Cosmology

4/2/2007

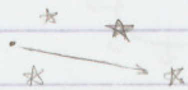
1963 There are only 2 ½ facts in cosmology.

1 — The sky is dark at night ⇒ implies boundary in space and time

2 — There is redshift $\propto$ distance

2½ — Universe appears to be evolving

Olber's paradox



sky should be as bright as sun

Distances + radial velocities (Hubble)

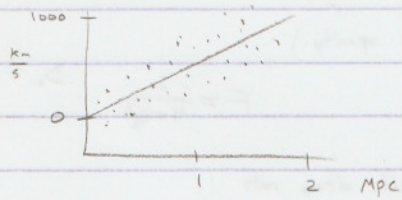
$v_r = \frac{dD}{dt} > 0$ (most) → moving away

$v_r = Kr + \underbrace{X\cos\alpha\cos\delta + Y\sin\alpha\cos\delta + Z\sin\delta}_{\text{our motion (negative peculiar velocity)}}$

α — R.A. δ — dec

$(X, Y, Z)\cdot\hat{n}$

Hubble: $\simeq 550 \frac{km}{s \cdot Mpc}$



a nearby 12th mag $v_r = 3379 \frac{km}{s}$ took ~week to get

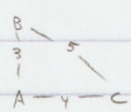
Lundmark law $v_r \sim Ar^2$ Hubble $v_r = Hr$ could be $\vec{v} = H\vec{r}$ cant measure transverse motions yet

→ isomorphic (maintains form under rotation and translation)

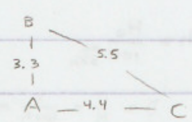
$\vec{v}' = R\vec{v}$ $\vec{r}' = R\vec{r}$ $\vec{v}' = H\vec{r}'$

└ rotation matrix

galaxy A $\vec{r}' = \vec{r} - \vec{r}_A$ $\vec{v}' = \vec{v} - \vec{v}_A$ $\vec{v}' = H\vec{r}'$



expand



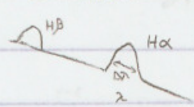
everything expands by same factor

our S.S. ~370 $\frac{km}{s}$ local group ~600 km/s (relative to the universe)

Supernovae

S And — in Andromeda, thought to be a nova 6th mag so Andromeda G close by ?!

Type I Type II — H lines :



$\lambda = (1+z)\lambda_{H_\alpha}$ └ 6563Å $\frac{\Delta\lambda}{\lambda} = 0.1$ for SN

└ no H lines

$V_{SN} \sim$ 10,000 — 15,000 km/s

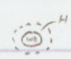
Type II

core collapse of Red S.G.

core ~WD → NS

Type I

Novae:



H layer around WD slowly building up → explodes and repeats

$I_{b,c}$ — no H lines when observed

Blue SG ◎ thin/no layer of H (H has been ejected previously)

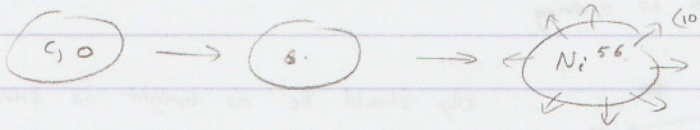
Ia $M \to 1.4 M_\odot$ Chandrasekhar limit $\to$ supernova from WD

might be off center ignition

detonation — shockwave, supersonic

deflagation — like burning paper

WD burn like deflagation, but might have off center detonation

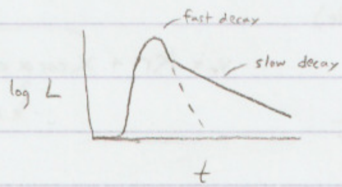


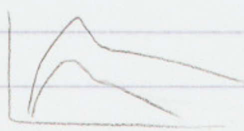
radiation just expands it (no light for us)

$Ni^{56} \to Co^{56} \to Fe^{56}$ radiation from radioactive decay is what we see

~9 d ~77 d

exponential decay

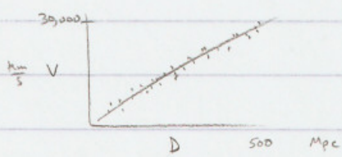




bright (more Ni) $\to$ slow (more opacity) ... by metals

faint $\to$ fast

$F = \frac{L}{4\pi D_L^2}$ D_L - luminosity distance



$\mathcal{M}$ magnitude at fiducial decay rate

$\log v = a(m - \mathcal{M}) + b$ (abs mag; apparent mag) $a = 0.2010 \pm 0.0035$

$v \propto r$ $\Delta \log v = 1 \to 10\times$ in $r \to \frac{1}{100}$ in $F \to 5$ mag $\to \frac{1}{5} = 0.2$

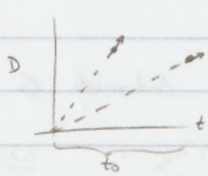
$v \sim A r^2$ $(v - v_A) \sim A(r - r_A)^2$ not isomorphic

HST Key Project $72 \pm 8 \frac{km}{s\ Mpc}$ from WMAP (assuming flat) 71 ± 3.5

Sandage 50 de Vaucouleurs 100 $h = \frac{H_0}{100 \frac{km}{s\ Mpc}}$

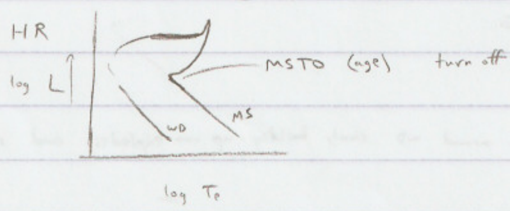
$V_r = 1500 \frac{km}{s}$ $D = 15\ h^{-1}$ Mpc $v = HD$ $D = \frac{v}{H} = \frac{v}{100h}$ Mpc

$[H] = \frac{1}{t}$ $100 \frac{km}{s \cdot Mpc} = \frac{10^7\ cm/s}{3.085678 \times 10^{24} cm} = 3.2 \times 10^{-18}\ s^{-1} = \frac{1}{9.78\ Gyr}$



$t_0 = \frac{1}{H_0}$ $H_0 t_0 \sim 1$ What is t_0?

Globular Cluster (old) $L \sim M^{4}$ (3-4) $\frac{M}{L} \sim M^{-3} \sim t \sim L^{-3/4}$



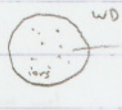
Suppose all distances are 10% higher

$L_{MSTO} \to +20\%$ $\quad t_{MSTO} \to -15\%$ $\quad$ by $t \sim L^{-3/4}$

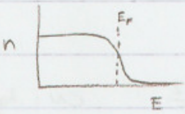
$H_0 \to -10\%$ $\quad \therefore \quad H_0 t_0 \to -25\%$

$t_{MSTO} \sim \cancel{18\,Gyr} \to 11.7 \pm 1.4$ Gyr $\quad t_0 \sim 12.2 \pm 1.5$ Gyr

using oldest WD in GC $\quad 12.7 \pm 0.7$ Gyr $\quad t_0 \sim 13.2$ Gyr



degenerate e^-



ions ~ gas in sea of denerate e^- $\quad$ high specific heat

solid with lattice — low specific heat

after crystallization — WD cools very rapidly

Solid objects — radioactive decay $\quad$ radiogenic

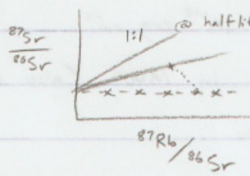
non radiogenic

$^{87}Rb \to {}^{87}Sr$

^{86}Sr

like K $\quad$ like Ca

chemically seporated



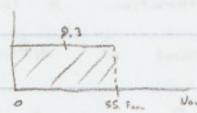
Allende metiorite $\quad 4.554$ Gyr

Earth $\quad 3.8$ Gyr (oldest rocks)

Moon $\quad 4.2$ Gyr

Age of elements

$^{187}Re \quad {}^{187}Os$ $\quad$ using production ratios $\quad 9.3 \pm 1.5$ Gyr age in Earth



$$\frac{t_0 + \overset{4.554}{t_{SS}}}{2} = 9.3 \qquad t_0 = 14 \pm 3 \text{ Gyr}$$

Elem. 14 ± 3
WD 13.2 ± 0.7
TO 12.2 ± 1.5
$\Big\} \sim 13.2$

$H_0 \simeq 72 \pm 8$ (11%) $\quad 950$ Gyr $\frac{km}{s \cdot Mpc}$

$$H_0 t_0 = \frac{950}{978} = 0.97 \pm 0.13$$

WMAP $\quad 71 \pm 3.5 \ \frac{km}{s \cdot Mpc}$

13.7 ± 0.2 Gyr

4/4/2007

Number counts

$N = \frac{4\pi}{3} \rho D^3$ (number density) $\quad$ flux limit $S = \frac{L}{4\pi D^2}$ $\quad D = \left(\frac{L}{4\pi S}\right)^{1/2}$ $\quad N \propto S^{-3/2}$

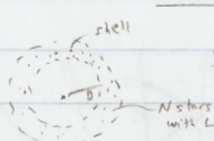
$\log N = -\frac{3}{2} \log S + \text{const}$ $\quad \therefore \quad \boxed{\frac{d \log N}{d \log S} = -\frac{3}{2}}$

$\tau_v \simeq 0.1$ $\qquad \tau(b) = \frac{\tau(v)}{\sin b}$

"zone of avoidance" MW dust extinction

cosmological principle: universe is homogeneous + isotropic

olbers' Paradox



$$S_*(\text{shell}) = \frac{L_*}{4\pi D^2}(4\pi D^2\, dD) N = L_* N\, dD \qquad \int_0^\infty L_* N\, dD = \infty$$

infinite flux

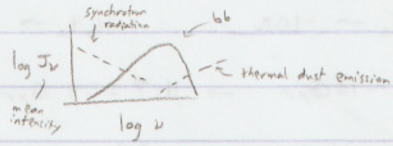
Steady state universe $\quad$ perfect cosmological principle $\quad$ Univ. doesn't change with time

→ number counts (of say quasars) vary with redshift disproving steady state

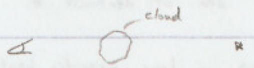
Cosmic Microwave Background

1965 Penzias + Wilson

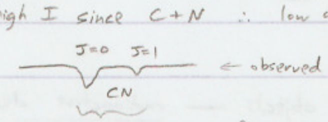
① Incredibly good blackbody 2.7 K



ignoring/masking out bright sources
like galactic plane, ecliptic, etc. → sources of synchrotron, thermal dust



CN, CH, CH^+ $\hbar\omega = \frac{I\omega^2}{2}$ $\omega = \frac{\hbar 2}{I}$ CN: high I since C + N ∴ low ω



CN absorbs photons from CMB

could be used to see how CMB evolves with redshift close to peak of CMB bb

energy density $u_\nu = \frac{4\pi}{c} J_\nu$ $\int u_\nu d\nu \sim 10^{-12}$ erg cm^{-3} $u = \frac{4\sigma}{c} T^4$

close to energy density of stars in MW (also cosmic rays)
say $1 L_\odot$ in 3pc

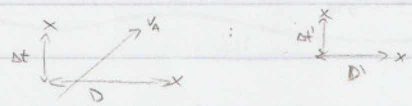
Supernova duration is longer at higher redshift → time dilation

QM is same at different redshift ← assumed some people are looking for variations of fine structure constant

G = constant? as far as we can tell, yes

4/9/2007

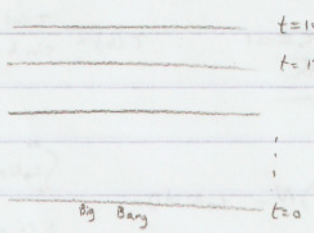
Spacetime



Distance – spatial separation at a common time

$V_r = HD$ $\vec{V} = H\vec{r} - \vec{V}_{obs}$ observer in motion wont see Hubble law since in motion

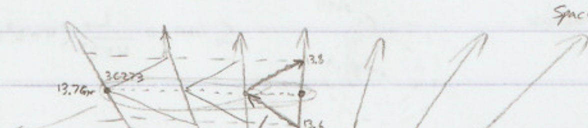
comoving observers observe Hubble law $V_r = HD$ + get age of Univ.



Homogeneous + Isotropic

density is ~constant along time slices

Spacetime in G.R.



light ray ∴ distance is 100 Mlyr

distance to 3C273 @ 13.7 Gyr
is sum of each steps

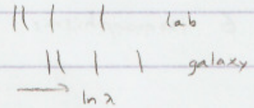
$$\frac{d D_{AB}(t)}{dt} = H(t) D_{AB}$$ — distance between A + B

$$D_{xy}(t) = \underbrace{a(t)}_{\text{scale factor}} D_{xy}(t_0)$$

$$\frac{d D_{xy}(t)}{dt} = \frac{da(t)}{dt} D_{xy}(t_0) = \frac{\dot a}{a} a D_{xy}(t_0) = \underbrace{\frac{\dot a}{a}}_{H(t)} D_{xy}(t)$$

$H(t) = \frac{\dot a}{a}$ logarithm of the time derv. of a

steady state $a(t) = e^{H(t-t_0)}$ for $H = \text{const}$



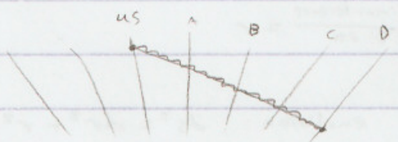
what we observe is redshift

$$\frac{\lambda_{obs}}{\lambda_{em}} = 1 + z \qquad -1 < z < \infty$$

blueshifts

$$\cancel{1+z = \sqrt{\frac{1+v/c}{1-v/c}}}$$

as we'll show later



$$\frac{\lambda_{us}}{\lambda_D} = \frac{\lambda_{us}}{\lambda_A}\frac{\lambda_A}{\lambda_B}\frac{\lambda_B}{\lambda_C}\frac{\lambda_C}{\lambda_D}$$

no long steps $A \to B \to C \to D$

$$\frac{\lambda_{us}}{\lambda_A} = 1+z \simeq 1 + \frac{v}{c} + \dots = 1 + \frac{H D_{usA}(t_A)}{c} = 1 + H(t_{us} - t_A) + \dots = \frac{a(t_{us})}{a(t_A)}(1 + \dots)$$

$\frac{D}{c} = t - t_0$ ↗

quadratic terms

$$\frac{\lambda_{us}}{\lambda_D} = \frac{a(t_{us})}{a(t_{em})}\left(1 + N \ddot a \left(\frac{t_{us} - t_{em}}{N}\right)^2\right) \longrightarrow N \to \infty$$

so quadratic terms drop

$$\boxed{\frac{\lambda_{obs}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})}}$$ cosmological redshift

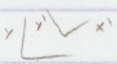
including our velocity

$$\frac{\lambda_{obs}}{\lambda_{em}} = \underbrace{\frac{a(t_{obs})}{a(t_{em})}}_{\text{cosmological}} \underbrace{\left(1 - \frac{\vec v_{obs}\cdot \hat n}{c}\right)}_{\text{our motion}} \underbrace{\left(1 + \frac{\vec v_{em}\cdot\hat n}{c}\right)}_{\text{emitter motion}} \underbrace{\left(1 - \frac{\Delta\Phi_{em}}{c^2}\right)\left(1 + \frac{\Delta \Phi_{obs}}{c^2}\right)}_{\text{gravitational redshift of emitters / observer}}$$

Metric

$ds^2 = dx^2 + dy^2$ in matrix form $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{matrix} g_{xx} & g_{xy} \\ g_{yx} & g_{yy} \end{matrix}$

special relativity $\eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$



$x' = a + x\cos\theta - y\sin\theta \qquad y' = b + x\sin\theta + y\cos\theta$

$$dx'^2 + dy'^2 = dx^2\cos^2\theta + dx^2\sin^2\theta + dy^2(\cos^2\theta + \sin^2\theta) + dx\,dy(\cos\sin - \sin\cos) = dx^2 + dy^2$$

rotations leave metric in same form

$\left.\begin{matrix}\text{translations} \\ \text{rotations}\end{matrix}\right\}$ isomorphism

$ds^2 = dr^2 + r^2 d\theta^2$ same as $dx^2 + dy^2$ in flat space

$ds^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ this has curvature

sphere: surface of a ball
↳ not homogeneous / isotropic

$ds^2 = d\theta^2 + \sin^2\theta \, d\varphi^2 \longrightarrow x^2+y^2+z^2=1$ unit sphere

3 isomorphisms $\quad \overleftrightarrow{R}$ rotational matrix $\quad SO(3)$ special orthogonal group in 3-space
↳ not including parity

3-sphere $\quad x^2+y^2+z^2+w^2=1 \quad \vec{R} \quad SO(4) \quad 6$ rotations

Now into 3D — $ds^2 = \underbrace{d\psi^2 + \underbrace{\sin^2\psi}_{r^2} \overbrace{\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)}^{\text{celestial sphere}}}$ $\quad$ 6 isomorphisms

θ: polar distance $\quad \varphi$: right ascension

tangential distance along surface $\quad$ (ψ is radial distance)

$r = \sin\psi$

$\dfrac{\text{circumference}}{2\pi} = r$

$dr = \cos\psi \, d\psi \qquad d\psi = \dfrac{dr}{\cos\psi}$

$$ds^2 = \frac{dr^2}{1-r^2} + r^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

euclidian: $ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$
in spherical

general form for metric: $\quad ds^2 = \dfrac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$

$k = +1$ sphere S^3
$\;\;= 0$ flat E^3
$\;\;= -1$ hyperbolic H^3

or: $\quad ds^2 = d\psi^2 + \begin{pmatrix} \sin^2\psi \\ \psi^2 \\ \sinh^2\psi \end{pmatrix} \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$

for $H^3 \qquad x^2+y^2+z^2-w^2 = -1 \qquad$ to understand it, we can embed it in Minkowski space (not space time)

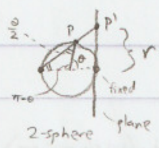
Lorentz transformations — $6 \begin{cases} 3 & \text{boost} \\ 3 & \text{rotations} \end{cases}$ $\qquad$ (saddle shape not correct, only use to show that around circle $\theta > 2\pi$)

w: extra made up dimension just to understand it

Geography maps

– conformal – preserves shape – Mercator
(not area)

$ds^2 = f(x,y,z)\left(dx^2+dy^2+dz^2\right)$



$\left.\right\} r = 2\tan\left(\frac{\theta}{2}\right)$
diameter

$dr = \cancel{2}\,\frac{1}{\cancel{2}} \sec^2\left(\frac{\theta}{2}\right) d\theta \qquad dr = \sec^2\left(\frac{\theta}{2}\right) d\theta$

$ds^2 = d\theta^2 + \sin^2\theta \, d\varphi^2$

$\dfrac{r^2 \, d\varphi^2}{\sec^4\frac{\theta}{2}} = \cos^4\frac{\theta}{2} \; 4\tan^2\frac{\theta}{2} \, d\varphi^2$

$= 4\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} \, d\varphi^2 = \left(\sin\theta \, d\varphi\right)^2$

$$ds^2 = \frac{dr^2}{\sec^4\frac{\theta}{2}} + \frac{r^2 \, d\varphi^2}{\sec^4\frac{\theta}{2}} = \frac{1}{\sec^4\frac{\theta}{2}} \underbrace{\left(dr^2 + r^2 \, d\varphi^2\right)}_{\text{plane in polar coords.}}$$

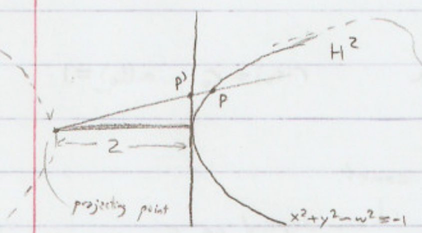
<u>conformal</u>

stereographic map

$\sec^2 x = 1 + \tan^2 x \qquad \left(\frac{r}{2}\right)^2 = \tan^2\frac{\theta}{2} \qquad 1 + \left(\frac{r^2}{2}\right) = \sec^2\frac{\theta}{2}$

$$ds^2 = \frac{1}{\left(1+\frac{r^2}{4}\right)^2}\left(dr^2 + r^2 d\varphi^2\right) = \frac{1}{\left(1+\frac{x^2+y^2}{4}\right)^2}\left(dx^2+dy^2\right)$$

stereographic metric $\qquad ds^2 = \dfrac{dx^2+dy^2+dz^2}{\left(1 + k\frac{(x^2+y^2+z^2)}{4}\right)^2}$



asimptote (0=45)
maximum $r \to 2$

sphere (finite) $\to$ infinite on plane

hyperboloid (infinite) $\to$ finite on plane

Metrics with spacetime

$$ds^2 = c^2 dt^2 - a(t)^2 R_0^2\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right)$$

$$d\eta = \frac{c\,dt}{a(t)} \qquad ds^2 = \underset{\hookrightarrow \eta}{a(t)^2}\left(d\eta^2 - R_0^2\left(d\psi^2 + S(\psi)^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right)\right)$$

conformal time η

light travels: $ds^2 = 0$

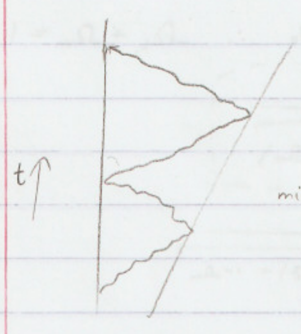
galaxy: fixed r, θ, φ (position) $\qquad d\varphi = d\theta = 0$

$\hookrightarrow D_p(t_0) \to R_0 r$

$$0 = c^2 dt^2 - a(t)^2 R_0^2 \frac{dr^2}{1-kr^2}$$

$$\int_{t_e}^{t_0} \frac{c\,dt}{a(t)} = R_0 \int_0^{r_s} \frac{dr}{\sqrt{1-kr^2}} \qquad \frac{\Delta t_0}{a(t_0)} = \frac{\Delta t_e}{a(t_e)} \qquad \boxed{\frac{\Delta t_0}{\Delta t_e} = \frac{a(t_0)}{a(t_e)} = \frac{\lambda_{obs}}{\lambda_{em}}}$$

measure $t_0 + 1$ 1 day later $\to$ get diff answer ?! $\to$ need to change t_e



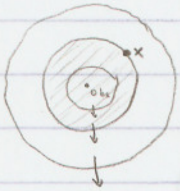
$\oint p\,dq =$ adiabatic invariant (action)

$\frac{h\nu}{c} = p \propto \frac{1}{a}$ size

$$\frac{\nu_{em}}{\nu_{obs}} = \frac{\lambda_{obs}}{\lambda_{em}} = 1 + z = \frac{a(t_0)}{a(t_e)}$$

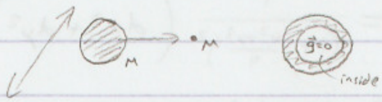
momentum scales as $\dfrac{1}{\text{scale factor}}$ (for anything)

4/11/2007



comoving particle

Newton's Iron Ball Theorem



no gravity within shell

Birkhoff's Theorem $\leftarrow \frac{v}{c} \ll 1 \iff$ Pressure $\ll \rho c^2$ $\therefore$ no pressure

cosmo: "Dust" $\rightarrow$ material with no pressure

$$\frac{1}{2}v^2 - \frac{GM}{r} = \text{const} \qquad r = r_0 a \qquad v = r_0 \dot{a} \qquad r(t_0) = r_0 \quad a(t_0) = 1$$

$$M = \frac{4\pi}{3}\rho_0 r_0^3$$

$$\frac{1}{2}r_0^2 \dot{a}^2 - G\frac{4\pi}{3}\rho_0 r_0^3 \frac{1}{r_0 a} = \text{const.}$$

can depend on r_0

$$r_0^2\left(\frac{\dot{a}^2}{2} - \frac{4\pi}{3}\rho_0\frac{G}{a}\right) = \text{const}' \times r_0^2$$

$$\dot{a}^2 = \frac{8\pi\rho_0 G}{3a} + \text{const}'' \qquad \text{const}''=0 \qquad \dot{a}^2 = \frac{8\pi G\rho_0}{3a}$$

$$\frac{da}{dt} = \sqrt{\frac{8\pi G\rho_0}{3}}\, a^{-1/2}$$

$$a^{1/2}\, da = \sqrt{\frac{8\pi G\rho_0}{3}}\, dt$$

$$\frac{2}{3}a^{3/2} = \sqrt{\frac{8\pi G\rho_0}{3}}\, t \qquad \boxed{a(t) = \left(\frac{t}{t_0}\right)^{2/3}}$$

"Dust"
const = 0
$\rightarrow$ critical density

$$\frac{1}{2}v^2 - \frac{GM}{r} = 0 \qquad \frac{1}{2}H_0^2 r_0^2 = \frac{4\pi}{3}G\rho_0 \cancel{r_0^3}\frac{1}{r_0} \qquad \boxed{\rho_{crit} = \frac{3H_0^2}{8\pi G}}$$

Critical Density

Dust only $\left.\begin{array}{l}\rho < \rho_c \\ \rho = \rho_c\end{array}\right\}$ expand forever $\qquad \rho < \rho_c \rightarrow$ negative curvature H^3

$\rho > \rho_c$ — recollapse $\rightarrow$ positive S^3 $\qquad \rho = \rho_c \rightarrow$ zero curvature E^3

omega curvature

$$\frac{1}{2}\dot{a}^2 = \frac{4\pi}{3}\frac{G\rho_0}{a} + \text{const} \qquad \dot{a}^2 = \frac{8\pi G\rho_0}{3a}\frac{H_0^2}{H_0^2} + \text{const}' = H_0^2\left(\frac{\rho_0}{\rho_{crit}}\frac{1}{a} + \Omega_k\right)$$

$$\Omega = \frac{\rho}{\rho_{crit}} \qquad \Omega_m = \frac{\rho_0}{\rho_{crit}}$$ "matter" $\iff$ "dust"

$$\dot{a}^2 = H_0^2\left(\Omega_m\frac{1}{a} + \Omega_k\right) \qquad t = t_0 \quad a(t_0) = 1 \quad \dot{a}(t_0) = H_0 \quad \therefore \quad \Omega_k + \Omega_m = 1$$

$$t_0 = \int dt = \int \frac{da}{\left(\frac{da}{dt}\right)} = \int_0^1 \frac{da}{\dot{a}} \qquad t_0 = \frac{1}{H_0}\int_0^1 \frac{da}{\sqrt{\frac{\Omega_m}{a} + (1-\Omega_m)}}$$

$$a = \frac{1}{1+z} \qquad da = -\frac{dz}{(1+z)^2} \qquad H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)^2\sqrt{\Omega_m(1+z) + 1 - \Omega_m}}$$

$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z^2)\sqrt{1+\Omega_m z}}$$

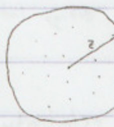
$\rho_{crit} = \dfrac{3H^2}{8\pi G}$ $\quad H = 100\ h\ \dfrac{km}{s \cdot Mpc}$ $\quad 1\,Mpc = 3.085678 \times 10^{24}\ cm$

10^{-24} = yocto-

$\rho_{crit} = \dfrac{3}{8\pi\left(\frac{2}{3}\times 10^{-7}\right)}\left(\dfrac{10^7}{3.08\times 10^{24}}\right)^2 h^2 = 1.88\times 10^{-29}\ h^2\ g/cm^3$ $\quad$ 18.8 yoctogram/m^3

$11.2\ h^2$ H atoms/m^3 $\quad \rho_{crit} \sim 5-6$ H/m^3 $\quad \rho_{crit} = 2.8\ h^2 \times 10^{11}\ M_\odot/Mpc^3$

$\rho c^2 = 1.88\times 10^{-29}\ \ 9\times 10^{20}\ \ 6\times 10^{11}$ $\quad \rho_{crit} c^2 = 10539\ h^2\ eV/cm^3$



Z_{lim} $\quad N_{gal}$ $\quad F_{gal}$

$L_{gal} \sim F_{gal}\ 4\pi\underbrace{\left(\dfrac{cz}{H_0}\right)^2}_{d^2}$ $\quad N_{gal} L_{gal} \sim \left(\dfrac{c Z_{lim}}{H_0}\right)^2 4\pi F_{gal} N_{gal}$

$Vol = \dfrac{4\pi}{3}\left(\dfrac{cz}{H_0}\right)^3$ $\quad$ luminosity density $\quad \underset{\text{emissivity}}{4\pi j} \sim \dfrac{N_{gal} L_{gal}}{Vol} \simeq \dfrac{N_{gal} F_{gal}}{c Z_{lim}} H_0$

$4\pi j_v \simeq (1.6 \pm 0.2)\ 10^8\ h\ L_\odot/Mpc$

$\Omega_{lum} = \dfrac{M/L}{M_\odot/L_\odot} \dfrac{1}{1750\ h}$

solar neighborhood stars

$M/L \sim 3.3$ $\quad \Omega_{lum} \simeq \dfrac{3.3}{1200} \simeq 0.25\%$ $\quad$ also $\Omega_{baryons} \simeq 4\%$

Suppose $\Omega_m = 2$ $\quad$ from Kepler's : $t = A(\underset{\text{excentric anomaly}}{E} - \sin E)$ $\quad a = B(\underset{e=1}{1} - \cos E)$

$dt = A(1-\cos E)\,dE$ $\quad da = B \sin E\ dE$

$\dot{a} = \dfrac{da/dE}{dt/dE} = \dfrac{B \sin E}{A(1-\cos E)}$ $\quad \ddot{a} = \dfrac{-B}{A^2}\dfrac{1}{(1-\cos E)^2}$

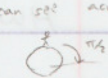
deceleration parameter $q = -\dfrac{a\ddot{a}}{\dot{a}^2}$

$g = -\dfrac{GM}{r^2}$ $\quad g = r_0 \ddot{a}$ ← acceleration

$\dfrac{4\pi}{3}\dfrac{G\rho_0 r_0^3}{r_0^2 a^2} = r_0 \ddot{a}$ $\quad \ddot{a} = \dfrac{4\pi G \rho_0}{3a^2}$

$q = \dfrac{1}{2}\Omega_m$

angle you can see across univ.



can't see multiple images

$q = \dfrac{1}{1+\cos E}$ $\quad \Omega_m = 2 \to q = 1 \to \cos E \to 0 \to E = \dfrac{\pi}{2}$

$t_0 = A\left(\dfrac{\pi}{2} - 1\right)$ $\quad t_{BC} = A(2\pi)$ (big crunch) $\quad E = 2\pi$ big crunch $\quad \dfrac{t_{BC}}{t_0} = \dfrac{2\pi}{0.57} \simeq 11$

Now add pressure

$$\frac{P}{\rho c^2} = \text{Equation of state (actually } P(\rho) \text{ is equation)} = w \qquad \frac{P}{\rho c^2} = w$$

$w=0$ "Dust"

radiation $P_{rad} = \frac{1}{3}\rho c^2$ $\quad w = \frac{1}{3}$

expand – do work → lose energy

Work $= P\,dV$ $\quad P = w\rho c^2$

$dV = 4\pi r^2 dr$ $\quad V = \frac{4\pi}{3} r^3$ $\quad E = V\rho c^2 = \frac{4\pi}{3}\rho c^2 r^3$

$W = w\rho c^2 4\pi r^2 dr$ $\quad \frac{W}{E} = 3w\frac{dr}{r}$ $\quad \frac{dE}{E} = -3w\frac{dr}{r}$ energy lost by expanding

$E \sim \underset{\text{mass}}{M} \sim a^{-3w}$

$r_0\ddot{a} = -\frac{GM}{r_0^2 a^2}$ this no longer applies in general relativity

$\frac{1}{2}r_0^2\dot{a}^2 - \frac{GM}{r_0 a} = \text{const}$ still applies

Acceleration equation $\boxed{\ddot{r} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) r}$ $\quad q = \frac{1}{2}\frac{\rho + \frac{3P}{c^2}}{\rho_{crit}}$

trace of stress energy tensor

"pressure has weight"

$q = \frac{1}{2}\Omega\left(1 + \frac{3P}{\rho c^2}\right) = \frac{1}{2}\Omega(1+3w)$

$\dot{a} = H_0\sqrt{\Omega_{k0} + \frac{\Omega_{m0}}{a} + \frac{\Omega_{r0}}{a^2}}$ $\quad \frac{GM}{r} \sim \frac{a^{-3w}}{a} \sim \frac{1}{a^2}$ for $w = \frac{1}{3}$

Radiation dominated, flat universe $\quad \Omega_r = 1 \quad \Omega_k = \Omega_m = 0$

$\dot{a} \sim \frac{1}{a}$ $\quad a\,da \sim dt$ $\quad a^2 \sim t$ $\quad \boxed{a(t) = \left(\frac{t}{t_0}\right)^{1/2}}$ valid for early universe $\lesssim 10^4$ yrs

@ z_{eq} $\quad \Omega_m = \Omega_r$ $\quad z_{eq} \sim 3300$

$\Omega_k = 1$ $\quad \Omega_m = \Omega_r = 0$ $\quad \boxed{a(t) = \left(\frac{t}{t_0}\right)}$ Curvature only

---

HW 2

$$\chi^2 = \sum_{i=1}^{2}\left(\frac{\ln\left(\frac{x_i/x_{23?}}{f(t)}\right)}{0.25}\right)^2$$

n data points

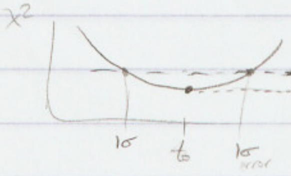
m parameters

@ the correct parameter value

χ^2 is distributed like χ^2 with n degrees of freedom

$\Delta\chi^2 = \chi^2_{true} - \min\chi^2$ is dist like χ^2 with m degrees of freedom (df)

$\{p\}$ set of parameters that minimize χ^2

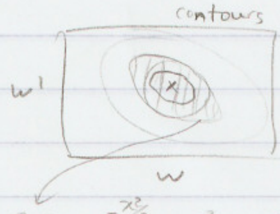


$\Delta\chi^2 = 1$ for 1 degree of freedom

($=4$ for 2σ error)

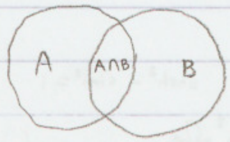
contours of χ^2

2 df



$\Delta\chi^2 = 6$ $\quad e^{-\chi^2/2} = e^{-3} \sim 5\%$ 95% confidence interval

4/16/2007



Bayes' Theorem

$$P(A\cap B) = \underbrace{P(A|B)}_{\text{prob. of A given B}}\ \underbrace{P(B)}_{\text{prob. of B}}$$
$$= P(B|A)\,P(A)$$

A = prior distribution of the prob of parameters $\{p\}$ $\quad P(A)$ = "the prior"

B = "the data"

$\underbrace{P(B|A)}_{\text{(likelihood)}}\ \underbrace{P(A)}_{\text{(prior)}}$ → apostoriori

$$\prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-(x_i - f_i\{p\})^2/\sigma_i^2}$$

$P(A)$ is of the same form (if it is from prior experiment)

nuisance parameters → (noises)

likelihood $L(\{p\},\{n\})$

$$\hat{L}(\{p\}) = \max_{\{n\}} L(\{p\},\{n\})$$

$$\hat{L}(\{p\}) = \int L(\{p\},\{n\})\,d\{n\}$$ not correct

need to include prior $f_p\{n\}$ $\therefore \int L(p,n) f(n)\,dn$

Density + Pressure

$$w = \frac{p}{\rho c^2} \qquad w=0 \text{ dust} \qquad w=\tfrac{1}{3} \text{ photons} \qquad w=-1$$

$$G_{\mu\nu} - \underbrace{\Lambda}_{\text{cosmological constant}} g_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$T_{\mu\nu} = \begin{pmatrix} \rho c^2 & & 0 \\ & p & \\ 0 & & p \\ & & & p \end{pmatrix}$$ isotropic (no shear, directions)

$$ds^2 = c^2dt^2 - dx^2 \ldots = c^2dt'^2 - dx'^2 - \ldots$$

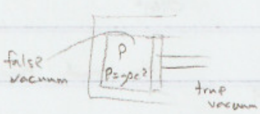
same metric under Lorentz transformation

$g_{\mu\nu}$ is Lorentz invariant ($g=0$ is one case)

can't tell how fast you are moving relative to the vacuum

using $w=-1$ Lorentz transformation does not change it

ρ = const under expansion



Do work $= (\rho c^2)\Delta V$

$w=-1$ false vacuum remains false vacuum under expansion

$$\rho \sim \rho_0 a^{-3(1+w)} \xrightarrow{w=-1} \rho = \rho_0 \text{ constant vacuum energy}$$

$$M \sim \rho a^3 \sim \rho_0 a_0^3 a^{-3w}$$

$$\frac{GM}{a} \sim \frac{a^3}{a} \overset{w=-1}{\sim} a^2$$

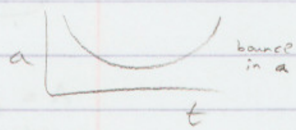
$$\frac{1}{2}\dot{a}^2 = \frac{\text{"}GM\text{"}}{a} + \text{const} \qquad \dot{a}^2 = H_0^2\left(\Omega_\Lambda a^2 + (1-\Omega_\Lambda)\right)$$

$\dot{a}^2 = H_0^2 a^2 \qquad a \propto e^{\pm H_0 t}$

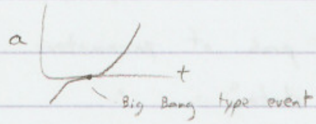
$H = H_0 \sqrt{\Omega_\Lambda}$

$\cosh^2 - \sinh^2 = 1$

$a \sim \cosh(Ht)$ is a solution $\longrightarrow \quad \dot{a}^2 = H^2 \sinh(Ht)^2 = H_0^2 (\Omega_\Lambda \cosh^2(Ht)) - H_0^2 \Omega_\Lambda$



with $a \sim \sinh(Ht)$



$H_0 \sqrt{\Omega_\Lambda}$

$H = 71 \times \sqrt{0.73}$

$t_{exp} \sim \frac{978 \text{ Gyr}}{\sim 58}$ e-folding timescale (?)

$$\boxed{\dot{a}^2 = H_0^2 \left(\frac{\Omega_m}{a} + \frac{\Omega_r}{a^2} + \Omega_\Lambda a^2 + \Omega_k \right)}$$

a^{-1-3w} ; curvature: $1 - \Omega_m - \Omega_r - \Omega_\Lambda$

$$\dot{a}^2 = H_0^2 a^2 \underbrace{\left(\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda + \frac{\Omega_k}{a^2} \right)}_{\text{now these are densities}}$$

$\Omega_{m,0} = \frac{\rho_{m,0}}{\rho_{crit,0}} \qquad \frac{\rho_{m,0}}{a^3} = \rho_m$

$$\dot{a}^2 = H_0^2 \frac{1}{\rho_{crit,0}} a^2 \left(\rho_m + \rho_r + \rho_{vac} + \rho_{curv} \right) \qquad \rho_{crit,0} = \frac{3H_0^2}{8\pi G}$$

$$\dot{a}^2 = 8\pi G a^2 \left(\rho_m + \rho_r + \rho_{vac} + \rho_{curv} \right) \qquad \frac{\dot{a}}{a} = H \qquad H^2 = 8\pi G \left(\rho_m + \rho_r + \rho_{vac} + \rho_{curv} \right)$$

$$t_0 = \int_0^1 \frac{da}{\dot{a}} = \frac{1}{H_0} \int_0^1 \frac{da}{\left(\Omega_m/a + \Omega_r/a^2 + \Omega_\Lambda a^2 + \Omega_k \right)^{1/2}}$$

$$\int \frac{da}{Ha} \qquad da = d\left(\frac{1}{1+z}\right) = \frac{1}{(1+z)^2} dz \qquad \int \frac{dz}{(1+z)^2 \frac{1}{1+z} H(z)} = \int_0^\infty \frac{dz}{(1+z) H(z)} = t_0$$

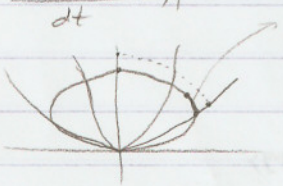
Observing at multiple z

z

$4R_0 \qquad t_e \qquad t_{obs} = t_0 + \Delta t_{obs} \qquad 1 + z = \frac{a(t_0)}{a(t_e)}$

what is $\frac{dt_e}{dz}$? $\qquad d(1+z) = \frac{a(t_0)}{a(t_0)^2} da(t_e) \qquad \frac{dz}{dt_e} = (1+z)^2 \frac{da}{dt_e}$

$\frac{d \ln(1+z)}{dt} = H \qquad \frac{dt}{dz} = \frac{1}{(1+z)H}$ How much time passes between an interval of z



$$\int_{t_e}^{t_{obs}} \frac{c\,dt}{a} = 4R_0$$

$a(t_e) = a(t_e)(1 + H(t_e)\Delta t + \dots)$

$a(t_0) = a(t_0)(1 + H(t_{obs})\Delta t + \dots)$

$$\frac{c}{a_e}\left(\Delta t_e - \frac{1}{2} H(t_e) \Delta t_e^2 + \dots\right) = \frac{c}{a_0}\left(\Delta t_{obs} - \frac{1}{2} H(t_{obs}) \Delta t_{obs}^2\right)$$

$$\Delta t_e = \frac{a_e}{a_0}\left(\Delta t_{obs} - \frac{1}{2} H(t_0) \Delta t_{obs}^2\right) + \frac{1}{2} H(t_e) \underbrace{\left(\frac{a_e}{a_0} \Delta t_{obs}\right)^2}_{\sim \Delta t_e^2} \qquad 1 + z = \frac{d\Delta t_0}{d\Delta t_e}$$

$$1+z = \frac{a_0/a_e}{1-\left[H_0 + \frac{a_e}{a_0}H(t_e)\right]\Delta t_0} = \frac{a_0}{a_e} + \left[\frac{a_0}{a_e}H(t_0) - H(t_e)\right]\Delta t_0$$

$$\frac{d(1+z)}{dt} = (1+z)H_0 - H(z)$$

$\Omega = 0 \quad a = t/t_0 \quad H = \frac{1}{t} = (1+z)H_0 \quad \therefore \frac{d(1+z)}{dt_0} = 0$ redshift doesn't change

$\Omega_m = 1 \quad H(z) = (1+z)^{3/2} H_0 \quad \frac{d(1+z)}{dt_0} = H_0\left(1+z - (1+z)^{3/2}\right)$

ex: $z = 3 \quad \frac{d(1+z)}{dt_0} = -4H_0$

$\Omega_\Lambda = 1 \quad H = \text{const} \quad \frac{d(1+z)}{dt_0} = zH$

4/18/2007

Flatness – Oldness Problem

$E_{tot} = \frac{1}{2}v^2 + \frac{GM}{r} = \text{const} \quad v^2 - \frac{8\pi G \rho R^2}{3} = H^2R^2 - \frac{8\pi G\rho R^2}{3} = \text{const}$

$\rho_{crit} = \frac{3H^2}{8\pi G} \quad * \left(\frac{3}{8\pi G}\right) \Rightarrow \rho_{crit}R^2 - \rho R^2 = \text{const} \quad R^2(\rho_{crit} - \rho) = \text{const}$

$\rho a^2\left(\frac{\rho_{crit}}{\rho} - 1\right) = \text{const} \quad \rho a^2(\Omega^{-1} - 1) = \text{const}$

matter dominated $\rho \sim a^{-3}$ radiation dominated $\rho \sim a^{-4}$

$\rho_0 \sim 9\times10^{-30}\ \text{gm/cc} \quad a = 1$ now $\quad \Omega_m \sim 0.3 \quad \Omega_\Lambda \sim 0.7 \quad \rho_{crit} = 10539 h^2\ \text{eV/cc}$

$u = aT^4 \quad a = \frac{4\sigma}{c} \quad \int B_\nu d\nu = \frac{\sigma}{\pi}T^4 \quad a = \frac{4 \cdot 5.67\times10^{-5}}{3\times10^{10}} \simeq 7\times10^{-15}$

$T_0 = 2.72528 \pm 0.00068\,K \quad u \simeq 4\times10^{-13}\ \text{ergs/cc} = 0.24\ \text{eV/cc}$

$\Omega_\gamma h^2 \simeq 2.3\times10^{-5}$

$\Omega_r h^2 = 4.165\times10^{-5}$ (including neutrinos) (we know Ωh^2 more accurately than Ω since that is what we measure)

$\Omega_r = 8.3\times10^{-5}$

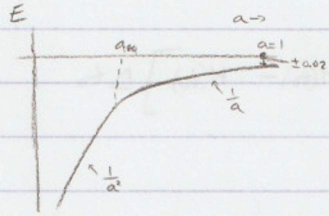
Radiation dominated era $\quad a = 10^{-4} \quad \rho_m = \rho_0 \underbrace{\Omega_m 10^{12}}_{3\times10^{11}} \quad \rho_{DE} = \underbrace{\rho_0\Omega_\Lambda}_{\text{negligible}} \quad \rho_r = \rho_0 \underbrace{\Omega_r 10^{16}}_{8.3\times10^{11}}$

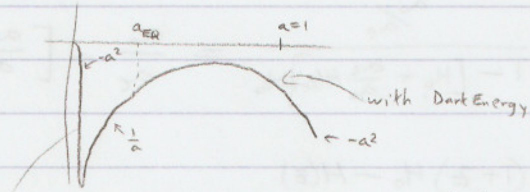
$$a_{EQ} = \frac{1}{1+z_{EQ}} = \frac{\Omega_r h^2}{\Omega_m h^2} = \frac{4.165\times10^{-5}}{0.15} \simeq \frac{1}{3500}$$

Can say $\rho \sim a^{-3}$ for $a > \frac{1}{3500}$ $\quad \rho \sim a^{-4}$ for $a < \frac{1}{3500}$

Planck time $a \sim 10^{-31} \quad \rho \sim \frac{10^{124}}{3500}\Omega_m\rho_0 \quad \rho a^2 \sim 10^{58}\rho_0 a_0^2$

$\Omega^{-1} - 1 = 0 \pm 0.02 \quad \therefore$ @ Planck time $|(\Omega^{-1}-1)| < 2\times10^{-60}$ extremely flat





Carnival game: roll bowling ball up hill to a target location

large vacuum energy density $\rho_{vac} >> \rho_0$ (inflationary scenario)

$\underbrace{30}_{10^{30}}$ decades of expansion needed to solve flatness-oldness problem

expansion during inflation e^N $N > 69$ prior could be uniform in N

$\rho a^2 (\Omega^{-1} - 1) = \text{const}$ $\quad e^{-2N} \, 10^{58} (\Omega^{-1} - 1) = (\Omega^{-1} - 1)$ should have uniform $\ln(\Omega^{-1} - 1)$

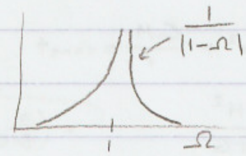
ρa^2 during inflation

logarithmic prior for $\Omega^{-1} - 1$

$0.999 < \Omega < 0.9999$
$0.99 < \Omega < 0.999$
$0.9 < \Omega < 0.99$
$1.1 > \Omega > 1.01$
$1.01 > \Omega > 1.001$
$1.001 > \Omega > 1.0001$

surviving parts of prior (likelihood eliminated rest)
equal probabilities

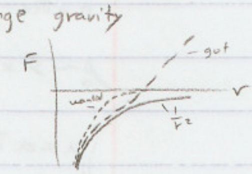
Prob(Ω)



if inflation happened, $\Omega = 1$ (flat)

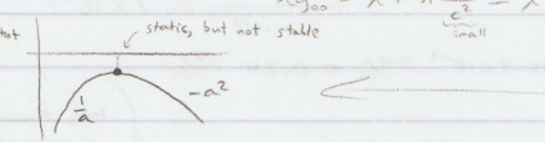
1917 Einstein matter + Λ static U ($H_0 = 0$)

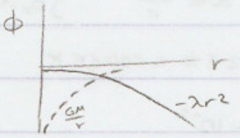
$\nabla^2 \phi = 4\pi G \rho \;\rightarrow\; \nabla^2 \phi - \lambda \phi = 4\pi G \rho$ $\quad \phi \sim \frac{e^{-\sqrt{\lambda} r}}{r}$ shorter range gravity



$g_{00} \simeq 1 + \frac{2\phi}{c^2}$ time part of metric $\quad \nabla^2 g_{00} - \lambda g_{00} = 8\pi G \rho$

$\lambda g_{00} = \lambda + \lambda \underbrace{\frac{2\phi}{c^2}}_{\text{small}} \simeq \lambda$





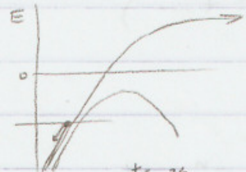
negatice energy — spherical geometry $\quad H_0 = 0 \Rightarrow \Omega \rightarrow \infty$

$\rho_{vac} = \frac{1}{2} \rho_{matter}$

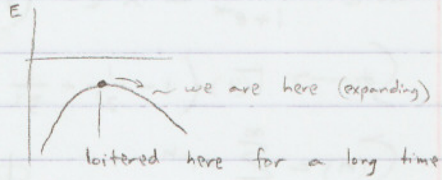
de Sitter - 1917 Λ, no matter

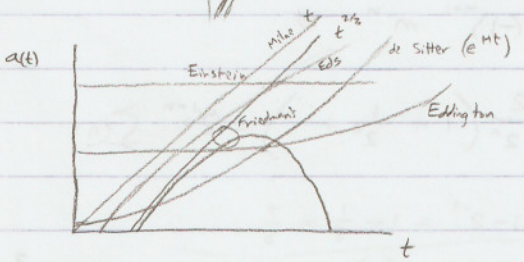
exponential expansion flat space $\Omega_\Lambda = 1$ $H = \text{const}$ $\rho_{vac} = \frac{3H^2}{8\pi G}$

Freidman
1922-23
several models



Eddington





Einstein - de Sitter $\Omega_m = 1$ $a(t) \sim t^{2/3}$ Milne $\Omega = 0$ $a \sim t$

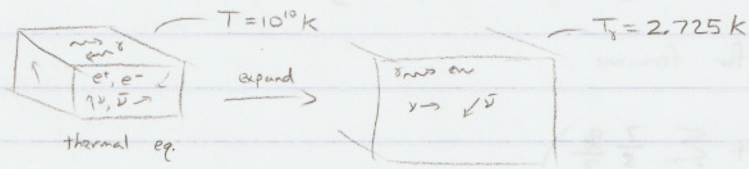
Thermal History of the Universe

neutrinos ν weak interactions crossection $\sigma_{\nu\nu} \sim E^2$ (energy)

once $E \gtrsim$ energy of W, Z bosons $\rightarrow$ weak behaves like electro

$n_\nu \sim a^{-3} \sim T^3$ $\quad \underbrace{n_\nu \sigma v_c}_{\text{interaction rate}} \sim T^5$ $\quad \frac{3H^2}{8\pi G} \propto \frac{aT^4}{c^2}$ $\quad \underbrace{H \sim T^2}_{\text{expansion rate}}$

$n_\nu \sigma v > H \Rightarrow$ thermal equilibrium $T \gtrsim 10^{10}\,K \sim 1\,MeV$ (~1 sec. after Big Bang)



e^-, e^+ annihilate
$T \sim 100\text{-}200$ keV

neutrinos have already decoupled (weak f. is very weak now)

in each cell of phase space $\frac{d^3p\, d^3q}{h^3}$ — number of cells in phase space

helicity - spin projected in direction of motion
helicity cant change for massless particles
(with mass - Lorentz transf.)

$$\frac{d^3p\, d^3q}{h^3}\, \underbrace{g_s}_{\text{spin statistical weight}} \frac{1}{e^{E/kT} \pm 1} = n$$

ex. photons $n_\gamma = \frac{1}{c^3}\frac{2\nu^2 d\nu}{e^{h\nu/kT} - 1}$ $\left(\frac{\#}{\text{cc Hz ster}}\right)$

g_s: e^+, e^- : 2
γ : 2 (1, -1, no 0)

$E = \sqrt{p^2c^2 + m^2c^4}$ for $e^\pm$

$kT \gg mc^2$ to simplify $E \simeq pc$ $\quad x = \frac{pc}{kT}$

$$u_{e^\pm} = g_s \int \frac{4\pi p^2 dp}{h^3} \frac{E}{e^{pc/kT} + 1} \qquad u = g_s 4\pi \left(\frac{kT}{hc}\right)^3 kT \int \frac{x^3 dx}{e^x + 1}$$

photons $u_\gamma = 2 \times 4\pi \left(\frac{kT}{hc}\right)^3 kT \int \frac{x^3 dx}{e^x - 1}$

$x^3 e^{-x}(1 + e^{-x} + e^{-2x} \ldots)$

$\Gamma(4)\left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \ldots\right)$

$\Gamma(4)\,\underset{\text{zeta}}{\zeta(4)}$

$$u = g_s\, 4\pi \left(\frac{kT}{hc}\right)^3 kT \int \frac{x^3\,dx}{e^x+1}$$

$$x^3 e^{-x} \frac{1}{1+e^{-x}} = x^3 e^{-x}\left(1 - e^{-x} + e^{-2x} - e^{-3x} + \dots\right)$$

$$\int dx \longrightarrow \Gamma(4)\left(1 - \frac{1}{2^4} + \frac{1}{3^4} - \frac{1}{4^4} + \dots\right)$$

$$\zeta(n) = \sum_{m=1}^{\infty} m^{-n} \qquad \phi(n) = \sum_{m=1}^{\infty} (-1)^{m+1} m^{-n}$$

$$\zeta(n) - \phi(n) = 2/2^n + 2/4^n + \dots = \frac{2}{2^n}\left(1 + \frac{1}{2^n} + \dots\right) = 2^{1-n}\,\zeta(n)$$

$$\phi(n) = \left(1 - 2^{1-n}\right)\zeta(n) \qquad n=4 \quad 1 - 2^{-3} = 1 - \frac{1}{8} = \frac{7}{8}$$

$$u_e = \frac{7}{8}\frac{g_s}{2} a T^4 \quad \text{for } e^\pm$$

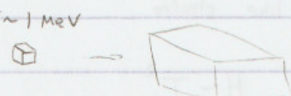
number density $\propto \int \frac{x^2\,dx}{e^x \pm 1}$

$n_\gamma \propto \Gamma(3)\,\zeta(3) \qquad n_e \propto \Gamma(3)\left(1 - \frac{1}{2^3} + \frac{1}{3^3} - \dots\right) \qquad n_e = \frac{3}{4} n_\gamma$

4/23/2007

Isentropic / Adiabatic expansion

$T \sim 1\,\text{MeV}$



start at $T=0$ then heat it (to calculate entropy)

$$dS = \frac{dQ}{T} \qquad \frac{du}{dT} = 4\,\frac{7}{8}\frac{g_s}{2} a T^3 \qquad S = \int \left(\frac{du}{dT}\right)\frac{1}{T}\,dT$$

$$S = \frac{4}{3}\frac{7}{8}\frac{g_s}{2} a T^3 \quad \text{for fermions}$$

$$S = \frac{4}{3} a T^3 \underbrace{\left(\sum_{\text{bosons}} \frac{g_s}{2} + \sum_{\text{fermions}} \frac{7}{8}\frac{g_s}{2}\right)}$$

$g_*(T) \longrightarrow 106.75$ @ $T \to \infty$ based on current Standard Model

fermions $n_f = \frac{3}{4}\frac{g_s}{2} n_\gamma$

$n_\nu \quad T_\nu \qquad T_\nu = T_{\nu 0}(1+z) \qquad T_{\nu 0}$

$n_e \quad T_\nu \qquad$ entropy gets dumped into photons

$n_\gamma \quad T_\nu \xrightarrow{\text{expand}} \quad n_\gamma \quad T_{\gamma 0} > T_{\nu 0}$

$$S = \frac{4}{3} a T_\nu^3 \left(\underset{\gamma}{1} + \underset{e^-}{\frac{7}{8}} + \underset{e^+}{\frac{7}{8}}\right) = \frac{4}{3} a T_\nu^3\, \frac{11}{4}$$

after expansion $\frac{4}{3} a T_{\gamma 0}^3 = \frac{4}{3} a T_{\nu 0}^3\, \frac{11}{4}$

T of CMB

$$T_{\nu 0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma 0} = 1.95\,K$$

current energy density of radiation including massless neutrinos $g=1$ $\Sigma g_s = 6$ $\nu_e\ \nu_\mu\ \nu_\tau$ $\bar\nu_e\ \bar\nu_\mu\ \bar\nu_\tau$

$$u_{\gamma\nu} = aT_{\gamma 0}^4 \left(1 + \frac{7}{8}\, 3 \left(\frac{4}{11}\right)^{4/3}\right) = 1.68\, aT_{\gamma 0}^4$$ neutrinos $\sim \frac{2}{3}$ of energy density as γ

$$n_\nu = n_\gamma\, \underbrace{3\ \frac{3}{4}}_{9/4}\ \underbrace{\frac{4}{11}}_{n \propto T^3} = \frac{9}{11} n_\gamma$$ not exact, neutrinos still interact ~1% correction

Energy density: $g_* \simeq 3.36$

$$S = \frac{g_{*s}}{2}\frac{4}{3} aT_{\gamma 0}^3 = \frac{4}{3} aT_{\gamma 0}^3 \left(\underbrace{1}_{\gamma} + \underbrace{3\,\frac{7}{8}\,\frac{4}{11}}_{\nu}\right)$$

$\frac{g_{*s}}{2} \to g_{*s} \simeq 3.91$ effective g after ~1s $\gamma+\nu$ ($e^{\pm}$ annihilate)

Planck units

$\alpha = \frac{e^2}{\hbar c}$ fine structure constant $\quad \frac{Gm^2}{r^2} \sim \frac{e^2}{r^2}$

$\frac{Gm^2}{\hbar c} = 1$ Defines Planck mass $\quad m_{Pl} = \sqrt{\frac{\hbar c}{G}}$

$$m_{Pl} = \left(\frac{10^{-27}\ 3\times 10^{10}}{\frac{2}{3}\, 10^{-7}}\right)^{1/2} = \left(\frac{9}{2}\, 10^{-10}\right)^{1/2} = 2.1\times 10^{-5}\ gm$$

$E_{Pl} = m_{Pl} c^2 \qquad \hbar = t_{Pl} E_{Pl} \qquad \omega_{Pl} = \frac{E_{Pl}}{\hbar} \qquad 2.1\times10^{-5}\ 9\times10^{20} \sim 20\times10^{15}$ ergs

$120\times10^{26}\ eV = 10^{19}\ GeV$

2×10^{16} ergs $= kT_{Pl} \qquad k = \frac{1}{7}\times10^{-15}$

$T_{Pl} = 14\times10^{31}\,K \simeq 10^{32}\,K$

$$\rho = \frac{1}{c^2}\frac{g_*}{2} aT_{Pl}^4 \qquad \frac{\overbrace{50}^{g_*/2} \times \overbrace{7\times10^{-15}}^{a}\ \overbrace{10^{128}\times 4}^{T^4}}{9\times10^{20}} \simeq 150\times10^{93} \simeq 10^{95}\ gm/cc$$

$$\rho a^2 \left(\frac{1}{\Omega} - 1\right) = \text{const.}$$

$\frac{4}{3}\frac{g_*}{2} aT_{Pl}^3 = S_{Pl} \qquad S_0 = \frac{4}{3}\frac{g_{*s}}{2} aT_{\gamma 0}^3$ ratio is $(1+z)^3$

$$(1+z)^3 = \frac{S_{Pl}}{S_0} = \frac{\overset{106.75}{g_*}}{\underset{3.91}{g_{*s}}}\left(\frac{T_{Pl}}{T_{\gamma 0}}\right)^3 = 10^{96.6} \qquad 1+z = 10^{32.2}$$

$$\rho_{Pl} = 10^{95} \qquad \rho_0 = 10^{-29} \qquad \frac{\rho_{Pl} a_{Pl}^2}{\rho_0 a_0^2} = 10^{\overset{95+29}{126} - \overset{\text{from } z}{64}} = 10^{62}$$ related to flatness-oldness problem

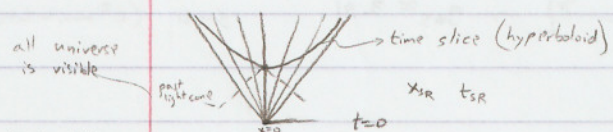
Observing angular sizes and fluxes of distant objects

Milne Model $\Omega_r = \Omega_m = \Omega_\Lambda = 0 \qquad \Omega_{tot} = 0$ hyperbolic

$H = \frac{1}{t} \qquad a = t/t_0$ because there is no gravity we can use Minkowski space $\rightarrow$ simplifies things

$$ds^2 = c^2 dt^2 - a(t)^2 R_0^2 \left(d\psi^2 + \sinh^2\psi \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right) \right)$$

$$R_0 = \frac{c}{H_0} \frac{1}{\sqrt{|1-\Omega_{tot}|}} \qquad ds^2 = c^2 dt^2 - \underbrace{\left(t/t_0\right)^2}_{a(t)^2} \underbrace{c^2 t_0^2}_{R_0^2} \left(\quad \right) = c^2 dt^2 - c^2 t^2 \left(\quad \right)$$



$x_{SR} \quad t_{SR} \qquad t_{SR} = t \cosh\psi \qquad x_{SR} = ct \sinh\psi \qquad y_{SR}, z_{SR}$

Note: $\frac{v}{c} = \tanh\psi$

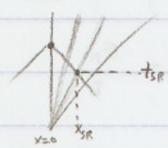
$$v_{SR} = \frac{x_{SR}}{t_{SR}} = c \tanh\psi \qquad \frac{D = ct\psi}{t} \Big) \; v = c\psi \text{ cosmic velocity}$$

special relativity velocity

$dy_{SR} = ct \sinh\psi \, d\theta = x_{SR} \, d\theta$ ← observed angle $\qquad D_A \, d\theta$ — physical transverse size

└ physical size of object $\qquad D_A = x_{SR}$

redshift: $\quad t_{SR} = t_0 - \frac{x_{SR}}{c}$ (retarded time)



$$v_{SR} = \frac{x_{SR}}{t_0 - \frac{x_{SR}}{c}}$$

$$1+z = \sqrt{\frac{1 + v_{SR}/c}{1 - v_{SR}/c}}$$ appropriate since no grav and Minkowski space

$$1+z = \left(\frac{c + \frac{x_{SR}}{t_0 - x_{SR}/c}}{c - \frac{x_{SR}}{t_0 - \frac{x_{SR}}{c}}} \right)^{1/2} = \left(\frac{c\left(t_0 - \frac{x_{SR}}{c}\right) + x_{SR}}{c\left(t_0 - \frac{x_{SR}}{c}\right) - x_{SR}} \right)^{1/2} \qquad (1+z)^2 = \frac{ct_0}{ct_0 - 2x_{SR}} = \frac{1}{1 - \frac{2x_{SR}}{ct_0}}$$

$$1 - \frac{2x_{SR}}{ct_0} = \frac{1}{(1+z)^2} \qquad \frac{2x_{SR}}{ct_0} = 1 - \frac{1}{(1+z)^2} \qquad x_{SR} = D_A(z) = \frac{c}{H_0} \frac{1}{2} \frac{2z + z^2}{(1+z)^2}$$

$$\boxed{D_A(z) = \frac{c}{H_0} z \frac{1 + \frac{1}{2}z}{(1+z)^2}}$$ How big an object is

$$L = 4\pi R^2 \sigma T_{em}^4 \qquad R = D_A \theta \qquad T_{obs} = \frac{T_{em}}{1+z}$$

occupation $\frac{1}{e^{h\nu/kT} - 1}$ is invariant $\quad \nu \rightarrow \frac{\nu}{1+z}$

$$F = \iint_0^\infty B_\nu(T_{obs}) \, d\nu \, d\Omega = \pi\theta^2 \frac{\sigma}{\pi} T_{obs}^4 = \theta^2 \sigma T_{obs}^4$$

$$F = \frac{L}{4\pi D_L^2} \qquad D_L = \left(\frac{L}{4\pi F} \right)^{1/2} \qquad \boxed{D_L = D_A (1+z)^2}$$

Luminosity Distance

True for all models

$$ds^2 = c^2dt^2 - a(t)^2 R_0^2 \left(d\psi^2 + \sinh^2\psi \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right) \right)$$

$$ds = a(t_{em}) R_0 \sinh\psi \, d\theta \implies \boxed{D_A = a(t_{em}) R_0 \sinh\psi}$$

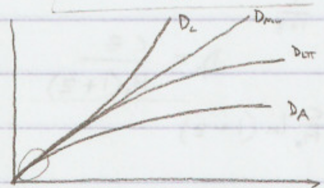
$$c\,dt = a(t) R_0 \, d\psi \quad \underset{ct}{} \implies \psi = \ln(1+z) \qquad a(t_{em}) = \frac{1}{1+z} \qquad R_0 = ct_0 = \frac{c}{H_0}$$

$$D_A = \frac{1}{1+z}\frac{c}{H_0}\left(\frac{\overbrace{1+z - \frac{1}{1+z}}^{e^\psi - e^{-\psi}}}{2} \right) = \frac{c}{H_0}\frac{2z+z^2}{2(1+z)^2} = \frac{cz}{H_0}\frac{1+\frac{1}{2}z}{(1+z)^2}$$

same as before different method

$$\Omega = 0 \qquad D_A = \frac{cz}{H_0}\frac{1+\frac{1}{2}z}{(1+z)^2} \qquad D_L = \frac{cz}{H_0}\left(1+\frac{1}{2}z\right) \qquad \boxed{D_{now} = \frac{c}{H_0}\ln(1+z)}$$

light travel time $\boxed{D_{LTT} = c(t_0 - t_{em})} = \frac{c}{H_0}\frac{z}{1+z}$



all $\frac{cz}{H_0}$ for small z

Open formula

QROMO — to evaluate integrals

4/25/2007

Many distances $\quad D_{now} = \frac{c}{H_0}\ln(1+z) \qquad D_L = \frac{c}{H_0} z \left(1+\frac{1}{2}z\right) \qquad D_A = \frac{D_L}{(1+z)^2} \qquad D_{LTT} = \frac{c}{H_0}\frac{z}{1+z}$

$$D = \frac{cz}{H_0} + \ldots$$

these are the ones we can measure (actually, it's redshift)

Einstein de Sitter Model

$\Omega_m = 1 \qquad \Omega_r = \Omega_v = 0$

$$ds^2 = c^2dt^2 - a(t)^2\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)\right] \qquad a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

$$\boxed{D_A = a(t_{em})\, r} \quad \longleftarrow ds = a(t)\, r \, d\theta \qquad \frac{1}{1+z} = \left(\frac{t_{em}}{t_0}\right)^{2/3} \qquad t_{em} = \frac{t_0}{(1+z)^{3/2}}$$

$$c\,dt = a(t)\, dr \qquad dr = \frac{c\,dt}{a(t)}$$

$$\boxed{r = \int_{t_{em}}^{t_0} \frac{c\,dt}{a(t)}} = \int_{\frac{1}{1+z}}^{1} \frac{c\,da}{a\,\dot a} = \int_{1/1+z}^{1} \frac{c\,da}{a H_0\sqrt{\Omega/a}} = \frac{c}{H_0}\int_{\frac{1}{1+z}}^{1}\frac{da}{\sqrt{a}} = \frac{c}{H_0}\, 2\sqrt{a}\,\Big|_{\frac{1}{1+z}}^{1}$$

$$r = \frac{2c}{H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

$$\boxed{D_{now} = r = \frac{2c}{H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)}$$

$$\boxed{\begin{aligned} D_A &= \frac{2c}{H_0}\left(\frac{1}{1+z} - \frac{1}{(1+z)^{3/2}}\right) \\ D_L &= \frac{2c}{H_0}\left(1+z-\sqrt{1+z}\right) \end{aligned}}$$

$z \to \infty \qquad D_{now} = \frac{2c}{H_0}$ $\qquad$ infinite universe with horizon

$$D_{LTT} = ct_0\left(1 - \frac{1}{(1+z)^{3/2}}\right) = \frac{2}{3}\frac{c}{H_0}\left(1 - \frac{1}{(1+z)^{3/2}}\right)$$

Model $\Omega_r = 1 \implies a(t) = \left(\frac{t}{t_0}\right)^{1/2}$

flat $\Omega_m = \Omega_\Lambda = 0$

$$r = \frac{c}{H_0}\int_{\frac{1}{1+z}}^{1} \frac{da}{a\sqrt{\Omega_r/a^2}} = \frac{c}{H_0}\left(1 - \frac{1}{1+z}\right) = D_{now}$$

$$D_A = \frac{c}{H_0}\frac{z}{(1+z)^2} \qquad D_L = \frac{cz}{H_0} \qquad D_{LTT} = ct_0\left(1 - \frac{1}{(1+z)^2}\right) = \frac{1}{2}\frac{c}{H_0}\left(1 - \frac{1}{(1+z)^2}\right)$$

Model flat $\Omega_v = 1 \implies a(t) = e^{H(t-t_0)}$

$$r = \frac{c}{H_0}\int_{\frac{1}{1+z}}^{1} \frac{da}{a^2} = \frac{cz}{H_0} \qquad \frac{1}{a}\Big|_1^{\frac{1}{1+z}} = 1+z-1 = z \qquad D_{now} = \frac{cz}{H_0}$$

no horizon, can see infinitely far away (though objects will be very faint)

$F \propto \frac{1}{D_L^2} \propto \frac{1}{z^4}$

$$D_A = \frac{cz}{H_0(1+z)} \qquad D_L = \frac{cz}{H_0}(1+z)$$

$$D_{LTT} = \frac{c}{H_0}\ln(1+z)$$

$$\dot a = H_0\sqrt{\underbrace{\frac{\Omega_m}{a}}_{w=0} + \underbrace{\frac{\Omega_r}{a^2}}_{w=\frac{1}{3}} + \underbrace{\Omega_v a^2}_{w=-1} + \Omega_k}$$

$w = -\frac{1}{3}$ for flat model with $a(t) = t/t_0$

$\Omega_k = 0$

$\ddot a = 0$ $\quad \dot a = \text{const}$

$D_{now} = \frac{c}{H_0}\ln(1+z)$ no horizon

$$D_A = \frac{c}{H_0}\frac{\ln(1+z)}{1+z} \qquad D_L = \frac{c}{H_0}(\ln(1+z))(1+z) \qquad D_{LTT} = \frac{c}{H_0}\frac{z}{1+z}$$

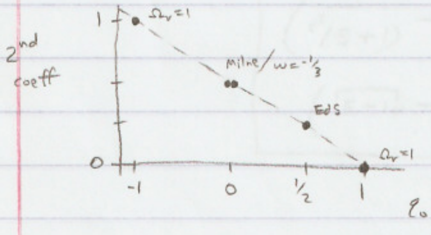
Taylor expansion of D_L

Milne $\quad D_L = \frac{cz}{H_0}\left(1 + \frac{1}{2}z\right) \qquad \Omega_r = 1 \quad D_L = \frac{cz}{H_0}$

EdS $\quad D_L = \frac{cz}{H_0}\left(1 + \frac{1}{4}z - \frac{1}{16}z^2 \dots\right) \qquad \Omega_v = 1 \quad D_L = \frac{cz}{H_0}(1+z)$

$w = -\frac{1}{3}$ flat $\quad \ln(1+z) = z - \frac{1}{2}z^2 + \frac{1}{3}z^3 = z\left(1 - \frac{1}{2}z + \frac{1}{3}z^2\right)$

$$z\left(1 - \frac{1}{2}z + \frac{1}{3}z^2\right)(1+z) = z\left(1 + \frac{1}{2}z - \frac{1}{6}z^2\right) \qquad D_L = \frac{cz}{H_0}\left(1 + \frac{1}{2}z - \frac{1}{6}z^2 \dots\right)$$



$$ds^2 = c^2 dt^2 - a(t)^2 R_0^2 \left(d\psi^2 + \begin{Bmatrix} \sin^2\psi \\ \psi^2 \\ \sinh^2\psi \end{Bmatrix} (d\theta^2 + \sin^2\theta \, d\varphi^2) \right)$$

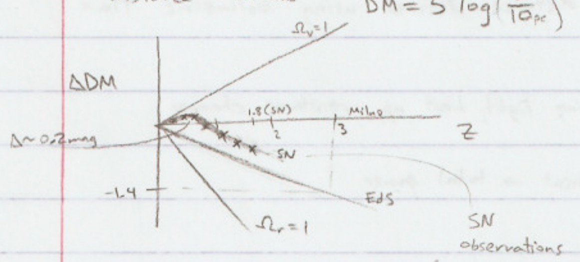
$\psi = \int \frac{c\,dt}{a R_0}$ indep of curvature $\qquad D_A = a(t_e) R_0 \begin{Bmatrix} \sin\psi \\ \psi \\ \sinh\psi \end{Bmatrix}$ $\qquad \sin\psi \simeq \psi - \frac{1}{3!}\psi^3$, $\sinh\psi \simeq \psi + \frac{1}{3!}\psi^3$

curvature effects only come in third order

first order term $\frac{cz}{H_0}$ $\qquad$ low redshift: only need H_0 and q_0

second order term depends on q_0

Distance Modulus $\quad DM = 5\log\left(\frac{D}{10_{pc}}\right)$



$$\Delta DM = 5\log\left(\frac{1+\frac{1}{4}z - \frac{1}{16}z^2 \cdots}{1+\frac{1}{2}z}\right) \quad \frac{\text{EdS}}{\text{Milne}}$$

@ $z=3$ $\quad$ EdS $D_L = \frac{4c}{H_0}$

Milne $D_L = 7.5\frac{c}{H_0}$

$\Omega_\Lambda = 1$: $\quad \Delta M = 5\log\left(\frac{1+z}{1+\frac{1}{2}z}\right)$

Different H_0 → slide up/down in ΔDM plot

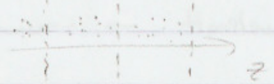
	z	m_{pk}	Decay time
low z sample	0.001		
	0.01	— calibrate m_{pk}, decay rate	
	0.1		

Calculating D_L

$$\psi = \frac{c}{R_0}\int_{1/(1+z)}^{1} \frac{da}{a\dot{a}} \qquad D_A = a(t_{em}) \begin{Bmatrix} \sin\psi \\ \psi \\ \sinh\psi \end{Bmatrix} R_0 \qquad R_0 = \frac{c}{H_0}\frac{1}{\sqrt{|1-\Omega_{tot}|}}$$

$$D_A = a(t_{em}) \frac{c}{H_0}\frac{1}{\sqrt{|1-\Omega_{tot}|}} \begin{pmatrix} \sin\psi \\ \psi \\ \sinh\psi \end{pmatrix}$$

$$\psi = \frac{H_0\sqrt{|1-\Omega_{tot}|}}{c}\frac{c}{H_0}\int_{1/(1+z)}^{1} \frac{da}{a\,\dot{a}/H_0} = \sqrt{|1-\Omega_{tot}|}\underbrace{\int_{1/(1+z)}^{1} \frac{da}{a\sqrt{\frac{\Omega_m}{a} + \frac{\Omega_r}{a^2} + \Omega_v a^2 + \Omega_k}}}_{Z_+}$$



group SN by z
and evaluate integral $\quad$ then integrate from $SN_i \to SN_{i+1}$ (small distance) in each group $\quad$ (?)

$$D_L = (1+z)\frac{c}{H_0}\frac{1}{\sqrt{|1-\Omega_{tot}|}} \begin{Bmatrix} \sin \\ I \\ \sinh \end{Bmatrix}_{\Omega_{tot}<1}^{\Omega_{tot}>1} \left(\sqrt{|1-\Omega_{tot}|}\, Z_+\right) = (1+z)\frac{c}{H_0} Z_+ \begin{Bmatrix} \frac{\sin x}{x} \\ 1 \\ \frac{\sinh x}{x} \end{Bmatrix} \qquad x = \sqrt{|1-\Omega_{tot}|}\, Z_+$$

add $\frac{Z_+}{Z}$

$$\frac{\sin x}{x} = 1 - \frac{1}{6}\underbrace{|1-\Omega_{tot}|}_{-(1-\Omega_{tot})} z^2 + \frac{1}{120}(1-\Omega_{tot})^2 z^4 + \ldots \qquad \frac{\sinh x}{x} = 1 + \frac{1}{6}(1-\Omega_{tot})z^2 + \frac{1}{120}(1-\Omega_{tot})^2 z^4$$

$$+\frac{1}{6}(1-\Omega_{tot})z^2$$

$$J(x) = \begin{cases} \frac{\sin\sqrt{-x}}{\sqrt{-x}} & \text{if } x<0 \\ \frac{\sinh\sqrt{x}}{\sqrt{x}} & \text{if } x>0 \end{cases} \quad 1 + \frac{1}{6}x + \frac{1}{120}x^2 + \cdots$$

$$D_L = (1+z)\frac{c}{H_0} Z_t \, J\left((1-\Omega_{tot}) Z_t^2\right)$$ fast way to numerically get D_L and thus fit SN

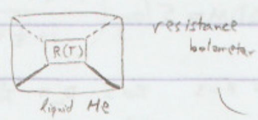
4/30/2007

K Correction

- adjustment for spectrum of source

$F_\nu \sim \nu^{-1}$ $\quad \nu F_\nu = \text{const} = \lambda F_\lambda$ Luminosity distance defined using bolometric flux

$$F_{bol} = \frac{L}{4\pi D_L^2}$$



shine light, heat up, resistance changes

measures heat → total power

$$\int_0^\infty F_\nu \, d\nu = F_{bol} \qquad \int_0^\infty L_\nu \, d\nu = L$$

ν_{obs} $\quad \Delta\nu_{obs}$

$$\text{signal} \propto \int_{\nu_{obs} - \frac{\Delta\nu_{obs}}{2}}^{\nu_{obs} + \frac{\Delta\nu_{obs}}{2}} F_\nu \, d\nu = \frac{\int_{\nu_e - \frac{\Delta\nu_e}{2}}^{\nu_e + \frac{\Delta\nu_e}{2}} L_\nu \, d\nu}{4\pi D_L^2}$$

$\nu_e = (1+z)\,\nu_{obs}$ $\quad \Delta\nu_e = (1+z)\,\Delta\nu_{obs}$

$$\Delta\nu_{obs} F_{\nu_{obs}} = \frac{\Delta\nu_e L_{\nu_e}}{4\pi D_L^2} \qquad F_{\nu_0} = \frac{(1+z) L_{\nu_e}}{4\pi D_L^2} \qquad F_{\lambda_0} = \frac{\frac{1}{1+z} L_{\lambda_e}}{4\pi D_L^2}$$

$$\frac{\Delta\nu_0}{\nu_0} = \frac{\Delta\nu_e}{\nu_e} = \frac{\Delta\lambda_0}{\lambda_0} = \frac{\Delta\lambda_e}{\lambda_e} \qquad \nu_0 F_{\nu_0} = \frac{\nu_e F_{\nu_e}}{4\pi D_L^2}$$

$$V = M_V + 5\log\left(\frac{D_L(z)}{10_{pc}}\right) + \underbrace{K(\nu_V, z)}_{\text{K correction}}$$

$$K(\nu, z) = -2.5\log\left(\frac{\nu(1+z) L_{\nu(1+z)}}{\nu L_\nu}\right)$$

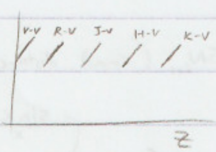
$\nu(1+z) \to \nu_e$

$K = 0$ for $z = 0$

can get rid of K if you measure at redshifted wavelength → measure at $\nu(1+z)$ rather than ν

R = 0.7
I = 0.9
J = 1.25
H = 1.6
K = 2.2

OH nightglow in atm



small K corrections by shifting bands

$2.4\,\mu m \to \frac{1}{2}\,eV \to 6000\,K \cdot k$

$\frac{h\nu}{kT} = 20 \to 10^{-8.7}$

from atm or zodiacal? not low enough

$$V = -2.5 \log \frac{F_\nu(V)}{F_\nu^\circ(V)} \qquad K = -2.5 \log \frac{F_\nu(K)}{F_\nu^\circ(K)}$$

F_ν° — zeroth mag. flux

need to know $\frac{F_\nu^\circ(V)}{F_\nu^\circ(K)} = \begin{cases} 6 & \text{for Vega (Johnson)} \\ 1 & \text{for AB} \end{cases}$ "Cheaters mag" → corresponds to add/subtract 2 mags

observe to K ~ 24 → actually 22

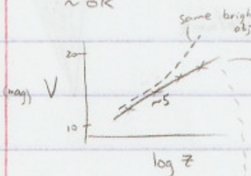
errors → $\times 1 \pm 5\%$

need to understand spectrum of detector or of source → difficult

Evolution

Feb 1970 Sandage Physics Today Cosmology: A search for 2 Numbers

$H_0 = 80 \pm 30\%$ ~OK $\quad q_0 = 1 \pm 0.6$ wrong

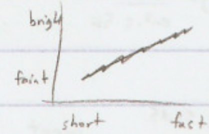


→ $q_0 = 1$ → straight line $D_L = \frac{cz}{H_0}$

$q_0 \sim -1 \quad \frac{cz}{H_0}(1+z) = D_L \quad$ we think $q_0 \simeq -0.6$

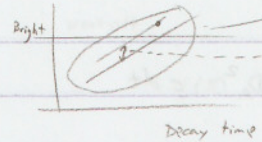
objects far away were brighter (long ago)

SN Type Ia

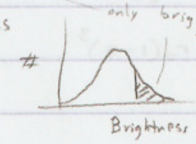


N_i^{56} mass → parameter

or



Malmquist bias



only bright stuff is seen as high redshift

Axions

$\xrightarrow{\gamma} A$ with $\uparrow \vec{B}$

photons into axions in presence of magnetic field

$$F_\nu = e^{-\lambda(t_0 - t_{em})} \frac{L}{4\pi D_L^2}$$

λ — decay rate $\quad \gamma \to A$

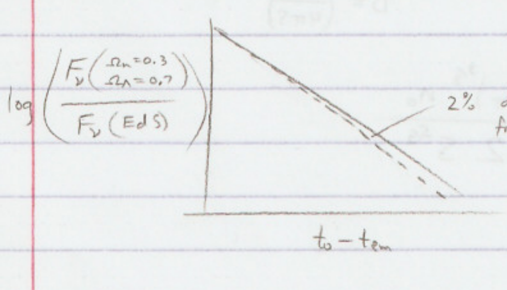
$$I_\nu^{CMB} = (1 \pm \epsilon) B_\nu(T_0)$$

$\epsilon \sim 10^{-4}$

Problem with model

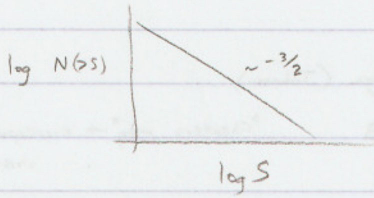
read notes

Distant supernova have no apparent reddening

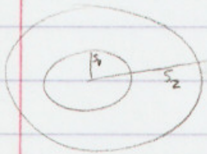


Number Counts

flux $\rightarrow S$



Euclidian dist^n of sources



$S_2 = \frac{1}{4} S_1 \qquad V \sim \left(\left(\frac{S_1}{S_2}\right)^{1/2}\right)^3 \sim S^{-3/2}$

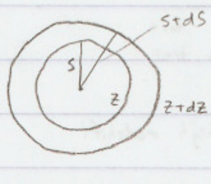
n_o objects of luminosity L both constant (conserved)

j – emissivity $\left[\frac{erg}{cm^3 \cdot s \cdot Sr}\right]$ $\qquad 4\pi j = n_o L$ for fixed comoving volume

energy density $u = \int \frac{n_o L \, dt}{1+z}$ ... expansion $\qquad \int n_o L \, dt \rightarrow$

$u = \frac{4\pi J}{c} \qquad [J] = \frac{erg}{cm^2 \cdot s \cdot Sr}$ $\qquad$ need to know $a(t)$

$$\nu_o u_{\nu_o} = \int \frac{n_o \nu_e L_{\nu_e}}{1+z} dt$$



want $\frac{\partial N}{\partial S}$

$n = n_o(1+z)^3 \qquad$ volume : $dV = D_A^{\;2}(z) \, c \, dt$ (thickness)

takes into account 4π for flat or else for curved

$$dN = n_o(1+z)^3 D_A^2 c \, dt$$

$$S = \frac{L}{4\pi D_L^2} \qquad \frac{dS}{dt} = \frac{2L}{4\pi D_L^2} \frac{dD_L}{dz} \frac{dz}{dt}$$

$$\frac{dN}{dS} = \frac{2\pi D_L^3}{L} n_o(1+z)^3 D_A^2 c \, dt \left(\frac{1}{\frac{dD_L}{dz}\frac{dz}{dt}dt}\right)$$

using first order approx. $\quad \frac{dz}{dt} = H_o \qquad D_A \simeq \frac{cz}{H_o} \simeq D_L \qquad \frac{dD_L}{dz} \simeq \frac{c}{H_o}$

$$\frac{dN}{dS} = \frac{n_o}{L} 2\pi \underbrace{\left(\frac{cz}{H_o}\right)}_{D}^5 \frac{c\,dt}{\left(\frac{c}{H_o}\right) H_o \, \cancel{dt}} = \frac{2\pi n_o}{L} D^5 \qquad D = \left(\frac{L}{4\pi S}\right)^{1/2}$$

$$= \frac{2\pi n_o}{L}\left(\frac{L}{4\pi S}\right)^{5/2} = \frac{4\pi n_o}{L}\left(\frac{L}{4\pi S}\right)^{5/2} = \frac{(L/4\pi)^{3/2} n_o}{2 S^{5/2}}$$

Euclidian $\boxed{\left(\frac{dN}{dS}\right)_E = \frac{(L/4\pi)^{3/2} n_o}{2 S^{5/2}}}$

Now add cosmological corrections

converting $D_A \to D_L$

$$\boxed{\frac{dN}{dS} = \left(\frac{dN}{dS}\right)_E \left[(1+z)^3 (1+z)^{-4} \frac{dz}{dD_L} \frac{c\,dt}{dz} \right]}$$

mean intensity $\quad J = \int_0^\infty S \frac{dN}{dS} dS$

consider Milne model

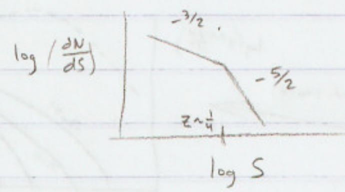
$\Omega = 0 \qquad D_L = \frac{cz}{H_0}\left(1 + \frac{1}{2} z\right) = \frac{c}{H_0}\left(z + \frac{1}{2}z^2\right) \qquad \frac{dD_L}{dz} = \frac{c}{H_0}(1+z)$

$t_e = \frac{t_0}{1+z} = \frac{1}{H_0}\frac{1}{(1+z)} \qquad \frac{c\,dt}{dz} = \frac{c}{H_0}\frac{1}{(1+z)^2}$

$$\frac{dN}{dS} = \left(\frac{dN}{dS}\right)_E \left[(1+z)^{-1} \frac{H_0}{c}(1+z)^{-1} \frac{c}{H_0} \frac{1}{(1+z)^2} \right] = \left(\frac{dN}{dS}\right)_E (1+z)^{-4}$$

$D_L \sim z^2$ for large z $\quad \therefore \quad S \sim z^{-4}$

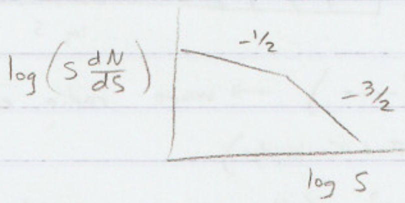
$$\frac{dN}{dS} = \left(\frac{dN}{dS}\right)_E S$$



in euclidian $J \to \infty$

$\frac{dN}{dS} \propto S^{-5/2}$ Olber's paradox

can also plot



$S\frac{dN}{dS} \to \infty$ as $S \to 0$

infinite faint sources

(no horizon in Milne)

EdS

$\Omega_m = 1 \qquad D_L = \frac{2c}{H_0}\left(1 + z - \sqrt{1+z}\right) \qquad t_e = \frac{t_0}{(1+z)^{3/2}} = \frac{2}{3}\frac{1}{H_0}(1+z)^{-3/2}$

$\frac{dD_L}{dz} = \frac{c}{H_0}\left(2 - \frac{1}{\sqrt{1+z}}\right) \qquad \frac{c\,dt}{dz} = \frac{c}{H_0}(1+z)^{-5/2}$

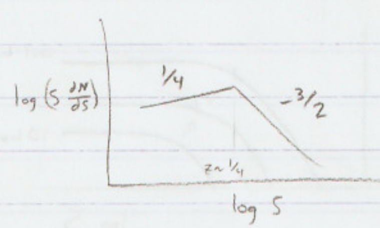
$$\frac{dN}{dS} = \left(\frac{dN}{dS}\right)_E \left(\frac{1}{(2\sqrt{1+z} - 1)(1+z)^3}\right) \simeq \left(\frac{dN}{dS}\right)_E \left(\frac{1}{\underbrace{(2+z-1)}_{(1+z)}(1+z)^3}\right)$$

for small z

for small z, $\frac{dN}{dS}$ indep of models but useful for large z (but evolution takes place then and complicates)

$D_L \sim z$ (large z) $\quad S \sim z^{-2} \quad z \sim S^{-1/2}$

$$\frac{dN}{dS} \sim \left(\frac{dN}{dS}\right)_E \left(\frac{1}{S^{-1/4}\, S^{-3/2}}\right) \sim \left(\frac{dN}{dS}\right)_E S^{7/4}$$



finite # of sources seen since we have horizon

$$N(>0) = \int_0^\infty \frac{dN}{dS} dS = \frac{1}{3} \underbrace{D_H^3}_{\text{horizon distance}} n_0 \quad \text{(per steradian)}$$

$$D_H = \int_0^1 \frac{c\,da}{a\dot{a}}$$

5/2/2007

1963 $2\frac{1}{2}$ facts steady state, flat, $H = $ const

$a \sim e^{Ht}$ $n \sim$ const

$$\frac{dN}{dz} = n_0 (1+z)^3 D_A^2(z) \frac{c\,dt}{dz} \qquad \frac{dN}{dS} = \left(\frac{dN}{dS}\right)_E \left(1 + \mathcal{O}(z^2) + \dots\right)(1+z)^4$$

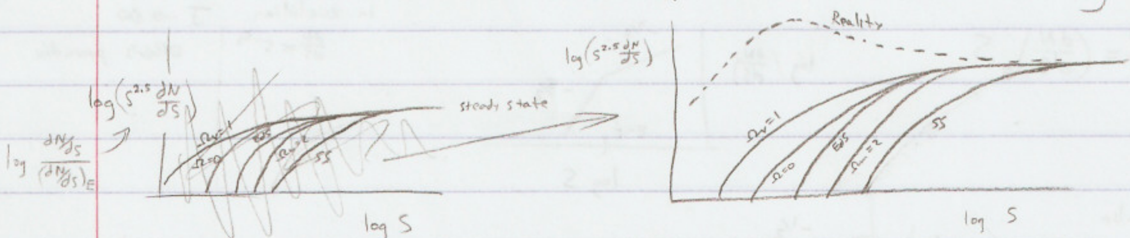
Steady state $\frac{dN}{dS} = \left(\frac{dN}{dS}\right)_E (1+z)^{-7} \left(1 + \mathcal{O}(z^2) + \dots\right)$ since $n \to$ constant (no $(1+z)^3$)

S_1 – Euclidean flux @ $\frac{c}{H_0}$

$$\frac{dN}{dS} = \left(\frac{dN}{dS}\right)_E \left[\frac{32}{\left(1+\sqrt{1+4\sqrt{S_1/S}}\right)^5 \sqrt{1+4\sqrt{S_1/S}}}\right]$$

$S >> S_1$

$$= \left(\frac{dN}{dS}\right)_E \left[\frac{1}{\left(1 + 1\sqrt{S_1/S}\right)^5 \left(1 + 2\sqrt{S_1/S}\right)}\right] = \left(\frac{dN}{dS}\right)_E \left[1 - 7\sqrt{S_1/S} + \dots\right]$$



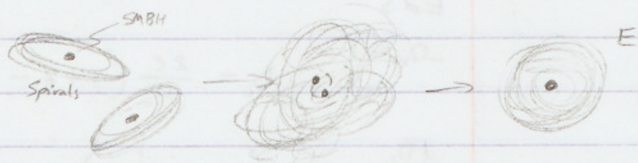
Used radio sources to test (- - - -) $\rightarrow$ more radio counts far away

(using 3C catalog Third Cambridge)

Abandon $n = $ const and $n = n_0 (1+z)^3$ (this was an asumption)

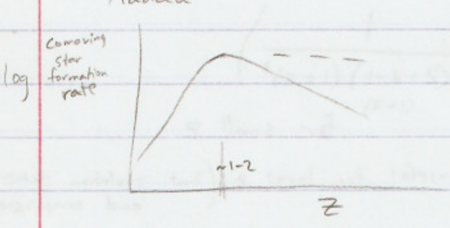
Radio sources $\rightarrow$ mergers between galaxies

# of collisions $\propto$ density^2



it appears that $n \propto (1+z)^9$ (then drops)

Madau

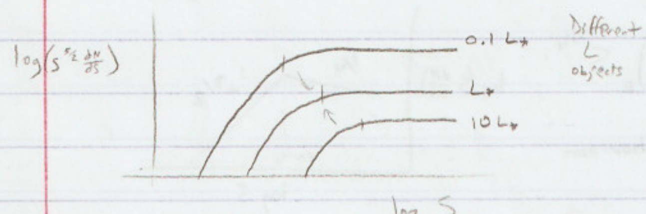


but a guess is $(1+z)^6 \propto n$

comoving $n_{cm} \simeq e^{6 H_0 (t_0 - t)}$

$\simeq (1+z)^6$ for $z << 1$

$\approx$ const for $z >> 1$



$0.1 L_*$ unlikely

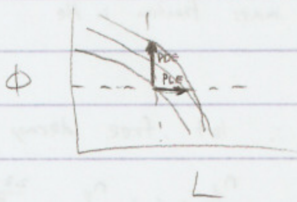
$10 L_*$ practically unseen

$$\left(\frac{dN}{dS}\right)_E = \frac{n_0\left(\frac{L}{4\pi}\right)^{3/2}}{2\,S^{5/2}}$$

$n=(1+z)^3\,\Phi(L,z)\,dL$ $\quad \Phi(L,z)=\Phi_0\left(\frac{L}{L_*}\right)^{-\alpha} e^{-L/L_*}$ $\quad \Phi_0=\Phi_0(z)$ density evolution

pure density evolution (PDE)

or $L_* = \text{function}(z)$, $\Phi_0 = \text{const}$ pure luminosity evolution (PLE)



Source counts disproves steady state model

<u>Big Bang Nucleosynthesis</u> (BBNS)

n, p, e^-, e^+, ν_e $\quad n \to p + e^- + \bar{\nu}_e$ $\quad p + e^- \to n + \nu_e$ $\quad e^+ + n \to p + \bar{\nu}_e$

$(m_n - m_p)c^2 = 1.3\,\text{MeV}$ $\quad kT = 1\,\text{MeV}$ $(t = 1\text{s})$

weak interactions (low E) $\sigma \propto E^2$ $\quad \nu_e + n \to p + e^-$ $\quad \sigma = \dfrac{2\pi\hbar^3 \nu_e (E_\nu + Q)^2}{f\,\tau_n\, m_e^5 c^9}$ $\quad Q = (m_n - m_p)c^2$

$\langle n\sigma v\rangle \sim T^5$ $\quad H \sim T^2$ $\quad$ as $E\downarrow$ $T\downarrow$ reactions stop (freeze out) ~1s after B.B. $\quad f = 1.634$ τ_n = neutron mean life

$\dfrac{n_n}{n_p} = e^{-Q/kT}$ (same volume and phase factors) $\quad \dfrac{n_n}{n_n + n_p} = 0.14$ ~2s

an aside {

$T = 10^{13}\,\text{K}$ $\quad kT = 1\,\text{GeV}$ $\quad n_{\bar{p}} \simeq n_p \simeq n_\gamma$ $\quad \dfrac{n_p - n_{\bar{p}}}{n_\gamma} \simeq 10^{-8}$ $\quad$ by $10^{12}\,\text{K}$ all $\bar{p}$ annihilate

Baryons $\dfrac{n_B}{n_\gamma} \sim$ constant for $z < 10^{8.5}$ $\quad \left(\dfrac{n_p - n_{\bar{p}}}{s}\right) = \text{const}$ (s: entropy density)

Baryogenesis $\longrightarrow$ $\dfrac{n_p - n_{\bar{p}}}{s} = \text{const}$ $\longrightarrow$ BBNS $\longrightarrow$ H, D, He, Li

}

CPT (C: charge conjugation, P: parity, T: time reversal)

P can be violated (not conserved)

CP not conserved

if CPT is conserved

T cannot be conserved (arrow of time)

this will imply matter > antimatter

$\dfrac{n_n}{n_n + n_p} = 0.14$ $\longrightarrow$ free decay $\longrightarrow$ assembly into nuclei

↳ this ~~#~~ will depend on freeze out temperature

first step:

$n + p \to d + \gamma$ strong interaction (2.2 MeV) $\quad$ "$d + d \longrightarrow {}^4He + \gamma$" (28 MeV)

this is slow interaction (depends on EnM) so have to do it in steps

tritium $\quad t + t \to {}^4He + n + n$

$t + {}^4He \to {}^7Li$ (small amount) $\quad p + {}^4He \to \cancel{{}^5Li}$ $\quad n + {}^4He \to \cancel{{}^5He}$ $\quad$ barrier at A=5

also at A=8 $\quad {}^8Li$ β (0.8s) $\quad {}^8Be$ 10^{-16}s $\quad {}^8B$ EC (0.8s) (if e^- are close by) $\quad$ decay times

mostly: H, D, ^3He, ^4He, some ^7Li

stronger weak interaction $\longrightarrow$ lower freeze out temperature $\longrightarrow$ more free decay $\longrightarrow$ less He if everything made to ^4He

n p p p p p p

n p p p p p p

$\hookrightarrow \alpha$

$10\,p$'s + $1\,\alpha$'s

$\frac{n_n}{n_n + n_p} = 0.14$

mass fraction in He $\quad \frac{4}{14} \simeq 0.28$ (10+4) $\quad$ 28% by mass in Helium

slightly higher than observed

$\therefore$ let free decay for 2 min $\therefore$ 0.24 by mass in He $\longrightarrow$ observed

$$\frac{n_d}{n_n} \sim (\;) \frac{n_p}{n_\gamma} e^{\frac{2.2\,\mathrm{MeV}}{kT}}$$

factor(T) $\quad \sim 10^{-9}$ $\longrightarrow$ must be $\sim e^{22} \sim 10^9$ $\rightarrow$ then $n + p \rightarrow d + \gamma$ can go

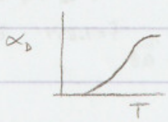
$p \rightarrow p$

$d + d \rightarrow {}^4He \qquad \frac{d}{p} = ?$

recombination coeff $\quad \alpha_D(T)$

$dn_D = -\alpha_D(T)\, n_D^2\, dt$

$$\frac{dX_D}{dt} = -\underbrace{2}_{2\ d\ \text{combine}} \alpha_D(T)\, \underbrace{X_D}_{\text{deuterium fraction}}^2\, \underbrace{n_B}_{\#\ \text{baryons}}$$



$T \sim t^{-1/2} \qquad n_B \sim t^{-3/2}$

$$\frac{dX}{X^2} = d\left(\frac{1}{X}\right) \qquad X_D^{-1} = 2 n_B(t_1) \int_{t_1}^{\infty} \alpha\Big(\overbrace{T_1\left(\tfrac{t}{t_1}\right)^{-1/2}}^{T}\Big)\left(\frac{t}{t_1}\right)^{-3/2} dt + \frac{1}{X_D(t_1)}$$

$$\frac{dn_D}{dt} = -2\,\alpha(T)\, n_D^2 \qquad \int d\left(\frac{1}{n_D}\right) = 2\int \alpha(T)\, dt$$

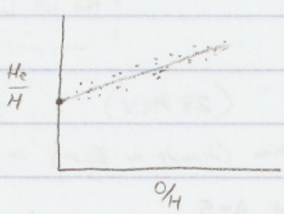
measuring X_D can allow us to get n_B:

$$\frac{D}{H} \sim 3\times 10^{-5} \longrightarrow \Omega_B h^2 \simeq 0.022 = 0.00367\,\eta_{10} \qquad \eta_{10} = 10^{10}\,\frac{n_B}{n_\gamma}$$

$\eta_{10} \simeq 6$

$n_\gamma \simeq 411/cc \qquad n_B = 1/4\ /m^3$

^4He is almost right, D is right, $^7Li \sim 2$-$3\times$ too high



1980 $\quad$ BBNS (✓) $\quad$ Hubble law (✓) $\quad$ Homo/Iso (✓) $\quad$ CMB + spectrum, dipole

Horizon problem $\qquad$ Flatness-Oldness problem

Monopole problem $\longrightarrow$ predicted by symmetry breakings

Evolution of Diffuse Background 5/7/2007

Radiative transfer

$$\frac{\partial I_\nu}{\partial s} = j_\nu - \alpha_\nu I_\nu \qquad I_\nu:\ erg/cm^2/sec/sr/Hz$$

$$d\tau_\nu = \alpha_\nu ds \qquad \frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \qquad \text{in LTE: } S_\nu = B_\nu(T)$$

n stars each with $L_\nu = 4\pi^2 R^2 B_\nu(T_*)$

$$j_\nu = \frac{nL_\nu}{4\pi} = n\pi R^2 B_\nu(T_*) \qquad \alpha_\nu = \underbrace{n\pi R^2}_{\text{crossection of star}} \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} = B_\nu(T_*)$$

$$I_\nu = e^{-\tau_\nu} I_\nu(0) + (1 - e^{-\tau_\nu}) S_\nu \qquad \tau_\nu >> 1 \to \text{Olber's paradox}$$

$$nL = 1.6h \times 10^8\ L_\odot/Mpc^3 \qquad * \to \odot \implies n = 1.6 \times 10^8\ h\ Mpc^{-3}$$

$$\pi R^2 n = \frac{3 \times 49 \times 10^{20} \times 1.6h \times 10^8}{3.08^3 \times 10^{72}} = 8h \times 10^{-44} \sim 10^{-43}\ cm^{-3}$$

optical depth 1 @ 10^{43} cm $= 10^{25}$ lyrs

Electron scattering

$$n = n_e(0)(1+z)^3 \qquad ds = c\,dt \qquad \tau_e = n_e(0)\,\sigma_T \int (1+z)^3 \frac{c\,da}{\dot a} = n_e(0)\,\sigma_T \frac{c}{H_0} \int \frac{(1+z)^3\,da\,x_e}{\sqrt{\Omega_m/a + \Omega_r/a^2 + \Omega_\nu a^2 + \Omega_k}}$$

(x_e: ionized fraction)

25% in He $\qquad 12p + 1\alpha = \frac{14e^-}{16\text{ baryons}} = \frac{7}{8}$

$$\underbrace{\frac{7}{8}\left(\frac{1}{4}\right)10^{-6}}_{n_e(0)} \underbrace{\frac{2}{3}\times 10^{-24}}_{\sigma_T} \times \underbrace{3000 \times 3.08 \times 10^{24}\,h^{-1}}_{c/H_0} \int \frac{x_e(1+z)^3\,da}{\sqrt{\ }}$$

$\frac{7}{8}\left(\frac{1}{4}\right)10^{-6}$ — $m^3 \to cm^3$; $\frac{7}{8}$: fraction of e^-; $\frac{1}{4}$: fraction of baryons

$$\tau_e \simeq \frac{1}{7}\times 10^{-2}\,h^{-1} \int \frac{x_e(1+z)^3\,da}{\sqrt{\Omega_m/a + \Omega_\nu a^2}} \qquad \text{high redshift:} \int \frac{x_e\,da}{\sqrt{\Omega_{m0}}\,a^{2.5}} = \frac{x_e}{\sqrt{\Omega_{m0}}}\frac{2}{3}a^{-1.5}$$

$$\tau_e \simeq 10^{-3} x_e \frac{(1+z)^{3/2}}{\sqrt{\Omega_m h^2}} \qquad @\ z\sim 100\ \ \tau_e \sim 1 \quad \text{but @ } z\sim 100 \text{ universe was neutral}$$

$$\tau_e \sim \frac{\Omega_b h^2}{\sqrt{\Omega_m h^2}}(1+z)^3 x_e \qquad \tau_e \sim 60\text{–}100\ @\ z\sim 1000 \quad \text{universe was ionized}$$

optical depth since reionization can affect CMB fluctuations

Now consider expanding universe (I_ν will change with expansion)

Consider comoving density of photons

$$\frac{I_\nu}{2h\nu\left(\frac{\nu}{c}\right)^2} = \frac{1}{e^{h\nu/kT} - 1} \quad \text{for B.B.} \qquad \text{this wont change with expansion}$$

$$\frac{\partial}{\partial z}\left(\frac{I_{\nu(1+z)}}{(1+z)^3}\right) = c\frac{dt}{dz}\left[\frac{j_{\nu(1+z)}}{(1+z)^3} - \alpha_{\nu(1+z)}\frac{I_{\nu(1+z)}}{(1+z)^3}\right]$$

population of galaxies

evolution of n_o

$n_o \; L_\nu \qquad j_\nu(z) = j_\nu(0)(1+z)^3$

if $\alpha \sim 0$

$$I_\nu = \int j_{\nu(1+z)}(0) \; c\, dt$$

@ z=0 (now)

$(1+z)^3$ cancel out

$$u = \int \frac{L \; dt}{(1+z)} \qquad \nu u_\nu = \int \frac{\nu(1+z) \, L_{\nu(1+z)}}{(1+z)} dt = \int \nu L_{\nu(1+z)} \, dt$$

$n L = \mathcal{L}$ luminosity density $\qquad j = \frac{\mathcal{L}}{4\pi}$

$$I = \frac{\mathcal{L}}{4\pi} \int \frac{c\, dt}{(1+z)} \qquad \mathcal{L} = 2h \times 10^8 \, L_\odot / Mpc^3$$

$\Omega_{mo} = 1 \quad \Omega_k = 0 \quad$ EdS $\qquad I = \frac{\mathcal{L}}{4\pi} \frac{c}{H_0} \int \frac{dz}{(1+z)^{3.5}} = 8 \; nW/m^2/sr$

Suppose you have hot IGM

X-ray background – spectrum as if coming from hot gas $\quad kT_{app} \simeq 40\,keV$
(fairly isotropic)

free-free emission $\qquad j_\nu = A n_e n_i \, e^{-h\nu/kT} / \sqrt{T} \simeq n_e(0)^2 (1+z)^3 \frac{e^{-h\nu/kT(z)}}{\sqrt{T(z)}}$

$$kT_{app} \simeq \frac{kT(z)}{(1+z)} \qquad I_\nu = A \, n_e(0)^2 \frac{c}{H_0} \int \frac{(1+z)\, e^{-\frac{h\nu(1+z)}{kT(z)}}}{\sqrt{T(z)} \sqrt{1+\Omega_m z}} dz$$

$z = 2 \quad T \to 120\,keV$? $\quad \to$ not enough electrons

$\langle n_e^2 \rangle \neq \langle n_e \rangle^2 \qquad \langle n_e^2 \rangle \gg \langle n_e \rangle^2 \quad$ clump electrons

Suppose you have neutrino decaying: $\quad \nu_H \to \nu_L + \gamma \qquad E_\gamma \simeq \frac{m_H c^2}{2}$

$$j_\nu = \frac{\rho_\nu c^2}{4\pi \tau \, 2} \delta\left(\nu - \frac{m_\nu c^2}{2h}\right)$$

decay time; half goes into light

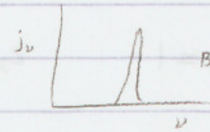
$$\rho_\nu = \rho_\nu(0)(1+z)^3 \qquad \text{Assume } \Omega_{mo} = 1$$

$$I_\nu = \frac{\rho_\nu(0) c^2}{8\pi\tau} \frac{c}{H_0} \int \frac{(1+z)^3 \, \delta\left(\nu(1+z) - \frac{m_\nu c^2}{2h}\right)}{(1+z)^3} \frac{da}{\sqrt{\Omega_m/a + \Omega_\nu a^2}}$$

$j(0) = \frac{j(z)}{(1+z)^3}$

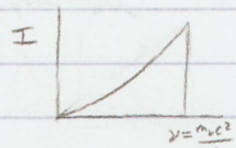
$$c\,dt = \frac{c}{H_0} \frac{dz}{(1+z)^{2.5}}$$

$$I_\nu = \frac{\rho_\nu(0) c^2}{8\pi\tau} \frac{c}{H_0} (1+z)^{-2.5} \Bigg|_{(1+z) = \frac{m_\nu c^2}{2h\nu}}$$



bluer than line
∴ not produced by decay (?)

step function



if neutrino is dark matter, τ must be very large or too many photons produced
(as long as its blue light)

Why is CMB black?

411 $\quad\gamma$ $\qquad$ electron scattering changes direction $\quad e^- + \gamma \to e^- + \gamma$

$\frac{1}{4}\times 10^{-6}$ $\quad e^-$ $\qquad$ but its already isotropic

$\frac{1}{4}\times 10^{-6}$ $\quad p$

$e^- + p + \gamma \to e^- + p$ $\quad$ not very effective in early universe
(not many e^-, p)

electron scattering can change E if e^- are moving

Kompaneets

photon density per mode $\quad n = \dfrac{I_\nu}{2h\nu\left(\frac{\nu}{c}\right)^2}$

$$y = \tau_e \frac{kT_e}{mc^2}$$

to change spectrum need e^- moving

$$\frac{\partial n}{\partial y} = x^{-2} \frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n + n^2\right)\right] \qquad x = \frac{h\nu}{kT_e}$$

non linear equation $\quad$ steady state $\left(\frac{\partial n}{\partial y} \to 0\right)$ $\quad$ wont give black body

photon number $\quad N = \int x^2 n \, dx \qquad \dfrac{\partial N}{\partial y} = \int x^2 \dfrac{\partial n}{\partial y} dx = \int \dfrac{\partial}{\partial x}\left(x^4\left(\dfrac{\partial n}{\partial x} + n + n^2\right)\right) dx$

$$\frac{\partial N}{\partial y} = x^4\left(\frac{\partial n}{\partial x} + n + n^2\right)\Bigg|_{\text{low end} = 0}^{\text{high end} = 0} = 0$$

no change in total # of photons

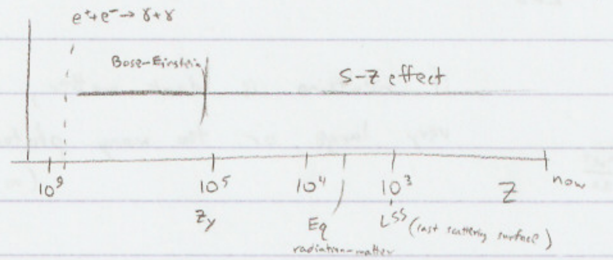
$$\therefore \quad n = \frac{1}{e^{x+\mu} - 1} \qquad \text{Bose-Einstein distribution}$$

$$\frac{\partial n}{\partial x} = \frac{-e^{x+\mu}}{(e^{x+\mu} - 1)^2} \qquad \frac{\partial n}{\partial x} + n + n^2 = \frac{\cancel{-e^{x+\mu}} + \cancel{e^{x+\mu}} \cancel{-1} \cancel{+1}}{(e^{x+\mu} - 1)^2} = 0$$

$$\tau_e \sim 10^{-3} z^{3/2} x_e \qquad \frac{kT}{mc^2} = \frac{3(1+z)}{6\times 10^9} = \frac{1+z}{2\times 10^9}$$

$y \sim \dfrac{z^{5/2}}{2\times 10^{12}}$ $\quad z \sim 10^4$ $\quad$ but now this is radiation dominated so $\Rightarrow$ might not be valid

$$(1+z)\frac{\partial y}{\partial z} = 1 \qquad z_y = \frac{10^5}{\sqrt{\Omega_B h^2/0.0224}}$$



$\mu \to$ very small, Bose-Einstein spectrum not observ

Suppose

$T_e \gtrsim T_\gamma$ slightly hotter $\quad f = \frac{T_e}{T_\gamma} \qquad n = \frac{1}{e^{fx}-1}$ photons (BB @ T_γ)

$$n^2 + n + \frac{\partial n}{\partial x} = \frac{1}{(e^{fx}-1)^2} + \frac{1}{(e^{fx}-1)} - \frac{f e^{fx}}{(e^{fx}-1)^2}$$

$$= \frac{1 + e^{fx} - 1 - f e^{fx}}{(e^{fx}-1)^2} = (1-f)\frac{e^{fx}}{(e^{fx}-1)^2} = (1-f^{-1})\frac{\partial n}{\partial x}$$

$$\frac{\partial n}{\partial y} = x^{-2}\frac{\partial}{\partial x}\left(x^4 \frac{\partial n}{\partial x}\left(1 - \frac{T_\gamma}{T_e}\right)\right) \qquad \frac{\partial n}{\partial \tau_e} = x^{-2}\frac{\partial}{\partial x}\left(x^4\frac{\partial n}{\partial x}\right)(T_e - T_\gamma)$$

$$y_D = \frac{k(T_e - T_\gamma)}{mc^2}\tau_e \qquad \Delta n = x^2\frac{\partial}{\partial x}\left(x^4\frac{\partial n_{BB}}{\partial x}\right) y_D$$

Distortion of spectrum from BB.

hot gas in cluster of galaxies

T_e large $\frac{\partial n}{\partial x}$ dominates over $n + n^2$ so get ↑ Sunyaev–Zeldovich effect

FIRAS $\quad |y_D| < 1.5 \times 10^{-5} \qquad |\mu| < 9 \times 10^{-5}$

energy:

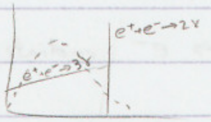
$$u = \int x^3 n \, dx \qquad \frac{\partial n}{\partial y_D} = x^{-2}\frac{\partial}{\partial x}\left(x^4\frac{\partial n}{\partial x}\right)$$

$$\frac{\partial u}{\partial y_D} = \int x \frac{\partial}{\partial x}\left(x^4\frac{\partial n}{\partial x}\right)dx = -\int x^4 \frac{\partial n}{\partial x}dx = \int 4x^3 n \, dx = 4u$$

$$\frac{\partial u}{\partial y_D} = 4u \qquad \frac{\Delta u}{u} < 6 \times 10^{-5} \qquad (y_D < 1.5 \times 10^{-5})$$

limit on how much energy can be added

explosions cant have made large scale structure (gravity can)



$$\frac{\partial n}{\partial y} = x^{-2} \frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n + n^2\right)\right]$$

a measure of expansion $\leftarrow (1+z)\frac{\partial y}{\partial z}$ $\qquad H = \frac{\dot a}{a} = -\frac{(\dot{1+z})}{(1+z)} = \frac{dz}{(1+z)\,dt}$

radiation dominated

$$(1+z)\frac{\partial y}{\partial z} = \left(\frac{c}{H}\right) n_e(0)(1+z)^3 \sigma_T \frac{k T_{\gamma 0}(1+z)}{mc^2} \qquad \rho = \frac{3H^2}{8\pi G} \qquad Ht = \frac{1}{2} \qquad \rho c^2 = 1.68\, a T_\gamma^4$$

neutrinos at diff. Temp.

$$\frac{3c^2}{32\pi G t^2} = 1.68\, a T_\gamma^4 \qquad \left(\frac{3c^2}{32\pi G\, 1.68 a}\right)^{1/4} t^{-1/2} = T_\gamma$$

$$\frac{27\times 10^{20}}{100\left(\frac{2}{3}\times 10^{-7}\right)1.68\,(7.5\times 10^{-15})} = \frac{81}{25.6} 10^{\overset{40}{20+5+15}} \qquad T = 1.3\times 10^{10}\,\mathrm{K}\; t^{-1/2} \qquad t > 100\,\mathrm{s}$$

$$n_p(0) \simeq 0.23\times 10^{-6} \frac{\Omega_B h^2}{0.022}$$

$$H \sim z^2 \quad \left(a \sim t^{1/2} \quad z \sim t^{-1/2}\right) \qquad (1+z)\frac{\partial y}{\partial z} \propto \overset{\frac{1}{H}}{z^{-2}}\, \overset{n}{z^3}\, \overset{T}{z} \sim z^2$$

$$(1+z)\frac{\partial y}{\partial z} = 1 \;\to\; z_y \sim 10^{5.1}$$

$$N = \int x^2 n\, dx \quad \#\text{ of photons} \qquad n = \frac{1}{e^{x+\mu}-1} \quad \text{Bose-Einstein Dist.}$$

$$= \int \frac{x^2\,dx}{e^{x+\mu}-1} = \int \frac{x^2 e^{-x} dx\, e^{-\mu}}{1-e^{-(x+\mu)}} = \int x^2 e^{-x} e^{-\mu}\left(1 + e^{-(x+\mu)} + e^{-2(x+\mu)} + \dots\right) dx$$

$$x' = x + \mu \qquad \int \frac{(x'^2 - 2x'\mu + \mu^2)e^{-x'}}{1-e^{-x'}} dx' = \Gamma(3)\zeta(3) - 2\mu\,\Gamma(2)\zeta(2) + \dots$$

change in dist$_n$ because of change in Temp

energy density $\quad U = \int x^3 n\, dx = \Gamma(4)\zeta(4) - 3\mu\,\Gamma(3)\zeta(3) + \dots$

$$N = N_0\left(1 - \frac{2\Gamma(2)\zeta(2)\mu}{\Gamma(3)\zeta(3)}\right)\left(\frac{T}{T_0}\right)^3 = N_0 \qquad U = U_0\left(1 - \frac{3\Gamma(3)\zeta(3)\mu}{\Gamma(4)\zeta(4)}\right)\left(\frac{T}{T_0}\right)^4$$

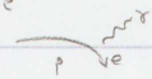
\# of photons conserved

$$\frac{T}{T_0} \simeq 1 + \frac{1}{3}\frac{2\Gamma(2)\zeta(2)\mu}{\Gamma(3)\zeta(3)} \qquad U = U_0\left(1 + \underbrace{\left(\frac{4}{3}\frac{2\Gamma(2)\zeta(2)}{\Gamma(3)\zeta(3)} - \frac{3\Gamma(3)\zeta(3)}{\Gamma(4)\zeta(4)}\right)}_{\sim 0.7}\mu\right)$$

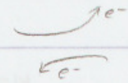
Adding energy but not photons

photons scatter of e^- → e^- changes momentum (accelerates) → e^- radiates

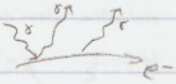
free-free



rare, not enough p and e^-



e-e scattering
no dipole radiation



$\Delta p = \frac{h\nu}{c}$ $\Delta t = \frac{1}{\nu}$ Double photon Compton Scattering

$\Delta p = m \Delta v$ $\frac{\Delta v}{\Delta t} = \frac{h\nu}{mc}\frac{1}{1/\nu} = \frac{h\nu^2}{mc} = a$ $P = \frac{2}{3}\frac{e^2 a^2}{c^3}$

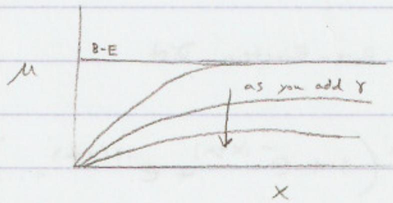
$\Delta E = P\Delta t = \frac{1}{\nu}\frac{2}{3}\frac{e^2}{c^3}\frac{h^2\nu^4}{m^2c^2} \simeq \frac{e^2h^2}{m^2c^5}\nu^3$ per scattering

$\frac{\Delta E}{h\nu} = \frac{e^2 h}{m^2c^5}\nu^2 = \frac{e^2}{hc}\frac{(h\nu)^2}{(mc^2)^2}$ not significant at low T $\frac{e^2}{hc} \sim 10^{-2}$

$j_\nu \sim \nu^0 e^{-h\nu/kT}$ like free-free

$\frac{\partial n}{\partial \tau} \propto$ low freq. dominated process (γ emitted @ low freq)

$\frac{\partial n}{\partial \tau} \underset{\text{prop.}}{\propto} \frac{1}{x^3}\underset{\text{alpha}}{\alpha}\left(\frac{kT}{mc^2}\right)^2\left(1 - \frac{n}{n_{BB}}\right)$



this happens $z \sim 2\times10^6$

thermalization $z_{th} = \frac{4.2\times10^5}{[\Omega_B h^2]^{0.4}} = 1.9\times10^6$ for $\Omega_B h^2 = 0.0224$

$t = \frac{1.8\times10^{20}}{T^2} = \frac{1.8\times10^{20}}{(1.9\times10^6\times2.73)^2} = \frac{1.8\times10^{20}}{25\times10^{12}} = \frac{1}{12}\times10^8 \sim 2$ months

before: BB after: B-E until $z_y \simeq 10^5$

$(z_y \sim 10^5)$: $t \sim 300\times2$ months $= 60$ yrs

$0 < t < 2$ months BB

2 months $< t <$ 1 century B-E

1 century $< t <$ now Sunyaev-Zeldovich distortion

if $E \to e^-$ which scatter of γ

$z_{cool} \simeq 7$ if you add E to e^- it can get rid of it all → after that, not all energy can get transfered to CMB (not enough time)

(say $z \sim 3$)

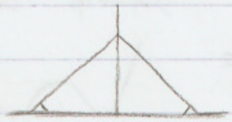
Inflation

Flatness-Oldness Problem: $\Omega \sim 1$ now but $\rho a^2\left(\frac{1}{\Omega}-1\right) = \text{const}$ so $\Omega = 1 \pm \epsilon$ in past $\epsilon \sim 10^{-60}$

Horizon Problem: we can see $D_{Hor} = \int_{\frac{1}{1090}}^{1} \frac{c\,da}{a\dot{a}}$ — recombination

gas at last scattering surface: $D_{Hor}(LSS) = \int_{0}^{\frac{1}{1090}} \frac{c\,da}{a\dot{a}}$

$D_{Hor}(LSS) \ll D_{Hor}(\text{now})$ $\quad \frac{\Delta T_{cmb}}{T_0} \sim 10^{-5}$



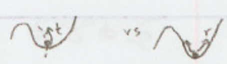
$\leftarrow V(\phi_1, \phi_2)$ potential field

a typical potential $m^2\phi^2$

Rotations in ϕ space leave V unchanged

high E → symmetric

low E → tangential, radial — perturbations



$\frac{\partial^2 V}{\partial(\Delta\phi_r)^2} = \text{large}$ ⟹ large mass

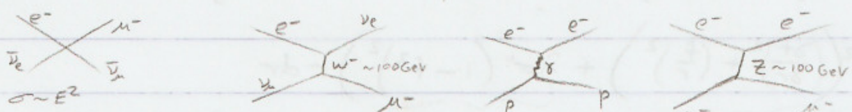
$\frac{\partial^2 V}{\partial(\Delta\phi_t)^2} = 0$ ⟹ zero mass

asymmetric solutions

Electro-weak Unification → potential like above

high E → γ, Z same

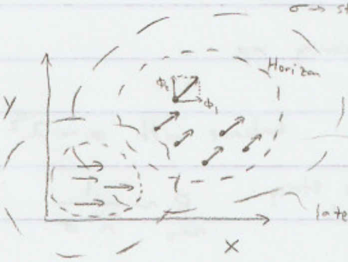
low E → γ, Z different



$\sigma \sim E^2$ → ∞ as $E \to \infty$

as $E \to \infty$ σ → stops growing

$e^+ + e^- \to Z$



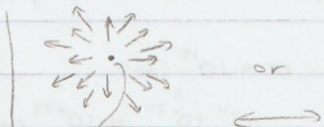
$\partial_\mu \phi \partial^\mu \phi$ $\quad (\nabla\phi)^2$ kinetic term

same direction to minimize

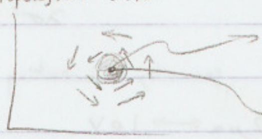
later time (bigger horizon) now have excess kinetic energy

→ topological defect

length of vectors is the same



or



→ large gradient

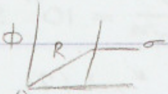
$(\nabla\phi)^2 \simeq V(0)$

→ $\phi = 0$

cosmic string out of page

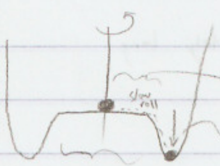
these are just rotations (which are invariant)

V is min when $\phi_1^2 + \phi_2^2 = \sigma^2$



$\left(\frac{\sigma}{R}\right)^2 = V(0)$ $\qquad E = R^2 V(0) \simeq \sigma^2$ per unit length

in 3D → cosmic string (topological defect) is now a point → a <u>monopole</u>



← inflation

energy released (radiate photons)

symmetry broken when off center (topological defects)

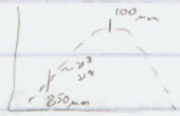
exponential growth due to vacuum energy

→ this would reduce density of magnetic monopoles (~1 per observable universe)

5/14/2007

SCUBA

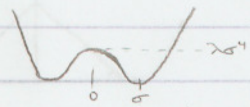
see sources $\sim 2\,mJy$ at $850\,\mu m$



$\nu^{-1} \to 0$ K-correction

Inflation and Spontaneous Symmetry Breaking

$$V(\phi_1, \phi_2) = \lambda\left[\sigma^2 - (\phi_1^2 + \phi_2^2)\right]^2$$



energy density $\qquad \hbar = c = 1 \qquad \frac{1}{L} \sim k \qquad \hbar k \sim p \sim mv \qquad \left[\frac{1}{L}\right] \sim [m]$

$\left[\frac{erg}{cm^3}\right] \to [m^4] \leftarrow [E] = [m] \qquad$ length $[L] = [m^{-1}]$

$$[\lambda] = 1 \qquad [\phi] = [m]$$

$\partial_\mu \phi\, \partial^\mu \phi$ kinetic energy term $\sim \left(\frac{\partial x}{\partial t} \cdot \frac{\partial x}{\partial t}\right) = v^2$

$[(m^{-1})^{-1} m \quad (m^{-1})^{-1} m] \to [m^4]$

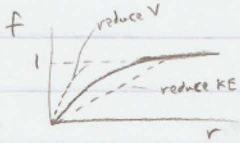
$$\phi_1 = \sigma f(r) \frac{x}{\sqrt{x^2+y^2}} \qquad \phi_2 = \sigma f(r) \frac{y}{\sqrt{x^2+y^2}}$$

spatial variables

cosmic string solution (pointing away)

Energy per unit length (string tension) of cosmic string

$$T = 2\pi \int \Bigg(\underbrace{\sigma^2\left(\left(\frac{\partial f}{\partial r}\right)^2 + \left(\frac{f}{r}\right)^2\right)}_{KE} + \underbrace{\lambda\sigma^4\left(1 - f^2\right)^2}_{V} \Bigg) r\,dr$$



$f(\infty) \to 1 \qquad f(0) \to 0$ to prevent blowing up

$$\frac{\partial f}{\partial r} \sim a\sigma \qquad a^2\sigma^4 + \lambda\sigma^4 \qquad \text{balance with } a \sim \lambda^{1/2}$$

$\sim \lambda^{1/2}\sigma$ length scale of cosmic string: $\quad R \sim \frac{1}{\lambda^{1/2}\sigma}$ radius

$$T \sim \pi R^2 \lambda\sigma^4 \sim \pi \frac{1}{\lambda\sigma^2}\lambda\sigma^4 \sim \pi\sigma^2 \qquad \text{string tension}$$

GUT symmetry breaking $\quad \sigma \sim 10^{15}\,GeV$

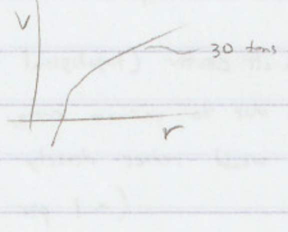
$\lambda = 1.2\,\mu m \Longleftrightarrow 1\,eV \qquad 10^{-4}\; 10^{-24} = 10^{-28}\,cm$ length scale

(10^{-4}: $\mu m \to cm$)

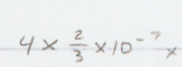
$$\sigma \sim \frac{10^{15}}{6\times10^{23}} = \frac{1}{6}\times10^{-8}\,g \sim 10^{-9}\,g \qquad \frac{10^{-9}}{10^{-28}} = 10^{19}\,g/cm \qquad \text{GUT scale cosmic string}$$

$$T = 10^{40}\,\frac{dyne}{cm} = 10^{31} \text{ metric tons} \longleftarrow \;\times c^2\ (?) \quad \text{or } \sigma^2\,(?)$$

QCD

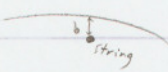


cosmic string is infinitely long



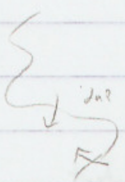
$\theta = \frac{4GM}{bc^2}$

Sun: $\frac{4 \times \frac{2}{3} \times 10^{-7} \times 2 \times 10^{33}}{7 \times 10^{10} \times 9 \times 10^{20}} = \frac{16}{3 \cdot 63} 10^{-4} = \frac{1}{12} 10^{-4} = 1.7''$



$\theta = \frac{8\pi G (M/L)\, b}{bc^2} = \frac{8\pi G \mu}{c^2}$ ... M/L indep of how close you get to string

see: not seen: $\frac{G\mu}{c^2} < 10^{-6}$



So string will move

$\rightarrow$ differential doppler shift of order $\frac{8\pi G\mu}{c^2}\left(\frac{v}{c}\right)$

---

for monopoles: $V = \lambda\left[\sigma^2 - (\phi_1^2 + \phi_2^2 + \phi_3^2)\right]^2$

$\phi_1 \sim \sigma f(r) \frac{x}{r} \quad \phi_2 \sim \sigma f \frac{y}{r} \quad \phi_3 \sim \sigma f \frac{z}{r}$

$M \simeq 4\pi \int \left(\sigma^2\left(\left(\frac{\partial f}{\partial r}\right)^2 + \left(\frac{f}{r}\right)^2\right) + \lambda\sigma^4(1-f^2)^2\right) r^2\, dr$

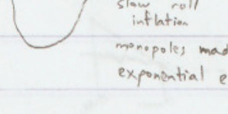
$R \sim \frac{1}{\lambda^{1/2}\sigma} \qquad \frac{4\pi}{3} R^3 \lambda \sigma^4 = \frac{4\pi}{3}\lambda^{-1/2}\sigma = M \quad \sim 10^{16}\ \text{GeV}$

time ? $\quad kT \sim \sigma \quad aT^4 \sim \rho \sim \frac{3H^2}{8\pi G} \sim t^{-2} \quad \therefore\ t \sim T^{-2} \sim \sigma^{-2}$

distance between monopoles at that time: ct (point-like topological defects) $\quad \therefore\ \rho \sim \frac{\sigma}{(ct)^3} \sim \frac{\sigma}{\sigma^{-6}} \sim \sigma^7$

$\rho_0 \sim \sigma^4$ too high $\quad kT = 10^{15}\ \text{GeV} \quad t \sim 10^{-36}\ \text{sec} \quad 10^{78}/cc$

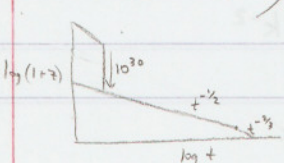
$\rightarrow \Omega_{monopoles} \sim 10^{14}$ too high $\quad \rightarrow 10^{-6}/cc \ \sim 10^{-8}/cc$ Some annihilate



$V' = \frac{\partial V}{\partial \phi} \quad \left|\frac{V'}{V}\right| \rightarrow$ very small

$10^5 : 1 \quad 10^{14} \rightarrow 10^{-1}$

Flatness - Oldness $\quad \rho a^2\left(\frac{1}{\Omega} - 1\right) = \text{const}$ need ρa^2 smaller $\Rightarrow a$ smaller than expected

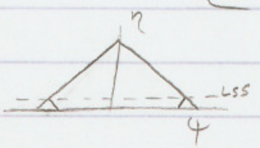


$10^{30} : 1$

Horizon problem

conformal time $\qquad \eta = \int \frac{dt}{a} \qquad D_{now} = \int \frac{c\,dt}{a}$

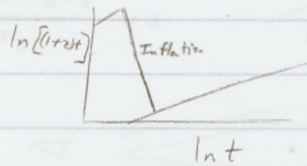
$$ds^2 = a^2 \left[d\eta^2 - \left(d\psi^2 + \begin{pmatrix} \sin \\ \sinh \end{pmatrix} \psi \right)^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right]$$



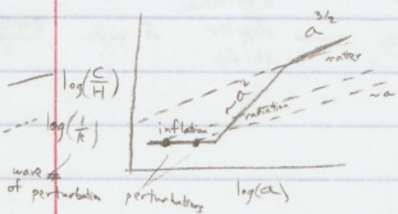
$\eta_{LSS} << \eta_0$ expect sky to be uniform in patches of $2°$ (?)

(last scattering surface)

$$\int \frac{dt}{a} = \int (1+z)\,dt = \int (1+z)\,t\, d(\ln t)$$



Quantum Fluctuations → megasized → large scale structure (LSS)



fluctuations get very large

pressure/density gradients not important

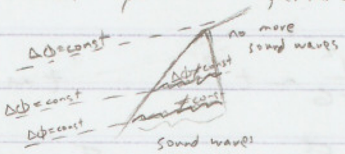
→ not enough time to smooth out perturbations

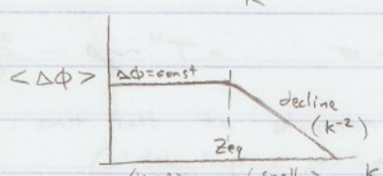
perturbations are ~indep universes

$$\rho a^2 \left(\frac{1}{\Omega} - 1 \right) = \text{const}$$

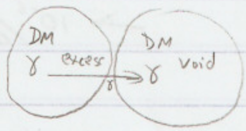
$$-\rho a^2 \frac{\Delta\Omega}{\Omega} = -\rho a^2 \frac{\Delta\rho}{\rho_{crit}} \qquad (\Delta\rho) a^2 = \text{const}$$

gravitational potential perturbation $\qquad \Delta\phi = \frac{G\Delta m}{R} = \frac{\frac{4\pi}{3} G \Delta\rho R^3}{R} = \frac{4\pi G}{3} (\Delta\rho) R^2$





$\Delta\phi = \text{const} \qquad \frac{\Delta\rho}{\rho} \propto L^{-2}$ homogeneous on large scales



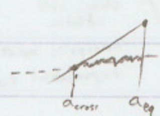
photons move over, DM contrast doesnt grow (until matter dominates)

dark matter

$H^2 \propto \rho \propto T^4 \propto a^{-4}$

comoving k_{CM} $\qquad$ Length Scale $\quad L = \frac{a_{cross}}{k_{cm}} = \frac{c}{H} \propto a^2_{cross}$

$a_{cross} \sim \frac{1}{k}$ $\qquad$ Lose growth from a_{cross} to a_{eq}



radiation dominated $\qquad \frac{\Delta\rho}{\rho} \propto a^2 = \left(\frac{a_{cross}}{a_{eq}} \right)^2 \qquad \therefore$ decline is k^{-2}

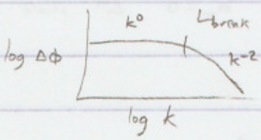
Final Exam June 15 9–12

Inhomogeneities

Length scale of perturbations $L = \frac{a_{cross}}{k_{cm}} = \frac{c}{H} \propto a_{cross}^2$ $\qquad a_{cross} \propto \frac{1}{k_{cm}}$

Factor of lost growth $\qquad$ radiation dominated $\qquad$ growing mode $\sim a^2$

$\left(\frac{a_{cross}}{a_{eq}}\right)^2 \sim k_{cm}^{-2}$



a_{eq}

$\rho_m = \Omega_m h^2$ 18.8 yoctograms/m^3 $\qquad \rho_r = \Omega_r h^2$ 18.8 yg/m^3

$$a_{eq} = \frac{\Omega_m h^2}{\Omega_r h^2} = \frac{0.27 \times 0.5}{4.165 \times 10^{-5}} \simeq (3300)^{-1}$$

$$\rho = \rho_0 \left(0.27 \times (1+z_{eq})^3 + 0.73 + 4.165 \times 10^{-5} \times (1+z_{eq})^4\right)$$

$\rho_m = \rho_r \quad \rho_\Lambda \simeq 0$

$$= \rho_0 \,(0.27)\,(1+z_{eq})^3\, 2 = \rho_0\, 0.02\ 10^{12} = 2 \times 10^{10} \rho_0$$

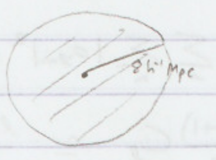
$H = 1.4 \times 10^5 \times H_0$ @ a_{eq}

$$\left(\frac{c}{H}\right)(1+z_{eq}) = \frac{c}{H_0} \frac{3300}{1.4 \times 10^5} = \frac{3.3}{140} \frac{c}{H_0} \simeq \frac{1}{42} \frac{c}{H_0}$$

$L_{break} \sim \frac{1}{42} \frac{c}{H_0} \simeq 100$ Mpc

Scales > 100 Mpc $\to k^0$ $\qquad$ scales < 100 Mpc $\to k^{-2}$

Measure of inhomogeneity / anisotropy $\quad \sigma_8$



$\langle\rho\rangle(x,y,z)$

average density in $8 h^{-1}$ Mpc $= R$

$$\left(\frac{\langle\rho\rangle - \bar{\rho}}{\bar{\rho}}\right)^2 = \sigma_8^2$$

$\bar{\rho}$ true average density

$$R_{ls} = \int_{a_{ls}}^{1} \frac{c\, da}{a \dot{a}} = 3.3 \frac{c}{H_0} \qquad \frac{100 \text{ Mpc}}{13000 \text{ Mpc}} \simeq \frac{1}{130} \text{ rad}$$

expect a change in CMB at this scale of order $l \sim 220$?
closer to $l \sim 120$

COBE $\quad l < 20 \quad$ (@ k^0 part)

$$\frac{\Delta T}{T} = \frac{1}{3} \frac{\Delta\Phi}{c^2} \qquad g_{00} = 1 + \frac{2\Delta\Phi}{c^2} \qquad \frac{\Delta\tau}{\Delta t} = 1 + \frac{\Delta\Phi}{c^2}$$

proper time run faster when grav. potential is faster

$$\frac{\Delta T}{T} = -\frac{\Delta a}{a} = \frac{2}{3}\left(\frac{\Delta\tau}{\Delta t}\right)$$

$a \propto t^{2/3}$

Standard grav redshift $\underset{(\text{minus})}{-}$ this effect

gives you $\quad \frac{\Delta T}{T} = \frac{1}{3} \frac{\Delta\Phi}{c^2}$

expect. $\frac{\Delta T}{T} = \frac{\Delta\Phi}{c^2}$

Gauge problem in General Relativity → freedom to change coordinates
(like in E&M we can add gradient of scalar to potential without changing anything)

equal power on all scales $\frac{\Delta T}{T}$ indep. of ℓ

$$\frac{\Delta T(\theta,\varphi)}{T} = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\varphi)$$

$a_{00}=0$ ΔT should cancel out over all sky

rotation $a_{\ell' m'} = \sum_{\ell' m' \ell m} D_{\ell\ell' m m'}\, a_{\ell m}\, \delta_{\ell\ell'}$ you mix up m, m' but not ℓ's

$D_{\ell m m'}(\alpha,\beta,\gamma)$ $\sum |a_{\ell m'}|^2 = \sum |a_{\ell m}|^2$

$$C_\ell = \langle |a_{\ell m}|^2 \rangle$$

$\overbrace{(+)(-)}^{2\theta}$ ...... $\frac{2\pi}{2\theta}$ waves around sky $=\ell$ $\therefore\ \ell \simeq \frac{\pi}{\theta}$

$\ell = 220$ $\theta = \frac{180^\circ}{220} = 0.8^\circ$ peak of CMB power spectrum

we care about $\underline{\Delta\ell \propto \ell}$ $\ell=220 \to \theta = 0.8^\circ$, $\ell = 221 \to \theta = 0.798^\circ$ there is spread when going to ℓ-space

what is variance of T on sky?

$$\left|\frac{\Delta T}{T}\right|^2 = \left\langle \sum_\ell \sum_m \sum_{\ell'} \sum_{m'} a_{\ell m} a^*_{\ell' m'} Y_{\ell m} Y^*_{\ell' m'} \right\rangle_{4\pi}$$ ⇝ averaging over 4π

$$\int Y_{\ell m} Y^*_{\ell' m'}\, d\Omega = \delta_{\ell\ell'}\,\delta_{mm'}$$

$$\left\langle\left|\frac{\Delta T}{T}\right|^2\right\rangle = \frac{1}{4\pi}\sum_\ell\sum_m |a_{\ell m}|^2 = \frac{1}{4\pi}\sum_\ell (2\ell+1) C_\ell$$ using $\Delta\ell = \ell$

$$\left\langle\left|\frac{\Delta T}{T}\right|^2\right\rangle = \frac{\ell(2\ell+1)}{4\pi} C_\ell \simeq \frac{\ell(\ell+1)}{2\pi} C_\ell$$

with equal power on all k scales: $C_\ell \propto \frac{1}{\ell(\ell+1)}$

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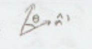
2 directions, averaged so 

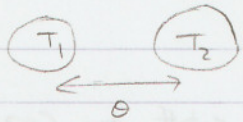
Angular Correlation Function $$C(\theta) = \left.\frac{\langle \Delta T(\hat n)\,\Delta T(\hat n')\rangle}{T_0^2}\right|_{\angle \hat n \hat n' = \theta}$$

$\theta = 0$

$C(0) = \left\langle\left(\frac{\Delta T}{T}\right)^2\right\rangle$ this is the variance

$$\left.\iint Y_{\ell m}(\theta,\varphi)\, Y^*_{\ell m}(\theta',\varphi')\, d\Omega\, d\Omega'\right|_{\angle\Omega\Omega'=\theta} = P_\ell(\cos\theta)$$ $P_\ell(1) = 1$

$$C(\theta) = \frac{1}{4\pi}\sum_\ell (2\ell+1)\, C_\ell\, P_\ell(\cos\theta)$$



$$\left\langle \left| \frac{T_1 - T_2}{T_0} \right|^2 \right\rangle = \left(\frac{\Delta T_1}{T}\right)^2 + \left(\frac{\Delta T_2}{T}\right)^2 - \frac{2\Delta T_1 \Delta T_2}{T^2}$$

$C(\theta)$
$\downarrow$
this is an older CMB studying method

$$= 2\left[C(0) - C(\theta)\right]$$

Beam with some profile

$\rightarrow$ averages out

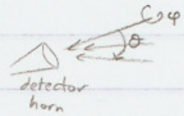
consider sky <u>flat</u> next to the beam

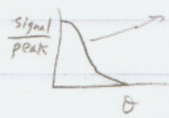
$e^{ik\theta}$ wavelength on sky $\frac{2\pi}{k}$ 2π radians in sky with ℓ waves $\rightarrow k=\ell$ $\rightarrow e^{i\ell\theta}$

integrate over beam function (flat sky)

$$B_\ell = \frac{\iint e^{i\ell\theta} B(\theta,\varphi)\, d\theta\, d\varphi}{\iint B(\theta,\varphi)\, d\theta\, d\varphi}$$

$B_\ell = 1$ for $\ell = 0$
$B_\ell \rightarrow$ small for large ℓ





$\rightarrow$ this is $B(\theta,\varphi)$ not exactly a gaussian

Assume gaussian

$$B(\theta,\varphi) = \exp\left[-\frac{1}{2\sigma^2}(\theta^2 + \varphi^2)\right]$$

$$B_\ell = \frac{\iint \cancel{\exp\left(\frac{-\varphi^2}{2\sigma^2}\right)} \exp\left(\frac{-\theta^2}{2\sigma^2} + i\ell\theta\right) d\theta\, d\varphi}{\iint \cancel{\exp\left(\frac{-\varphi^2}{2\sigma^2}\right)} \exp\left(\frac{-\theta^2}{2\sigma^2}\right) d\theta\, d\varphi}$$

$$-\frac{1}{2}\left(\left(\frac{\theta}{\sigma}\right)^2 - 2i\ell\sigma\frac{\theta}{\sigma}\right) = -\frac{1}{2}\left(\frac{\theta}{\sigma} - i\ell\sigma\right)^2$$
$-\ell^2\sigma^2$
$-\frac{1}{2}\ell^2\sigma^2$ $\quad -\frac{1}{2}\ell^2\sigma^2$

$\theta' = \theta - i\ell\sigma^2 \quad \rightarrow \quad B_\ell = \underbrace{e^{-\frac{1}{2}\ell^2\sigma^2}}_{\text{flat sky}} = \underbrace{e^{-\frac{1}{2}\ell(\ell+1)\sigma^2}}_{\text{real sky(?)}}$

$a_{\ell m}^{obs} = a_{\ell m}^{true} B_\ell \qquad C_\ell^{obs} = B_\ell^2 C_\ell^{true}$

$C^{obs}(\theta) = \frac{1}{4\pi}\sum_\ell (2\ell+1) C_\ell B_\ell^2 P_\ell(\cos\theta)$ $\qquad \ell \simeq \frac{1}{\sigma} \rightarrow$ ability to measure stuff is cutoff

$B = e^{-\frac{1}{2}\frac{\theta^2}{\sigma^2}} = \frac{1}{2} \qquad \frac{1}{2}\theta^2/\sigma^2 = \ln 2 \qquad \theta = \sqrt{2\ln 2}\,\sigma \qquad \text{FWHM} = 2\theta = \sqrt{8\ln 2}\,\sigma$

$\sigma = \frac{\text{FWHM}}{\sqrt{8\ln 2}} \rightarrow 2.36$

COBE $FWHM = 7^\circ = \frac{1}{8}$ radian

$\sigma = \frac{1}{2.36}\frac{1}{8} = \frac{1}{19}$ rad $\quad \ell \simeq 19$ is where you get cutoff for COBE

WMAP $FWHM = 13' \rightarrow 4.5 \times 7 \times 19 = 31 \times 19 \simeq 500 \rightarrow \ell$

bands L S C X K Q V W

K: 23; Ku Ka: 33; Q: 41; V: 61; W: 94 GHz

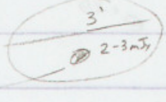
22 → water line; oxygen line

WMAP publishes out to $\ell \sim 1000$

but B_ℓ not gaussian and can be corrected by looking at point source (Jupiter)

5/21/2007

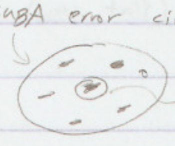
SCUBA



13'' beam size

$$\sigma_{position} \simeq \frac{\sigma_{beam}}{SNR} \simeq \frac{FWHM}{2 \times SNR}$$ → signal to noise ratio

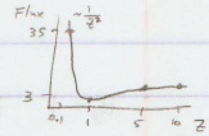
SCUBA error circle



in VLA

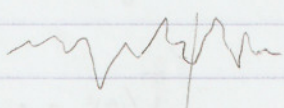
→ cm radio source

median redshift ~ 2.4



Inhomogeneity $\qquad$ Density Fluctuations

random process

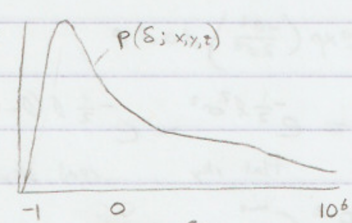


random function from bag of function $\rho(x,y,z)$

1 point x,y,z what is probability density function? PDF

→ know, draw multiple functions from bag

$$\delta = \frac{\rho(x,y,z)}{\bar{\rho}} - 1$$



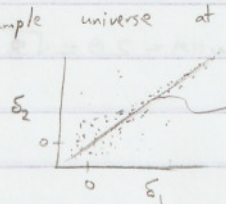
not a Gaussian random process

$$1 = \int p(\delta; x,y,z)\, d\delta$$

$p(\delta; x,y,z) = p(\delta)$ → stationary random process ⟵ homogeneous (this is what we mean when we say that)

PDF indep of position

Sample universe at 2 points



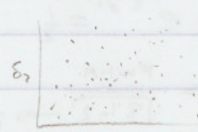
if 2 points close together

very far apart:

↓

independence

$p(\delta_1 \delta_2) = p(\delta_1)\, p(\delta_2)$ $\quad$ (as $|\vec{x}_1 - \vec{x}_2| \rightarrow \infty$)



(in general does not mean uncorrelated)

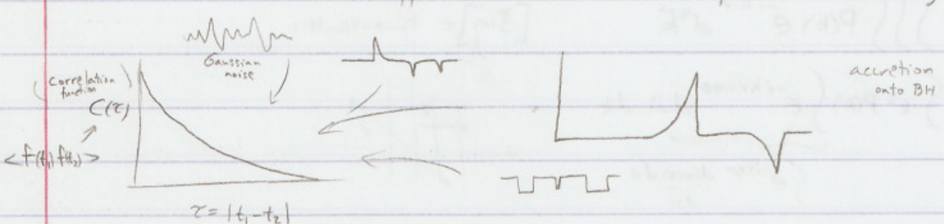
for Gaussian uncorrelated → independent

density far away shouldn't depend on density here (independent)

$p(\delta_1, \delta_2; \vec{x}_1, \vec{x}_2) = p(\delta_1, \delta_2; |\vec{x}_1 - \vec{x}_2|)$ stationary, isotropic

Homogeneous + Isotropic: density fluctuations are an isotropic stationary random process

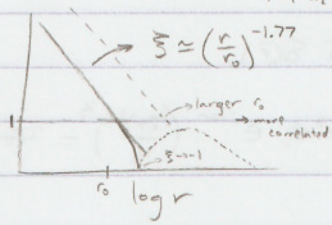
if δ is small ~ can approximate with Gaussian, but δ is generally large

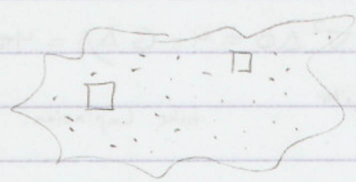


same correlation function if process is Gaussian, correl. funct gives you all info

cross product function

$\langle \delta(\vec{x}_1)\, \delta(\vec{x}_2) \rangle = \xi(r)$ $\hookrightarrow |\vec{x}_1 - \vec{x}_2|$ can measure density at various places and calculate ξ





n points

$\frac{n(n-1)}{2}$ pairs

can sort them in bins by radius $DD(r)$ # of pairs with r separation

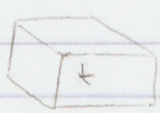
Monte Carlo a bunch of Poisson random points: $RR(r)$ (# pairs)

typical (but not optimum): $\xi(r) = \frac{DD(r)}{RR(r)} - 1$

$r_0 = 5\, h^{-1}\, \text{Mpc}$ typically

$\langle \delta_G(\vec{x}_1)\, \delta_Q(\vec{x}_2) \rangle = \xi_{GQ}(r)$ — galaxy – quasar correlation function

galaxy – quasar

Consider box  V_u $\delta(\vec{r}) = \frac{\Delta \rho}{\bar{\rho}} = \frac{(2\pi)^{3/2}}{V_u^{1/2}} \sum \delta_k e^{i \vec{k} \cdot \vec{r}}$ periodic 'tiles' of universe

variance $\text{var}(\delta(0)) = \frac{(2\pi)^3}{V_u} \sum |\delta_k|^2$ Plane waves e^{ikr}

units $[\delta(\vec{x})] = $ dimensionless $[\delta_k] = L^{3/2}$

$$\xi(\vec{r}) = \langle \delta(\vec{r}')\, \delta(\vec{r}' + \vec{r})^* \rangle = \frac{(2\pi)^3}{V_u} \sum_k \sum_{k'} \delta_k \delta_{k'}^* \langle e^{i \vec{k} \cdot \vec{r}' - i \vec{k}' \cdot (\vec{r}' + \vec{r})} \rangle$$

— need $k = k'$ $\langle e^{ikx} \rangle = 0$ unless $k = 0$

average over the box $(\vec{r}')$

$$= \frac{(2\pi)^3}{V_u} \sum_k |\delta_k|^2 e^{-i \vec{k} \cdot \vec{r}}$$

technically $\vec{k}$

$P(k) = \langle |\delta_{\vec{k}}|^2 \rangle \qquad [P(k)] = L^3$

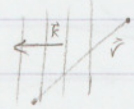
$\Delta k_x \underbrace{L_x}_{\text{box length}} = 2\pi \qquad \Delta k_y L_y = 2\pi \qquad \Delta k_z L_z = 2\pi \qquad \Delta k_x \Delta k_y \Delta k_z = \dfrac{(2\pi)^3}{L_x L_y L_z}$

volume in k-space

$\therefore V_u \to \infty \Rightarrow$ turns to integral

$$\xi(\vec{r}) = \iiint P(k)\, e^{-i\vec{k}\cdot\vec{r}}\, d^3\vec{k} \qquad [\xi(r)] = \text{Dimensionless}$$

$$= \int k^2 P(k) \underbrace{\int e^{-ikr\cos\theta}\, d\Omega}_{\int e^{-ikr\mu}\, \frac{d(\cos\theta)}{d\mu}\, d\varphi} dk$$



$2\pi\, 2\, \dfrac{\sin(kr)}{kr} \qquad \dfrac{\sin x}{x} = j_0(x)$ or $\mathrm{sinc}(x)$

$$\xi(\vec{r}) = 4\pi \int k^2 P(k)\, \frac{\sin(kr)}{kr}\, dk \qquad \xi(r) \xleftrightarrow[\text{Fourier Transform}]{} P(k)$$

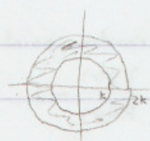
Fluctuations of Potential

$\nabla^2 \phi = 4\pi G \rho \qquad \nabla^2 \Delta\phi = 4\pi G \Delta\rho = 4\pi G \bar{\rho}\, \delta(\vec{r})$

$\Delta\phi = \dfrac{(2\pi)^{3/2}}{V_u^{1/2}} \sum_k \phi_k\, e^{ikr}$ take Laplacian: $-k^2 \phi_k\, e^{ikr} \left(\dfrac{(2\pi)^3}{V_u}\right)^{1/2} = 4\pi G \bar{\rho} \left(\dfrac{(2\pi)^3}{V_u}\right)^{1/2} \delta_k\, e^{ikr}$

$\phi_k = -\dfrac{4\pi G \bar{\rho}}{k^2}\, \delta_k$

Constant power on all scales :



$k^3 P_\phi(k) = \text{const}$ for all k

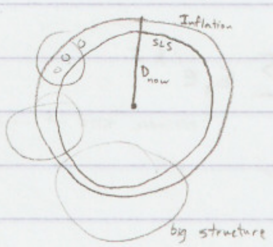
$\Rightarrow$ equal power on all scales (EPAS)

$P_\phi(k) = \langle |\phi_k|^2 \rangle$ power spectrum

$P_\phi(k) = \dfrac{16\pi^2 G \bar{\rho}^2}{k^4} P(k) \qquad \therefore$ need $P(k) = A k^1$ $\quad n=1$

WMAP: $n = 0.951 \pm 0.017$

COBA: $n = 1.2 \pm 0.3$

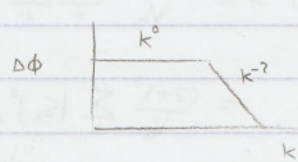


$D_{now} = \displaystyle\int \frac{c\, da}{a \dot{a}}$

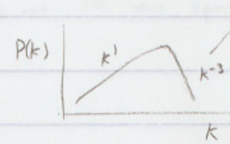
$\dfrac{\Delta T}{T} = \dfrac{1}{3} \dfrac{\Delta\phi}{c^2} \qquad \Delta T_\ell = \text{indep of } \ell \text{ if } n=1$

Potential pertubations dont evolve (sound waves ?)

Density perturbations do



$\Delta\phi = k^{3/2} |\phi_k|$



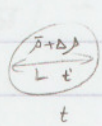
$\langle \delta \rangle = \text{indep of } k$

Growth function for linear density perturbations 5/23/2007

Linear Growth Function

$$\frac{\Delta\rho}{\rho} = \delta(x) = \frac{(2\pi)^{3/2}}{V_u^{1/2}} \sum \underbrace{\delta_k}_{\text{Gaussian random}} e^{i\vec{k}\cdot\vec{x}} \qquad \text{Sum of plane waves}$$

if ? uncorrelated, and Independent of direction – isotropic Gaussian random process



$L > \frac{c_s}{H}$ $\underbrace{\text{pressure gradients}}_{\text{this can cause expansion, pressure itself cannot}}$ not important

↳ can slow down expansion in GR (acceleration eqn)

$$\dot{a} = H_0\sqrt{X} \qquad X = \frac{\Omega_m}{a} + \Omega_v a^2 + \frac{\Omega_r}{a^2} + \Omega_k$$

$$t = \int_0^{a_{final}} \frac{da}{\dot{a}} \qquad t' = t + (a'-a)\frac{1}{H_0\sqrt{X}} - \underbrace{(\Omega_k' - \Omega_k)}_{\text{energy term}} \int_0^a \frac{1}{H_0}\left(-\frac{1}{2}\right)\frac{da}{X^{3/2}}$$ Taylor expansion?

evolving to common time so $t' = t$

$$\therefore \quad \Delta a \propto \sqrt{X} \int_0^a \frac{da}{X^{3/2}} \qquad \frac{\Delta\rho}{\bar{\rho}} = \frac{3\,\Delta a}{a} \quad \text{volume change}$$

Linear Growth Function: $D(a) \propto \frac{\sqrt{X}}{a}\int_0^a \frac{da}{X^{3/2}}$ Normalized to 1 now

$\Omega_m = 1 \quad \Omega_v = \Omega_r = \Omega_k = 0$

$$X = \frac{1}{a} \qquad D(a) = a^{-3/2}\int_0^a a^{3/2}\,da = \frac{2}{5}a^{5/2}a^{-3/2} = \frac{2}{5}a$$

$\therefore$ growth of perturbations $\propto a$

$\Omega_k = 1$

$X = 1 \quad D(a) = \frac{1}{a}\int_0^a da = \frac{a}{a} = \text{const}$ no growth in perturbations

perturbations are frozen in

$\Omega_v = 1$

$X = a^2 \quad D(a) = \frac{a}{a}\int_0^a a^{-3}\,da$ Diverges

$\frac{\dot{D}}{D} \simeq H\,\Omega_m^{0.6}$ an approximation

when $\Omega_m \ll 1$ growth slows down

$\bar{\rho}_0 = \rho_{crit,0}(\Omega_{m_0} + \Omega_{v_0})$ $\qquad$ $\bar{\rho} = \rho_{crit,0}\left(\frac{\Omega_{m_0}}{a^3} + \Omega_{v_0}\right) = \rho_{crit}(\Omega_m + \Omega_v)$

$$\Omega_m = \frac{\Omega_{m0}/a^3}{(\Omega_{m0}/a^3 + \Omega_{v0})}\Omega_{tot} \qquad \rho a^2\left(\frac{1}{\Omega_{tot}} - 1\right) = \text{const}$$

$$\frac{1}{2} = \frac{\Omega_{m0}/a^3}{\Omega_{m0}/a^3 + \Omega_{v0}}$$ → matter is half of total ($\Omega_m = \Omega_v$)

$$\frac{1}{2}\Omega_{v0} = \frac{1}{2}\Omega_{m0}/a^3 \qquad a = \left(\frac{\Omega_{m0}}{\Omega_{v0}}\right)^{1/3} \sim \left(\frac{1/4}{3/4}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3} \simeq 0.7$$

not so long ago

after this point we are vacuum dominated $\qquad Z \sim 1/2$

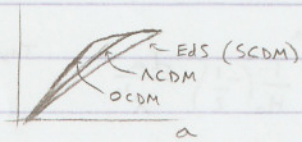
open universe (OCDM) $\quad \left(\frac{1}{\Omega_{m0}} - 1\right) = a^{-1}\left(\frac{1}{\Omega_m} - 1\right) \qquad \rho a^2 \propto a^{-1}$

$\Omega_m = 1/2 \qquad \frac{1}{\Omega_m} - 1 = 1$

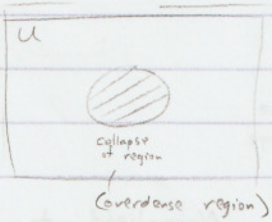
$= a^{-1}$

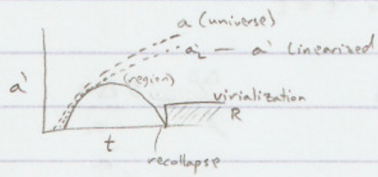
$\therefore \quad a = \dfrac{1}{\frac{1}{\Omega_{m0}} - 1}$ $\qquad$ if $\Omega_{m0} \sim 1/4 \qquad a \sim \frac{1}{3}$

$\to z \sim 2$



Press-Schecter Method

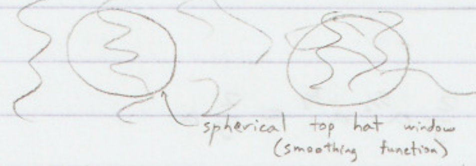




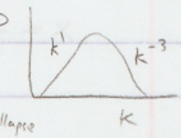
δ_c linearized density contrast @ collapse

$$\delta_c = \left(\frac{a}{a_L}\right)^3 - 1 \qquad \delta_v = \left(\frac{a}{R}\right)^3 - 1$$

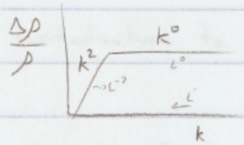
virialized radius



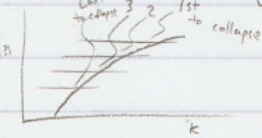
→ $\delta > \delta_c$ → collapse into cluster



$$\left(\frac{\Delta\rho}{\rho}\right)_k \propto \left(k^3 P(k)\right)^{1/2}$$



more accurate (?) ⟶



Bottom-Up Collapse
→ smallest scales form/collapse first

$$r = \frac{r_{max}}{2}(1 - \cos\eta)$$

$\eta = 0 \quad r = 0 \quad t = 0$
$\eta = 2\pi \quad r = 0 \quad t = t_c$ collapse

$\Omega_m = 1$

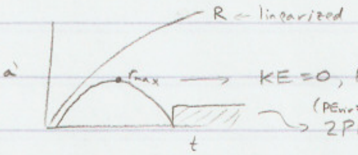
equivalent eccentric anomaly (E) → conformal time $\qquad d\eta = \frac{dt}{a}$

$$t = \frac{t_c}{2\pi}(\eta - \sin\eta) \qquad M = E - e\sin E$$

mean anomaly

$$r \simeq \frac{r_{max}}{4}\eta^2 \qquad t \simeq \frac{t_c}{2\pi}\frac{1}{6}\eta^3 \qquad R = \frac{r_{max}}{4}\left(\frac{12\pi}{t_c}\right)^{2/3} t^{2/3}$$

linearized
→ no collapse



$\longrightarrow$ KE = 0, PE

$\rightsquigarrow \overset{(PE_{vir}=)}{2PE} \rightarrow KE = -PE \qquad KE = -\frac{1}{2}(PE)_{vir}$ virial equation when $PE \rightarrow 2PE$

$$r_{vir} = \frac{1}{2} r_{max}$$

$$\left(\frac{R(t_c)}{\frac{1}{2}r_{max}}\right)^3 = \left(\frac{1}{4}\cancel{r_{max}}(12\pi)^{2/3}\frac{1}{\frac{1}{2}\cancel{r_{max}}}\right)^3 = \frac{1}{8}(12\pi)^2 = 18\pi^2 \simeq \underline{177}$$

in papers

$\rho > 177\rho_{universe} \rightarrow$ collapse

any thing with density contrast larger than this has collapsed

$$\rho_{in} \propto r^{-3} \qquad \rho_{out} \propto a^{-3} \propto t^{-2}$$

$$\frac{\rho_{in}}{\rho_{out}} \propto \frac{t^2}{r^3} = \frac{(\eta - \sin\eta)^2}{(1-\cos\eta)^3} = \frac{\left(\frac{1}{6}\eta^3 - \frac{1}{120}\eta^5 + \dots\right)^2}{\left(\frac{1}{2}\eta^2 - \frac{1}{24}\eta^4 + \dots\right)^3} = \frac{\eta^6}{36}\frac{8}{\eta^6}\left(1 - \frac{1}{10}\eta^2 + \frac{1}{4}\eta^2 + \dots\right)$$

$$= \frac{1}{4.5}\left(1 + \underbrace{\frac{3}{20}\eta^2}_{\text{linear growth function}} + \dots\right)$$

$D = \frac{3}{20}\eta^2 \propto a$ as expected $(\Omega_m = 1)$

$$D = \frac{3}{20}\left(\frac{12\pi t}{t_c}\right)^{2/3} \qquad @\ t_c: \quad D = 0.15\,(12\pi)^{2/3} = 1.68647\dots = \delta_c$$

$\delta_c \simeq 1.69$ if evolving linearly, but actually $\underline{\delta_c \simeq 177}$

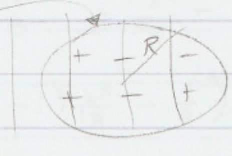
non linear

This has been Press-Schecter Method

Correlation function $\xi(r) = \int P(k)e^{-ikr}d^3r = 4\pi\int k^2 P(k)\frac{\sin kr}{kr}dk$

covariance @ separation 0 is variance

Smoothing



$$\delta_k^{sm} = W_R(k)\,\delta_k$$

smoothing window

$\mu = \cos\theta$

$$\frac{1}{\frac{4\pi}{3}R^3}\int_{r<R} e^{i\vec{k}\cdot\vec{x}}d^3\vec{x} = W_R = \frac{3}{4\pi R^3}\int_0^{2\pi}\int_{-1}^{1}\int_0^R r^2\,d\mu\,dr\,d\phi\; e^{ikr\mu}$$

$$\frac{3}{4\pi R^3}\int_0^R (2\pi)\,2r^2\frac{\sin kr}{kr}dr = \frac{3}{R^3}\int_0^R r\frac{\sin(kr)}{k}dr = \frac{3\left[\sin(kR) - kR\cos(kR)\right]}{(kR)^3} = W_R(k)$$

$$W_R(k) = 1 - \frac{(kR)^2}{10} + \dots$$

variance $\sigma_R^2 = 4\pi\int P(k)\,|W_R(k)|^2 k^2\,dk \quad \leftarrow$ a formula for σ_8

$\frac{\sin(kr)}{kr} = 1$ for $r = 0$ (to get variance from ξ)

$\sigma_8 \sim 1 < 1.69$ 1 standard deviation $\Rightarrow$ collapse — most things in U collapsed

$z = 1$ $\quad$ .3 standard deviation $\Rightarrow$ collapse (EdS) but in ΛCDM (or OCDM) dont need 3 standard deviations

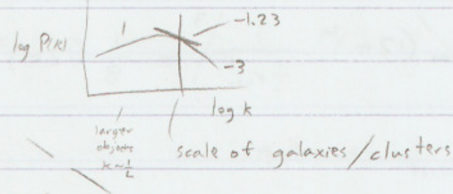
$\therefore$ more clusters forming long ago

$\sigma_{25} \sim \frac{1}{3}$

superclusters
$\rightarrow$ not collapsed

empirically $\xi(r) \propto r^{-1.77}$

$\underbrace{k^3 P(k)}_{k^2 dk \;\; \widehat{k^\alpha}} = k^3 k^\alpha = k^{3+\alpha} \qquad k \sim \frac{1}{r} \qquad r^{-(3+\alpha)} = r^{-1.77} \quad \therefore \; \alpha = -1.23$



Probability that objects of size M or larger have collapsed

$P(>M) \qquad M = \frac{4\pi}{3}\rho_0 R^3 \qquad P(>M) = \frac{1}{\sqrt{2\pi}} \int_{\delta_c/\sigma_k}^{\infty} e^{-\frac{x^2}{2}} dx$

$n(M) = \frac{\rho_0}{M}\left(\frac{\partial P}{\partial M}\right)$

Press-Schecter formula for number density of galaxies

once things collapsed $\rightarrow$ density contrast is 177, this keeps growing as universe expands. If now you see density contrast $(8) 177$ then $8 = 2^3 = a^3 \quad \therefore \; z = 1$ object formed approx. at this z

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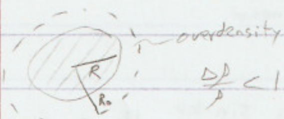
5/30/2007

Peculiar Velocity



$\frac{\Delta\rho}{\rho} < 1 \rightarrow$ can take linearized growth rate $\qquad \frac{\Delta\rho}{\rho} \propto D(t)$

$\frac{\rho}{\bar\rho} = \left(\frac{R_0}{R}\right)^3 = 1 + \frac{\Delta\rho}{\rho}$

$\dot{\left(\frac{R_0}{R}\right)} = \frac{\dot R_0}{R} - \frac{R_0 \dot R}{R^2} = \frac{H R_0}{R} - \frac{R_0(HR_0 - v_{pec})}{R^2}$

$= HR_0\left(\frac{1}{R} - \frac{R_0}{R^2}\left(1 - \frac{v_{pec}}{HR_0}\right)\right) = \frac{HR_0}{R}\left(1 - \frac{R_0}{R}\left(1 - \frac{v_{pec}}{HR_0}\right)\right)$

$\qquad H\left(1+\frac{\Delta\rho}{\rho}\right)^{1/3} \qquad \left(1+\frac{\Delta\rho}{\rho}\right)^{1/3}$

first order (?)

$H\left(-\left(\frac{\Delta\rho}{\rho}\right)^{1/3} - \frac{v_{pec}}{HR_0}\right)$

$\frac{1}{3} H R \frac{\Delta\rho}{\rho} \Omega_m^{0.6} = v_{pec}$

$\frac{\Delta n_g}{\bar n_g} = b \frac{\Delta\rho}{\rho} \qquad$ if $n_g \propto \rho^2 \Rightarrow b^2$

(binary collision process)

$\uparrow$ this is what we know (# gal.)

$\frac{\Omega_m^{0.6}}{b} = \beta$

$\frac{\Delta\rho}{\rho} \sim \sqrt{k^3 P(k)}$

peculiar acc. of MW $\quad g_{pec} = \sum \frac{G\Delta M}{R^2}$ $\quad$ but can't see all distribution of large scale structure (Great Attractor beyond MW)

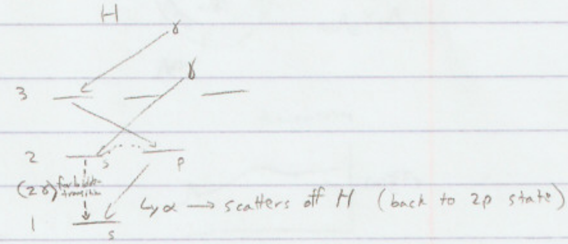
$\vec{g}\,\frac{1}{H} \sim v_{pec}$

can use CMB, to get v_{pec} (Dipole)

Baryon Oscillations

surface of last scattering $\quad z = 1089 \pm 1$

$$\frac{d(\underbrace{nx}_{\#\text{ ions}})}{dt} = -R(T)(nx)^2 \frac{\overbrace{\Lambda}^{\text{transition prob. }(2s\to 1s)\ (?)}}{\Lambda + \underbrace{\beta_u(T)}_{\text{conversion rate }(2p\to 2s)\ (?)}}$$



$$\frac{dt}{dz} = -3.09\left(\Omega_m h^2\right)^{-1/2} z^{-5/2}$$

$$d\left(\frac{1}{n_e}\right) = \alpha(T)\,dt \qquad \underbrace{n_e}_{\#\text{ density}} \propto \underbrace{dt^{-1}}_{\text{time spent}} \qquad \tau \propto c\,\underbrace{n_e \sigma\, dt}_{\text{cancel out}}$$

$$\tau(z) = 0.37\left(\frac{z}{1000}\right)^{14.25} \qquad \tau(1089) \sim 1$$

what is sound speed in baryon + photon fluid?

$$c_s^2 = \frac{dP}{d\rho} \qquad P = \underbrace{P_{baryon}}_{\text{neglibible}} + P_{photon} = \frac{1}{3}aT^4 = \frac{1}{3}aT_0^4(1+z)^4$$

$$\rho = \rho_{baryon} + \rho_{photon} = \left(\Omega_b h^2\right)\left(18.8\times10^{-30}\ \mathrm{g/cm^3}\right)(1+z)^3 + \frac{aT^4}{c^2}$$

$\hookrightarrow \rho_{crit}$ with $h=1$

$$\frac{aT_0^4}{c^2} = \left(\Omega_\gamma h^2\right)\left(18.8\times10^{-30}\right)$$

$$dP = \frac{4}{3}aT_0^4(1+z)^3\Delta z = c^2\frac{4}{3}\left(\Omega_\gamma h^2\right)\left(18.8\times10^{-30}\right)\frac{\Delta z}{1+z}(1+z)^4$$

$$d\rho = 3\left(\Omega_b h^2\right)\left(18.8\times10^{-30}\right)(1+z)^2\Delta z + 4\left(\Omega_\gamma h^2\right)\left(18.8\times10^{-30}\right)(1+z)^3\Delta z$$

$$c_s^2 = \frac{c^2\frac{4}{3}(1+z)^3}{\frac{\Omega_b}{\Omega_\gamma}3(1+z)^2 + 4(1+z)^3} = \frac{c^2}{3}\,\frac{1}{1+\frac{\Omega_b}{\Omega_\gamma}\frac{3}{4}\frac{1}{1+z}} \qquad z\to\infty \quad c_s = \frac{c}{\sqrt{3}}$$

$$\Omega_b h^2 = 0.022 \qquad \Omega_\gamma h^2 = \frac{4.165\times10^{-5}}{1.68} \qquad \frac{3}{4}\,\frac{2.2\times10^{-2}\times1.68}{4.165\times10^{-5}} \simeq \frac{2}{3}\times10^3 = 667$$

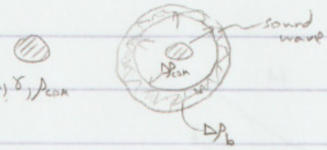
$$c_s = \frac{c}{\sqrt{3}}\left(1+\frac{667}{1+z}\right)^{-1/2}$$

$$D = \int_{BB}^{LS}\frac{c_s\,dt}{a} \qquad = \int_0^{1/1090}\frac{da}{a\dot{a}}\,\frac{c}{\sqrt{3}}\,\frac{1}{\sqrt{1+\frac{3}{4}\frac{\Omega_b}{\Omega_\gamma}\frac{1}{1+z}}}$$

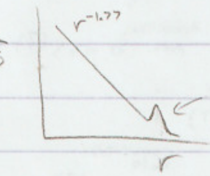
comoving distance that sound waves can travel

$$\Delta D = \int_0^{\frac{1}{1090}} \frac{\frac{c}{\sqrt{3}}\, da}{a H_0 \sqrt{\frac{\Omega_m}{a} + \frac{\Omega_r}{a^2} + \Omega_v a^2 + \Omega_k}\ \sqrt{1 + \frac{3}{4}\frac{\Omega_b}{\Omega_\gamma} a}}$$ sound travel time

$$\simeq \frac{c}{H_0}\frac{1}{\sqrt{3}} \int_0^{\frac{1}{1+z}} \frac{da}{\sqrt{\Omega_m a}} = \frac{c}{H_0}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{\Omega_m}} \int_0^{\frac{1}{1090}} \frac{da}{a^{1/2}} = \frac{c}{\sqrt{3}\ \Omega_m^{1/2} H_0} \frac{2}{\sqrt{1+z_{LS}}}$$



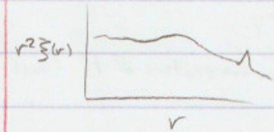
baryons move from dark matter



correlation function of galaxies

want peak @ distance where sound can travel

Measurement:



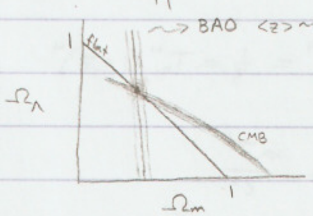
what you measure is velocity: $H_0 D \sim \Omega_m^{-1/2}$ (with $z \sim 0$)

At other z, we calculate distance between galaxies using angular size distance (angular correlation)

physical distance $= aD$ $\quad \theta = \frac{aD}{D_A}$ $\quad D = \int \frac{c\, da}{a\dot{a}}$ $\quad D \to D + \Delta D$ → correlated

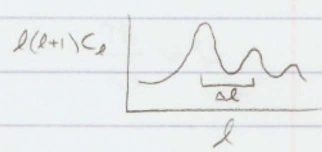
this is a way of measuring $H(z)$ $\quad D = \int \frac{c\, da}{aHa} = \int \frac{c}{H} dz$

$$\Delta D = \frac{\Delta z}{H}$$



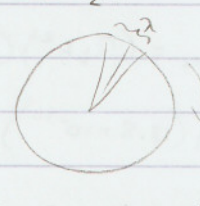
Baryon Acoustic Oscillation (what we have been doing)

Also: $l_a = \Delta l$ $\quad \Delta D = \frac{1}{2}\lambda$



l_a: 1/3 % precision

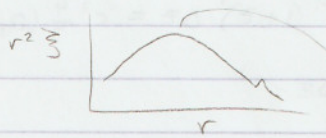
ΔD: 3 %

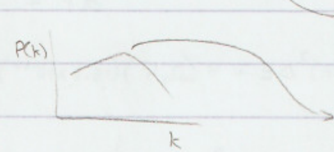


circumference $= 2\pi D_A(1089)(1+z)$ (comoving)

$$l = \frac{\text{circ.}}{\lambda}$$

$$l = \frac{\pi D_A(1089)(1090)}{\Delta D}$$





due to horizon size @ z_{eq}

$\rho_m = \rho_r$

$$D = \int_0^{a_{eq}} \frac{c\, da}{a\dot{a}} \qquad v = H_0 D$$

$H \propto \rho^{1/2}$

$$(\Omega_m h^2)(1+z)^3 = (\Omega_r h^2)(1+z)^4 \qquad 1 + z_{eq} = \frac{\Omega_m h^2}{\Omega_r h^2}$$

$$= \int_0^{a_{eq}} \frac{c}{H_0} \frac{da}{a\sqrt{\underbrace{\frac{\Omega_m}{a}}_{\text{drop}} + \frac{\Omega_r}{a^2}}} = \frac{c}{H_0\sqrt{\Omega_r}} \int_0^{a_{eq}} da = \frac{c}{H_0\sqrt{\Omega_r}} a_{eq}$$

$$v = \frac{c}{\sqrt{\Omega_r}} \frac{\Omega_r h^2}{\Omega_m h^2} \frac{h}{\sqrt{h^2}}$$

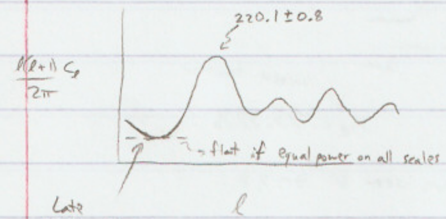
$$\underbrace{v}_{\text{what we measure}} = \frac{c}{\underbrace{\Omega_m h}_{\text{don't know}}} \underbrace{\sqrt{\Omega_r h^2}}_{\text{know}} \sim \frac{1600 \text{ km/s}}{\Omega_m h} \sim 7000 \text{ km/s}$$

$\Gamma = \Omega_m h \qquad v = \frac{c\sqrt{\Omega_r h^2}}{\Gamma}$

$\Gamma = 0.2 \pm 0.015$ numbers quoted 0.22, 0.18 — not put on diagram since we dont know h (10%)

10%

6/4/2007

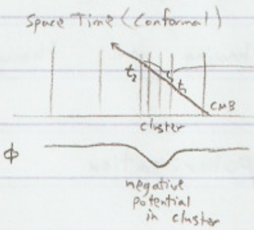


Information from CMB

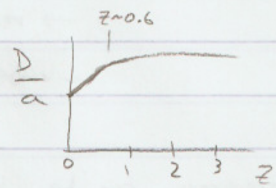
Late Integrates Sachs-Wolfe Effect

(Grav potential perturbation $\phi \simeq$ const $\quad \frac{G\Delta M}{r} \propto \frac{D(t)}{a(t)}$ growth function

in ΛCDM $\quad D(t)$ stops growing when $\Omega_v \simeq \Omega_m$



→ energy: $-\Delta\phi(t_1) + \Delta\phi(t_2)$



$$\Delta\phi\,\Delta\left(\frac{D}{a}\right) = \Delta\phi\left(\dot{\frac{D}{a}}\right) \times \frac{L}{c}$$

length; (transit time through cluster)

$z \sim 0.6$

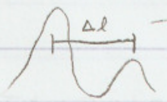
$(1.6)^3 = 4.096 \qquad \rho_m = 4 \cdot \rho_{m0} > \rho_v$ matter dominated

~2400 Mpc

ISW effect stronger for larger scales
→ low l are higher than EPAS

Can be used to study correlation between large scale structure and CMB

Peak comes when grav. potential perturbation + density perturbations are in phase



$\frac{1}{2}\lambda$ accross sky for sound wave

$\Delta l \simeq 300 \qquad 1.2°$ for λ

want sound to have travel only half $\quad 0.6°\, D_A(z_{LS})(1+z_{LS}) = \int_0^{a_{LS}} c_s \frac{da}{a\dot{a}}$

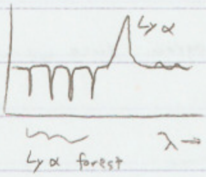
distance sound traveled

Heights of Peaks / Valleys determined by $\frac{\Omega_B}{\Omega_m}$

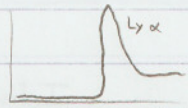
Low l flat plateau gives potential perturbation during inflation

Overall slope gives n in $P(k) \propto k^n \qquad n = 0.951 \pm 0.017$

$n=1 \Rightarrow$ EPAS during inflation



if U were neutral:
(1 neutral in 10^4 or more)



Gunn-Peterson Trough

ionized fraction $x_e < 99.99\%$

seen at $z \sim 6$

no Quasars seen @ $z \sim 7, 8, \ldots$

Column density of e^-? Optical Depth?

Total optical depth can be determined from CMB

e^- @ $z \sim 10$ scatter/blur light from CMB

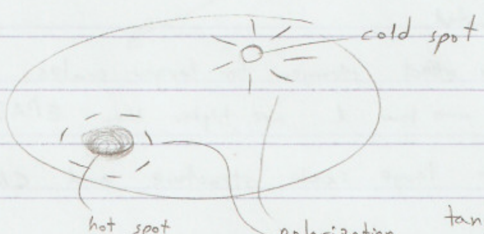
→ reduces T fluctuations, suppression at high ℓ's $e^{-2\tau}$

~ constant reduction for most ℓ not like having $n<1$, though close

→ n-τ degeneracy

Degeneracy can be broken since e^- scattering produces polarization

→ want to look for polarization



correlation
tangential – hot spot
radial – cold spot

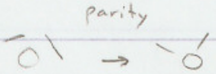
spin 2 rotate 180° get back same thing

| polarization 90° = polarization 270°

↳ spin spherical harmonics

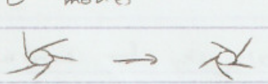
E modes

parity



(?)

B modes



(?)

e^- scattering can only produce E mode polarization $\sim 0.1 \times \tau \times \Delta T$

DAISY in south pole tried to measure this ($\ell \sim 100$)

found $\tau = 100\%$ at LS, U was ionized back then; this was known

WMAP:

$\langle \ell(\ell+1) C_{EE} \rangle = 0.089 \pm 0.03\ \mu K^2$ amplitude is 10% compared to angular power

$\tau \sim 10\%$ for $\ell \sim 5$

CMB allows us to measure things after inflation ended

C_{BB} → B mode polarization (just as difficult to measure as E mode)
↳ produced during inflation by QM fluctuations
(not e^- scattering)

From CMB: A, n, $\frac{\Omega_B}{\Omega_m}$, ℓ_A, τ, $\frac{\Omega_\gamma}{\Omega_m}$ 6 parameters

amplitude

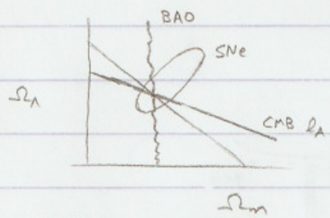
Ω_γ important at this time

Early ISW Effect : photons fly off which affects potential perturbations

$\Omega_\gamma h^2$ well known to 0.1%

$\Omega_B h^2$ to ~ 5-6% $\Omega_{CDM} h^2$ to ~ 7-8% Don't get Ω_K from CMB

cold dark matter



Galaxy Cluster Models

$\rho(r)$, Φ $\nabla^2\Phi = 4\pi G\rho$ $\Phi = -\int \frac{G\rho \, d^3x}{|\vec{x}-\vec{x}'|}$

Cluster velocity dispersion independent of location (core/edges)

⇒ isothermal distribution

$$\rho = \rho_0 e^{-\Phi/\sigma^2}$$

try power law solution $\rho = \rho_1 r^\alpha$

$$\Phi = -\sigma^2 \ln\left(\frac{\rho_1 r^\alpha}{\rho_0}\right) \qquad \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \underbrace{\frac{\partial}{\partial r}\left(-\sigma^2 \overbrace{\ln\left(\frac{\rho_1 r^\alpha}{\rho_0}\right)}^{\alpha \ln r + \ln\rho_1 - \ln\rho_0}\right)}_{-\sigma^2\alpha\frac{1}{r}}\right] = \frac{-\sigma^2\alpha}{r^2} \propto \rho \quad \therefore \alpha = -2$$

$-\sigma^2 \alpha r$

$$\nabla^2\Phi = 4\pi G\rho$$

$$\frac{2\sigma^2}{r^2} = 4\pi G \rho_1 r^{-2} \qquad \rho_1 = \frac{\sigma^2}{2\pi G}$$

$$\therefore \rho = \frac{\sigma^2}{2\pi G}\frac{1}{r^2}$$

$$M(<R) = \int_0^R 4\pi r^2 \rho \, dr = \frac{2\sigma^2}{G} R$$

circular orbit

$$\frac{V_c^2}{R} = \frac{G M(<R)}{R^2} \qquad M(<R) = \frac{V_c^2 R}{G}$$ same as before with $V_c^2 \to 2\sigma^2$

2 axes perpendicular to radial axis

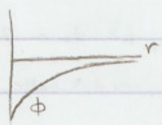
this has been Singular Isothermal Sphere

NFW: Navarro – Frank – White Model 1995 MNRAS 275 720

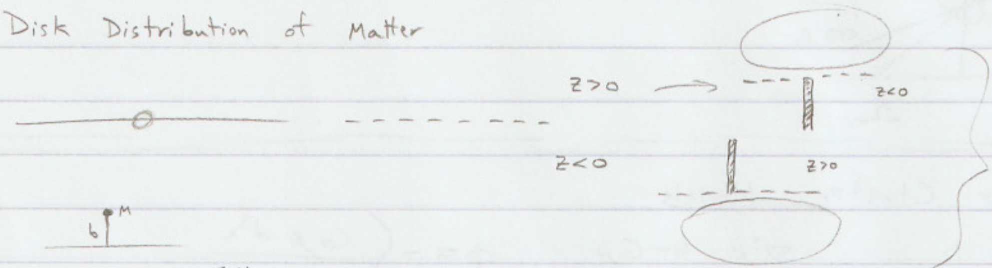
$$\rho = \frac{1500\,\rho_0}{x(1+5x)^2} \qquad x = \frac{r}{r_{200}} \qquad \rho_0 = \langle \rho \rangle$$

r_{200}: radius within which overdensity is 200 (approx of 177)

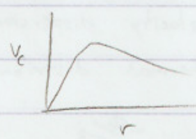
$$\Phi = -240\pi G \rho_0 r_{200}^2 \frac{\ln(1+5x)}{x}$$



Disk Distribution of Matter



$$\Phi = \frac{-GM}{\left(x^2+y^2+(|z|+b)^2\right)^{1/2}}$$ Kuzmin Disk



$\Sigma \sim \frac{1}{r}$ $\quad V_c \sim$ constant

Mestel Disk

6/6/2007

Clusters of Galaxies   by Mike Jura

Measuring Masses

X-ray Emission — thermally emitted

$\mathcal{E}_x \propto n_e^2 T^{1/2}$ (free-free emission)

$$F_x = \frac{n_e^2 T^{1/2}}{\underbrace{D^2}_{\text{distance}}} \underbrace{(\theta D)^3}_{\text{volume}}$$

θ — angular radius

characteristic $T \sim 10^8 K$ (10 keV)

can measure T as function of location — turns out to be isothermal

doesn't appear to be in clumps, filaments (could have complicated n_e)

$n \propto n_e(R)$ $n_e = n_e(0)\left[1 + \frac{r^2}{r_c^2}\right]^{-3\beta/2}$ isothermal   can use to fit T, $n_e(0)$, r_c, β

Can derive mass of gas $n_e(0) \simeq 0.01\,cm^{-3}$ $M_{gas} \simeq 10^{13} M_\odot$

$V = \left(\frac{\gamma k T}{\mu}\right)^{1/2}$ sound speed $V \sim 3\times10^8\ cm/s$ $\frac{3\times10^{24}cm}{3\times10^8\ cm/s} = 10^{16}s <$ Hubble time

→ should be in hydrostatic equilibrium

$\frac{dP}{dr} = -\rho g$ can derive g (since we know n, T and $P = P(n,T)$)

$g = \frac{G m_t}{r^2} \Longrightarrow m_t(r)$ total mass (substantially larger than baryonic mass)

$M_{gas} \sim 10\% M_t$

Can also use x-rays to study Zeldovich–Sunyaev effect $\left(y \propto \int n_e k T_e\, ds\right)$

SZ effect, not ZS

Optical Spectroscopy

— estimate velocities of galaxies, mass of galaxies → kinetic energy

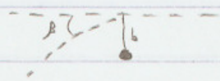
$\frac{1}{2}\langle PE\rangle = \langle KE\rangle$ $\frac{1}{2}\frac{GM_t^2}{R_c} = \langle KE\rangle$ $M_t >$ baryonic mass $M_{t_{opt}} \sim M_{t_{xray}}$ (within factor of 2 or so)

Mass of galaxy : rotation curve of spirals } include DM
velocity dispersion of ellipticals
light from stars (+ model)

Still have more DM in cluster than in galaxies

$\frac{M}{L} \sim 30$ typical galaxy $\frac{M}{L} \sim 300$ cluster

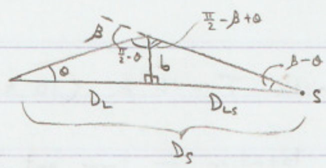
Gravitational Lensing



$$\boxed{\beta = \frac{4GM}{bc^2}}$$

Einstein Ring

Source directly behind Lens



$b = \theta D_L$

$b = (\beta - \theta) D_{LS}$

$\theta D_L = (\beta - \theta) D_{LS}$

$$\beta = \frac{4GM}{bc^2} = \frac{4GM}{\theta D_L c^2}$$

$$\beta\theta = \frac{4GM}{D_L c^2}$$

$D_{LS}\beta = \theta(D_L + D_{LS}) = \theta D_S$ $\qquad \beta = \frac{\theta D_S}{D_{LS}}$ $\qquad \frac{\theta^2 D_S}{D_{LS}} = \frac{4GM}{D_L c^2}$

$$\boxed{\theta = \left(\frac{4GM}{c^2}\frac{D_{LS}}{D_S D_L}\right)^{1/2}}$$

weak lensing outside ring → 'banana shapes'

strong lensing inside → multiple images

Can use this to get mass → agrees with M_{top}, M_{txray} (within factor of ~2)