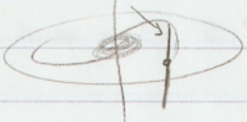
use computer control to track

→ field of view rotates

Ecliptic Coordinates

planets move at ~constant ecliptic latitude

Galactic Coordinates

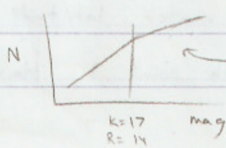


pole is ⊥ to orbital motion of stars

this is centered on Earth

0 galactic longitude → center of galaxy

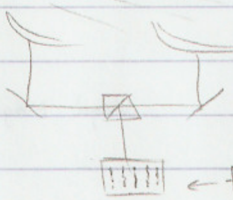
Extragalactic objects



no more stars from Galaxy

Light and Photometry

Light is ... a causal connection between 2 points in space and time



photon with discrete energy can follow very complicated paths

interference pattern → photon interferes with itself

wave – interact as a wave

particle – emitted + absorbed in discrete energy packets

$\lambda \cdot f = c$ $\quad 3\times10^{8}$ m/s

visible light 400 nm – 700 nm
7.5×10^{14} Hz
3 eV

Planck ⇒ energy per photon

$E = hf$

$h = 6.625\times10^{-27}$ erg·s

wavelengths of light – detection

Type	λ	ϵ		
gamma rays	< 1 pc	> 1.2 MeV	nuclear energies	
soft gamma rays	0.001 – 0.01 nm	120 keV – 1.2 MeV		
X-rays	0.01 – 1 nm	1.2 – 120 keV	auger electrons → 2 kev	
soft x-rays	1 – 10 nm	120 – 1200 eV		
EUV	10 – 100 nm	12 eV – 120 eV	ionizing atoms	use mirrors at grazing angle
UV	100 – 400 nm	3 eV – 12 eV		
visible	400 – 700 nm	1.7 – 3 eV	electron transitions in atoms; destroy molecules	
	700 nm – 1 μm	1.2 – 1.7 eV		
near IR	1 – 3 μm	0.4 – 1.2 eV	band gap of semiconductors	thermal e^-
mid IR	3 – 20 μm	0.06 – 0.4 eV	very cold semiconductors; bolometers (ΔT)	
thermal IR	20 – 1000 μm (1 mm)	1.2×10^{-4} – 0.06 eV	bolometer	
millimeter	1 – 3 mm; gap due to water in atm	1.2×10^{-4} – 3.6×10^{-4} eV	huge # of e^- → free e^- in metals (antenae)	
radio	10 – 100 m	$< 1.2\times10^{-5}$ eV		

Photometry

flux: ergs/s/cm² cgs units ergs/s $= 10^{-7}$ Watts

magnitudes: Hipparchus 2nd century BC 1st – 6th magnitude by eye qualitative for 2000 yrs

1856 – Pogson – quantitative measurements

5 mag diff = 100 times brighter 1 mag diff = 2.5119 times brighter

$\text{mag} = -\underset{\uparrow \text{exact}}{2.5} \log(\text{flux}) + \text{const}$ $\qquad m_1 - m_2 = -2.5 \log\left(\frac{f_1}{f_2}\right)$

$\Delta m = 1 \qquad 10^{\left(\frac{-1}{2.5}\right)} = 2.5119$

Sun: −26.8 Vega: 0.03

color

V_{mag} = V filter $\quad R_{mag}$ = red filter $\quad B_{mag}$ = blue filter

$$-2.5 \log\left(\frac{f_R}{f_B}\right) = M_R - M_B + C_R + C_B$$

Vega arbitrarily chosen to have no color $\quad M_B - M_V = 0 \quad$ Vega Magnitudes

$\underset{short}{M_B} - \underset{long}{M_V} > 0$ red $\quad < 0$ blue

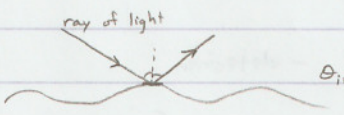
AB Magnitude system : if $\frac{f_1}{f_2} = 1$ then $M_1 - M_2 = 0$

10/2/2007

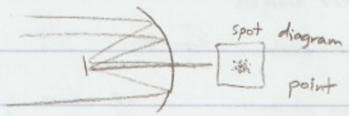
Read Chpt $\underset{Intro}{1}$, $\underset{Optics}{4.1 - 4.4}$, $\underset{Optics}{5.3}$ "Astronomical Optics" Schroeder 2^{nd} edition

Optics (Geometric)

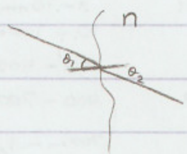
law of reflection



$\theta_{in} = \theta_{out}$



point spread function



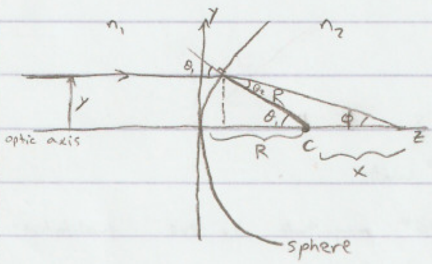
Snell's law transparent or translucent material

speed of light in material: $v = \frac{c}{n}$ — index of refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$\nu_1 = \nu_2$ peak + valley conservation $\quad \lambda_1 \nu_1 = \frac{c}{n_1} \quad \lambda_2 \nu_2 = \frac{c}{n_2} \quad \nu_1 = \frac{c}{n_1\lambda_1} = \frac{c}{n_2\lambda_2} = \nu_2$

$$\lambda_1 n_1 = n_2 \lambda_2 \qquad \lambda_1 = \lambda_2\left(\frac{n_2}{n_1}\right)$$



R: radius of curvature

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \sin(\theta_1 - \theta_2) = \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2$

$\phi = \theta_1 - \theta_2$

$$\sin\theta_1 = \frac{y}{R} \qquad \sin\theta_2 = \frac{n_1 y}{n_2 R} \qquad \frac{\sin\theta_2}{x} = \frac{\sin\phi}{R} = \frac{\sin(\theta_1 - \theta_2)}{R}$$

paraxial approximation $\quad y \ll R \implies \theta_1, \theta_2$ are small

$$x = \frac{R}{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \left(\frac{y}{R}\right)^2} - \sqrt{1 - \left(\frac{y}{R}\right)^2}}$$

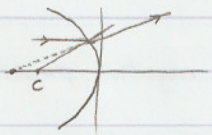
with approximation: $\quad x = \frac{R}{\frac{n_2}{n_1} - 1} = \frac{n_1 R}{n_2 - n_1}$ indep. of y

focus

when $y \sim R$, x changes $\rightarrow$ spherical aberration

focal length $\quad f = R + x = \boxed{\frac{n_2 R}{n_2 - n_1} = f}$

Convex system



$R<0$ center to left of surface

f is negative $\Rightarrow$ virtual focus

Ex:

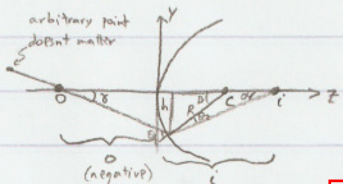
$R=10\,cm$ $\quad n_1=1$ $\quad n_2=1.5$ $\quad f=\frac{1.5 \cdot 10}{0.5}=30\,cm$

$n_2=1.05$ $\quad f=\frac{1.05\,(10)}{1.05-1}=200\,cm$

large n $\Rightarrow$ short focal length for same R

(large n_2-n_1)

more general system



$\theta_2=\beta-\alpha \qquad \theta_1=\beta+\gamma$

$n_1 \sin(\beta+\gamma)=n_2\sin(\beta-\alpha)$

$\sin\gamma=-\frac{h}{o}$

o, i are conjugate points

light starting at one ends at the other regardless of ray

$$\boxed{\frac{n_2}{i}-\frac{n_1}{o}=\frac{1}{R}(n_2-n_1)}$$ paraxial approx (small h)

ellipse is a perfect conjugate system

if $o\to\infty$ $\quad \frac{n_2}{i}=\frac{n_2-n_1}{R}$ $\quad i=f$ $\quad$ focus is conjugate to ∞

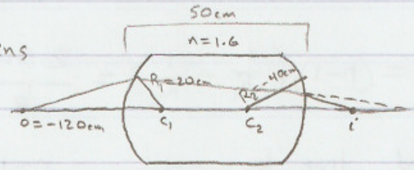
$o=-30\,cm$ $\quad n_1=1$ $\quad n_2=1.5$ $\quad R=10\,cm$ $\quad i=?$ $\quad \frac{1.5}{i}+\frac{1}{30}=\frac{1.5-1}{10}$ $\quad i=90\,cm$

if $o=-5\,cm$, $i=-10\,cm$ virtual image

(image won't form on paper)

$$\frac{n_2}{i}-\frac{n_1}{o}=\frac{n_2-n_1}{R}=\frac{n_2}{f}=\text{power of surface}$$ high power = short f

Real lens



where is i?

$f_1=\frac{1.6\,(20)}{1.6-1}=53.3\,cm$ $\qquad P_1=\frac{n}{f_1}=\frac{1.6}{53.3}=0.03\,cm^{-1}$

$\frac{n_2}{i_1}-\frac{n_1}{o_1}=P_1$ $\quad i_1=73.85\,cm$ $\quad$ second surface $\quad o_2=+23.85\,cm$ $\quad n_1=1.6$ $\quad n_2=1.0$

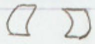
$f_2=\frac{n_2R_2}{n_2-n_1}=66.67\,cm$ $\qquad P_2=\frac{n_2}{f_2}=\frac{1}{66.67}=0.015\,cm^{-1}$

$\frac{n_2}{i_2}-\frac{n_1}{o_2}=P_2$ $\quad i_2=12.18\,cm$

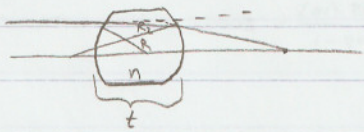
opposing surfaces on lens help each other

similar surfaces hurt each other

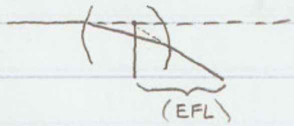
 more power than either surface  more negative power

 less |power| than either

general lens



Effective focal length (EFL)



front + back EFL not identical

$$\frac{n}{i_1} - \frac{1}{o} = P_1 = \frac{n-1}{R_1}$$

$$i_1 = \frac{nR_1}{n-1} \qquad o_2 = i - t = \frac{nR_1 - nt - t}{n-1}$$

$$\frac{1}{EFL} = \frac{n-1}{R_1} + \frac{1-n}{R_2} - \frac{t}{n}\left(\frac{n-1}{R_1}\right)\left(\frac{1-n}{R_2}\right) \qquad P_{system} = P_1 + P_2 - \frac{t}{n} P_1 P_2$$

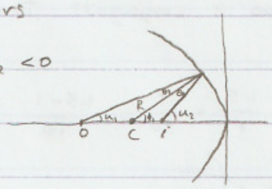
thin lenses $P_{system} = P_1 + P_2$ $\quad \frac{1}{EFL} = \frac{n-1}{R_1} + \frac{1-n}{R_2} = \boxed{(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}}$

Lens maker equation

Powered Mirrors

$o, i, \phi, \theta_1, u_1, R, u_2 < 0$

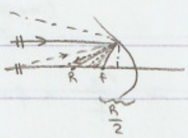
θ_2 – positive



$\phi = u + \theta_1 \qquad \theta_1 = \phi - u \qquad u_2 = \phi - \theta_2$

$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$ general definition of focus

if $o = R$ $\quad i = R$ $\quad \frac{1}{f} = \frac{2}{R}$ $\quad \boxed{f = \frac{R}{2}}$



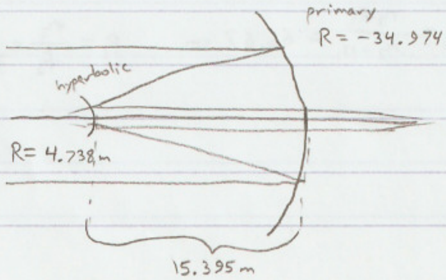
mirror of radius R, lens with radii R

$P = \frac{1}{f} = \frac{2}{R}$   $P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2(n-1)}{R}$

mirror has more power than standard lens by $n-1$ $\qquad n \sim 1.5$

$P_{system} = P_1 + P_2 - \frac{t}{n} P_1 P_2$

for mirror systems: Keck Schmidt – Cassegrain



$R = -34.974\,m$ $\qquad D = 10\,m$ effective

$P_1 = -\frac{2}{R_1} = 0.0572\ m^{-1} \qquad f = 17.487\,m$

$$\boxed{\text{Focal ratio} = \frac{\text{focal length}}{\text{diameter of illuminated surface}} = F/\#}$$

Keck $\frac{17.487}{10} = 1.748$ (difficult)

$F/2$ = focal length is twice the diameter (difficult to make)

$F/200$ = focal length is $200\times$ diameter
need larger dome, mirror is close to flat

2^{nd} mirror $P_2 = \frac{2}{R_2} = -0.427\ m^{-1}$ $f_2 = -2.37m$

secondary undoes what first mirror does

$P = P_1 + P_2 - \frac{d}{n} P_1 P_2$ (positive #)

$-0.365 + 0.372$

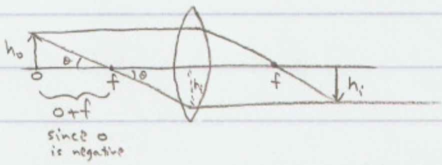
separation cancels the power

low power $\rightarrow$ long f $\leftarrow$ this is what we want

$P = 0.006686\ m^{-1} \rightarrow EFL = 149.6m$

$F/\# = 14.96$

lateral magnification

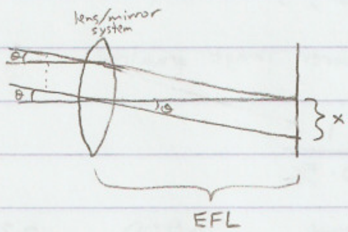


$\frac{h_i}{f} = \frac{h_o}{f+o}$

$$\boxed{\frac{h_i}{h_o} = \frac{f}{f+o} \equiv \text{magnification}}$$

m negative $\rightarrow$ inverted

plate scale



$\tan\theta = \frac{x}{EFL}$

paraxial $\tan\theta = \theta$ $x = f\theta$

$$\boxed{\Delta x = EFL \cdot \Delta\theta}$$

SHARC pixels $\Delta x = 18.5\mu m$

wanted $\Delta\theta = 0.''020$ "/pixel

$= 9.7\times10^{-8}$ radians

$EFL = \frac{\Delta x}{\Delta\theta} = \frac{18.5\ \mu m}{9.7\times10^{-8}} = 191\ m$

Keck had 149m

ex/ $\Delta\theta = 0.''2$ coarser pixels
$\Delta x = 18\ \mu m$

$F/\# = 1.85$ (for keck) EFL = 18.5m hard to make

10/4/2007

Detectors

IR (largest) 2000 × 2000 pixels Optical 16000 × 16000 Mid-IR 256 × 256

Stars have size FWHM ~ 1" 1" can resolve unless pixel size $\lesssim$ 1"

Nyquist's theorem: 2 pixels per resolution element
pixels = ½ FWHM or smaller

$0.''6$ in IR $0.''19$ in FWHM at best (very good image quality)

smallest scales @ keck ~$0.''15$ take advantage of 10–15% seeing conditions ($0.''3$)

Optical detectors ~0."25 to sample 0."5 seeing

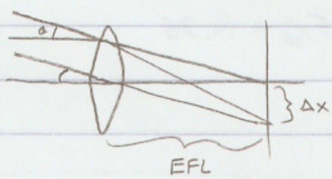
Field of view = # pixels * plate scale

spectographs are often coarser

AO gets to diffraction limit $\Delta\theta_{FWHM} \sim \frac{\lambda}{D}$ 1.2μm 3-5μm

NIRC2 infrared camera has plate scales of 0."01, 0."02, 0."04 per pixel
(1024 pixels) field of view: 10" 20" 40"

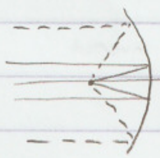
set plate scale, EFL



lens converts angle into position

$\Delta x = EFL \cdot \Delta\theta$

Focal ratio $F/_{\#} = \frac{EFL}{Diameter}$



} small diameter, lare $F/_{\#}$, good image quality

all mirror, small $F/_{\#}$, poorer image quality

Plate Scale $\frac{\Delta\theta}{\Delta x} = \frac{1}{EFL} = \frac{1}{D \cdot F/_{\#}}$

Δx in μm   D in m $\Delta\theta$ in arcseconds $\frac{\Delta\theta('')}{\Delta x(\mu m)} = \frac{0.206}{D(m)\, F/_{\#}}$

(fast: beam focuses quickly, small focal length)

keep $\frac{\Delta\theta}{\Delta x}$ fixed: instruments are much slower (better) on smaller telescopes

@ 24 in camera has 1" pixels @ keck same camera has 0."07 pixels ... too small

difraction limited camera - as D↑ $\frac{\Delta\theta}{\Delta x}$↓ so same $F/_{\#}$
are same size and difficulty
on larger telescopes

Summary

$n = \frac{c}{v}$ single surface $\frac{n_2}{i} - \frac{n_1}{o} = \frac{1}{R}(n_2 - n_1) = P$ (power)

thin lens $\frac{1}{i} - \frac{1}{o} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f} = P$

thick lens $P = P_1 + P_2 - \frac{d}{n} P_1 P_2$ mirrors $f = \frac{R}{2}$

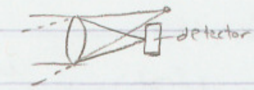
lateral magnification $m = \frac{h_i}{h_o} = \frac{f}{f + o}$ (o normally negative) $\Delta x = EFL \cdot \Delta\theta$ $F/_{\#} = \frac{EFL}{D}$

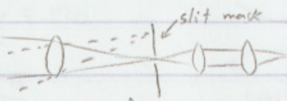
Optical Planes — Stops + Aperture

Field of view FOV

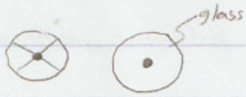
Field stop – sets FOV ex: detector, slits – in spectographs (sets which objects through)

all go in focal plane coronagraphic mask

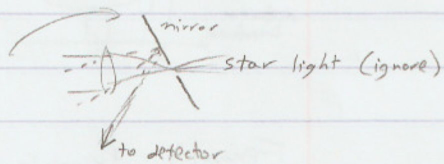




field stop at intermediate focus



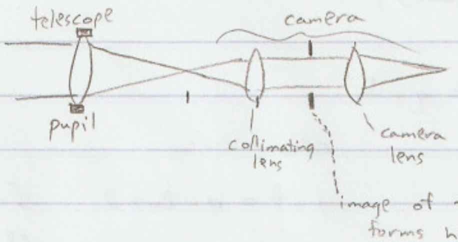
or mirror with hole

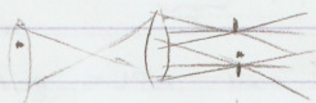


Aperture stop – sets how much light gets into system → often called pupils

usually primary mirror eye – pupil

exit pupils – image location for a reimaged pupil (conjugate plane to telescope and image)





can put masks in the arc to block out light from parts of telescope (say, if damaged)

⇒ equivalent to telescope

deformable mirrors can be placed @ exit pupil

can put pupil stop to get less light from telescope (to avoid beams and supports)

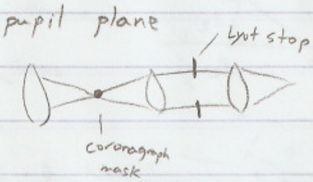
cold stop – pupil stop that is very cold, limits light ↳ warm so too much IR light

Lyot stops – Bernard Lyot (imaged corona of sun)

field stops make diffraction effects in pupil plane

image the exit pupil:

bright ring in exit pupil (from center)



← usually present in coronographic systems

Fermat's Principle

light follows path that minimizes travel time

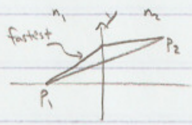
today we say optical path length

$$d(OPL) = c\,dt = n \cdot \underbrace{ds}_{\text{physical distance along ray}} = n \cdot v \cdot dt$$

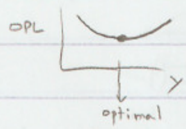
most general version: light follows path that is a stationary value of OPL (max, min, or inflection point)

$$OPL = c\int_{t_1}^{t} dt = \int n(s)\,ds \quad \text{(path integral)}$$

small change in path — no change in OPL

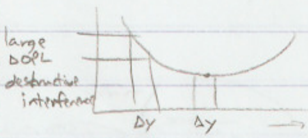


$\longrightarrow$ OPL(y)



$$OPL = \int f(y, z, \dot{z})\,dy$$

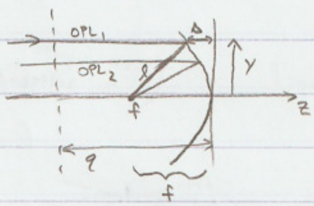
$$\frac{\partial f}{\partial y} - \frac{d}{dz}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0 \quad \text{Lagrange equations}$$



$\longrightarrow$ ΔOPL small constructive interference

Fermat's Principle and Reflecting Surface

ideal shape for infinite conjugate concave mirror



want shape where all OPL are the same

$\Longrightarrow$ want $\Delta(y)$

on axis: $OPL = q + f$

off axis: $OPL = q - \Delta + \ell$

$OPL_1 = OPL_2$

Pythagorean theorem $\ell^2 = (f-\Delta)^2 + y^2$

$q + f = q - \Delta + \ell \qquad \ell = f + \Delta$

$\ell^2 = (f+\Delta)^2$

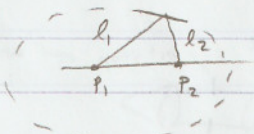
$(f-\Delta)^2 + y^2 = (f+\Delta)^2$

$y^2 = \cancel{f^2} + 2\Delta f + \cancel{\Delta^2} - \cancel{f^2} + 2\Delta f - \cancel{\Delta^2} = 4f\Delta$

$\Delta = \dfrac{y^2}{4f}$ Parabola

perfect for ∞ to point conjugate

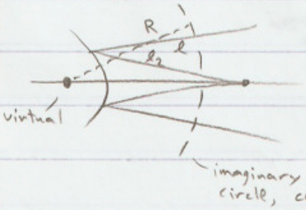
what is mirror surface for one point to another



want $\ell_1 + \ell_2 = \text{constant}$

$\Longrightarrow$ ellipse

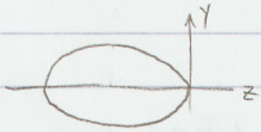
ellipse maps to real conjugate points



want $\ell + \ell_2 = \text{constant}$

$\Longrightarrow$ hyperbola

imaginary point to real point



optical expression

$$y^2 - 2Rz + (1-e^2)z^2 = 0$$

R – radius of curvature

e is eccentricity $\quad -e^2 = k$ conic constant

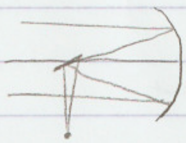
$e=0 \quad k=0$ sphere $\qquad e<0 \quad k>0$ prolate ellipsoid

$0<e<1 \quad -1<k<0$ oblate ellipsoid

$e=1 \quad k=-1$ paraboloid $\qquad e>1 \quad k<-1$ hyperboloid

simplest telescope (mirror)

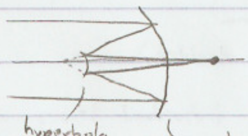
single parabola



Newtonian telescope

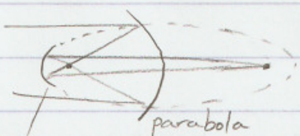
long focal length – lots of magnification in short body

Cassegrain telescope



most large telescopes

Gregorian telecope



field aberations – off axis light not focused perfectly

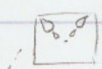
depend on θ, y (size of illuminated area) $\qquad f(\theta, y, y^2, \theta^2, \theta y, \dots) = C_1\theta + C_2 y + C_3\theta y + \dots$

perturbation theory

each constant has name/corresponds to
– spherical aberration
– astigmatism

5 free parameter (R, k, separation)

some determine focal length



aplanatic system – no spherical aberration, no coma

aplanatic Cassegrain telescope: Ritchey-Chretian telescope

Hubble, Keck, TMT

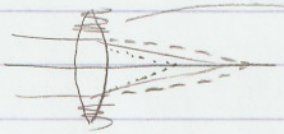
$$k_1 = -1 - \frac{2(1+\beta)}{m^2(m-\beta)} \qquad k_2 = -\left(\frac{m+1}{m-1}\right)^2 - \frac{2m(m+1)}{(m-\beta)(m-1)^3}$$

primary slightly hyperbolic

m – magnification

β – angle in FOV

Chromatic Aberration



paraxial approx:

different colors focus at different places

$\rightarrow n(\lambda)$

ex/ crown glass

$n(656.3\text{nm}) = 1.51461 \qquad n(587.8) = 1.51707 \qquad n(486.1) = 1.52262$

$$p = \frac{1}{f} = (n-1)\underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{\text{const}} \qquad \Delta p \propto \Delta n(\lambda)$$

$\Delta P(486.1 - 656.3) = 0.019 \sim 2\%$

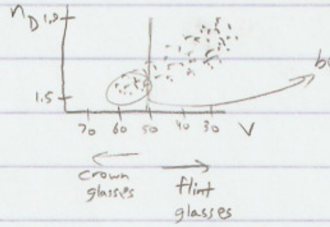
$$V = \frac{n_D - 1}{n_F - n_C} \qquad \frac{n_{mid} - 1}{n_{long} - n_{short}} \qquad \frac{1}{V} = \text{dispersive power}$$

Abbe number

large $\rightarrow$ more dispersive like prism

when using Fraunhofer C, D, F

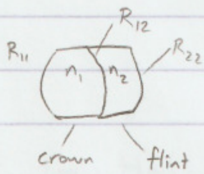
$H\beta$ 589.2nm (sodium doublet) $H\alpha$



better for individual lenses

achromatic doublet

no color effect



perfect on axis performance at 3 wavelengths

$p = P_1 + P_2$ thin lens

$$= (n_1 - 1)\left(\frac{1}{R_{11}} - \frac{1}{R_{12}}\right) + (n_2 - 1)\left(\frac{1}{R_{21}} - \frac{1}{R_{22}}\right)$$

$$= (n_1 - 1)K_1 + (n_2 - 1)K_2$$

want $$\frac{dp}{d\lambda} = 0 = \frac{dn_1}{d\lambda}K_1 + \frac{dn_2}{d\lambda}K_2$$

$$\frac{dn_1}{d\lambda} \approx \frac{n_L - n_s}{\lambda_L - \lambda_s} \qquad \frac{dn_2}{d\lambda} \approx \frac{n_{2L} - n_{2s}}{\lambda_L - \lambda_s}$$

$$= \left(\frac{n_{1L} - n_{1s}}{\lambda_L - \lambda_s}\right)\left(\frac{n_{1m} - 1}{n_{1m} - 1}\right)K_1 + \left(\frac{n_{2L} - n_{2s}}{\lambda_L - \lambda_s}\right)\left(\frac{n_{2m} - 1}{n_{2m} - 1}\right)K_2 \qquad (n_{1m} - 1)K_1 = P_1$$

$$= \frac{n_{1L} - n_{1s}}{n_{1m} - 1}\frac{P_1}{\Delta\lambda} + \frac{1}{V_2}\frac{P_2}{\Delta\lambda} = \frac{1}{\Delta\lambda}\left(\frac{P_1}{V_1} + \frac{P_2}{V_2}\right) = 0 \qquad \text{so } \frac{P_1}{V_1} + \frac{P_2}{V_2} = 0$$

$$P_1 = \frac{-P_2 V_1}{V_2} \qquad \text{total power: } p = P_1 + P_2 \qquad P_1 = p\left(\frac{-V_1}{V_2 - V_1}\right) \qquad P_2 = p\left(\frac{-V_2}{V_2 - V_1}\right)$$

specify V_1, V_2, desired p

ex/ Crown $V = 64.55$

Flint $V = 37.97$ $P_c = p(2.42)$ $P_F = p(-1.42)$

2 Beam Interference

10/9/2007

2 sources of EM waves of equal frequency

$\vec{E}_1, \vec{E}_2$ at location $\vec{r}$ and time t

different propagation vectors $\vec{k}_1, \vec{k}_2$

different phase shift ϵ_1, ϵ_2

$\vec{E}_1(\vec{r}) = \vec{E}_1 \cos(\vec{k}_1 \cdot \vec{r} - \omega t + \varepsilon_1)$ $\quad$ $\vec{E}_2(\vec{r}) = \vec{E}_2 \cos(\vec{k}_2 \cdot \vec{r} - \omega t + \varepsilon_2)$

@ $\vec{r}$ $\quad \vec{E} = \vec{E}_1 + \vec{E}_2$

power per unit area (irradiance)

$$I = \varepsilon_0 c \langle \vec{E} \rangle^2 = \epsilon_0 c \langle \underset{\substack{I_1 \\ \shortparallel \\ E_1^2}}{\vec{E}_1^2(\vec{r})} + \underset{\substack{I_2 \\ \shortparallel \\ E_2^2}}{\vec{E}_2^2(\vec{r})} + \underbrace{2 \vec{E}_1(\vec{r}) \cdot \vec{E}_2(\vec{r})}_{I_{12} \text{ interference term}} \rangle$$

$$I_{12} = 2 \left(\vec{E}_1 \cdot \vec{E}_2 \right) \cos(\vec{k}_1 \cdot \vec{r} - \omega t + \varepsilon_1) \cos(\vec{k}_2 \cdot \vec{r} - \omega t + \varepsilon_2)$$

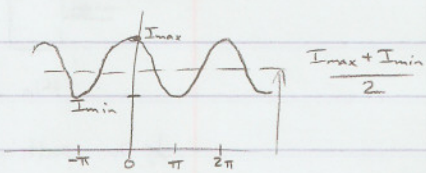
$$\boxed{I_{12} = \vec{E}_1 \cdot \vec{E}_2 \cos(\delta)} \qquad \boxed{\delta = \underbrace{\overset{\text{path difference}}{(\vec{k}_1 - \vec{k}_2) \cdot \vec{r}} + \overset{\text{intrinsic shift}}{(\varepsilon_1 - \varepsilon_2)}}_{\text{phase difference}}}$$

if $\vec{E}_1 \parallel \vec{E}_2$ $\quad \vec{E}_1 \cdot \vec{E}_2 = E_1 \cdot E_2$

$$I = I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2} \cos(\delta)}_{\text{interference term}}$$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ for $\delta = 0, 2\pi, 4\pi, \ldots$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ for $\delta = \pi, 3\pi, 5\pi, \ldots$



$I_{max} - I_{min} = 4\sqrt{I_1 I_2}$

"Fringe Contrast" or "Visibility" $\equiv \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$ always between 0 and 1

ex/ $E_1 = 2 \cos(\vec{k}_1 \cdot \vec{r} - \omega t + \frac{\pi}{3})$ $\quad E_2 = 5 \cos(\vec{k}_2 \cdot \vec{r} - \omega t + \frac{\pi}{4})$ $\frac{V}{m}$

where path difference = 0 $\quad I_1 = \frac{1}{2} \epsilon_0 c E_1^2 = \frac{1}{2} \epsilon_0 c (2)^2 = 5.31 \times 10^{-3} \frac{W}{m^2}$

$I_2 = 3.32 \times 10^{-2} \frac{W}{m^2}$ $\quad \delta = \frac{\pi}{3} - \frac{\pi}{4}$ $\quad$ interference function of position

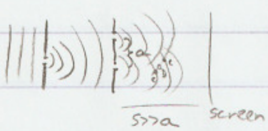
$I_{12} = 2\sqrt{I_1 I_2} \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 2.56 \times 10^{-2} \frac{W}{m^2}$

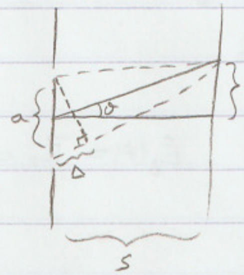
$I_{max} = 6.50 \times 10^{-2} \frac{W}{m^2}$ $\quad I_{min} = 1.19 \times 10^{-2} \frac{W}{m^2}$

visibility $= \dfrac{6.5 - 1.19}{6.5 + 1.19} = 0.69$ $\quad$ If $|E_1| = |E_2|$ $\quad I_{max} = 4E_1^2$, $I_{min} = 0$ $\quad$ visibility = 1

Young's Double Slit

Thomas Young 1802 monochromatic source of light, plane parallel





if $a \ll s$ $\Delta \sim a \sin\theta$

if $\Delta = m\lambda$ constructive

if $\Delta = (m \pm \frac{1}{2})\lambda$ destructive

$I_1 = I_2 = I_0$ $\delta = \frac{2\pi\Delta}{\lambda}$ $I = I_0 + I_0 + 2I_0 \cos\delta$

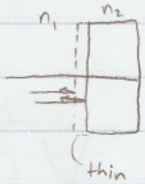
$$I = 2I_0(1+\cos\delta) = 4I_0\cos^2\frac{\delta}{2} = 4I_0\cos^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$$

if $a \ll s$ θ small $\sin\theta \sim \tan\theta = \frac{y}{s}$

$I = 4I_0\cos^2\left(\frac{\pi a y}{s\lambda}\right)$ maxima $\frac{\pi a y}{s\lambda} = m\pi$ Δy between maxima $\Delta y = \frac{s\lambda}{a}$

Interference in Dielectric Coatings

(transparent coatings in glass)



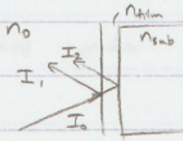
reflectance @ boundary $r = \left(\frac{1-\frac{n_2}{n_1}}{1+\frac{n_2}{n_1}}\right)^2$

few percent, but adds up in large system

thin coating - 2 reflections destructively interfere

$\frac{1}{2}\lambda$ path difference $\rightarrow$ $\frac{1}{4}\lambda$ coating (antireflection coating)

For good cancellation want r_1, r_2 to be equal

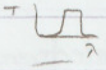


$I_1 \ll I_0$ $I_1 = I_0\left(\frac{1-\frac{n_f}{n_0}}{1+\frac{n_f}{n_0}}\right)^2$

$(I_0 - I_1)$

$$I_2 \simeq I_0\left(\frac{1-\frac{n_s}{n_f}}{1+\frac{n_s}{n_f}}\right)^2$$

most filters have many layes with different index to get



perfect $I_1 = I_2$ $\frac{n_f}{n_0} = \frac{n_s}{n_f}$

$\Rightarrow n_f = \sqrt{n_0 n_s}$

many glasses $n_s > 1.5$ $n_0 = 1$ $n_f = 1.22$

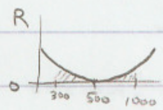
nearest practical coating: MgF_2 $n_f = 1.38$

"(quarter wave MgF_2)" some benifit for $\Delta = \frac{\lambda}{4}$ to $\frac{3\lambda}{4}$ range of destruction optical path difference $OPD = \Delta = \frac{\lambda_0}{2}$

$$\frac{\lambda_{max}}{4} = \frac{\lambda_0}{2} \qquad \lambda_{max} = 2\lambda_0 \qquad \frac{3\lambda_{min}}{4} = \frac{\lambda_0}{2} \qquad \lambda_{min} = \frac{2\lambda_0}{3}$$

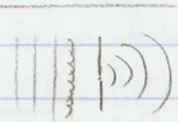
$\frac{\lambda_{max}}{\lambda_{min}} = 3$ broad band coating   difficult to make broad filters

ex/ $\lambda_0 = 500nm$ $\lambda_{min} = 333nm$ $\lambda_{max} = 1000\,nm$



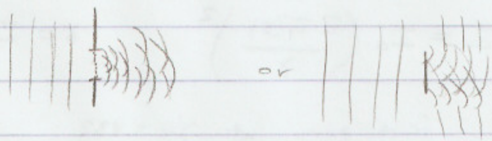
bit of red + blue – looks purple   physical thickness $= \frac{\lambda}{4 n_f}$

Diffraction



Huygen's principle

spherical wave



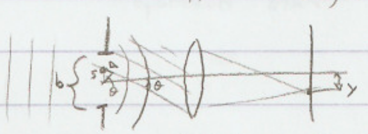
or

diffraction is interference from continuous source   aperture or obstruction or irregularity in material

"Far field diffraction" Fraunhofer   source + image plane far apart (plane waves)

"Near Field Diff." Fresnel Diff.
(silent s)

Fraunhofer from a single slit



perfect lens   focal plane of lens

↳ converts θ to position

r – distance traveled

r_0 – distance to point on optic axis $(s=0)$

$r = r_0 + \Delta = r_0 + s \cdot \sin\theta$

Electric field from position s above optic axis evaluated at p

$$dE_p = \left(\frac{dE_0}{r}\right) e^{i(kr - \omega t)} = \left(\underbrace{\frac{dE_0}{r_0 + s\cdot\sin\theta}}_{\sim r_0}\right) e^{i(kr_0 + k\sin\theta \cdot s - \omega t)}$$

far away (so inverse square difference is small)

$dE_0 = E_L\, ds$

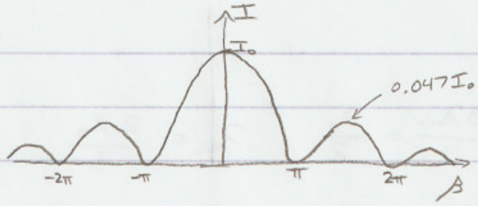
└ amplitude per unit length (constant)

$$dE_p(s) = \frac{E_L}{r_0} e^{ikr_0} e^{(iks \cdot \sin\theta)} e^{-i\omega t} ds$$

$$E_p = \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} \int_{-b/2}^{b/2} e^{ik\sin\theta \cdot s} ds$$

$\beta = \frac{1}{2} kb\sin\theta \qquad \mathrm{sinc}(r) = \frac{\sin r}{r}$

$$I = \frac{\epsilon_0 c}{2} |E_p|^2 = I_0 \frac{\sin^2\left(\frac{1}{2}kb\sin\theta\right)}{\left(\frac{1}{2}kb\sin\theta\right)^2} = I_0\, \mathrm{sinc}^2\beta$$

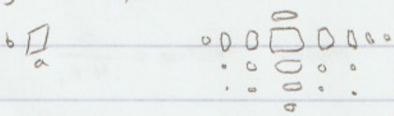


min @ $\beta = \pi$

$\frac{1}{2} k b \sin\theta_{min} = m\pi \qquad \sin\theta_{min} \simeq \frac{y_{min}}{f}$

$$\boxed{y_{min} = \frac{m \lambda f}{b}}$$

rectangular aperture



$I(x,y) = I_0 \, \mathrm{sinc}^2\beta \, \mathrm{sinc}^2\alpha$

$\beta = \frac{1}{2} k b \sin\theta \qquad \alpha = \frac{1}{2} k a \sin\theta$

circular aperture
(diameter D) integrate over circular disk

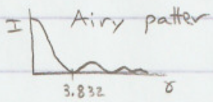
$$I = I_0 \left(\frac{2 J_1(\gamma)}{\gamma} \right)^2 \qquad \gamma = \frac{\pi D}{\lambda} \sin\theta$$

$J_1(\gamma)$ – Bessel function of 1st kind

$$J_1(\gamma) = \frac{\gamma}{2} - \frac{(\gamma/2)^3}{1^2 \cdot 2} + \frac{(\gamma/2)^5}{1^2 \cdot 2^2 \cdot 3} - \ldots$$

first zero at $\gamma = 3.832$

$\lim_{x \to 0} \frac{J_1(x)}{x} \to$ constant



$3.832 = \frac{\pi D}{\lambda} \sin\theta \qquad D \sin\theta = 1.22 \lambda \qquad \boxed{\sin\theta = 1.22 \frac{\lambda}{D}}$ first null

$\sin\theta \sim \theta$ FWHM $\sim 1 \frac{\lambda}{D}$

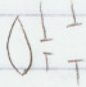
optical system can resolve 2 sources if 1st nulls overlap

$$\boxed{\Delta\theta = 1.22 \frac{\lambda}{D}}$$

Human eye: Bright condition $D \sim 2$mm $\lambda = 550$ nm $\Delta\theta = 0.000332$ radians $= 1.2$ arcmin

Dark conditions $D \sim 8$mm $\Delta\theta = 0.3$ arcmin

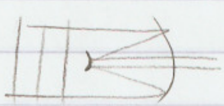
actually can only see ~ 1 arcmin resolution

1)  optical quality of lens is bad (at edges)

→ can change it "custom wavefront technology"

2) detectors in eye spaced at closest 0.'s  0.'s can't resolve

single circular aperture



central obscuration airy rings get brighter

diameter of secondary $d = \epsilon D$ (fraction)

FWHM indep. of secondary

dark ring moves inward

$$I(\rho) = I_0 \frac{1}{(1-\epsilon^2)^2} \left[\frac{2 J_1(\gamma)}{\gamma} - \epsilon^2 \frac{2 J_1(\epsilon\gamma)}{\epsilon\gamma} \right]^2$$

secondary size sets field of view

ϵ	EE_{1st}	EE_{2nd} (encircled energy)
0	0.838	0.91
0.2	0.71	0.90
0.4	0.58	0.89
0.6	0.37	0.71

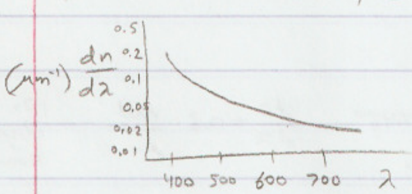
Hubble
$\epsilon = 0.33 \quad EE_{1st} = 0.65$

Spectroscopy

10/11/2007

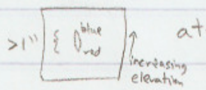
dispersion – $\frac{d\lambda}{d\theta}$ angular dispersion

prism – transmissive optic with variable dispersion $n(\lambda)$



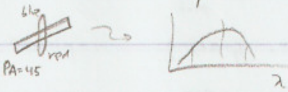
high UV dispersion, lower IR dispersion

typical materials



atmospheric dispersion

difficult to place slit @ arbitrary orientation

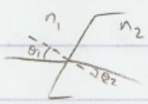


not getting blue + red

parallactic orientation: 
(not always possible)

atmospheric dispersion correctors (ADC's)  variable dispersion setup
– undoes the dispersion of atmosphere

Prism is to make spectrum goal: $\frac{d\lambda}{d\theta}$



@ λ_1: $n_1 \sin\theta_1 = n_2 \sin\theta_2$ @ λ_2: $n_1' \sin\theta_1 = n_2' \sin\theta_2'$

$n_1' = n_2' = 1.0$

$\therefore\ n_2 \sin\theta_2 = n_2' \sin\theta_2'$

$n_2' = n_2 + \Delta n_2$ $\quad n_2 \sin\theta_2 = n_2 \sin\theta_2' + \Delta n_2 \sin\theta_2$ $\quad n_2(\sin\theta_2 - \sin\theta_2') = \Delta n_2 \sin\theta_2'$

$$n_2\left(2\sin\left(\frac{\theta_2-\theta_2'}{2}\right)\cos\left(\frac{\theta_2+\theta_2'}{2}\right)\right) = \Delta n_2 \sin\theta_2'$$

$\lambda_1 \sim \lambda_2$ $\quad \lambda_2 = \lambda_1 + d\lambda$ $\quad \therefore\ n_2\big(\underbrace{(\theta_2-\theta_2')}_{d\theta}\cos\theta_2'\big) = \underbrace{\Delta n_2}_{dn}\sin\theta_2'$

$$\frac{n_2\, d\theta}{d\lambda} = \frac{dn_2 \tan\theta}{d\lambda} \qquad \frac{d\theta}{d\lambda} = \frac{dn_2}{d\lambda}\frac{\tan\theta_2}{n_2} \qquad \frac{dn}{d\lambda} \propto \frac{1}{\lambda^3} \text{ in optical}$$

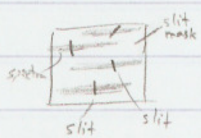
$\frac{dn}{d\lambda} \sim 0.1\mu m^{-1}$ $\quad n = $ 1.4 @ 400 nm, 1.45 @ 900 nm $\quad$ typical $\frac{d\theta}{d\lambda} = 0.1\ \frac{\tan\theta}{1.5}\left(\frac{rad}{\mu m}\right)$

@ $\theta = 45°$ $\quad \frac{d\theta}{d\lambda} = 0.06\ \frac{rad}{\mu m}$ $\quad$ ex/ $\Delta\lambda = 300\,nm$ $\quad \Delta\theta = 0.02$ rad $\sim 1°$

low dispersion from prism's
high throughputs (few light lost)

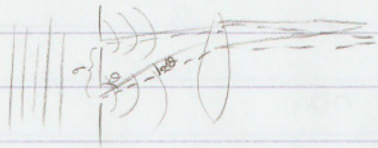
objective prisms – lots of tiny spectra in single image, short period of time

multiple slit spectographs



no:  (bad)

Diffraction Grating



$\Delta = m\lambda$ constructive interference

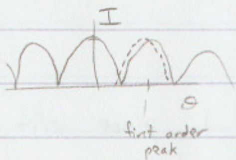
$$\Delta = \sigma \sin\theta \qquad \lambda = \frac{\sigma \sin\theta}{m}$$

$$\frac{d\lambda}{d\theta} = \frac{\sigma}{m}\cos\theta$$

$$\frac{d\theta}{d\lambda} = \frac{m}{\sigma\cos\theta}$$

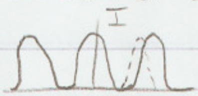
ex/ $\sigma = 2.5\,\mu m$ (400 lines/mm) $m=1$ $\theta = 45°$ $\frac{d\theta}{d\lambda} = 0.6\ \frac{rad}{\mu m}$ 10x prism dispersion even in 1^{st} order

2 slits



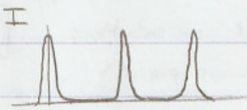
can't separate small $d\theta$

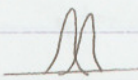
3^{rd} slit @ same spacing phase shift is 2Δ from 1^{st} slit



narrower

multiple slit: (thousands)

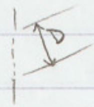




resolved

diffraction limit 2 wavelengths are separated is $\Delta\theta \simeq \frac{\lambda}{D}$ — total diameter of slits

$D = N\sigma\cos\theta$



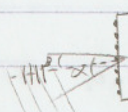
$$\Delta\theta = \frac{\lambda}{N\sigma\cos\theta}$$

$$\frac{m\,\Delta\lambda}{\sigma\cos\theta} = \frac{\lambda}{N\sigma\cos\theta} \qquad \boxed{\frac{\lambda}{\Delta\lambda} = mN}$$ Diffraction limit of grating

$\frac{\lambda}{\Delta\lambda} \equiv$ resolution of spectograph $= R$ $\qquad R_{diff} = mN$

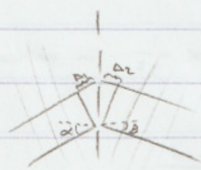
$\Delta\lambda$ — smallest distinguishable λ difference

or:



mirror

law of reflection doesn't hold (plane → spherical waves)



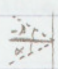
total path difference $\Delta = \Delta_1 + \Delta_2$

$= \sigma\sin\alpha + \sigma\sin\beta$

$$\boxed{\lambda = \frac{\sigma(\sin\alpha + \sin\beta)}{m}}$$ grating equation

if $\alpha = -\beta$: no interference $m=0$

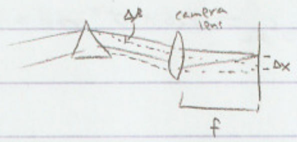
0^{th} order — all wavelengths propagate



Making a Spectograph

grating or prism converts $\Delta\lambda$ into $\Delta\theta$

need something to convert $\Delta\theta$ to Δx (lens)



$\Delta x = f \Delta\beta$ $\boxed{\frac{dx}{d\beta} = f_{camera}}$

linear dispersion $\frac{dx}{d\lambda} = \frac{dx}{d\beta}\frac{d\beta}{d\lambda} = f_{camera}\frac{d\beta}{d\lambda}$

from grating equation $\frac{d\lambda}{d\beta} = \frac{\sigma \cos\beta}{m}$ $\therefore$ $\boxed{\frac{dx}{d\lambda} = \frac{f_{cam}\, m}{\sigma \cos\beta}}$

ex/ $f_{cam} = 200\,mm$ $\frac{1}{\sigma} = 240\ \frac{grooves}{mm}$

$\alpha = 27°$ $\beta = 35°$ $m = 2$

$\lambda = \frac{\sigma(\sin\alpha + \sin\beta)}{m} = 2.14\,\mu m$ this light comes out in m=2 in this direction

$\frac{dx}{d\lambda} = 117194\ mm/mm = 117\ mm/\mu m$ $\frac{dx}{d\lambda} = 0.00853\ \frac{\mu m}{mm}$

so 1mm above line: $\lambda = \overset{2.14}{\lambda_0} + 0.00853\,\mu m = 2.149\,\mu m$

detector in plate $\Delta x = 27\,\mu m$ pixels $\Delta\lambda = 0.00023039\ \mu m/pixel$

resolution: $2\Delta\lambda = 0.00046078\,\mu m$

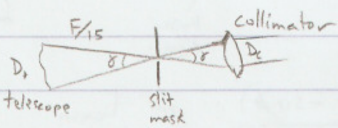
(Nyquist) # pixels $R_{sampling} = \frac{\lambda}{\Delta\lambda} = \frac{2.14}{0.00046} = 4644$

$\frac{\Delta\lambda}{\lambda} = \frac{\Delta v}{c}$ $\Delta v = \frac{c}{R} = 65\ km/s$

to achieve $R_{diff} = 4644$ in $m=2$ $R = mN \rightarrow N = 2300$ groves

$\frac{1}{\sigma} = 240\ \frac{groves}{mm}$ $\therefore$ need to illuminate 9.7 mm of grating $\sim 1cm$

need parallel/collimated light
→ need collimator for full system



$F_{collimator} = F_{telescope}$

need $D_c = 9.7\,mm$ $\therefore$ $f_{coll} = 15\,D_c = 146\,mm$

largest slit I can use will map to 2 pixels on detector

magnification of instrument $= \frac{f_{cam}}{f_{coll}} = \frac{200\,mm}{146\,mm} = 1.37$

2 pixel slit maps to $54\,\mu m$ @ detector $\overset{\Delta x}{2(27\,\mu m)}$

slit $< \frac{54\,\mu m}{1.37} = 40\,\mu m$

slit limited spectrograph is where slit maps to more than 2 pixels what is $40\mu m$ in $F/_{15}$ focal plane?

$$\frac{\Delta x}{\Delta\theta(sky)} = f_{telescope} = F/_{15} \cdot \underbrace{10m}_{Keck} \qquad \Delta\theta = \frac{\Delta x}{f} = \frac{40\mu m}{\underbrace{150 \times 10^6 \mu m}_{(10 \cdot 15) m}} = 2.6 \times 10^{-7} \text{ rad} = 54 \text{ milliarcsec}$$

this $\Delta\theta$ is diffraction limit of telescope at $2.14\mu m$

For 10x larger slit, you need 10x larger grating

$\Delta\theta = 1''$ Beam diameter $\Rightarrow$ 180mm

$f_{cam} = 200mm \qquad F/_{camera} = \frac{200}{180} = 1.1$ really hard

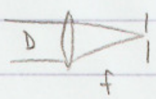
10/16/2007

more about making spectograph

generally need: $D = 10m \quad F/_{\#} = 15$

$R = ?$ resolution $\quad \Delta\lambda = ?$ passband $\quad$ slit width $\Delta\theta = ?$

detector pitch Δx (# μm per pixel)



$\Delta \overline{x}$ slit width (in μm)

$$\Delta\theta = \frac{\Delta \overline{x}}{f} \qquad \boxed{\Delta \overline{x} = \Delta\theta\, D \cdot F/_{\#}}$$

$f = (D \cdot F/_{\#})$

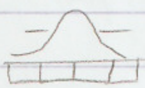
physical size of slit

slit focal plane

(can add mirror/lenses to change $F/_{\#}$ and chang $\Delta\overline{x}$)

Slit width $\xrightarrow{\text{Maps to}}$ n pixels

2, 3, 4 usually



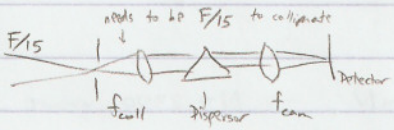
Nyquist theorem requires $n \geq 2$

Magnification of camera + collimator optics

$$m = \frac{f_{cam}}{f_{coll}} = \frac{n\Delta x}{\Delta\overline{x}} = \frac{n\Delta x}{\Delta\theta\, D \cdot F/_{\#}} = m$$

physical size of detector

physical size of slit



$$R = \frac{\lambda}{\Delta\lambda} \qquad \Delta\lambda = \frac{d\lambda}{dx} n\Delta x = \frac{\sigma \cos\beta}{m f_{cam}} n\Delta x \qquad \lambda = \frac{\sigma(\sin\alpha + \sin\beta)}{m}$$

linear dispersion

order

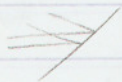
$$R = \frac{\cancel{\sigma}(\sin\alpha + \sin\beta)}{\cancel{m}} \frac{\cancel{m} f_{cam}}{n\Delta x\, \cancel{\sigma}\cos\beta} = \frac{f_{cam}}{n\Delta x} \frac{\sin\alpha + \sin\beta}{\cos\beta} = R$$

$$\boxed{f_{cam} = n\Delta x\, R \frac{\cos\beta}{\sin\alpha + \sin\beta}}$$

$$f_{coll} = f_{cam} \frac{\Delta\theta \; D \cdot F/\#}{n \Delta x} = \boxed{R \cdot \Delta\theta \cdot D \cdot F/\#_{tel} \cdot \frac{\cos\beta}{\sin\alpha + \sin\beta} = f_{coll}}$$

grating size: $\boxed{\frac{f_{coll}}{F/\#} = R \, \Delta\theta \, \frac{\cos\beta}{\sin\alpha + \sin\beta} \cdot D}$

ex/ $R = 100{,}000$ $D = 10\,m$ $\Delta\theta = 1''$

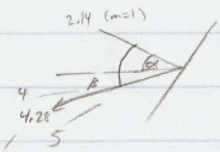
grating size $= (5m) \frac{\cos\beta}{\sin\alpha + \sin\beta}$ if $\alpha \sim \beta = 70°$  $\therefore$ grating size $\sim 0.9\,m$

if $\Delta x = 18\,\mu m$ $n = 3$ pixel $\implies$ $F/_{cam} = 1.05$ very fast Schmidt camera

if $D = 30\,m$, grating $\sim 2.7\,m$ $F/_{cam} = 0.37$

Orders

built spectograph where $\lambda = \frac{\sigma(\sin\alpha + \sin\beta)}{m} = 4.28$ when $m=1$



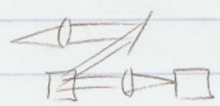
if sensitivity $1-25\,\mu m$

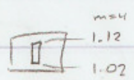
2.14 with m=2 1.42 with m=3
1.07 with m=4
} all of these detected in same place

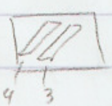
H band filter = 1.55 − 1.85 no light Y band = 0.98 − 1.10 µm → see 1.07 µm light

1 − order sorting filters

2 − second grating (cross dispersed echelles)

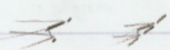


1 grating 

2 gratings 

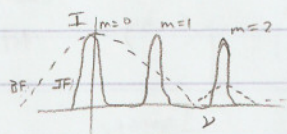
cross dispersion splits apart orders

How do we make majority of light in just one order?

path difference from each individual facet 

light prefers to go in direction with no path difference across facet

Interference Function $IF = \left(\frac{\sin N\nu}{N \sin\nu}\right)^2$



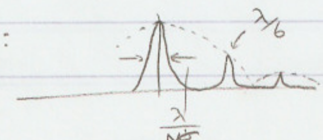
$\nu = \frac{\pi\sigma}{\lambda}(\sin\alpha + \sin\beta)$

N is number of facets

Blaze Function − interferometric intensity pattern of a facet

$BF = \left(\frac{\sin\gamma}{\gamma}\right)^2$ $\gamma = \frac{\pi b}{\lambda}(\sin\alpha + \sin\beta)$ b − size of facet

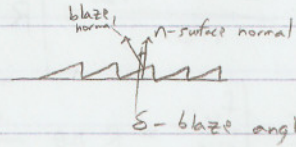
True intensity = IF · BF :

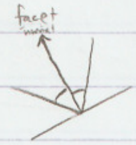


→ most photons will go to m=0

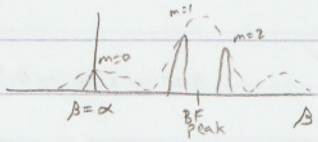
Blaze Function sets fraction of light in each order

Blazing a grating — tipping facets





If $\beta = \alpha - 2\delta$ then reflaction symmetric about facet normal
then you get maximum intensity



for a particular wavelength

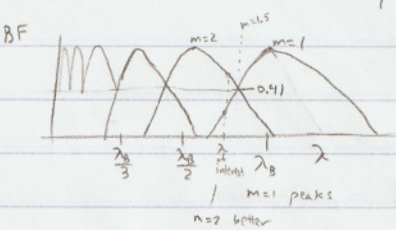
maximum light will occur
if $\alpha = \beta = \delta$

$\lambda_B = 2\sigma \sin\delta$

Littrow configuration

Quasi Littrow $\alpha = \beta = \delta$, but there is out of
plane angle γ (quasi-Littrow angle)

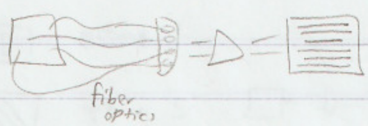
NIRSPEC's echelle is quasi-Littrow usually want γ small



all peaks occur @ same angle

OSIRIS grating
$\lambda_B = 6.5\ \mu m$ $\lambda_3 = 2.17\ \mu m$ (K) $\lambda_4 = 1.57\ \mu m$ (H)
$\lambda_5 = 1.3\ \mu m$ (J) $\lambda_6 = 1.08\ \mu m$ (Y)

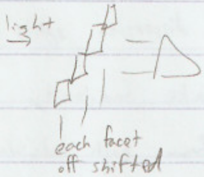
Multiple Objects

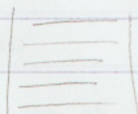


Integral field

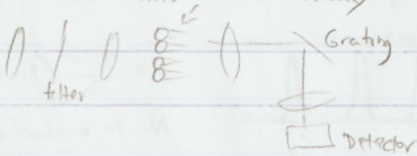
slicer mirror

tilt different angles



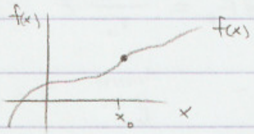


OSIRIS uses lenslet array



each lenslet produces 1 spectrum
tilt array so get multiple spectra

Fourier Analysis 10/18/2007



Taylor series - expand about x_0

$$f(x) \approx f(x_0) + (x-x_0) f'(x_0) + \frac{(x-x_0)^2}{2} f''(x_0) + \dots$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$ Maclaurin series: $x_0 = 0$

$$e^{i\theta} = \sum_{n=0}^{\infty} \frac{(i\theta)^n}{n!} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} \dots$$ Euler's formula: $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$

$$\boxed{\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta \qquad \frac{e^{i\theta} - e^{-i\theta}}{2i} = \sin\theta}$$

Fourier's Theorem (1807) - any singular valued function $f(x)$ on interval $[-\pi, \pi]$ may be replaced with $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

a_n, b_n - how much $f(x)$ looks like $\sin(nx)$, $\cos(nx)$

$$\int_{-\pi}^{\pi} \cos(nx)\cos(mx)\,dx = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases} \qquad \int_{-\pi}^{\pi} \cos(nx)\sin(mx)\,dx = 0$$

extend $[-\pi, \pi]$ to $[-l, l] \Longrightarrow u = \frac{\pi x}{l}$ $\qquad f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty}\left(a_n\cos\left(\frac{n\pi x}{l}\right) + b_n\sin\left(\frac{n\pi x}{l}\right)\right)$

wave number $k = \frac{nx}{l}$ $\qquad f(x) = \frac{a_0}{2} + \sum a_n\left(\frac{e^{i\frac{n\pi x}{l}} + e^{-i\frac{n\pi x}{l}}}{2}\right) + b_n\left(\frac{e^{i\frac{n\pi x}{l}} - e^{-i\frac{n\pi x}{l}}}{2i}\right)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \underbrace{\left(\frac{a_n}{2} + \frac{b_n}{2i}\right)}_{c_n} e^{i\frac{n\pi x}{l}} + \left(\frac{a_n}{2} - \frac{b_n}{2i}\right)e^{-i\frac{n\pi x}{l}} = \boxed{\sum_{n=-\infty}^{\infty} c_n e^{i\frac{n\pi x}{l}} = f(x)}$$

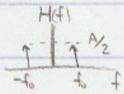
$$f(x) = \sum_{n=-\infty}^{\infty} c_k e^{ikx} \qquad c_k = \frac{1}{2l}\int_{-l}^{l} f(x) e^{-ikx}\,dx$$

$l.\ l \to \infty$ $\qquad f(x) = \int_{-\infty}^{\infty} C(k) e^{ikx}\,dk \qquad \boxed{C(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x) e^{-ikx}\,dx = F(k)}$

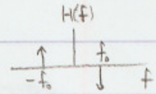
Fourier transform of $f(x) = F(k)$

$f = \frac{k}{2\pi}$ $\quad x = t$ $\qquad H(f) = \int_{-\infty}^{\infty} h(t) e^{-i2\pi ft}\,dt \qquad h(t) = \int_{-\infty}^{\infty} H(f) e^{i2\pi ft}\,df$ $\qquad$ frequency - time

ex/ $h(t) = A\cos(2\pi f_0 t)$ $\qquad H(f) = \int_{-\infty}^{\infty} A\cos(2\pi f_0 t) e^{-i2\pi ft}\,dt = \frac{A}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$

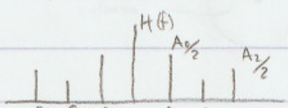


$h(t) = A\sin(2\pi f_0 t)$ $\qquad H(f) = \frac{A}{2}\delta(f + f_0) - \frac{A}{2}\delta(f - f_0)$



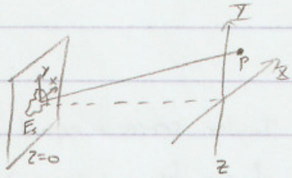
$h(t) = A_0\cos(2\pi f_0 t) + A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t)$

$$H(f) = \sum \frac{A_n}{2}\left(\delta(f - f_n) + \delta(f + f_n)\right)$$



Fourier Optics

Fraunhoffer diffraction



$$dE_p = \frac{dE_L}{r_0} e^{i(kr-\omega t)}$$

inverse square law

$I = |E|^2$

$E_s = \frac{\text{source strength}}{\text{unit area}}$

$E_s = \frac{E_L}{r}$

$$E_p = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_s(x,y)\, e^{i(kr-\omega t)}\, dx\,dy$$

r_0 is on axis distance

$r^2 = (X - x)^2 + (Y - y)^2 + (z-0)^2 \qquad r_0^2 = X^2 + Y^2 + z^2$

$r^2 = r_0^2 - 2xX - 2yY + x^2 + y^2$

Fraunhoffer limit $X \gg x \quad Y \gg y \qquad r^2 = r_0^2 - 2xX - 2yY$

$$r = r_0\left(1 - 2\frac{(xX + yY)}{r_0^2}\right)^{1/2} \approx r_0\left(1 - \frac{xX + yY}{r_0^2}\right)$$

$$E_p = \iint E_s\, e^{i\left(kr_0 - kr_0\left(\frac{xX+yY}{r_0^2}\right) - \omega t\right)} dx\,dy$$

$$= e^{i(kr_0 - \omega t)} \iint E_s\, e^{-ik\left(\frac{xX+yY}{r_0}\right)}$$

$$k_x = \frac{kX}{r_0} \quad k_y = \frac{kY}{r_0} \qquad E_p = e^{i(kr_0-\omega t)} \overbrace{\iint E_s\, e^{-i(k_x x + k_y y)} dx\,dy}^{\text{2D Fourier Transform}}$$

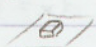
source function

$$I = |E|^2 = \left|\iint E_s\, e^{-i(k_x x + k_y y)} dx\,dy\right|^2$$

Irradiance = square of Fourier transform of aperture function

focal plane intensity is |fourier transform of aperture|^2

$\mathcal{F}(\text{square top hat}) = \text{sinc}(k_x)\,\text{sinc}(k_y) \quad I = \text{sinc}^2(k_x)\,\text{sinc}^2(k_y)$



$\mathcal{F}(\text{round top hat}) = \frac{J_1(x)}{x} \qquad I = \left(\frac{J_1(x)}{x}\right)^2$

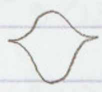
$\mathcal{F}(\text{Gaussian}) = \text{Gaussian}$

softening of aperture function removes ringing off edges

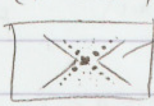
less transmitted at edges

shaped pupil coronographs — remove Airy rings (to detect planets for example)

Sperger–Kasdin

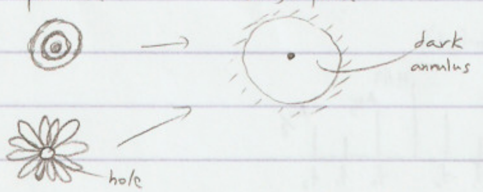


← telescope shape $\xRightarrow{\mathcal{F}}$



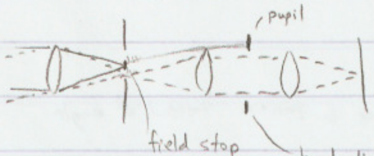
dark wedges (if perfect cancellation)

aperture



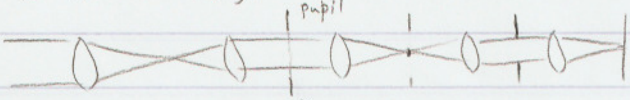
Lyot Coronographs

Bernhard Lyot (1939) — solar corona without eclipse



bad light diffraction of field stop is at outside edge of pupil

to reduce ringing:



Apodized — variable transmission

gray scale transmission softens ringing in 2^{nd} pupil plane

much better suppression, high contrast

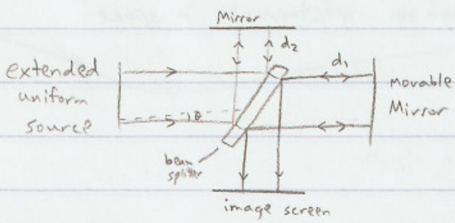
Optical Interferometry

1-Wavefront Division — sample different positions same front (double slit, 2 Telescopes)

2-Amplitude Division — split beam into 2+ paths and recombine (antireflection coating)

Michelson Interferometer (Albert Michelson 1881)

Michelson-Morley exp - 1887 Diameter of Betelgeuse - 1919 @ Mt. Wilson



$I = 4I_0 \cos^2\left(\frac{\delta}{2}\right)$ $\delta = \frac{2\pi}{\lambda}\Delta$ (OPD) $\Delta = 2(d_2 - d_1)$

for different θ: $\Delta = 2(d_2-d_1)\cos\theta$

constructive interference occurs $\Delta = m\lambda = 2(d_2-d_1)\cos\theta$

$$\Delta\theta = \frac{\lambda}{2(d_2-d_1)\sin\theta}$$

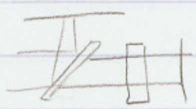
ex/ $\theta = 0$ move d_1 by 0.73 mm, $\Delta m = 300$ fringes

$m\lambda = 2(d_2-d_1)$ $\boxed{\lambda = \frac{2\Delta d}{\Delta m}}$

$\lambda = \frac{2(0.73\,mm)}{300} = 487\,nm$

index piece of glass + measure n

$\Delta = 2(n-1)t$ (thickness)



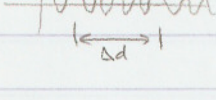
glass — changes Δ

ex/ glass 0.005 mm thick, 487 nm laser
see 10.5 fringes go by (as inserting glass) $\rightarrow$ $\Delta = 10.5\lambda = 5.15\times10^{-6}$ m

$n - 1 = \frac{\Delta}{2t}$ $n = 1.51$

can measure n for glasses by filling empty chambers

2 λ's of light



$$\lambda' - \lambda = \frac{\lambda\lambda'}{2\Delta d}$$

ex/ $\lambda = 589\,nm$ $\Delta\lambda = 6\,nm$

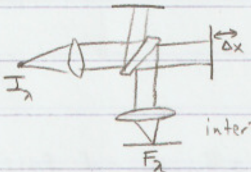
$\Delta d = 28.9\,\mu m$

(5000x more motion than $\Delta\lambda$)

Fourier Transfer Spectrometer (FTS)

- scanning Michelson Interferometer

Twyman – Green Interf



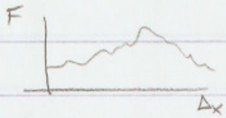
interference in position instead of angle

$$F_\lambda = I_\lambda \cos^2\left(\frac{\delta}{2}\right) = \frac{1}{2} I_\lambda (1+\cos\delta) = \frac{1}{2} I_\lambda \left(1+\cos(2k\Delta x)\right) \qquad k = \frac{2\pi}{\lambda}$$

$$F_\lambda(x) = \frac{I_\lambda(\lambda)}{2} + \frac{I_\lambda(\lambda)}{2}\cos(2k\Delta x)$$

$$F(\Delta x) = \frac{1}{2}\int I(k)\cos(2k\Delta x)\,dk$$

at particular spacing Δx
intensity is fourier coefficient
at corresponding k or λ



$\overset{\mathcal{F}}{\Longrightarrow}$ spectrum of source

benefit – source is bright ($\frac{1}{2}I$)
downside – have to scan

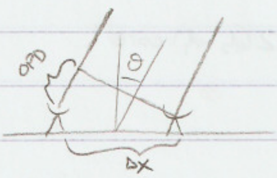
$$R = \frac{\lambda}{\Delta\lambda} = 4\,\frac{\Delta x(\text{range})}{\lambda}$$

$\Delta x = 10\text{cm}$
$\lambda = 400\text{nm}$ $\quad R = 4\times10^5$

good for mid-IR high res spectographs in space

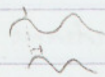
10/23/2007

2 locations in same wavefront



$\text{OPD} = \Delta = \Delta x\cos\theta$ (?, or $\sin\theta$)

have to record phase
→ radio waves
can't record phase in optical

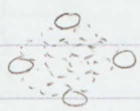


$\phi = \frac{2\pi}{\lambda}\Delta x\,\theta$ for small angle
phase difference

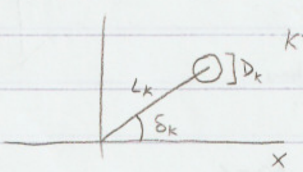
for optical have to use mirrors to get photons together

n-telescopes

sparse aperture interferometry



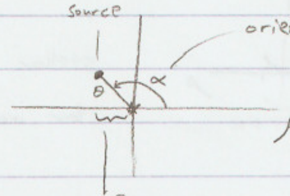
telescope field:



k^{th} telescope
phase shift $= \phi_k$

coordinates for sky

angles on sky



orientation to baseline

$\rho (\sim x)$ baseline

$\rho = \theta \cos\alpha$ — dot product of source + telescope orientation

field from source

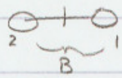
$$E(\theta,\alpha) = dE(\theta,\alpha) \sum_{k=1}^{n} D_k e^{i\left(\frac{2\pi L_k \theta}{\lambda}\right)\cos(\delta_k - \alpha)}$$

intensity of source

phase shift

2 telescopes $D_1 = D_2 = D$ $\quad L_k = \frac{B}{2}$

$\delta_1 = \pi \quad \delta_2 = 0$



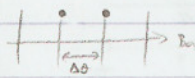
$$E(\theta,\alpha) = 2D\cos\left(\frac{2\pi L_k \theta}{\lambda}\right) dE(\theta,\alpha)$$

$$\text{Irradiance} = \left| \int E(\theta,\alpha)\, d\theta\, d\alpha \right|^2 = 4D^2 \left| \int dE(\theta,\alpha) \cos\left(\frac{2\pi L_k \theta}{\lambda}\right) d\theta\, d\alpha \right|^2$$

source function

→ cosine transform of source function

Irradiance $\propto$ to how much source function looks like cosine function on sky with frequency $\frac{2\pi L_k}{\lambda}$



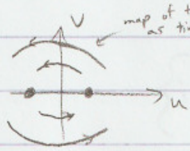
⟹ strong signal

•• → weak signal

one orientation – one fourier component

uv plane

2D $\mathcal{F}$ of xy



map of telescope as time passes

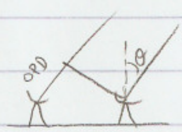
have to map uv plane

filled aperture has many baseline so can see all uv plane

you get very high angular resolution

vary separations run over time to map all of it

2 telescopes



constructive interference

$\phi + OPD = 0$

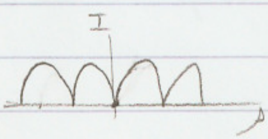
nulling interferometer puts in a π phase shift $\quad \phi + OPD = \pi \quad$ destructive interference

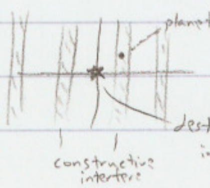
$$E \propto A(\theta,\alpha) = D e^{i\frac{\pi B\theta}{\lambda}\cos(\pi - \alpha)} e^{0} + D e^{i\frac{\pi B\theta}{\lambda}\cos(0-\alpha)} e^{i\pi}$$

$$I(\rho) = 4D^2 \sin^2\left(\frac{\pi B \rho}{\lambda}\right) \sim \frac{4D^2\pi^2 B^2}{\lambda^2}\rho^2$$

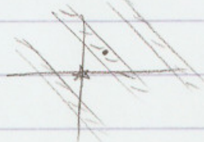
	1	2
D_k	D	D
L_k	$B/2$	$B/2$
δ_k	π	0
ϕ_k	0	π

1978 – Bracewell $\quad$ 2 telescopes arranged π out of phase $\quad$ (Bracewell Nuller)

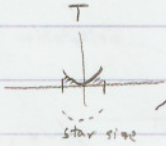




Dif. orientation:



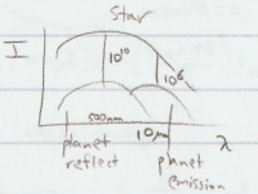
But: zoom in @ $\rho = 0$



stellar leakage
– star not point source @ these resolutions

so not as good as expected

<u>2 element nuller</u>



$B = 75\,m \qquad \lambda = 12\,\mu m$

null depths

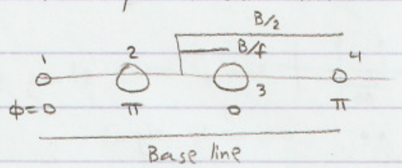
ρ (mas)	$\frac{I(\rho)}{I(peak)}$
0.5 mas	2.3×10^{-3}
1	9.1×10^{-3} ~ 1% of starlight is left (stellar leakage)
2	3.6×10^{-2}
4	1.4×10^{-1}

milli arcsec

solar diameter ~ 1% of orbital diameter

@ 10 pc solar diameter = 1 milliarcsec (mas)

Shao, Angel, Woolf 1994
– 4 telescope linear interferometer



	Diameter
1	D
2	MD
3	MD
4	D

OASIS Interferometers

Sum of 2 Bracewell Nullers

$$\underset{\text{amplitude}}{A(\rho)} = D\left\{ \underbrace{-2\sin\left(\frac{\pi B\rho}{\lambda}\right)i}_{} + \underbrace{2M\sin\left(\frac{2\pi B\rho}{\lambda f}\right)i}_{} \right\}$$

$\approx$ → null both nullers

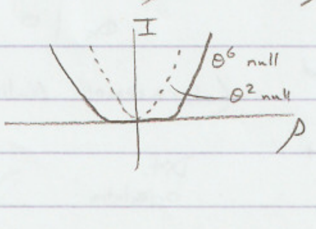
$$\sin\left(\frac{\pi B\rho}{\lambda}\right) = \frac{\pi B\rho}{\lambda} - \frac{1}{3}\left(\frac{\pi B\rho}{\lambda}\right)^3 \cdots$$

$$\sin\left(\frac{2\pi B\rho}{f\lambda}\right) = \frac{2\pi B\rho}{f\lambda} - \frac{1}{3}\left(\frac{2\pi B\rho}{f\lambda}\right)^3 \cdots$$

$$A = 2iD\left\{ \frac{\pi B\rho}{\lambda} + \frac{1}{3}\left(\frac{\pi B\rho}{\lambda}\right)^3 \cdots + \frac{2\pi MB\rho}{f\lambda} - \frac{M}{3}\left(\frac{2\pi B\rho}{f\lambda}\right)^3 \right.$$

if $\frac{2M}{f} = 1 \implies A(\rho) \approx 2iD\rho^3\left\{ \frac{1}{3}\left(\frac{\pi B}{\lambda}\right)^3 - \frac{M}{3}\left(\frac{\pi B}{M\lambda}\right)^3 \right\}$

$\therefore\ I \propto \rho^6$



$\implies$ stellar leakage reduced

ex/ 1-2-2-1 $M=2$ $B=75m$ $\lambda = 12\mu m$

$I(\rho) = 4D^2\left(2\sin(47.6\rho('')) - \sin(95.2\rho(''))\right)$

ρ (mas)	$\frac{I(\rho)}{I(peak)}$
0.5	2×10^{-11}
1	1.3×10^{-9}
2	8.1×10^{-8}
4	6×10^{-7}

→ enough to find Earth

EXNPS started NASA ORIGINS

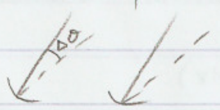
Terrestrial Planet Finder

idea now is to oscillate phase very quickly

stability/calibration rather than brute force null

0 budget, all for Moon + Mars TPF – no funding

differential phase measurements



phase shift between 2 stars – θ shift in sky

angular measurement at diffraction limit $\frac{\lambda}{B}$

Space Interferometry Mission (SIM)

$B = 9m$ optical

1° narrow field astrometry to $3\mu as$

μ-arcsec

goal: $1\mu as$

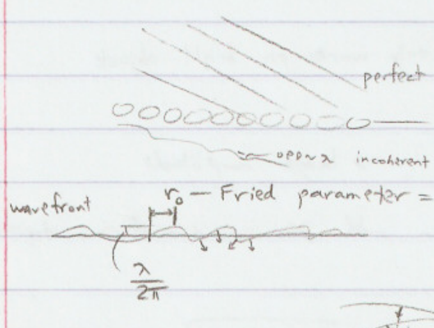
$\sim 15^\circ$ wide field astrometry $30\mu as$

goal $5.4\mu as$

astrometry angles in Galaxy

Galactic Distance Scale

Adaptive Optics (Chpt 14)

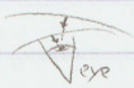


r_0 – Fried parameter = coherence length rms distance between 1 radian shifts

$\frac{\lambda}{2\pi}$

rms $\theta = \frac{\lambda/2\pi}{r_0}$

$r_0 \sim 10$-20 cm



change in position (can't tell with eye)

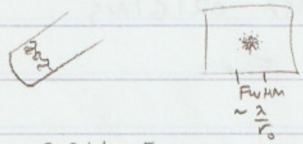
FWHM of speckle cloud $\theta(FWHM) = \frac{\lambda}{r_0}$

ex/ $r_0 = 12.5\text{cm}$ $\lambda = 500\text{nm}$ $\theta = 1''$ (eye resolution: $1'$)

100Hz time scale — intensity variations : twinkling

$\Delta\theta > 1''$ — average intensity — planets don't twinkle

Timescale



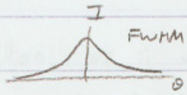
very short exposure

big telescope produces speckle cloud

accidentally be coherent spots across telescope in dif. directions

speckle FWHM $\sim \frac{\lambda}{D}$

long exposure – central limit theorem → Gaussian



seeing disk

seeing $= 1''$

turbulent layer moves by wind

timescale is set by wind $\tau_0 = \dfrac{0.3\, r_0}{v}$ coherence time

independent set of turbulence

$v = 20$ mph $= 900$ cm/s

$r_0 = 15\text{cm}$ (500nm) $\tau_0 = 0.006$ seconds 130 times per second (Hz)

@ $2.2\mu\text{m}$ $r_0 = 80\text{cm}$ $\tau_0 = 0.032$ seconds ~ 30 Hz

10/25/2007

No class next week !

r_0 — distance along wavefront for 1 radian shift of phase

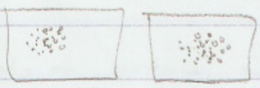
Speckle Interferometry

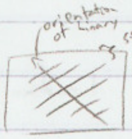
1 sec exposure — speckles smeared out

if exposure $\sim \tau_0$ → freeze atmosphere

only works on bright objects

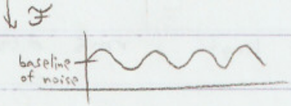
Binary star $\left(\frac{2\lambda}{D}\right)$:





Andrea thesis

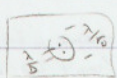
$\downarrow \mathcal{F}$



equal lum → larger amplitude

can add Fourier transforms together

early 90's - adaptive optics



$\text{FWHM} = \sqrt{\left(\frac{\lambda}{r_0}\right)^2 + \left(\frac{\lambda}{D}\right)^2}$

cant deconvolve them

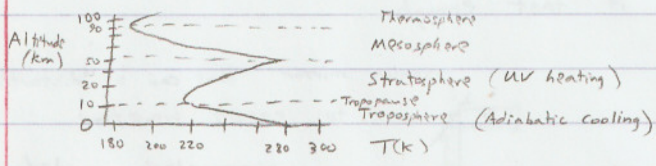
Adaptive Optics

Sources of wavefront error

index of refraction of air $n = 1 + \left[77.6 \times 10^{-6}\left(1 + 7.52 \times 10^{-3}\,\lambda^{-2}\right)\frac{P}{T}\right]$

P – millibar
T – kelvin
λ – μm

Pressure has strong vertical gradient, very little horizontal variation

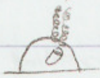


main variation horizontally is thermal

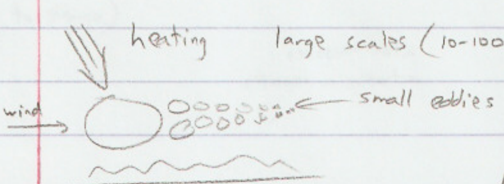
top of Trop $P = 200\,mb$
" Strat $P = 1\,mb$
" meso $P = 0.002\,mb$

virtually all seeing occurs below 20 km
Tropopause often dominates

ground layer for many sights truly dominates

— dome seeing  most domes heat up during day then give off at night

Keck + others are kept cold (expected night temp.)



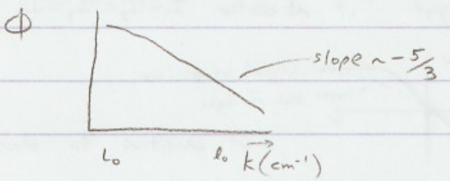
large scales (10–100 m) = L outer scale length

smallest scale set by viscosity (1 mm's)
$= \ell$

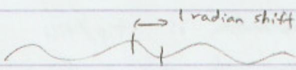
Reynold's num $Re \sim 1$

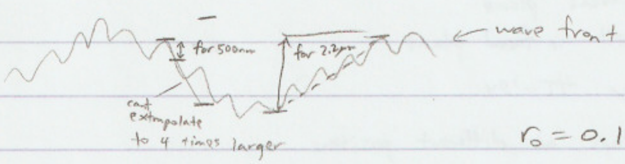
Kolmogorov Turbulence
↳ equal power on all scales

$\left(\frac{v\ell}{\nu} = Re\right)$ (ν – viscosity)



$r_0 = 15\,cm$ @ 500 nm ? @ 2.2 μm





zenith angle

$$r_0 = 0.185\,\lambda^{6/5}(\cos\gamma)^{3/5}\,\Sigma^{-3/5}$$

$$\Sigma = \int_0^\infty C_N^2(z)\,dz \qquad C_N^2 = \frac{\partial N}{\partial T} C_T$$

= measure of turbulence at height z

$r_0 \propto \lambda^{6/5}$

ex/ Σ fixed $\gamma = 0$ seeing 1" @ 500 nm

$\frac{\lambda}{r_0} = 1'' = 4.8 \times 10^{-6}$ rad $\therefore r_0 = 15\,cm$

$r_0(2.2) = r_0(500)\left(\frac{2.2 \times 10^{-6}}{5 \times 10^{-7}}\right)^{6/5} \approx 85\,cm$

$\theta(2.2) = 0.''74$

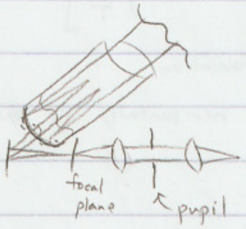
$10\,\mu m$: $\theta = 0.''55$

@ 20 μm large telescopes are diffraction limited

AO work for 1-10 μm , sweet spot: 1-2.4 μm

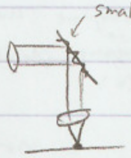
$$\frac{\theta(\lambda_1)}{\theta(\lambda_2)} = \left(\frac{\lambda_2}{\lambda_1}\right)^{1/5} \qquad \boxed{\theta \propto \lambda^{-1/5}} \quad \text{seeing}$$

AO Systems



want to push + pull on primary mirror
cant do it fast enough

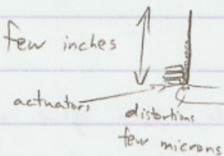
∴



smaller mirror that can be distorted to correct wavefront

piezoelectric deformable mirrors

Deformable Mirrors



surface map

piezo material (crystaline)

voltage deforms lattice – change in size (very rapid) (couple of kHz)

control theory usually sets control bandwidth ~10x slower than actuation

atm ~100 Hz
actuators ~ 1000 Hz

need Wavefront Sensor (WFS)

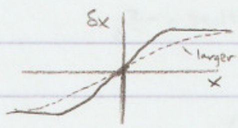
Shack-Hartmann WFS

Quad Cell

2x2 light sensitive pixels

$$\delta x = \frac{(I_2 + I_1) - (I_3 + I_4)}{I_1 + I_2 + I_3 + I_4}$$

shine spot if at center $I_1 = I_2 = I_3 = I_4$

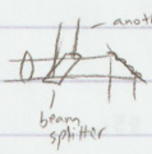


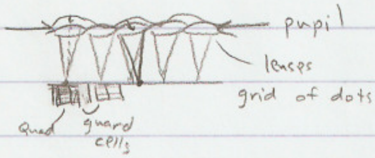
more sensitive to sharp targets

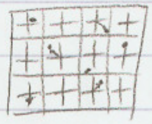
lowest order aberration is tip/tilt (global slope to wavefront)
generates motion of star in focal plane
Tip/tilt tracking – quad cell in focal plane
└ 60 inch Palomar TT-'94

Large telescopes we need slope at different positions

Shack-Hartmann system: breakup telescope into lots of little telescopes and meas. slope at each



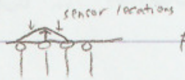




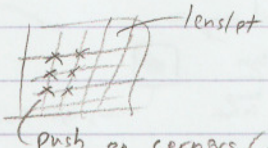
WFS camera want only 1 star per Quad

every quad cell produces displacement in x + y which is $\propto$ to wave front error

measures slope of wavefront



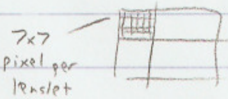
Fried geometry



push on corners (actuation sites)

extended objects – hard
underestimate slopes

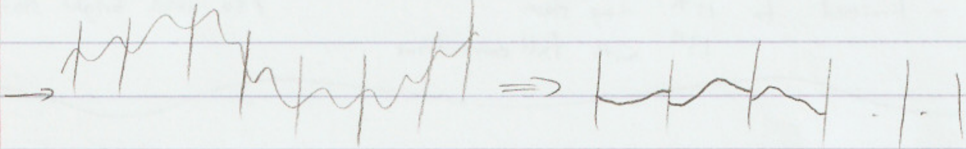
vision science early systems – 20 actuators (eye is nearly diff. limited so dont need that much)



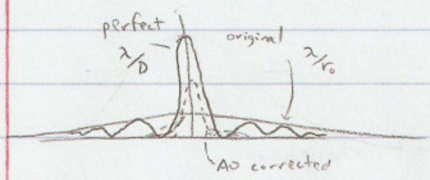
crosscorrelation between patterns
same thing for solar astronomers

keck diameter 1000 cm
want spatial sampling close to $r_0 \sim 40$ cm
13 elements across ~132 active sensors

AO correction is partial



Strehl ratio : $\dfrac{\text{peak height achieved in PSF}}{\text{peak height of perfect PSF}}$



1 µm SR < 1%

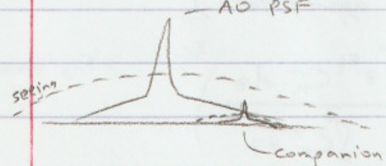
$SR \propto e^{-\left(\theta/\lambda\right)^2}$ residual WFE

SR ~ 10–60%

1 – SR in halo – λ/r_0 in size

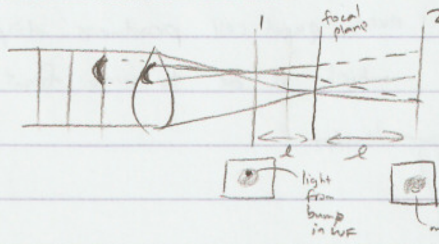
SR ~ amount of power in diffraction limited core

SR ~ 50% half light in extended Airy rings (halo)



benefit: put more power into core

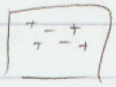
Curvature Sensor



can map to curvature in WF

rapidly take out of focus image

over image $\frac{I_1 - I_2}{I_1 + I_2}$



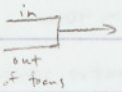
map to curvature in wavefront

bimorph mirror

2 pieces of metal

electrodes (location where can control curvature)

applying voltage:



signal to bimorph mirror

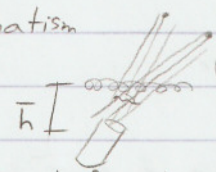
limitation of Shack Hartmann — every sensor looks through little telescope (so need bright stars)

Keck - limited to 11th mag star
13th with full corrections

11/6/2007

wavefront sensor → measure error
↓
Deformable mirror → flattens wavefront

Anisoplanatism



can pass through different atmosphere

r_0 = distance before 1 radian of phase shift occurs

isoplanatic angle θ_0 (angular size where correction is appropriate)

$$\boxed{\theta_0 = 0.31 \frac{r_0}{\bar{h}} \cos(\gamma)}$$

zenith angle

effective altitude ~5-10 km

$$\bar{h} = \left(\frac{\int_0^\infty z^{5/3} C_n^2 \, dz}{\int_0^\infty C_n^2 \, dz} \right)^{3/5}$$

5/3 moment of z

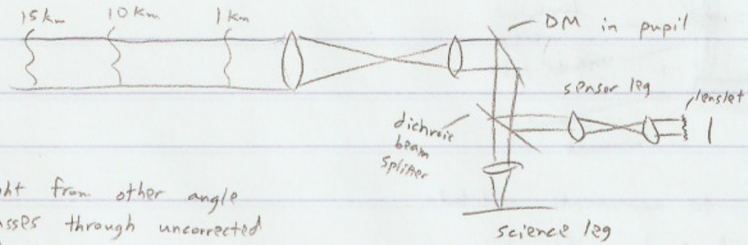
$\theta_0 = \infty$ if $\bar{h} = 0$

Ground Layer AO — corrects low level turbulence
modest correction over entire sky

seeing 0."8 ⟹ 0."5 FWHM

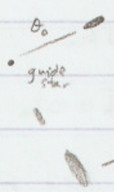
AO systems

usually single conjugate system — single DM (Deformable Mirror)



light from other angle passes through uncorrected turbulence

	R (650nm)	H	J	M	K (2.2μm)
θ_0	7"	9"	15"	20"	30"



— core has less light and elongated

$\theta > 2\theta_0$

— $\sqrt{2}$ worse than seeing

Guide Stars — 10th mag (R band)

now 13th mag for Shack-Hartmann

17th mag curvature (best at 12-13th)

Galactic Center — 13.3 mag (R ↗) (guide star) @ 33" → difficult AO target

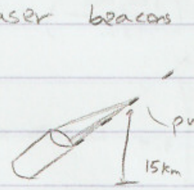
K ~ 8-9 mag @ 7" → easy - infrared WFS

guide stars are rare so cant study every galaxy, etc

<u>particular</u> target:

make your own <u>star</u>

Laser beacons   Rayleigh beacons   Rayleigh scattering is strongly backscattered (shine green laser on sky)



pulsed layer (few microsecond long)

15km $3\mu s = 1$ km in atm

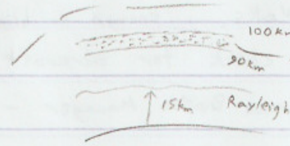
usually see just dot, but large telescopes see width/length of laser

small telescopes

Rayleigh beacons can be very bright → optical correction

range gating — high speed shutters

main problem <u>low altitude</u>



minerals in upper atm (meteors)

Sodium laser guide star

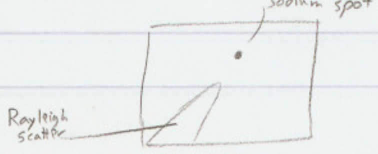
$\lambda = 589$ nm yellow line in Sodium (resonantly scatters)

excite this transition with laser, telescope will see faint star

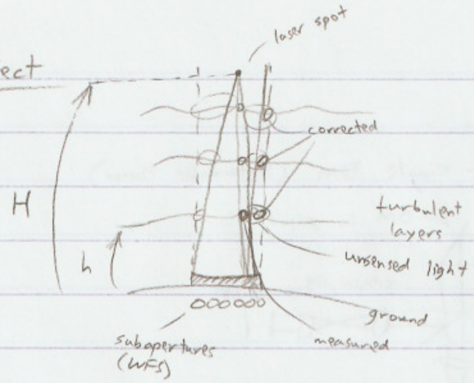
Na_2CO_3 sodium carbonate forms in lower atm

typical mag ~ 8-9th mag

centrally projected laser
secondary shields from Rayleigh



Cone Effect



scale sensor measurements to appropriate altitude

turbulence is magnified by $\left(1-\frac{h}{H}\right)$

$\bar{h} = 5km$ $\quad H = 90\ km$ $\quad \frac{1}{94\%}$ magnification

if $\bar{h} = 10\ km$ $\quad \frac{1}{88\%}$ magnification

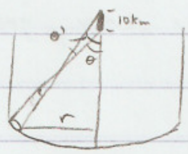
laser systems scale their measurement assuming typical $\bar{h}$

larger error occurs if $\bar{h}$ is different

laser guide systems (LGS) always poorer performance compared to NGS (of same mag.)

Spot Elongation

spot is streak of length θ



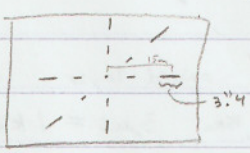
$\theta = \tan^{-1}\left(\frac{H}{r}\right)$

$\theta = \tan^{-1}\left(\frac{90{,}000}{15m}\right)$ 90km (TMT)

$\theta = 89.°990$ $\quad \theta' = \tan^{-1}\left(\frac{100{,}000}{15m}\right) = 89.°991$ 100km

$\Delta\theta = 3.''4$

Shack-Hartmann Sensor Data:



hard to sense 1 axis

: central projection cuts spot elongation in half

spot elongation becomes map of Na density

Small amount of cirrus kills system (~1 mag of extinction, incoming + outgoing)
1 mag 1 mag
2 mag

can also scatter light

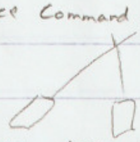
typical sodium lasers ~ 10 Watts enough to blind, set fire to things

$\therefore$ have spotters to check for aircraft

Space Command - Space Battle Manager - checks your targets

hitting foreign satellite - act of war

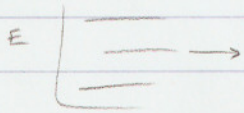
laser collisions



second nearby telescope can see other telescope's laser beam

LASER

Tunable lasers — need 589 nm

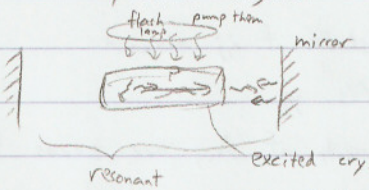


e^- pile up for unusual amount of time

Population Inversion — too many e^- in upper state

photon with energy corresponding to transition passes by it will increase chances of decay — stimulated emission

amplified signal

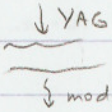


resonant cavity → coherent (in phase) radiation

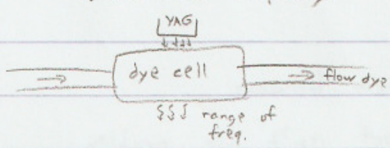
$\lambda = 940, 1064, 1120, 1320, 1440$ nm — lasing wavelengths for Nd:YAG laser

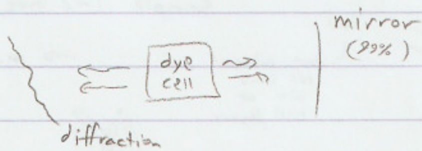
Dye lasers

dye's have complex molecules that can emit @ many frequencies



large bands of allowed wavelengths

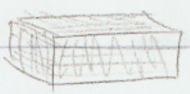




high cost, high complexity
maintenance issues

diffraction grating
→ set angle to send 589nm back into dye and stimulate more 589nm

Non-Linear Crystal



resonant crystal motions

crystal emits @ resonant freq.

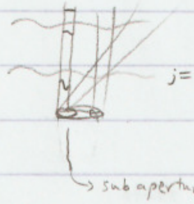
$f = 1$ Hz   if put in ½ Hz light, still will make 1 Hz light
frequency doubler

sum frequency crystals
emit @ $f = f_1 + f_2$
input freqs.

1064 nm : 2.82×10^{14} Hz   1320 nm : 2.27×10^{14} Add up : 5.09×10^{14} Hz
→ $\lambda = 589.1$ nm

Tomography

11/8/2007

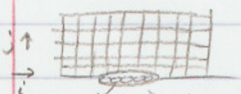


$j = \phi - n$

sub aperture $\quad y_i = \sum \phi_j$ along line of sight

phase shift

2 lines of sight will intersect at different altitudes



j i Y_k telescope

atm divided into grid

volume elements x_{ij} = phase shift of i,j element

subapertures – phase shifts Y_k

$$Y_k = \sum_i \sum_j A_{ij}^{(k)} x_{ij}$$

majority of A_{ij} are zeros for particular Y_k

big linear equation

$A_{ij}(k) = 0$ if light doesnt go through x_{ij}

$= 1$ if it does

have Y_k, predict $A_{ij}(k)$

$\longrightarrow$ solve for x_{ij}

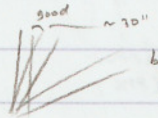
generally – systems try to solve for 2–5 layers

ex/ $\begin{cases} 0-1 \text{ km} \\ 1-2 \text{ km} \\ 2-5 \text{ km} \\ 5-10 \text{ km} \\ 10+ \text{ km} \end{cases}$

requires ~5 guide stars

inversion of matrix scales as D^4

– pushes limits on computations



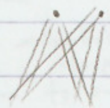
good ~30″

bad

dont intersect the same atm at most layers

Sample 1–2 arcminutes

$\rightarrow$ multiple laser beacons



cone effect isn't big problem

Thirty + mirror telescopes require multiple laser beacons

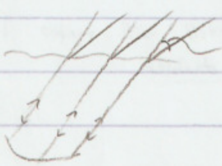
Multiple-Conjugate AO

MCAO – for Gemini telescope – first multiple laser beacon system

5 sodium stars 3 natural guide stars (for tip/tilt & focus) – can be very faint (17^{th} mag)

Aside

tip/tilt & focus cannot be sensed with a laser beacon

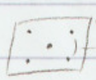


net/average tip/tilt

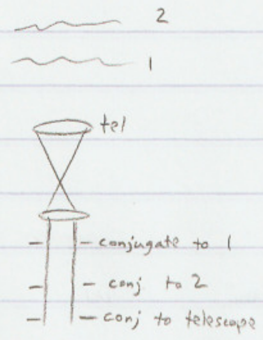
outgoing & returning light travels through same path so cant tell tip/tilt

out in

launched from behind secondary



42.″5 laser beacons (eye cant separate beams)

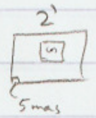


from tomography know ϕ @ tel, 1, 2

can put Deformable Mirrors at each conjugate position to correct for atm. at different layers

Gemini: 3 DMs (0 km, 4.5 km, 9 km)

diffraction limit of 30m at 1 μm is 9 mas
so want pixels ~ 5 mas
field of view ~ 2' = 120"



$\updownarrow$ 24,000 pixels

$\rightarrow$ 625,000,000 pixel^2

(largest IR: 4,000,000)

multiple guide stars for tomography, but several individual DM's for different directions
Multi-Object Adaptive Optics (MOAO)

this allows you to put pixels where you want and not get a huge CCD

IFU's integral field units are very pixel hungry
(0."005) $\underbrace{16 \times 16}_{256}$ spatial locations R~4000 $\Delta\lambda$ = 20% bandpass $\rightarrow$ ~2000 pixels per spectrum

512,000 pixels for 0."08 × 0."08

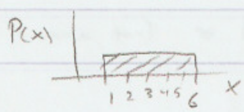
Statistics

Probability distributions

P(A) — probability of event A ex/ $P(2) = 0.1\bar{6} = 1/6$ 6-sided die

normalization condition $\sum_A P(A) = 1$ or $\int_{-\infty}^{\infty} P(x)\,dx = 1$

ex/ Equal probability between 1 & 6



$$P(x) = \begin{cases} C & 1 \le x \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_1^6 C\,dx = 1 \quad C x \Big|_1^6 = 1 \quad C = 1/5$$

Expectation Values

repeat many times and calculate an average

N trials, for each possible outcome A, n_A will be the number of times it occurs

$$\langle A \rangle = \frac{\sum \text{all results}}{N} = \frac{\sum_A A n_A}{N} = \sum_A A \underbrace{\left(\frac{n_A}{N}\right)}_{P(A)} = \sum_{\text{all } A} A\, P(A)$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x\, P(x)\,dx$$

ex/ die $\langle A \rangle = \sum_{i=1}^{6} i\left(\frac{1}{6}\right) = 3.5$ (cant occur)

ex/ $\langle x \rangle = \int_1^6 x\left(\frac{1}{5}\right) dx = 3.5$ (can occur)

$\langle v^2 \rangle = \int_{-\infty}^{\infty} v^2 P(v)\, dv$ second moment of distribution

$$\boxed{\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)\, P(x)\, dx}$$

$\alpha_n = \langle x^n \rangle$ n^{th} moment $= \int_{-\infty}^{\infty} x^n P(x)\, dx$

$\alpha_0 = 1$

ex/ $P(x) = 0.2$ $\quad 1 \le x \le 6$ $\quad \alpha_3 = 64.75$

$P(x) = 0.2$ $\quad 0 \le x \le 6$ $\quad \alpha_3 = 31.25$

central moments : $\boxed{\langle (x-\alpha_1)^n \rangle = \mu_n}$ n^{th} deviation from mean

$\mu_0 = \langle (x-\alpha_1)^0 \rangle = 1$

$\mu_1 = \langle (x-\alpha_1)^1 \rangle = \int (x-\alpha_1) P(x)\, dx = \alpha_1 - \alpha_1 = 0$

$\mu_2 = \langle (x-\alpha_1)^2 \rangle = \int x^2 P(x)\, dx - 2\alpha_1 \int x P(x)\, dx + \int \alpha_1^2 P(x)\, dx$

$= \alpha_2 - 2\alpha_1 \cdot \alpha_1 + \alpha_1^2 = \alpha_2 - \alpha_1^2$ ← the variance

expectation of deviation squared = variance

square root of variance $\sigma = \sqrt{\mu_2}$ dispersion, rms, sigma, standard deviation

central moments tell you about shape

ex/ die $\alpha_1 = 3.5$ $\quad \alpha_2 = 15.1\bar{6}$ $\quad \mu_2 = 2.91\bar{6}$ $\quad \sigma = 1.708$

$\frac{2}{3}$ inside 1σ (not always true) just for Gaussian

Binomial Distribution

success or fail 2 outcomes ex/ coin toss $s = f = \frac{1}{2}$

s = prob. of success $\quad f$ = prob. of failure $\quad s + f = 1$

Binomial is from N events $P_N(n)$ = prob. of n successes given N tries

$\underbrace{\underbrace{ssssss}_{n}ffff}_{N} \rightarrow$ prob: $s^n f^{N-n}$

rearrangements have same probability

how many patterns with n successes? (all with prob: $s^n f^{N-n}$)

number of rearrangements of N items is $N!$

in a particular arrangement there are $n!$ ways to rearrange the s's

total #: $\dfrac{N!}{n!(N-n)!}$ ← # rearrangements of N

$n!$ ← n successes; $(N-n)!$ ← failures $(N-n)$

$$\therefore \boxed{P_N(n) = \frac{N!}{n!(N-n)!} s^n (1-s)^{N-n}}$$ Binomial Distribution

$\langle n \rangle = N \cdot s$ mean $\qquad f = 1-s$

$\mu_2 = N \cdot s \cdot (1-s)$ variance $\qquad \boxed{\sigma = \sqrt{Ns(1-s)}}$

ex/ fair coin $s = \frac{1}{2}$ $\quad P_{50}(n) \quad \langle n \rangle = 25 \quad \mu_2 = 12.5 \quad \sigma = 3.54$ heads

expectation is 25 ± 3.5 heads

$s = 0.6 \quad f = 0.4$ (crooked) $\quad \langle n \rangle = 30 \quad \sigma = 3.46$

Poisson Distribution

special case of binomial $\qquad$ limit as $s \to 0$ but $Ns \to \lambda$ constant

so $N \gg n$

ex/ 1000 photons/sec

$s =$ likelihood of photon in particular nanosecond $= 10^{-6}$ $\quad$ 1 sec exposure $\quad Ns = 10^9 \, 10^{-6} = 10^3 = \lambda$

$$\frac{N \cdot (N-1) \cdot (N-2) \cdots 1}{(n \cdot (n-1) \cdots 1)((N-n)(N-n-1) \cdots 1)} = \frac{N \cdot (N-1) \cdots (N-n+1)}{n(n-1) \cdots 1}$$ ← n terms

$$P_N(n) = \frac{N \cdot s \cdot (N-1) \cdot s \cdots (N-n+1) s}{n!} (1-s)^{N-n}$$

$Ns \sim (N-n+1)s \qquad (1-s)^{N-n} \sim (1-s)^N \qquad$ Approximations

$$P_N(n) \cong \frac{(Ns)^n}{n!}(1-s)^N$$

$(1-s)^N$ & e^{-sN} have identical Taylor expansion (under Approx.)

$$\boxed{P_N(n) = \frac{\lambda^n}{n!} e^{-\lambda}}$$ Poisson Distribution

mean $\langle n \rangle = N \cdot s = \lambda \qquad$ variance $= \lambda \qquad \sigma = \sqrt{\lambda}$

Poisson (counting statistics) $\quad \sigma = \sqrt{Ns} \quad$ can get σ from 1 sample

Detector has intrinsic noise ex/ $40e^-$

IR - lots of background

exposure for 60 seconds   science target: $300\ e^-$ background: $10{,}000\ e^-$

BG noise = 100

$\sigma = \sqrt{S+B} = \sqrt{10300} = 103$ signal will be 300 ± 103

± 40 (device)

integration time: 1s   science: $5e^-$ B: $166e^-$

$\sigma = \sqrt{171} = 13e^-$ signal = $5 \underset{\text{photon}}{\pm 13} \underset{\text{detector}}{\pm 40}$

Background Limited (BLIP)

— Detector noise < $\sqrt{\text{Background}}$ $\sigma_{\text{Detector}} < \sigma_{\text{Photon}}$

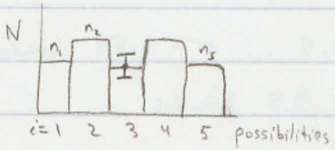
5% = 0.05

$\frac{\sqrt{N}}{N} = 0.05 \Rightarrow N = 400$

27% : 108 people of 400

$\therefore$ 27% ± 2%

Statistics on a Histograms



$n_i = \#$ in ith bin

$N = \sum_i n_i$

for ith: Success is an ith event   failure is any other (binomial problem)

$P_N(n_i)$ $\langle n_i \rangle = s \cdot N$ $n_i \approx sN$ $s \approx \frac{n_i}{N}$ estimator

$\sigma_i = \sqrt{N s(1-s)} = \sqrt{n_i \left(1 - \frac{n_i}{N}\right)}$

if $n_i \ll N$, s is small but $s \cdot N = n_i$ Poisson $\therefore$ $\sigma_i = \sqrt{n_i}$

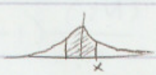
$\boxed{\sigma_{\text{Poisson}} \geq \sigma_{\text{Binomial}}}$

Normal or Gaussian Distribution

$P(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ $\langle x \rangle = 0$ $\mu_2 = 1$ $\sigma = 1$

general: $\boxed{P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\alpha)^2}{2\sigma^2}}}$ $\langle x \rangle = \alpha$

$\sigma = \sigma$



error function = $\text{erf}(x) = 2\int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt$

erf(1) = area under curve out to 1σ on each side

$\text{erf}(\infty) = 1$ $\text{erf}(1) = 68.27\%$ $\text{erf}(2) = 95.4\%$ $\text{erf}(3) = 99.73\%$

FWHM $P(x_{1/2}) = \frac{1}{2}P(0)$ $\text{FWHM} = 2x_{1/2} = 2\sigma\sqrt{2\ln(2)} = 2.355\sigma$

Central Limit Theorem

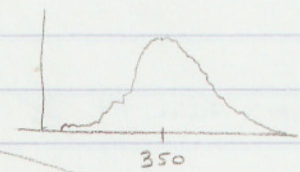
lots of independent data different distributions $x_1, x_2, \dots x_n$

$$\frac{\sum_i (x_i - \alpha_i)}{\sqrt{\sum_i \sigma_i^2}} = y$$ new random variable

y is Gaussian distribution in limit of n large

ex/ roll 100 die $y = \sum$ dice roll

$\bar{x} = 350$ $\mu_y = 170$



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Propagation of Errors

ex/ Linear function $y = \sum_i a_i x_i$ (weight: a_i; random quantity: x_i)

$\bar{y} = \langle y \rangle = \text{mean} = ?$ $V(y)$ (variance) $\sigma_y = ?$

$$\bar{y} = \int_{-\infty}^{\infty} y P_y \, dy = \int \sum a_i x_i P_i(x_i) \, dx_i = \sum_i a_i \int x_i P(x_i) \, dx_i = \sum a_i \bar{x}_i$$

$$\bar{y} = \sum_i a_i x_i$$

$$V(y) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j V_{ij}$$ covariance matrix

$$V_{ij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x_i - \alpha_i)(x_j - \alpha_j) \underbrace{P(x_i, x_j)}_{\text{joint probability}} \, dx_i \, dx_j$$

if x_i's are independant

then $P(x_i, x_j) = 0$ if $i \neq j$

if $i = j$ $P(x_i, x_i) = P^2(x_i)$ $V_{ij} = \sigma_i^2$ $i = j$ or $= 0$ if $i \neq j$

$$V(y) = \sum_i a_i^2 V(x_i) \qquad \boxed{\sigma(y) = \sqrt{\sum_i a_i^2 \sigma_i^2}}$$

if $a_i = 1$ $\sigma(y) = \sqrt{\sum_i \sigma_i^2}$

ex/ 25 measurements of quantity with $\sigma = 3$

$$\sigma(\text{sum}) = \sqrt{\sum_{i=1}^{25} 3^2} = \sqrt{25(9)} = 5(3) = 15$$

if all σ_i are same then $\sigma(\text{sum}) = \sqrt{N}\,\sigma_i$

mean of set of data: $\sigma(\text{mean}) = \frac{\sqrt{N}}{N} \sigma_i = \boxed{\frac{\sigma_i}{\sqrt{N}} = \sigma_{\text{mean}}}$

$\sqrt{4\sigma_d^2+\sigma_t^2} \Leftarrow 2d+t \neq d_1+d_2+t \Rightarrow \sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2}$ (independent)

(both desks are completely dependent (identical))

ex/ 2 desks & 1 table   D | D | T   will it fit into a room?

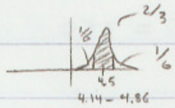
desk = 1.7 ± 0.1 m   table = 1.1 ± 0.3 m

total length: $2D + T$ $a_d = 2$ $a_t = 1$

$\overline{\text{leangth}} = 2\bar{d} + \bar{t} = 4.5\text{ m}$ worst element dominates

$$\sigma_{\text{length}} = \sqrt{a_d^2\sigma_d^2 + a_t^2\sigma_t^2} = \sqrt{4(0.1)^2 + 1(0.3)^2} = 0.36\text{ m}$$

$\therefore\ l = 4.5 \pm 0.36$ m



if room is 4.86 m, desks + tables fit 5/6 of the time

$$y = f(x_1, x_2, \ldots, x_n) \qquad \bar{y} = f(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$$

Taylor expand about mean

$$y = \bar{y} + \left.\frac{\partial f}{\partial x_1}\right|_{\bar{x}_1}(x_1 - \bar{x}_1) + \left.\frac{\partial f}{\partial x_2}\right|_{\bar{x}_2}(x_2 - \bar{x}_2) + \ldots$$

$$= \bar{y} + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial x_i}\right|_{\bar{x}_k}(x_i - \bar{x}_i) = \underbrace{\bar{y} - \sum_{i=1}^{n} \left.\frac{\partial f}{\partial x_i}\right|_{\bar{x}_k}\bar{x}_i}_{z \text{ (constant)}} + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial x_i}\right|_{\bar{x}_k} x_i + \ldots$$

$\therefore$ has no variance

$$a_i = \left.\frac{\partial f}{\partial x_i}\right|_{\bar{x}'s}$$

$$V(y) = \sum_{i=1}^{n} a_i^2\, V(x_i) \quad \therefore \quad \boxed{\sigma_y^2 = \sum_{i=1}^{n}\left(\left.\frac{\partial f}{\partial x_i}\right|_{\bar{x}}\right)^2 \sigma_{(x_i)}^2}$$

ex/ $y = \dfrac{x_1}{x_2}$

$$V(y) = \left(\left.\frac{\partial y}{\partial x_1}\right|_{\bar{x}}\right)^2 V(x_1) + \left(\left.\frac{\partial y}{\partial x_2}\right|_{\bar{x}}\right)^2 V(x_2)$$

$$= \left(\frac{1}{\bar{x}_2}\right)^2 V(x_1) + \left(-\frac{\bar{x}_1}{\bar{x}_2^2}\right)^2 V(x_2) = \frac{\sigma_1^2}{\bar{x}_2^2} + \frac{\bar{x}_1^2}{\bar{x}_2^4}\sigma_2^2$$

$\bar{y} = \dfrac{\bar{x}_1}{\bar{x}_2}$

$$V(y) = \bar{y}^2\left(\frac{\sigma_1^2}{\bar{x}_1^2} + \frac{\sigma_2^2}{\bar{x}_2^2}\right) = \sigma_y^2$$

ex/ $x_1 = 12 \pm 2$ $x_2 = 6 \pm 1$ $\bar{y} = 2$ $\sigma_y = 2\sqrt{\frac{4}{144} + \frac{1}{36}} = 0.47$

$y = 2 \pm 0.47$

CCD Detectors

Optical photons 1.5 – 2.5 eV   metals often have binding energy of 1–few eV

$\to e^-$ photoelectron   work function = E needed to liberate e^-

$$E_{e^-} = h\nu - W.F.$$

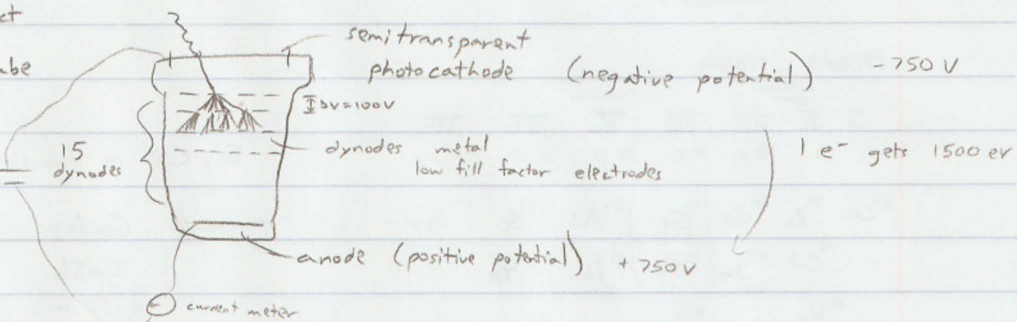
detecting current due to 1 e^- is challenging

Photoelectric Effect

Photomultiplier Tube

electron avalanche

very large gain



large spike in current with every photon

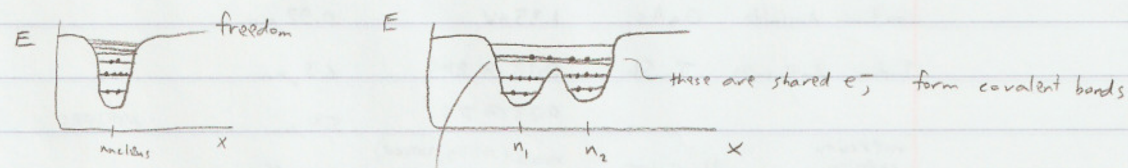
no spectral information — original energy of photon is lost

proportional counters can be made with $I \propto E_\gamma$

typical quantum efficiency ~20%   good for low signal situations
can be made very large

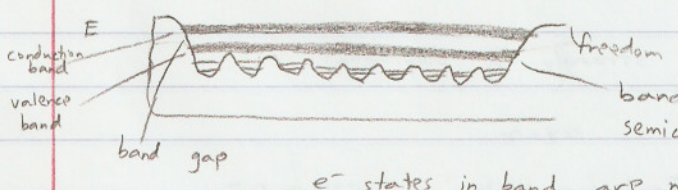
Semiconductors

Energy levels in atom



twice as many states as individual
small splitting of energies

many atom system



e^- states in band are not localized to individual atom

if valence e^- level is full $\to$ no conduction, insulators

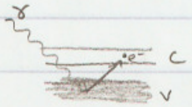
e^- in empty band can roam freely $\to$ conduction

high conduction material if conduction band ΔE overlaps with valence band $\to$ metals
$\to$ high electrical & thermal conductors

insulators $\to$ large separation between conductor & valence band   } few eV semiconductors: $\overset{30\times}{\downarrow}$ thermal energies separate bands ~ 1eV

normally nonconductive

but optical photon can elevate e^- to conduction band



can build up e^- in conduction band

Periodic Table

$\xleftarrow{\text{metals}}$ $\xrightarrow{\text{nonmetals}}$

I	II	III	IV	V	VI	VII
		^{13}Al	^{14}Si	^{15}P	^{16}S	^{17}Cl
^{29}Cu	^{30}Zn	^{31}Ga	^{32}Ge germanium	^{33}As	^{34}Se	Br
	^{48}Cd	^{49}In Indium	^{50}Sn Tin	^{51}Sb Antimony	^{52}Te	
	^{80}Hg		^{82}Pb			

Si, Ge — family IV semiconductor

compound GaAs — family III–V semiconductor
InSb

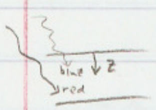
50-50 mix
HgCdTe — family II–VI semiconductor

different band gaps

11/15/2007

name	symbol	E_{gap}	$\lambda_c = \frac{hc}{E_{gap}}$ (cutoff λ)	
silicon	Si	1.12 eV @ 295K	1.11 μm	optical astronomy UBVRI ($\overset{\uparrow}{}$800nm + 1μm)
		0.5 eV @ 77K	2.3 μm	77K: liquid Nitrogen
germanium	Ge	0.7 eV @ 295K	1.85 μm	
Gallium Arsinide	GaAs	1.35 eV	0.92 μm	
Indium Antimonide	InSb	0.18 @ 295	6.9 μm	
		0.23 @ 77	5.4 μm	NIRSPEC
mercury cadmium telluride (mercadtel)	$Hg_x Cd_{1-x} Te$ (II II VI)	x=0.8 (80% Hg, 20% Cd) 0.1 eV @ 77	12.4 μm	
		x=0.5 0.5 eV @ 77	2.5 μm	Keck, VLT, JWST

absorption occurs in crystal, strongly λ dependent



$$I(z) = I(0)\, e^{-\alpha z} \qquad \alpha z = \tau$$

@ 300K for Si $\alpha = 5\,\mu m^{-1}$ for 400nm $\langle z\rangle \sim 0.2\,\mu m$

$\alpha = 0.1\,\mu m^{-1}$ for 800nm $\langle z\rangle \sim 10\,\mu m$

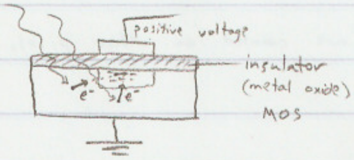
if 10 μm thick: absorb 65% of red, ~100% of blue

cool Si to 77K

$\alpha = 4\,\mu m^{-1}$ for 400nm $\langle z\rangle \sim 0.25\,\mu m$

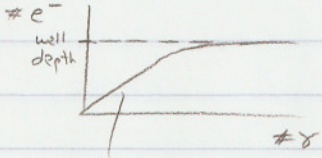
$\alpha = 0.005\,\mu m^{-1}$ for 800nm $\langle z\rangle \sim 2000\,\mu m$ impractically deep

Detector

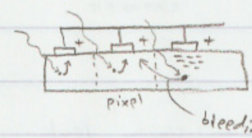


accumulate charge beneath gate (electrode)

unit cell can become saturated when repulsion from e^- matches attraction to gate

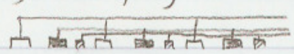


slope = conversion efficiency or quantum efficiency ~0.9



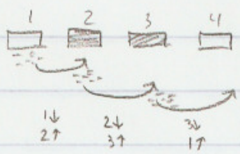
bleeding occurs when full wells spill over to neighbors

charge coupling – charge follows positive electrodes



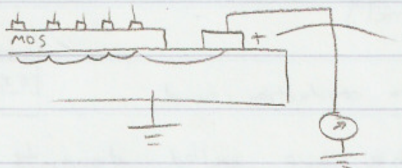
3 phase CCD

turn down voltage on original and turn on neighbor



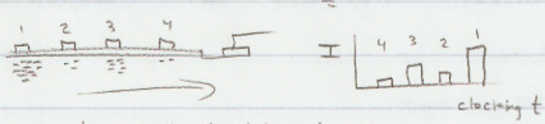
charge shuffling to track charge

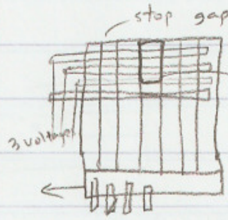
drift tracking – telescope stationary, let sky move but track charge to match sky



readout gate

initially grounded but then gets voltage to gather e^- and read them





stop gaps – insulating layers that prevent e^- motion

1 pixel

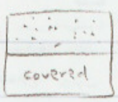
readout row with electrodes ⊥ to original direction

reading array: 1 – cycle all rows 1 down

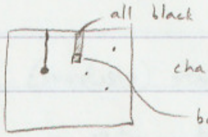
2 – cycle read out row 1000 times

3 – at each of these cycles one pixel of charge goes out output electronics

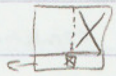
for low noise – read slowly

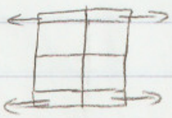
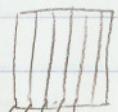


rapidly shuffle pattern down, slowly read while still accumulating charge on top charge can get left behind



charges read down those left behind are counted on other pixels (on top)

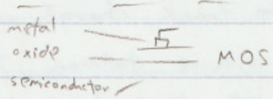
if bad pixel in readout row: 

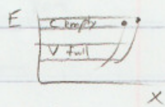
 multiple read out channels 

electron traps – slowly leak exponential decay
(impurities trap e^-)

blooming – fills beyond well 

Final 11/20/2007
Friday 7th 2-4 pm

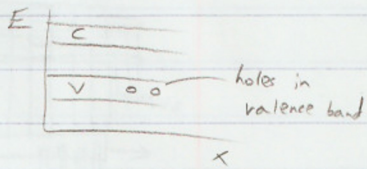


Contaminants in semiconductor (still neutral)
family V – too many electrons, will go to conduction band 

if you intentionally add contaminants they are called dopants
family V – n-type semiconductors
(too many negative charges

typical dopants – phosphorus, arsenic, antimony

p-type semiconductors – family III, too few e^-

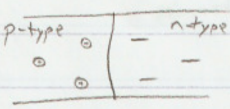


either e^- or holes can move

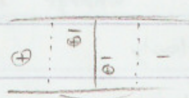
early CCD's used p-type semiconductor: gates drive away holes first so no conduction until light hits system (it will now have net charge)

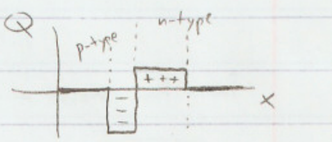
pn junction

(both still net neutral)

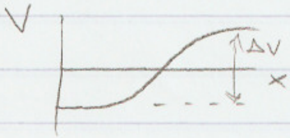


make contact
current will flow



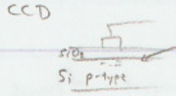


depletion zone
– no charge carriers



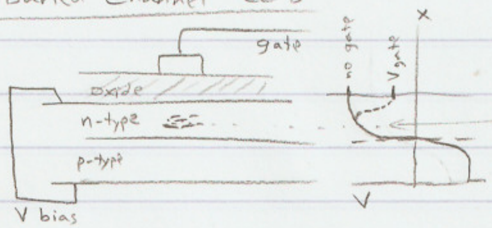
ΔV is set — built in voltage
— stops further flow

CCD



boundary is worst part of crystal

Buried Channel CCD



e^- will accumulate within n-type, not at boundary

can still move the e^- by cycling volage on gates

Dark Current

not photoelectrons → thermal electrons

e^- elevated to conduction band

appear as normal signal even without light

Locations

band gap $\sim 30 \times \langle E_{gap} \rangle$

1 – neutral semiconductor (Si)

2 – depleted Si

3 – boundary to oxide ⇐ dominates by $\times 100 - 1000$

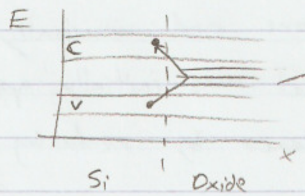
typical # is 100,000 e^-/sec in 30μm pixel @ room temperature

win exponentially with cooling

(7.3 Ian's book)

factor of 3 reduction for every 10°C

@ −100°C you get ~0.1 e^-/sec still huge for astronomy



intermediate states

if you set gate voltage negative, the boundary becomes negative and intermediate states fill → no jumping

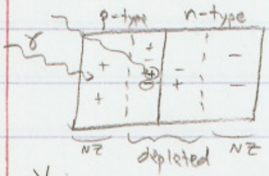
MultiPhase Pinned CCD (MPPN CCD) → these are ones used in Astronomy

eliminate boundary dark current
helps with persistence

modern CCD's: 0.1 e^-/hour

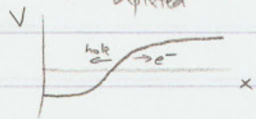
lots of clever tricks in Chpt 7

Photodiode

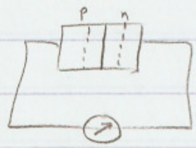


no current flow

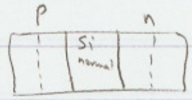
recombination occurs quickly in Neutral Zone

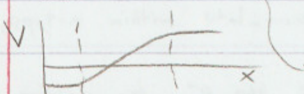


if photon strikes depletion zone — current flows



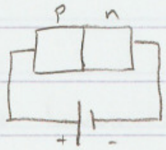
current flows $\propto$ flux on depletion zone





→ larger in size

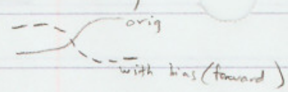
pn junction forms a diode



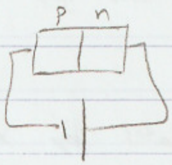
forward biasing reduces potential accross junction

and reduces size of depletion zone

can get current flow in conduction band from n to p



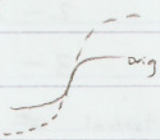
reverse bias:



voltage increases across gap

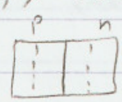
depletion zone increases

charge cannot flow



current will only flow in one direction in pn junction

apply reverse bias to pn junction and take it away



get junction with large depletion zone and unusually large voltage across it (essentially what was initially applied)

photo (electrons + holes) will restore initial balance by reducing voltage

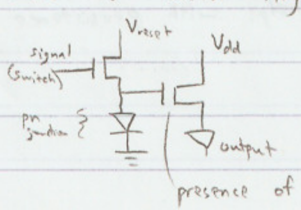
at future times V across junction tells us integrated charge

$$Q = \frac{CV}{e}$$

capacitance

basically a capacitor

measure V with high impedence output (often j-fet)



hit switch → pn junction charged

take way → integrate

$I \propto V$ on switch

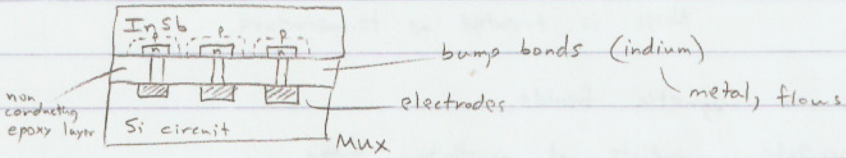
presence of voltage allows current to flow that was photovoltaic detector

IR 1–5 μm ↗

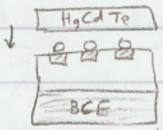
Si is transparent to red + IR especially when cooled

InSb, HgCdTe — difficult to make circuits

Hybrid detectors



Teledyne – HgCdTe



squeeze together — bonds form

cool differently so can shatter material if contraction isn't handled propery

stresses Si so its contraction matches HgCdTe

molecular beam epoxy is semiconductor growth atom by atom

MUX
(light sensitive to optical photons, works as photodiode $I \propto$ flux instantaneously
just Si circuitry → preliminary tests

11/27/2007

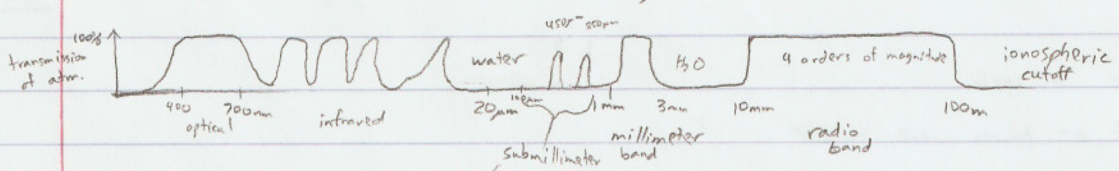
Radio Astronomy — Chpt 12

accidental start 1932 Karl Jansky

$f = 20.5$ MHz to lightning — found localized source with 24 hr period

plane of MW galaxy – thermal in nature — ISM – thermal

Hay – 1942–43 Sun detected during solar max — 1st nonthermal — magnetic

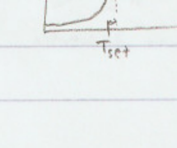


20 – 100 μm SOFIA
Balloons, space

→ very high, dry site

20 μm – 1 mm — bolometers & superconducting detectors dominate

(radiation heats detector and changes resistance



Radio Emission Sources

- any solid has temperature – radiates as blackbody usually IR (far IR)

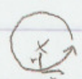
  Radio is in Rayleigh Jeans limit

  $$S_f = \frac{2kTf^2}{c^2} \propto T$$ often refer to intensity as temperature (Brightness Temp.)

  How hot would BB need to be to generate that signal

  Noise is treated as temperature

- Charged particles in magnetic fields



charges oscillate radiate at oscillatory freq.

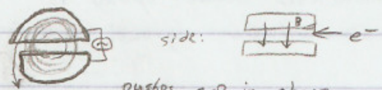
$$a_c = \frac{v^2}{r} \quad a_m = \frac{F}{m} = \frac{q\vec{v}\times\vec{B}}{m} = \frac{qvB}{m}$$
(circular)

$$a_c = a_m \quad \frac{v^2}{r} = \frac{qvB}{m} \quad r = \frac{mv}{eB}$$ gyro radius

$$f = \frac{\text{rate}}{\text{distance}} = \frac{v}{2\pi r} = \frac{eB}{2\pi m}$$ radius, velocity independent

cyclotron radiation

cyclotron



e^- loose E by radiation

relativistic e^- dont have same frequency (mass changes)

synchrotron – cyclotron for relativistic particles freq depends on Energy

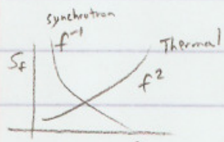
radiated power $\propto \gamma^4$ ← more radiation

we detect synchrotron emission (not cyclotron – too weak)

lots of relativistic particles in universe

freq depends on B + v of particles

$S_f \propto f^{-\alpha}$ $\alpha \sim 0.2 - 1.2$ depends on spectrum of particle velocities



- line emission

  most e^- levels are $\sim eV$

H $n=101 \rightarrow n=100$ transition $\lambda = 46\,cm$ tend to be weak

molecules $\underbrace{\text{rotate}}_{\text{millimeter radio}}$ + $\underbrace{\text{vibrate}}_{\text{IR}}$ H_2 in $2\mu m$ band – rotation-vibration state

$$f_{ij} = \frac{\hbar}{\pi I}\left(j(j-1) - i(i-1)\right)$$ for rotation

I: moment of inertia

requires net dipole moment in e^- charge strong radiators are asymmetric : CO, H_2O, H_2CO, CS, NH_3

want to know about H_2 but rely on tracers

ratio of $\frac{CO}{H_2}$ is not constant, often predictable

$C^{13}O$ is optically thin even when $C^{12}O$ is optically thick

in cloud cores, protostars chemistry is complicated → range of molecules

— 21 cm emission

H in lowest state

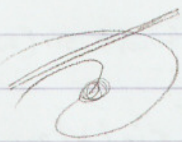
p, e^- spins : $\uparrow\downarrow$ or $\uparrow\uparrow$

($\uparrow\downarrow$) lower in energy

$\uparrow\uparrow \to \uparrow\downarrow \sim$ 21cm, 1420 MHz

predicted in 1943 — van de Hulst

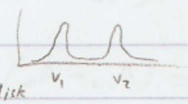
lifetime $\sim 10^6$ yrs



$10^{14} - 10^{22}$ atoms/cm^2

very bright source of line emission

line width is very small ⟶ can map kinematics of galaxy

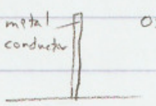


Oort equations — map velocity into differential motion of disk

convert angle + velocity into distance

Detectors in Radio

beyond 1mm individual photons dont matter



oscillatory (AC) voltage

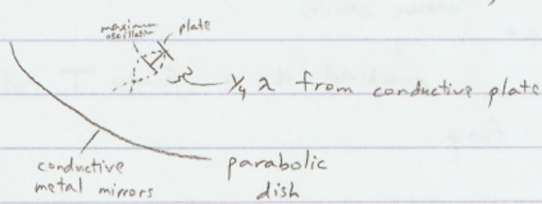
e^- oscillate + radiate

dipole radiator

measure an oscillatory current from a passing radio wave

pure dipole recievers at first

1930s radio dishes concentrate light into small dipole



small fluctuations in surface $\lambda/20$ dont matter much

optical: 20-30 nm

1 cm : 0.5 mm — easier to make

Largest steerable radio

└ Bonn — 100 m

Arecibo — 300m — 1000ft largest (not steerable)

can also use mesh of metal with hole << λ

"Underneath that dirty Puerto Rican telescope lies the nice metal mesh detecting surface." — James

AC signal from reciever
would like to amplify this but freqs. are still very high (100 GHz)
difficult to amplify
radio frequency amplifier are expensive and get modest gain
instead of amplifying we mix the signal
— heterodyne receiver — combine 2 different freq sources

$$\text{Signal} = A_{\text{object}} \sin(\omega_{\text{source}}) + A_{\substack{\text{local}\\\text{oscillator}}} \sin(\omega_{LO})$$

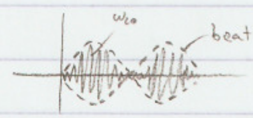
$$\omega_{\text{source}} = \omega_{LO} + \delta\omega$$

$$S = A_o \sin(\omega_{LO} + \delta\omega) + A_{LO} \sin \omega_{LO}$$

$$S = (A_{LO} - A_o)\sin(\omega_{LO}) + A_o\left(\sin(\omega_{LO}) + \sin(\omega_{LO} + \delta\omega)\right)$$

$$= (A_{LO} - A_o)\sin(\omega_{LO}) + 2A_o\left[\sin\left(\tfrac{1}{2}(2\omega_{LO} + \delta\omega)\right)\cos\left(\tfrac{1}{2}(\omega_{LO} - \omega_{LO} - \delta\omega)\right)\right]$$

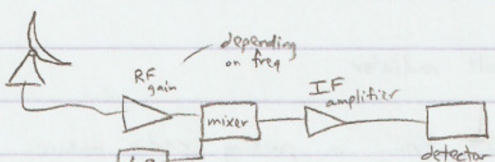
$$= \underbrace{(A_{LO} - A_o)\sin(\omega_{LO} t) + 2A_o \sin\left(\left(\omega_o + \frac{\delta\omega}{2}\right)t\right)}_{\text{high freq}} \underbrace{\cos\left(\frac{\delta\omega}{2} t\right)}_{\text{beat oscillation}}$$



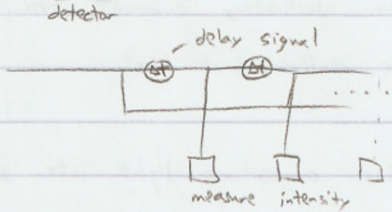
produce intermediate freq. (IF)

$S_{\text{beat}} \propto A_{\text{object}}$

$\delta f \sim 100\,\text{MHz}$ → much easier to amplify



autocorrelate signal



lots of power on $f = \frac{1}{2\Delta t}$

back in phase

proportional to how much signal looks like $f = \frac{1}{\Delta t}$ → one Fourier component

combined this is Fourier T of temporal signal

→ produces spectrum near LO freq.

Angular Resolution

no atmospheric distortion, telescopes close to perfect

$$\delta\theta = \frac{\lambda}{D} \quad \text{diffraction limit}$$

ex/ Bonn $D = 100m$ $\lambda = 1m$ $\delta\theta = 0.01 rad = 0.6°$

interferometry is very important must get D much larger

multiple telescopes give $\delta\theta = \frac{\lambda}{\text{separations}}$

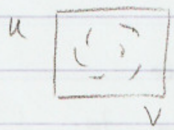
VLA in Socorro, NM 10's of km in area 100x angular resolution of Bonn

radio: can record phase so can post process signals later on

can correlate signals in post processing

VLBI — North America (Arecibo) + Australia + Europe $\delta\theta \sim 0.''001$

→ highest angular resolution



inverse FT into sky

Interferometers lose sensitivity to extended sources

clean algorithm — insert point sources to help deconvolve

11/29/2007

High Energy Detectors

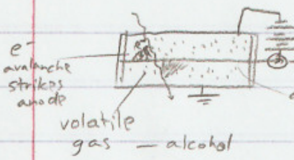
$E > 100eV$ → ionization

	λ	E	
soft x-rays	1–10nm	120eV – 1.2keV	ionizes any atom
x-rays	0.01 – 1nm	1.2keV – 120keV	ionizes e^- at any state
soft γ-rays	0.001 – 0.01nm (1pm)	120 – 1200 keV (1.2 MeV)	nuclear transitions
γ-rays	< 1pm	> 1.2 MeV	can reach TeV — nuclear transitions + unknown physics

Sun: $\lambda = 1nm$ → 5×10^9 photons/m^2 s $\lambda = 1fm$ ($10^{-15}m$) → few photons/m^2/day

low flux rates

Geiger Counter



gas saturates — every pulse looks about the same

gain $\sim 10^8$

2 Disadvantages: — no spectral information (all saturate)
— dead time $\sim 200\mu$sec (e^- recombine)
$\therefore$ limited to count rates of 100,000/sec

Advantages: — large spectral response
— can detect particles

Proportional Counter

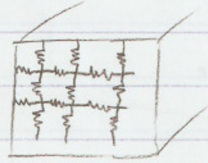
— current produced $\propto$ incident energy

turn down voltage → weaker signal → need more advanced electronics

— get some spectral info — 10–40% spectral resolution
ex/ 1 ± 0.4 MeV (40%)

— no true saturation $\therefore$ no dead time

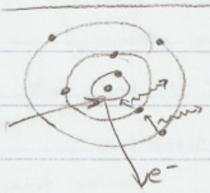
e^- in avalanche are local to anode
→ this allows multiple anodes in same device



can compare signal strength & timing to localize where it passed

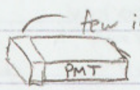
multiple sets of grids give directionality

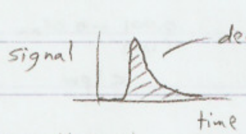
Scintillation Detectors



plastics

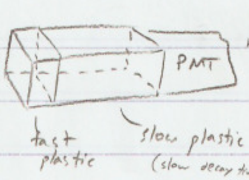
inner shell ionization → multiple transitions (including optical)





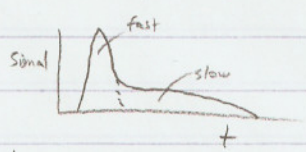
integrated signal $\propto$ Energy

problem: thin plastics — particle goes all way through
saturation problem



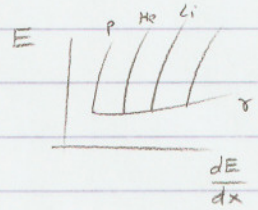
more plastic — extends energy range for proportionality

} phoswich



slow signal $\propto E$ total

fast signal $\propto \frac{dE}{dx}$

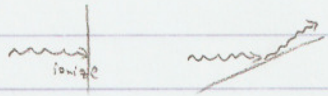


work to about 10 MeV

Forming Images

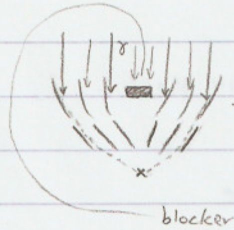
$E >$ few keV ionization always occurs

$E < 2$ keV can have reflection



grazing incidence mirror

~85° very large angles



nested shells of paraboloids with common foci

hyperbolic secondary

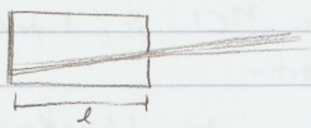
blocker (lead)

$\theta = \frac{\lambda}{D} \Rightarrow$ very high angular resolution

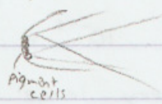
resolution: couple of arcsec (far away from diffraction limit even for space)
limited by optics

Directionality

pinhole camera

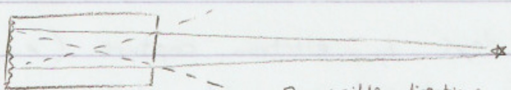


primitive eyes



modern:

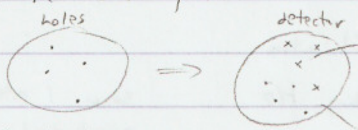
small pinhole
– less light, sharper image



rejection between detectors

2 possible directions, 2 detection so can use rejection to determine where source is

non-redundant pattern of holes



more holes → more light

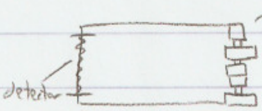
needs to be sparse

redundant pattern

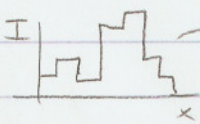
bad

can make lead pinhole camera

balloon experiments



non redundant set of absorbers



PSF for point source

by rotating can get many PSF's
→ unique solution for source distribution

Neutrino Detection

little neutral one – Pauli – 1930

β decay

missing momentum no charge, very little rest mass

ν – neutrino — no strong, EM interactions only weak force for interactions

Solar neutrino : 10^{-10} chance of being absorbed in sun

produced in vast quantities by nuclear reactions

primordial neutrinos from first few minutes → very low in energy

1st neutrino detector R. Davis Jr. 1968

$\nu_e + n \rightarrow p + e^-$

$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$ min $E = 0.814$ MeV

pp chain ν $E \sim 0.26$ MeV

Boron side chain
$${}^3He + {}^4He \rightarrow {}^7Be$$
$${}^1H + {}^7Be \rightarrow {}^8B + \gamma$$
$${}^8B \rightarrow {}^8Be + e^+ + \nu_e \text{ — 7.2 MeV}$$
$${}^8Be \rightarrow {}^4He + {}^4He$$

mean free path for ν_e in ${}^{37}Cl$ is 1 parsec

10^{-46} to 10^{-50} m^2 cross section

very large tanks 600 tons tetrachloroethene C_2Cl_4

${}^{37}Ar$ – noble gas becomes free

└ unstable decay halflife of 35 days by electron capture (emits 2.8 keV photon)

He bubbled through – Ar + He are collected in cold charcoal trap

wait for 2.8 keV photons

1 atom accuracy for every 10^{30} original Cl atom

integration times of 50-100 days before bubbling He

saw 1 atom/day
predicted 3 atoms/day

Solar neutrino problem : detect only 1/3 of ν_s predicted

SAGE (60 tonnes) Gallex (30 tonnes)

Gallium $^{71}Ga + \nu_e \rightarrow {}^{71}Ge + e^-$ energy treshold = 0.236 MeV

→ sensitive to pp ν_s

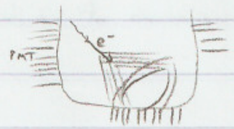
still got 1/3 of ν_s predicted

<u>scattering process</u>

ν scatters off e^-

80's — large water tanks with PMTs surrounding them

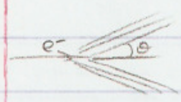
3000–8000 ton tanks waiting for p decay but detected ν



Cherenkov radiation e^- has high v (relativistic)

strong EM interactions

ring of PMTs light up — provides directional info



$\sin\theta = \frac{c}{nv}$ $V_{min} = \frac{c}{n} = 225{,}000$ km/s in H_2O

Kamiokande + IMB

SN1987a in LMC — 13 ν's arrived 1 day before light pulse (kept their lead for 200 kpc)

had 1 day head start

tight rest mass limit on ν

3 types of neutrino ν_e — normal matter

ν_μ, ν_τ > family mixing can occur (scattering with matter)

masses have to be comparable to get $\frac{1}{3}\nu_e, \frac{1}{3}\nu_\mu, \frac{1}{3}\nu_\tau$

SNO — heavy water tank

sensitive to all 3 families → confirmed mixing

small Δ masses between families

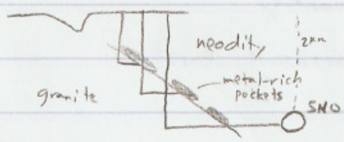
12/4/2007

Sudbury Neutrino Observatory

mine 6800 ft (2 km) deep Sudbury basin — impact site (2nd largest on Earth)

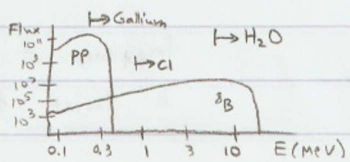
Diam ~ 250 km 1.85 billion yrs ago (paleoproterozoic)

magma rose through cracks rich in Ni, copper, Platinum, Gold



1000 tonnes D_2O spherical tank

9600 PMTs



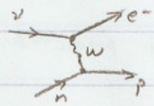
2 main reactions

Charged current reaction — flux of ν_e

$\nu_e + d \rightarrow p + p + e^-$

(e^-) leaves with most of E

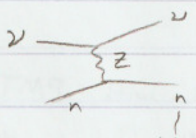
requires production of W particle



requires electron neutrino

Neutral current reaction — flux of all ν

$\nu + d \rightarrow p + n + \nu$



scattering

equal cross section for each of 3 atoms

liberated

add ^{35}Cl to tank $\quad n + {}^{35}Cl \rightarrow {}^{36}Cl \xrightarrow{\text{decay}} 4\gamma$'s — signature of neutral current

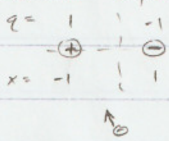
must remove U, Th from water

must have low radioactivity — neutron sources

sense about 30 events per day (each process)

Gravity Waves

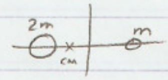
electric dipole moment $= \sum qx =$ EDM



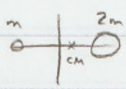
EDM $= -2$ $\qquad$ ⊖ ⊕ $= 2 =$ EDM

$$\text{luminosity} \propto \frac{d^2}{dt^2}(\text{EDM}) = \sum q \cdot \vec{a}$$

Mass dipole moment (gravitational) $\quad$ charge is mass



$\text{GDM} = \sum mx = m + 2m(-1) = -m$



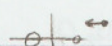
$\text{GDM} = 2m(1) + m(-1) = m$

Luminosity $\propto \sum ma = \sum$ forces $= 0$ for a closed system

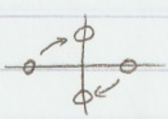
closed system $\quad \sum mx =$ center of mass $=$ fixed quantity

no net dipole radiation $\qquad$ higher order moments exist

need third mass m'

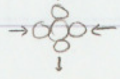


quadrupole radiation



frequency of quadrupole radiation is <u>twice</u> frequency of source

half full turn is back to original



distortion of source

only 1 charge, net force is always zero
because inertial mass equivalent to gravitational mass
→ no gravitational dipole radiation

gravity waves arise from extreme + nonlinear processes
no closed form terms

approximations for expected sources

<u>binary sources</u> (eccentricity helps)

$$L_g = \frac{2\times10^{-63}\, M_1^2 M_2^2 \left(1+30e^3\right)}{(M_1+M_2)^{2/3}\, \underbrace{P^{10/3}}_{\text{period}}} \quad [\text{Watts}]$$

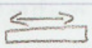
ex/ $M_1 = M_2 = 1.5\times10^{30}$ kg (neutron star) $e=0$ $P = 10^4$ s (2.78 hrs)

$L_g = 2\times10^{24}$ W ($L_\odot = 4\times10^{26}$ W)

$f = 2 f_{orbital} = 2\times10^{-4}$ Hz

flux = ? $F = \frac{L}{4\pi r^2}$ (actually not symmetric radiation)

if $r \sim 250$ pc $F = 3\times10^{-15}$ W/m²


1m

wave produces change in length: strain $= \frac{\Delta x}{x}$

strain $\simeq \frac{F(W/m^2)}{10^7}$ unitless

<u>rotating oblate spheroid</u>

$$L_g = \frac{G M^2 \omega^6 r^4 (A+1)^6 (A-1)^2}{64 c^5} \quad [\text{Watts}]$$

ω – angular rotation speed A – ratio of major to minor axis
r – radius of sources (geometric mean of radii)

ex/ millisecond pulsar P = 31 millisec ($\omega = 100$ rad/s)
$m = 1.5\times10^{30}$ kg $A = 0.99998$ $r = 15$ km
$L_g = 1.5\times10^{26}$ W freq = 30 Hz (not twice)
if dist = 1 kpc $F = 10^{-14}$ W/m² strain of 10^{-21} in 1m rod : 10^{-6} fm

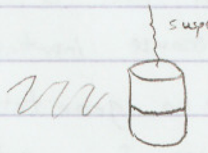
transient events may be 1000s stronger, but very rare

Weber 1969 first detector

1 ton cylinder of aluminum

looking for resonant oscillation

$f_{res} = 1660$ Hz



2 cylinders 1 km apart

strapped with piezo electric belts

$\rightarrow \boxed{PZ} \leftarrow$ change in $V \Rightarrow \Delta x$

or change in $\Delta x \Rightarrow$ voltage

1 event in 1969 $\frac{\Delta x}{x} = 10^{-16}$

cant confirm

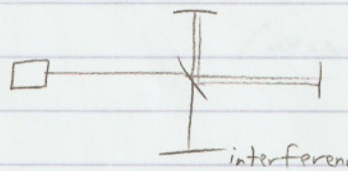
cooled versions become sensitive at 10^{-19} strain

might be possible to reach 10^{-21}

no repeatable detections

Laser Interferometric Systems

Michelson interferometer



great for quadropole detection

LIGO (4 km)

0.01 fm motions

multiple reflections make arms longer

average over many photons → lots of laser power

5 large systems (2 US, 2 Europe, 1 Australia)

Lousiana
Washington (state) > independent system

mirrors flat to ½ % of a wave

strains of order 10^{-21} have been achieved

— no gravity waves so far

Advanced Ligo

new coatings
more powerful laser
more massive mirrors

thermal lensing – heating of coatings
radiation pressure
laser stability
} sources of errors

LISA (scrapped) 3 spacecraft in equilateral triangle

20° behind earth's orbit

separation between spacecraft: 5,000,000 km

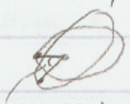
sensitive to known perturbations

Indirect Detection

binary pulsar system PSR 1913+16

orbital period: 7.75 hrs (2.8×10^4 s)

orbital precession: 4.2261 ± 0.0007 °/yr



precession of perihelion Mercury: 43" per century

rotational stability is 10^{12}

both objects have $m = 1.41 \pm 0.06\ M_\odot$

from orbital parameters: predict orbital period should slow down by -2.4×10^{-12} s/s (by grav radiation)

observed: $-2.30 \pm 0.22 \times 10^{-12}$ s/s

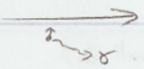
if one of sources was a star, then tidal torques could do it

no optical counterpart to this system

12/6/2007

Bremstrahlung

Cerenkov



— constructive interference since $v > \frac{c}{n}$

Friday 2–4pm

Dark Matter: WIMPS weakly interacting — weak force

Echelle Gratings

$$\lambda = \frac{\sigma(\sin\alpha + \sin\beta)}{m} \qquad \frac{d\lambda}{d\beta} = \frac{\sigma\cos\beta}{m} \qquad \frac{d\beta}{d\lambda} = \frac{m}{\sigma\cos\beta}$$

High Dispersion: large $\delta\beta$ for small $\delta\lambda$

echelle grating is used at large m → large angles

large blaze angle $> 45°$

ex/ $\frac{1}{\sigma} = 25.46\ \text{mm}^{-1}$ $\qquad \delta = 63°$ $\qquad \lambda_{blaze} = 2\sigma\sin(\delta)$ $= 70\ \mu m$ — 1st order λ for Littrow config.

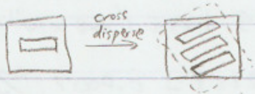
$R = mN \qquad N = \frac{L}{\sigma} = 25.46\,(L)$

at $1\ \mu m$ $\quad m = 70 \quad R = 1782 \cdot L \quad$ if $L = 120$ mm $\quad R_{diff} = 213{,}840$ very high

λ (70th order in Littrow) = 1 μm $\lambda(71) = 0.986 \mu m$ $\lambda(69) = 1.014 \mu m$

very strong order overlap

→ use cross disperser

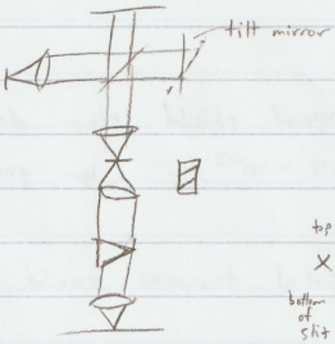


← rotate detector to have them horizontal

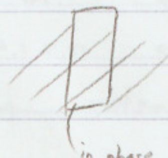
at angle – longer λ slightly more dispersed than shorter

Externally Dispersed Interferometer

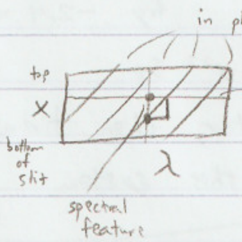
LLNL David Erskine 1997



so fringes tilt



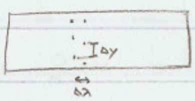
phase shift that depends on focal plane position



can see if feature has shifted slightly (planet detection)

boosts effective dispersion by factor of 6

(bright at



→ large Δy for small $\Delta\lambda$

1999 – Erskine · 12 m/s accuracy measured lunar orbit around Earth 4 m/s

Jian Ge – KPNO – 51 Peg

1st detection of new star 2.1 m at KPNO

Sloan – EDI spectrograph w/ fiber feeds started survey March 2006

Data Reduction

array of pixels i,j locations $t = t_{int}$ (integration time)

$$V_o(i,j,t) = \underbrace{B(i,j)}_{\text{bias}} + \underbrace{D(i,j)}_{\text{dark current}} \cdot t + t \cdot \underbrace{G(i,j)}_{\text{gain}} \times \Big(\underbrace{O(i,j)}_{\text{object}} + \underbrace{S}_{\text{sky}}\Big)$$

sky uniform accross angles of interest

bias frame

— short dark exposure $V_b(i,j,t) \simeq B(i,j)$

dark frame

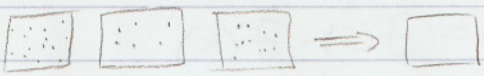
— long dark exposure $V_o - V_d = t \cdot G(i,j) \cdot \big(O(i,j) + S\big)$

$V_d = B + D \cdot t$

sky frame

— blank piece of sky $V_s = B(i,j) + D(i,j) \cdot t + t \cdot G(i,j) \cdot S$

$V_o - V_s = t \cdot G(i,j) \cdot O(i,j)$



different fields

super sky combination of frame but masking sources

noise improved by $\sqrt{N}$

ideally sky is much higher S/N than individual object frames

Gain $G(i,j)$ includes detector and optics

└ gain measurements needs photons that ideally came from ∞

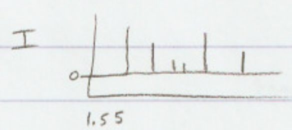
$V_s - V_D = t \cdot S \cdot G(i,j)$

IR μm^+ – sky is bright enough

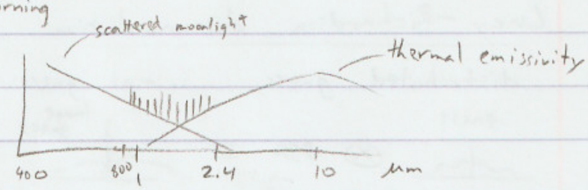
supersky ⟹ flat field — gain as function of position

dome flat — dramatically out of focus object (closed dome)

twilight flats — done in evening or morning



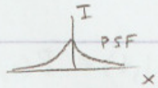
← sky OH lines



near IR – line dominated background

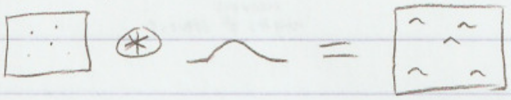
Deconvolution

convolution   look at star  Gaussian profile

Point Spread Function 

all stars have same PSF

map of point sources



Σ every pixel times overlapping pixels  do for each pixel

PSF, kernel, Green's function
all same thing

telescope, diffraction, atm, detector

Deconvolving : have image, kernel $\rightarrow$ want source function

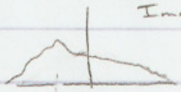
$$S \circledast K \Rightarrow I \qquad \mathcal{F}(S \circledast K) = \mathcal{F}(S) \times \mathcal{F}(K)$$

$$\therefore \mathcal{F}(I) = \mathcal{F}(S) \cdot \mathcal{F}(K) \qquad \therefore \mathcal{F}(S) = \frac{\mathcal{F}(I)}{\mathcal{F}(K)} \qquad S = \mathcal{F}^{-1}\left(\frac{\mathcal{F}(I)}{\mathcal{F}(K)}\right)$$

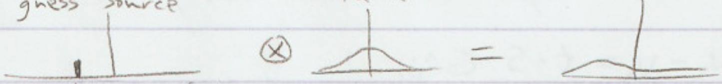
noise kills this
virtually never done

Clean algorithm

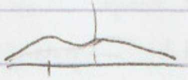
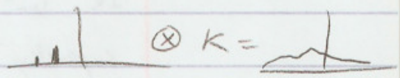
Image



guess source   kernel



small fraction of necessary flux

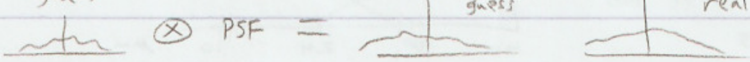
Image − Convolved guess = residual  new guess 

repeat until only noise is left

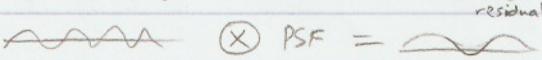
Lucy-Richardson deconvolution

distributed guess   initial guess can be the image

guess   image guess   real image



real − image guess = residuals   blurred residual



which pixels to blame for this?

add blurred residual to guess for next guess

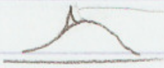
and repeat

seeing limited image

PSF – Gaussian   no high spatial freq. info $\mathcal{F}$(Gaussian) = Gaussian

deconvolving on seeing is useless

deconvolution works well with some high resolution power



PSF of Hubble   7% of light in diffraction limited core

For 3 yrs all HST data was deconvolved (Lucy-Richardson)

get PSF from stars (isolated are best)