Astro 271 Mike Jura
Notes: David Rodriguez

1/8/2007

intensity I_ν ergs cm^{-2} s^{-1} Hz^{-1} $ster^{-1}$ function of direction

detector size, band pass, beam opening size

Intensity is constant with distant (without source/sink)

Flux varies as $1/r^2$

Intensity is equivalent to surface brightness

Absorption

$I_\nu \rightarrow$ $\square$

$$dI_\nu = -I_\nu \underbrace{\alpha_\nu}_{\substack{\text{absorption}\\ \text{coefficient}\\ [cm^{-1}]}} \underbrace{ds}_{\text{path length [cm]}}$$

optical depth [dimensionless] $d\tau_\nu = \alpha_\nu ds$ $\quad \therefore \; dI_\nu = -I_\nu d\tau_\nu$

$$\therefore \; I_\nu = (I_\nu)_0 e^{-\tau_\nu}$$

$(I_\nu)_0$ — boundary value

$\tau_\nu < 1$ optically thin (transparent)

$\tau_\nu > 1$ optically thick (opaque)

mean free path $\ell_\nu = \alpha_\nu^{-1}$

line absorption

$$h\nu + X \rightarrow X^* \qquad \alpha_\nu = n\,\sigma_\nu$$

n — number density, σ_ν — crossection

opacity $\chi_\nu = \dfrac{\alpha_\nu}{\rho}$ ← mass density $\qquad \mu$ mean molecular weight $\quad \therefore \; \rho = n\mu$

$$\chi_\nu = \frac{\cancel{n}\sigma_\nu}{\cancel{n}\mu} = \frac{\sigma_\nu}{\mu} \quad [cm^2\, g^{-1}]$$

Sources

$$\frac{dI_\nu}{ds} = \underbrace{-\alpha_\nu I_\nu}_{\text{sink}} + \underbrace{\varepsilon_\nu}_{\text{source (emissivity)}} \qquad \varepsilon_\nu \; [erg\ cm^{-3}\ s^{-1}\ Hz^{-1}\ ster^{-1}]$$

$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{\varepsilon_\nu}{\alpha_\nu} \qquad \frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

S_ν ← source function $[erg\ cm^{-2} s^{-1}\ Hz^{-1}\ ster^{-1}]$

slab

$\underbrace{|\quad|}_{\tau_\nu} \;\triangleright\;$ solution $I_\nu = S_\nu(1 - e^{-\tau_\nu})$

At $\tau_\nu = 0$ $\quad I_\nu = 0$

$$I_\nu = S_\nu\left(1 - e^{-\tau_\nu}\right) \qquad \tau_\nu = \kappa_\nu L$$

optically thin limit $I_\nu = S_\nu\left(1 - 1 + \tau_\nu\right) = S_\nu \tau_\nu = \frac{\varepsilon_\nu}{\kappa_\nu}\kappa_\nu L = \varepsilon_\nu L$

distance through medium

optically thick limit $I_\nu = S_\nu$ (independent of L)

thermodynamic equilibrium $S_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1} = B_\nu(T)$ Planck function

Kirchoff's laws $\frac{\varepsilon_\nu}{\kappa_\nu} = B_\nu(T)$

ability to produce photons

ability to destroy photons

Mean Intensity $J_\nu = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\nu(\theta,\varphi)\sin\theta\, d\theta\, d\varphi$

Flux

$F_\nu = \int_0^{2\pi}\int_0^{\pi} I_\nu(\theta,\varphi)\cos\theta\sin\theta\, d\theta\, d\varphi$

measure of flow of energy accross a surface

"flux density" (in papers, etc)

$\left[\text{erg cm}^{-2}\text{ s}^{-1}\text{ Hz}^{-1}\right]$

$F = \int_0^\infty F_\nu\, d\nu$

flux (or total flux)

isotropic $I_\nu = (I_\nu)_0 \quad J_\nu = (I_\nu)_0 \quad F_\nu = 0$

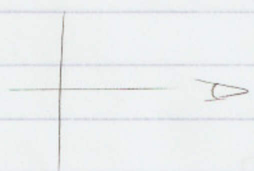
$\lambda\nu = c \qquad d\nu = \frac{c}{\lambda^2}d\lambda \qquad F_\lambda\left[\text{erg cm}^{-2}\text{ s}^{-1}\text{ ster}^{-1}\,\mu\text{m}^{-1}\right]$ (?)

Jansky $\text{Jy} = 10^{-23}\text{ erg cm}^{-2}\text{ s}^{-1}\text{ Hz}^{-1} = 10^{-26}\text{ W m}^{-2}\text{ Hz}^{-1}$

Magnitude scale $F_\nu = (F_\nu)_0\, 10^{-0.4\, m(\nu)}$

zero point

magnitude at a freq. or λ



Flux from a surface

$0 \le \theta \le \pi/2 \quad I_\nu = (I_\nu)_0$

$\pi/2 < \theta < \pi \quad I_\nu = 0$

$$F_\nu = (I_\nu)_0\int_0^{2\pi}\int_0^{\pi/2}\sin\theta\cos\theta\, d\theta\, d\varphi = 2\pi(I_\nu)_0\left(\frac{\sin^2\theta}{2}\right)\Big|_0^{\pi/2} = \pi(I_\nu)_0$$

Blackbody $F_\nu = \pi B_\nu = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1} \qquad F = \frac{2\pi h}{c^2}\int_0^\infty \frac{\nu^3\, d\nu}{e^{h\nu/kT} - 1} \qquad x = \frac{h\nu}{kT} \quad \nu = \frac{kT}{h}x$

$d\nu = \frac{kT}{h}dx$

$$F = \frac{2\pi h}{c^2}\left(\frac{kT}{h}\right)^4 \underbrace{\int_0^\infty \frac{x^3\, dx}{e^x - 1}}_{\frac{\pi^4}{15}} = \sigma T^4 \qquad \boxed{\sigma = \frac{2\pi^5}{15}\frac{k^4}{c^2 h^3}}$$

Line Absorption + Emission

— u

— L $\qquad \varepsilon_\nu = \frac{n_u h\nu}{4\pi} A_{uL}\, \phi(\Delta\nu)$

(Einstein A)

n_u – # density in upper level $\qquad A_{uL}$ – Einstein rate – transition probability

$h\nu$ – energy of photon $\qquad \phi(\Delta\nu)$ – line broadening coefficient

A_{uL} $[s^{-1}]$ rate at which you spontaneously go from $u \to L$

inverse of the mean lifetime in the upper level for this transition

$\Delta E \Delta t \geq \hbar/2$ $\qquad$ lines are broadened also due to doppler shifts, pressures, etc

due to neighbors

$\Delta\nu = \nu - \nu_0$

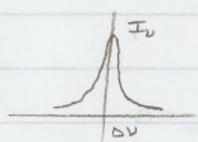
line center

$$\int_{-\infty}^{\infty} \phi(\Delta\nu)\, d(\Delta\nu) = 1$$

Consider doppler shift

$$\frac{\Delta\nu}{\nu} = -\frac{v_r}{c} \qquad f_{(v_r)} dv_r \propto e^{-\frac{1}{2}\frac{m v_r^2}{kT}} dv_r \qquad v_r^2 = \frac{(\Delta\nu)^2 c^2}{\nu^2}$$

$$\phi(\Delta\nu) = \sqrt{\frac{mc^2}{2\pi kT \nu^2}}\, \exp\left(-\frac{mc^2 (\Delta\nu)^2}{2kT\nu^2}\right)$$



line broadening can be used to determine temperature (if line broadening is from thermal)

$$\kappa_\nu = \frac{n_L h\nu}{4\pi} B_{Lu}\, \phi(\Delta\nu)$$

Stimulated emission term $-\frac{n_u h\nu}{4\pi} B_{uL}\, \phi(\Delta\nu)$

Thermodynamic Equilibrium $\qquad$ steady state $\qquad$ photon destruction rate = photon creation rate

$$\frac{A_{uL}}{4\pi} n_u h\nu\, \phi(\Delta\nu) + \frac{J_\nu n_u h\nu}{4\pi} B_{uL}\, \phi(\Delta\nu) = \frac{J_\nu n_L h\nu}{4\pi} B_{Lu}\, \phi(\Delta\nu)$$

$$A_{uL} n_u + J_\nu n_u B_{uL} = J_\nu n_L B_{Lu}$$

$$J_\nu = \frac{n_u A_{uL}}{n_L B_{Lu} - n_u B_{uL}} \quad \text{mean intensity}$$

$$J_\nu = \frac{\left(\frac{A_{uL}}{B_{uL}}\right)}{\left(\frac{n_L B_{Lu}}{n_u B_{uL}} - 1\right)} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$\boxed{\frac{A_{uL}}{B_{uL}} = \frac{2h\nu^3}{c^2}} \qquad \frac{n_u}{n_L} = \frac{g_u}{g_L} e^{-\frac{h\nu}{kT}}$$

$$\boxed{g_L B_{Lu} = g_u B_{uL}}$$

$$\kappa_\nu = \frac{n_L h\nu}{4\pi} B_{Lu}\, \phi(\Delta\nu)\left(1 - \underbrace{\frac{n_u B_{uL}}{n_L B_{Lu}}}_{\text{stimulated emission term}}\right)$$

lasers if $n_u > n_L \to \kappa_\nu < 0 \qquad \tau_\nu < 0$

$I_\nu \propto e^{-\tau_\nu} \to$ amplification

Low Energy Spectral Lines — 1/10/2007

$h\nu < kT$

21 cm line in H

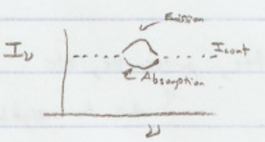
$F=1 \quad g=3$ $\qquad$ weight: $g = 2F+1$ $\qquad$ $\nu = 1.4\ \mathrm{GHz}$

$F=0 \quad g=1$

Einsten A: $A = 2.869 \times 10^{-15}\ s^{-1}$

$$I_\nu = I_{cont}\, e^{-\tau_\nu} + S_\nu \left(1 - e^{-\tau_\nu}\right)$$

$$I_\nu = \left(S_\nu - I_{cont}\right)\left(1 - e^{-\tau_\nu}\right) + I_{cont}$$



$S_\nu > I_{cont} \rightarrow$ emission

$S_\nu < I_{cont} \rightarrow$ absorption

$$S_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1} \simeq \frac{2\nu^2}{c^2} kT = \frac{2kT}{\lambda^2} \qquad (h\nu \ll kT)$$

$$I_{cont} = \frac{2kT_b}{\lambda^2}$$ ← brightness temperature

$T > T_b$ emission

$T < T_b$ absorption

Consider case $I_{cont} = 0$

$$I_\nu = S_\nu \left(1 - e^{-\tau_\nu}\right)$$

i) $\tau_\nu \gg 1$ (opaque) $\qquad I_\nu = S_\nu \qquad T_b = T$

ii) $\tau_\nu \ll 1 \qquad I_\nu = S_\nu \tau_\nu$

$$K_\nu = n_L B_{Lu} \frac{h\nu}{4\pi} \phi(\Delta\nu)\left(1 - e^{-\frac{h\nu}{kT}}\right) \qquad \tau_\nu = \int K_\nu\, ds$$

$N_L = \int n_L\, ds$ Column Density [atoms cm^{-2}]

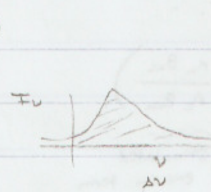
$$\tau_\nu = N_L B_{Lu} \frac{h\nu}{4\pi} \phi(\Delta\nu) \frac{h\nu}{kT}$$

$$\frac{n(F=1)}{n(F=0)} = \frac{3}{1} e^{-\frac{h\nu}{kT}} \simeq 3 \qquad N_L \simeq \frac{1}{4} N(H)$$ ← total H column density

$B_{Lu} = 3 B_{uL}$ (rate up, rate down) more likely to go up $\qquad \frac{3}{1} \qquad \frac{A_{uL}}{B_{uL}} = \frac{2h\nu^3}{c^2}$

$$\tau_\nu = \frac{N(H)}{4}\left(\frac{3 A_{uL} c^2}{2h\nu^3}\right)\frac{(h\nu)^2}{4\pi kT}\phi(\Delta\nu)$$

$$\int_{-\infty}^{\infty} \phi(\Delta\nu)\, d(\Delta\nu) = 1$$



$$I_\nu = S_\nu \tau_\nu = \frac{2\nu^2}{c^2} kT \; \frac{N(H)}{4} \left(\frac{3 A_{uL} c^2}{2h\nu^3}\right) \frac{(h\nu)^2}{4\pi kT} \phi(\Delta\nu)$$

$$= \frac{N(H)}{4} \; \frac{3 A_{uL}}{4\pi} h\nu \; \phi(\Delta\nu)$$

$$\int_{-\infty}^{\infty} I_\nu \, d(\Delta\nu) = \frac{N(H)}{4} \; \frac{3 A_{uL}}{4\pi} h\nu$$

optically thin $\rightarrow$ $I_\nu = S_\nu \tau_\nu = \frac{\varepsilon_\nu}{\kappa_\nu} \kappa_\nu L = \varepsilon_\nu L$

(Another way) $\int \varepsilon_\nu \, d(\Delta\nu) = n_u \frac{A_{uL}}{4\pi} h\nu$ $\qquad \int I_\nu \, d(\Delta\nu) = \int n_u \, ds \; \frac{A_{uL} \, h\nu}{4\pi} = \frac{3}{4} N(H) \frac{A_{uL}}{4\pi} h\nu$

$$I_\nu = \frac{2kT_b}{\lambda^2} \qquad \frac{2k}{\lambda^2} \int T_b \, d(\Delta\nu) = \frac{N(H)}{16\pi} 3 A_{uL} h\nu$$

$$\frac{\Delta\nu}{\nu} = \frac{v}{c} \quad \text{— radial velocity} \qquad d(\Delta\nu) = \frac{\nu}{c} dv$$

$$\boxed{N(H) = \frac{32\pi}{3} \frac{k}{A_{uL} h c \lambda^2} \int T_b \, dv} \qquad N(H) = 1.823 \times 10^{13} \int T_b \, dv \quad [\text{cgs}]$$

Flux $F = \Omega I$

M_{total} mass of hydrogen gas $\qquad$ Area $= \Omega D^2$ (solid angle of telescope beam; distance^2)

$m(H)$ mass of hydrogen atom

$$\boxed{M_{total} = N(H) \, \Omega \, D^2 \, m(H)}$$

Absorption – depth depends on T $\qquad \tau \sim \frac{N_L h\nu}{4\pi} B_{Lu} \frac{h\nu}{kT}$

CO molecular line $\qquad$ 1 Debye $= 10^{-18}$ statcoulomb cm

	$g = 2J+1$	E: $hcB\,J(J+1)$
$J=2$	5	$6\,hcB$
$J=1$	3	$2\,hcB$
$J=0$	1	0

^{12}CO $\quad B = 1.9313 \text{ cm}^{-1}$

$$A = \frac{64\pi^4 \nu^3 \mu^2}{3hc^3} \; \frac{J+1}{2J+3} \qquad J+1 \rightarrow J$$

μ – dipole moment of the molecule

0.1098 Debye $= \mu$

J $1 \rightarrow 0$ $\quad \nu = 115$ GHz $\quad \lambda = 2.7$ mm $\quad A_{J=0} = 7.1 \times 10^{-8} \text{ s}^{-1}$

Single Temperature Description of the level populations

partition function $\quad \zeta = \sum_i g_i e^{-E_i/kT}$

$$\zeta \simeq \int_0^\infty (2J+1) e^{-hcB\,J(J+1)/kT} dJ \qquad u = \frac{hcB}{kT} J(J+1) \qquad du = \frac{hcB}{kT}(2J+1)\,dJ$$

$$\zeta \simeq \frac{kT}{hcB}$$

$$N(J) = \frac{(2J+1)\, e^{-\frac{hcBJ(J+1)}{kT}}}{\zeta} N(CO) \qquad \zeta \leftrightarrow \varsigma$$

T_{mb} main beam temperature $\qquad I_\nu = \frac{2\nu^2}{c^2} k T_{mb}$

optically thin emission, no background $\qquad I_\nu = S_\nu \tau_\nu$

Consider high temperature case $\quad kT \gg h\nu \qquad S_\nu \simeq \frac{2\nu^2}{c^2} kT$

(Not always a good approx.)

$T_{mb} \simeq T \tau_\nu$

Consider $2 \to 1$ line $\qquad$ optically thin high temperature emission

$$\tau_{21} = B_{12} N(1) \frac{h\nu}{4\pi} \underbrace{\frac{h\nu}{kT}}_{(1-e^{-\frac{h\nu}{kT}})} \phi(\Delta\nu) \qquad \int \tau_{21}\, dv = B_{12} N(1) \frac{h\nu}{4\pi} \frac{h\nu}{kT} \frac{c}{\nu}$$

$d(\Delta\nu) = \frac{\nu}{c} dv$

$$B_{12} = \frac{5}{3} B_{21} = \frac{5}{3} A_{21} \frac{c^2}{2h\nu^3} \qquad A_{21} = \frac{64\pi^4 \nu^3 \mu^2}{3hc^3} \frac{2}{5} \quad \left(\frac{J+1}{2J+3} \quad J=1\right)$$

$$B_{12} = \frac{64}{9} \frac{\pi^4 \mu^2}{h^2 c}$$

$$N(1) \simeq \frac{3}{\zeta} N(CO) = \frac{3\, hcB}{kT} N(CO)$$

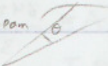
$h\nu(2\to1) = 4\, hcB \qquad N(1) = \frac{3}{4} \frac{h\nu}{kT} N(CO)$

$$\int \tau_{21}\, dv = \frac{\cancel{64}^{\,4}}{\cancel{9}_{\,3}} \frac{\cancel{\pi^4}^{\,3} \mu^2}{h^2 c} \frac{3}{4} \frac{h\nu}{kT} N(CO) \frac{(h\nu)^2}{4\pi kT} \frac{c}{\nu} = \frac{4}{3} \pi^3 \mu^2 h\, N(CO) \frac{\nu^2}{(kT)^2}$$

$$N(CO) = \frac{3k^2 T \int T_{mb}\, dv}{4\pi^3 \mu^2 h \nu^2}$$

Need $T \to$ temperature of gas

telescope beam



azimuthally symmetric Gaussian

$$d\Omega = 2\pi e^{-\frac{\theta^2}{\theta_0^2}} \theta\, d\theta$$

$$\Omega = \pi \theta_0^2 \qquad \theta_{FWHM} = 2\sqrt{\ln 2}\, \theta_0 \qquad A = \pi D^2 \frac{\theta_{FWHM}^2}{4 \ln 2}$$

$\theta = \theta_{HWHM}$ Half Width Half Maximum $\qquad \Omega(\theta_{HWHM}) = \frac{1}{2} \Omega(0)$

$$e^{-\left(\frac{\theta_{HWHM}}{\theta_0}\right)^2} = \frac{1}{2} \qquad \Omega(0) = 1 \qquad \left(\frac{\theta_{HWHM}}{\theta_0}\right)^2 = \ln 2 \qquad \theta_{HWHM} = \theta_0 \sqrt{\ln 2}$$

$$\theta_{FWHM} = 2\theta_{HWHM} = 2\sqrt{\ln 2}\, \theta_0$$

$$\therefore M(CO) = N(CO)\, m_{CO}\, A \qquad \pi D^2 \frac{\theta_{FWHM}^2}{4 \ln 2}$$

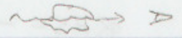
total mass of CO $\qquad$ mass of CO molecule

1/17/2007

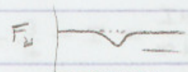
High Energy Line Photons

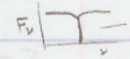
$h\nu \gg kT$

Absorption lines


Interstellar Medium
QSO Absorption

$$I_\nu = I_0 e^{-\tau_\nu} \qquad \frac{dI_\nu}{d\tau_\nu} = -I_\nu \qquad \tau_L = N_L \frac{h\nu B_{Lu}}{4\pi}\phi(\Delta\nu)$$


F_ν — optically thin


F_ν — optically thick

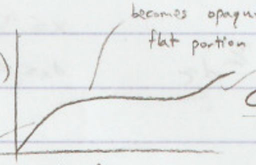
$$\frac{h\nu B_{Lu}}{4\pi} = \frac{\pi e^2}{mc} f$$ oscillator strength [dimensionless] $\qquad f \sim 1$ strong lines

Equivalent width: $\displaystyle\int_{-\infty}^{\infty} \frac{I_0 - I_\nu}{I_\nu} d(\Delta\nu) = W_\nu \qquad \frac{W_\nu}{\nu} = \frac{W_\lambda}{\lambda}$

$$W_\nu = \int_{-\infty}^{\infty} (1 - e^{-\tau_\nu})\, d(\Delta\nu)$$ If $\tau_\nu \ll 1$ throughout the line

$$\Rightarrow W_\nu = \int_{-\infty}^{\infty} \tau_\nu\, d(\Delta\nu) = N_L \frac{\pi e^2}{mc} f$$

$$W_\lambda = \frac{\pi e^2}{mc^2} f \lambda^2 N_L$$


$\log\left(\frac{W_\lambda}{\lambda}\right)$, $\log(N_L f \lambda)$, linear portion, becomes opaque, flat portion, square root portion

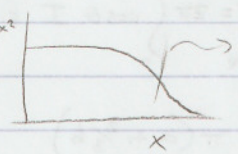
Curve of Growth

Thermal broadening:
$$\phi(\Delta\nu) = \frac{1}{\sqrt{\pi}\, b'} e^{-\frac{(\Delta\nu)^2}{b'^2}} \qquad \int_{-\infty}^{\infty} \phi(\Delta\nu)\, d(\Delta\nu) = 1$$

$$b = b' \frac{c}{\nu} \qquad \tau_L = \tau_0 e^{\frac{-(\Delta\nu)^2}{b'^2}}$$

$$W_\nu = \int_{-\infty}^{\infty} (1 - e^{-\tau_\nu})\, d(\Delta\nu) \qquad W_\lambda = \frac{b\lambda}{c}\int_{-\infty}^{\infty}\left(1 - e^{-\tau_0 e^{-x^2}}\right) dx = \frac{2b\lambda}{c}\int_0^{\infty}\left(1 - e^{-\tau_0 e^{-x^2}}\right) dx$$

$\tau_0 \gg 1$


$1 - e^{-\tau_0 e^{-x^2}}$, x

$\rightarrow \tau_0 e^{-x^2} \simeq 1 \qquad \frac{1}{\tau_0} = e^{-x_c^2} \qquad -\ln \tau_0 = -x_c^2 \qquad x_c = \sqrt{\ln \tau_0}$

critical x

$$W_\lambda \cong \frac{2b\lambda}{c}(\ln \tau_0)^{1/2}$$

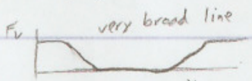
after τ_0 becomes large W_λ is roughly insensitive to τ_0 (flat portion)

$$\phi(\Delta\nu) = \frac{\gamma/4\pi}{(\Delta\nu)^2 + \gamma^2} \qquad \gamma = \sum_L \frac{A_{uL}}{4\pi}$$

Small compared to thermal broadening

$$\tau_L = N_L \frac{\pi e^2}{mc} \frac{\gamma/\pi}{(\Delta\nu)^2}$$

Damping portion of the Curve of Growth


F_ν, very broad line, ν

ex. Damped Lyman α QSOs

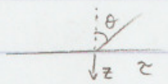
at $\tau = 0.5 \qquad W_\nu \sim \sqrt{N_L}$ Square root portion

can be used to study D/H ratio — Deuterium

D produced in Big Bang and declining

produced in solar flares

Stellar Atmosphere



$\mu = \cos\theta$ ν-suppressed

$$\frac{dI}{ds} = -\kappa I + \varepsilon \qquad \kappa\, ds = -\frac{d\tau}{\mu}$$

$$\mu \frac{dI}{d\tau} = I - \frac{\varepsilon}{\kappa} = I - S \qquad I(\mu,\tau) \quad S(\tau) \qquad \mu\frac{dI}{d\tau} = I - S$$

$$S = a + b\tau \qquad \frac{dI}{d\tau} - \frac{I}{\mu} = -\frac{S}{\mu} \qquad e^{-\tau/\mu}\frac{dI}{d\tau} - e^{-\tau/\mu}\frac{I}{\mu} = -\frac{S}{\mu}e^{-\tau/\mu}$$

$$\frac{d}{d\tau}\left(I e^{-\tau/\mu}\right) = -\frac{S}{\mu}e^{-\tau/\mu} \qquad \text{Now } \int_{\tau_1}^{\tau_2}$$

$$I(\tau_2,\mu)e^{-\tau_2/\mu} - I(\tau_1,\mu)e^{-\tau_1/\mu} = -\int_{\tau_1}^{\tau_2} \frac{S}{\mu}e^{-\tau/\mu}\, d\tau \qquad \begin{matrix}\tau_2 \to \infty \\ \tau_1 \to 0\end{matrix} \quad \text{infinite atmosphere}$$

finite I deep inside

$I(\infty) \to c$

$$I(0,\mu) = \int_0^\infty \frac{S}{\mu}e^{-\tau/\mu}\, d\tau$$

$$I = \int_0^\infty \frac{(a+b\tau)}{\mu}e^{-\tau/\mu}\, d\tau = a\int_0^\infty \frac{e^{-\tau/\mu}}{\mu}\, d\tau + b\int_0^\infty \frac{\tau}{\mu}e^{-\tau/\mu}\, d\tau \qquad x = \tau/\mu \quad dx = \frac{d\tau}{\mu}$$

$$I = a\int_0^\infty e^{-x}\, dx + b\mu\int_0^\infty x e^{-x}\, dx \implies I = a + b\mu$$

if $b=0$, isothermal wont matter what direction you look you always see $I = a$

$b \neq 0$ $\mu = 0$ $\theta = 90°$ sideways $I = a$ (constant Source function, isothermal surface)

$\mu = 1$ $\theta = 0°$ straight down $I = a + b$ (Source function @ $\tau = 1$)

$$F = \int_0^{2\pi}\int_0^{\pi} \cos\theta\, I(\theta,\varphi) \sin\theta\, d\theta\, d\varphi \qquad F = 2\pi\int_0^{\pi/2} \cos\theta\, I \sin\theta\, d\theta \qquad \begin{matrix}\mu = \cos\theta \\ d\mu = -\sin\theta\, d\theta\end{matrix}$$

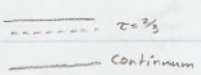
atmosphere

$$F = 2\pi\int_0^1 \mu I\, d\mu = 2\pi\int \mu(a + b\mu)\, d\mu = \pi\left(a + \tfrac{2}{3}b\right)$$

observe at $\tau = 2/3$

Lines occur higher than continuum

$\tau = \int \kappa\, ds$



Read Notes for this part

Emission

$$\frac{dI}{d\tau} = -I + S \qquad I = S(1 - e^{-\tau}) \qquad \tau \ll 1 \qquad I = S\tau = \frac{\varepsilon}{\kappa}\kappa L = \varepsilon L$$

Ionized H lines $H^+ + e \to H^* \to H + h\nu$ Capture Cascade Sequence

$3 \to 2$ $H\alpha$

$4 \to 2$ $H\beta$

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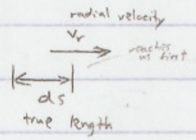
Retarded potentials

$$t_r = t - \frac{d}{c} \qquad \phi(\vec r, t) = \int \frac{\rho(\vec r', t_r)}{d} dV' \qquad \vec A(\vec r, t) = \frac{1}{c}\int \frac{\vec J(\vec r', t_r)}{d} dV'$$



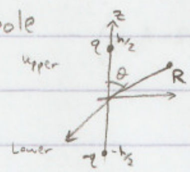
$\Rightarrow$ ds' assigned length

$$\frac{ds'}{c} = \frac{ds' - ds}{v_r} \qquad ds' = \frac{ds}{1 - v_r/c}$$

$$\vec v_r = \hat d \cdot \vec v \qquad \hat d = \frac{\vec d}{d}$$

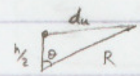
$$\phi = \frac{q}{d - \hat d \cdot \vec v/c} \qquad \vec A = \frac{q\,\vec v/c}{d - \hat d \cdot \vec v/c} \quad \text{for moving point charges}$$

Dipole



$$R \gg h \qquad q = q_0 \cos(\omega t) \qquad \vec p = q h \hat z \qquad p_0 = q_0 h \qquad \vec p = p_0 \cos(\omega t)\hat z$$

$$\phi = \phi_u + \phi_L$$



$$d_u^2 = R^2 + (h/2)^2 - 2R\frac{h}{2}\cos\theta$$

$$d_u = \left(R^2 - Rh\cos\theta + h^2/4\right)^{1/2}$$

$$\phi_u = \frac{q_0 \cos\left(\omega\left(t - \frac{d_u}{c}\right)\right)}{d_u}$$

$$\phi_L = \frac{-q_0 \cos\left(\omega\left(t - \frac{d_L}{c}\right)\right)}{d_L} \qquad d_L = \left(R^2 + Rh\cos\theta + h^2/4\right)^{1/2}$$

$R \gg h$

$$d_u = R\left(1 - \frac{h}{R}\cos\theta + \frac{h^2}{4R^2}\right)^{1/2} \simeq R\left(1 - \frac{h}{2R}\cos\theta\right) \qquad d_L \simeq R\left(1 + \frac{h}{2R}\cos\theta\right)$$

$$\cos\left(\omega\left(t - \frac{d_u}{c}\right)\right) = \cos\left(\left(\omega t - \frac{\omega R}{c}\right) + \left(\frac{\omega h}{2c}\cos\theta\right)\right)$$

$\lambda \nu = c \qquad \lambda = \frac{c}{\nu} = \frac{2\pi c}{\omega} \qquad \frac{\omega h}{2c} \sim \frac{h}{\lambda} \ll 1 \qquad \lambda \gg h$

$\cos(\text{small}) = 1 \qquad \sin(\text{small}) = \text{small}$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \longrightarrow \cos\left(\omega\left(t - \frac{R}{c}\right)\right) - \sin\left(\omega\left(t - \frac{R}{c}\right)\right)\frac{\omega h}{2c}\cos\theta$$

$$\cos\left(\omega\left(t - \frac{d_L}{c}\right)\right) = \cos\left(\omega\left(t - \frac{R}{c}\right)\right) + \frac{\omega h}{2c}\cos\theta \sin\left(\omega\left(t - \frac{R}{c}\right)\right)$$

$$\frac{1}{d_u} = \frac{1}{R\left(1 - \frac{h}{2R}\cos\theta\right)} \simeq \frac{1}{R}\left(1 + \frac{h}{2R}\cos\theta\right) \sim \frac{1}{R} \quad \text{first order}$$

$$\phi = -\frac{q_0}{R}\frac{h\omega}{c}\cos\theta \sin\left(\omega\left(t - \frac{R}{c}\right)\right) = -p_0\frac{\omega}{Rc}\cos\theta \sin\left(\omega\left(t - \frac{R}{c}\right)\right) = \phi$$

$$\vec I = \frac{dq}{dt}\hat z \qquad q = q_0 \cos(\omega t) \qquad \frac{dq}{dt} = -q_0 \omega \sin \omega t \qquad \vec A = \frac{1}{c}\int_{-h/2}^{h/2} \frac{-q_0 \omega \sin(\omega(t - d/c))}{d} dz\, \hat z$$

$$d \simeq R \qquad \vec A = -\frac{q_0}{c}\frac{\omega}{R}\sin\left(\omega\left(t - \frac{R}{c}\right)\right) h \hat z \qquad \vec A = -\frac{p_0 \omega}{cR}\sin\left(\omega(t - R/c)\right)\hat z$$

$$\hat z = \cos\theta \hat R - \sin\theta \hat\theta \qquad \vec A = -\frac{p_0 \omega}{cR}\cos\theta \sin\left(\omega\left(t - \frac{R}{c}\right)\right)\hat R + \frac{p_0 \omega}{cR}\sin\theta \sin\left(\omega(t - R/c)\right)\hat\theta$$

$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$ $\qquad \nabla\phi = \frac{\partial \phi}{\partial R}\hat{R} + \underbrace{\frac{1}{R}\frac{\partial \phi}{\partial \theta}}_{\text{small } \frac{1}{R^2}}\hat{\theta}$ (ignoring R^{-2} terms or higher)

$$\frac{\partial \phi}{\partial R} \simeq \frac{+p_0\omega^2}{Rc^2}\cos(\theta)\cos(\omega(t-R/c))$$

$$\frac{\partial \vec{A}}{\partial t} = -\frac{p_0\omega^2}{cR}\cos(\omega(t-R/c))\,(\cos\theta\,\hat{R} - \sin\theta\,\hat{\theta})$$

$$\vec{E} = \frac{-p_0\omega^2}{Rc^2}\left[\left(\cancel{\cos\theta\cos(\omega(t-R/c))} - \cancel{\cos\theta\cos(\omega(t-R/c))}\right)\hat{R} - \sin\theta\cos(\omega(t-R/c))\hat{\theta}\right]$$

$$\boxed{\vec{E} = \frac{-p_0\omega^2}{c^2R}\sin\theta\cos(\omega(t-R/c))\,\hat{\theta}} \qquad \left(\text{static}: \vec{E} \sim \frac{1}{R^3}\right)$$

$$\vec{B} = \nabla\times\vec{A} = \frac{1}{R}\left(\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial}{\partial\theta}(A_R)\right)\hat{\varphi} \simeq \frac{1}{R}\frac{\partial}{\partial R}\left(\frac{p_0\omega}{c}\sin\theta\sin(\omega(t-R/c))\right)$$

$$\boxed{-\frac{p_0\omega^2}{c^2R}\sin\theta\cos(\omega(t-R/c))\,\hat{\varphi} = \vec{B}}$$

$$\vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{B}) = \frac{p_0^2\omega^4}{4\pi R^2c^3}\sin^2\theta\cos^2(\omega(t-R/c))\,\hat{r}$$

$$\boxed{\langle\vec{S}\rangle = \frac{p_0^2\omega^4}{8\pi R^2c^3}\sin^2\theta\,\hat{r}} \qquad d\vec{a} = r^2\sin\theta\,d\theta\,d\varphi\,\hat{r}$$

Power radiated

$$\langle P\rangle = \int\langle\vec{S}\rangle\cdot d\vec{a} = \frac{p_0^2\omega^4}{8\pi c^3}\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\,d\theta\,d\varphi = \boxed{\frac{p_0^2\omega^4}{3c^3} = \langle P\rangle}$$

Larmor's formula $\boxed{P = \frac{2}{3}\frac{q^2\dot{u}^2}{c^3}}$ $\quad \dot{u}$ – acceleration

Ex: $x = h\cos(\omega t)\hat{z}$ $\quad \dot{x} = u = -h\omega\sin\omega t\,\hat{z}$ $\quad \ddot{x} = \dot{u} = -h\omega^2\cos(\omega t)\hat{z}$

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$\uparrow z$ $\rightsquigarrow \cdot_{e^-}$ $\qquad \vec{E} = \vec{E}_0\sin\omega t\,\hat{z}$ $\qquad \vec{x}$ – vector location of e^-

$m\ddot{\vec{x}} = q\vec{E}$ $\qquad \vec{u} = \frac{d\vec{x}}{dt}$ $\qquad \dot{\vec{u}} = \frac{q}{m}E_0\sin\omega t\,\hat{z}$ $\qquad \dot{u}^2 = \frac{q^2E_0^2}{m^2}\sin^2\omega t$

$P = \frac{2}{3}\frac{q^4}{m_e^2c^3}E_0^2\sin^2\omega t$ $\qquad \langle P\rangle = \frac{1}{3}\frac{q^4}{m_e^2c^3}E_0^2$ $\qquad \underbrace{\vec{S}}_{\text{flow of energy}} = \underbrace{c}_{\text{speed}}\underbrace{\frac{1}{4\pi}\vec{E}\times\vec{B}}_{\text{energy density}}$

$\langle P\rangle = \sigma\langle\vec{S}\rangle$ $\qquad \langle\vec{S}\rangle = \frac{c}{8\pi}E_0^2$

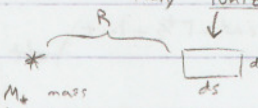
$\frac{1}{3}\frac{q^4\cancel{E_0^2}}{m_e^2c^3} = \sigma\frac{c}{8\pi}\cancel{E_0^2}$ $\qquad \boxed{\sigma = \frac{8\pi}{3}\frac{q^4}{m_e^2c^4}}$ $\quad e^-$ scattering crosssection

$\sigma = 6.65\times10^{-25}\,cm^2$

$\frac{q^2}{r_e} = m_ec^2 \Longrightarrow \sigma = \frac{8\pi}{3}r_e^2$

Eddington Limit

fully ionized pure H



n - density

M_* mass

L_* luminosity

$$|\vec{F}|_{\text{Gravity}} = \frac{GM_*}{R^2} m_{\text{blob}} \qquad m_{\text{blob}} = ds\, dA\, n\, m_H$$

inward force $\frac{GM_*}{R^2} n\, m_H\, ds\, dA$

flow of momentum into the blob $\frac{L_*}{4\pi R^2 c}$

the amount transferred into the blob : $\frac{L_*}{4\pi R^2 c} dA \underbrace{ds\, n\sigma}_{\text{increment of optical depth}}$

Gravitationally bound if

$$\frac{GM_*}{R^2} ds\, dA\, n\, m_H > \frac{L_*}{4\pi R^2 c} dA\, ds\, n\, \sigma$$

$$L_* < \frac{GM_* m_H 4\pi c}{\sigma} \qquad \varkappa = \frac{\sigma}{m_H} = 0.4 \text{ cm}^2 \text{ g}^{-1} \text{ for pure H}$$

$\vec{F}_{rad}$ radiative reaction force

$$-\int_{t_1}^{t_2} \vec{F}_{rad} \cdot \dot{\vec{u}}\, dt = \frac{2}{3}\frac{q^2}{c^3}\int_{t_1}^{t_2} \dot{\vec{u}} \cdot \dot{\vec{u}}\, dt = \frac{2}{3}\frac{q^2}{c^3}\left\{ \left.\left(\vec{u} \cdot \dot{\vec{u}}\right)\right|_{t_1}^{t_2} - \int_{t_1}^{t_2} \vec{u} \cdot \ddot{\vec{u}}\, dt \right\}$$

full cycle so zero

$$\int_{t_1}^{t_2} \left(\vec{F}_{rad} - \frac{2}{3}\frac{q^2}{c^3}\ddot{\vec{u}}\right) \cdot \vec{u} = 0 \qquad \vec{F}_{rad} = \frac{2}{3}\frac{q^2}{c^3}\ddot{\vec{u}} \qquad \text{Not a solid proof}$$

$$\vec{F}_{rad} = m\tau \ddot{\vec{u}} \qquad \tau = \frac{2}{3}\frac{q^2}{mc^3}$$

$$\vec{F} = -k\vec{x} = -m\omega_0^2 \vec{x} \qquad m\ddot{\vec{x}} = -m\omega_0^2 \vec{x} + m\tau \dddot{\vec{x}}$$

spring force

$x \sim \sin \omega t$

$u \sim \omega_0 \cos \omega_0 t$

$\ddot{\vec{u}} \approx -\omega_0^2 \vec{u}$ (weak damping)

$$\ddot{x} + \omega_0^2 x + \omega_0^2 \tau \dot{x} = 0$$

$$x = x_0 e^{\alpha t} \qquad \dot{x} = \alpha x \qquad \ddot{x} = \alpha^2 x \qquad \alpha^2 + \omega_0^2 \tau \alpha + \omega_0^2 = 0$$

$$\alpha = \frac{-\omega_0^2\tau \pm \sqrt{\omega_0^4\tau^2 - 4\omega_0^2}}{2} = \frac{-\omega_0^2\tau \pm \sqrt{-4\omega_0^2\left(1 - \frac{\omega_0^2\tau^2}{4}\right)}}{2} \simeq -\frac{\omega_0^2\tau}{2} \pm i\omega_0\left(1 - \cancel{\frac{\omega_0^2\tau^2}{8}}\right)$$

ignore

$t=0 \quad x = x_0 \quad \dot{x} = 0$

$$x(t) = x_0 e^{-\Gamma t/2} \cos \omega_0 t = x_0 e^{-\Gamma t/2}\left(\frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2}\right)$$

damped harmonic oscillator

$\Gamma = \omega_0^2 \tau$

To get spectrum → Fourier analyze

$$x_F(\omega) = \frac{1}{2\pi}\int_0^\infty x(t)\, e^{i\omega t}\, dt = \frac{1 x_0}{4\pi}\int_0^\infty \left(e^{i\omega t} e^{-\Gamma t/2 + i\omega_0 t} + e^{i\omega t - \Gamma t/2 - i\omega_0 t}\right) dt$$

$$x_F(\omega) = \frac{x_0}{4\pi}\left(\frac{1}{\Gamma/2 - i(\omega+\omega_0)} + \frac{1}{\Gamma/2 - i(\omega-\omega_0)} \right)$$

Near ω_0 $\quad x_F(\omega) \simeq \frac{x_0}{4\pi}\,\frac{1}{\Gamma/2 - i(\omega-\omega_0)}$

intensity $\quad x_F^2(\omega) \simeq \frac{x_0^2}{16\pi^2}\,\frac{1}{(\Gamma/2)^2 + (\omega-\omega_0)^2}$

$\Delta\nu = \frac{\omega-\omega_0}{2\pi}$ $\quad \phi(\Delta\nu) \sim \frac{\Gamma/4\pi^2}{(\Delta\nu)^2 + \left(\frac{\Gamma}{4\pi}\right)^2}$ $\quad \int_{-\infty}^{\infty} \phi(\Delta\nu)\, d(\Delta\nu) = 1$

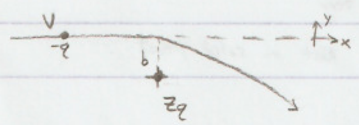
$\phi(\Delta\nu) = \frac{\delta/\pi}{(\Delta\nu)^2 + \delta^2}$ $\quad \delta = \frac{\Gamma}{4\pi}$

$\delta = \frac{\Gamma}{4\pi} = \sum \frac{A_{UL}}{4\pi}$

for small damping (small Γ)
spectrum is mostly around ω_0 (single wavelength)
(narrow)

Free-free Emission / Absorption

1/29/2007



characteristic collision time $\quad t_{col} = \frac{b}{v}$

total acceleration $\quad a = \frac{\Delta v}{t_{col}}$ — in y direction

$a_y = \frac{-Zq^2\, b}{(b^2+x^2)^{3/2} m_e}$ $\quad \Delta v = \int_{-\infty}^{\infty} a_y\, dt = -\frac{Zq^2 b}{m_e}\int_{-\infty}^{\infty}\frac{dt}{(b^2+v^2t^2)^{3/2}}$

$y = vt \quad dy = v\,dt \quad \Delta v = \frac{-Zq^2 b}{m_e v}\int_{-\infty}^{\infty}\frac{dy}{(b^2+y^2)^{3/2}}$ $\quad y = ub \quad \frac{-Zq^2 b}{m_e v}\,\frac{b}{b^3}\int_{-\infty}^{\infty}\frac{du}{(1+u^2)^{3/2}}$

$\frac{-2Zq^2}{m_e v b}\int_0^\infty \frac{du}{(1+u^2)^{3/2}}$ $\quad u = \tan\theta \quad du = \frac{d\theta}{\cos^2\theta} \quad 1+\tan^2\theta = \sec^2\theta$ $\quad \frac{-2Zq^2}{m_e v b}\int_0^{\pi/2}\frac{d\theta}{\cos^2\theta}\cos^3\theta = -\frac{2Zq^2}{m_e v b} = \Delta v$

Larmor formula $\quad P = \frac{2}{3}\frac{q^2 a^2}{c^3} = \frac{2}{3}\frac{q^2}{c^3}\frac{4Z^2q^4}{m_e^2 b^4}$

$a = \frac{\Delta v}{t_{col}} = \frac{\Delta v}{b/v} = \frac{-2Zq^2}{m_e b^2}$

$$P = \frac{8}{3}\frac{Z^2 q^6}{m_e^2 c^3 b^4}$$ Avg power radiated in collision

$$E = P\,t_{col} = \frac{8}{3}\frac{Z^2 q^6}{m_e^2 c^3 b^3 v}$$

to get spectrum, Fourier analysis acceleration

$$\ddot{x}_F = \int_{-\infty}^{\infty} \ddot{x}(t)\, e^{i\omega t}\, dt \qquad \omega_{max} = \frac{\pi}{t_{col}} = \frac{\pi v}{b} \quad \frac{dE}{d\omega} = \frac{E}{\omega_{max}}$$

$$\frac{dE}{d\omega} = \frac{8 Z^2 q^6}{3\pi c^3 m_e^2 v^2 b^2} \qquad \frac{dE}{d\nu} = 2\pi\,\frac{dE}{d\omega} = \frac{16\, Z^2 q^6}{3\, m_e^2 c^3 v^2 b^2}$$

Now an ensemble of particles (multiple collisions)

$$\underbrace{4\pi}_{\text{steradians}}\, \underbrace{\varepsilon_\nu}_{\text{emissivity}} = \underbrace{n_i n_e}_{\#\text{ ions, } e^-} \langle \sigma v \rangle \frac{dE}{d\nu}$$

$$d\sigma = 2\pi b\, db \qquad 4\pi\varepsilon_\nu = n_i n_e \int_{b_{min}}^{b_{max}} 2\pi b v\, db\, \frac{dE}{d\nu} = 2\pi n_i n_e v\, \frac{16 Z^2 q^6}{3 m_e^2 c^3 v^2} \ln\frac{b_{max}}{b_{min}}$$

$\frac{1}{b}db$

$b_{max} = \frac{v}{\omega}$ $\qquad b_{min}$ — quantum effects (deBroglie wavelength); breakdown $\Delta v \sim v$ acceleration large

$$4\pi\varepsilon_\nu = n_i n_e \frac{1}{v}\frac{32\pi^2 Z^2 q^6}{3\sqrt{3}\, m_e^2 c^3}\, g_{ff} \qquad \text{Gaunt factor } g_{ff} = \frac{\sqrt{3}}{\pi}\ln\frac{b_{max}}{b_{min}}$$

Maxwell-Boltzmann dist'n $\quad f(v)\,dv = 4\pi v^2 \left(\frac{m_e}{2\pi kT}\right)^{3/2} e^{-\frac{m_e v^2}{2kT}}\, dv \qquad \int_0^\infty f(v)\,dv = 1$

$$\frac{1}{2}m_e v^2 = h\nu \implies v_{min} = \left(\frac{2h\nu}{m_e}\right)^{1/2} \qquad \int \frac{1}{v} f(v)\, dv$$

$$y = \frac{1}{2}\frac{mv^2}{kT} \qquad dy = \frac{m}{kT} v\, dv$$

$$4\pi\varepsilon_\nu = n_i n_e \frac{32\pi^2 Z^2 q^6}{3\sqrt{3}\, m_e^2 c^3} g_{ff} \int_{v_{min}}^{\infty} v\, e^{-\frac{1}{2}\frac{m_e v^2}{kT}}\, 4\pi \left(\frac{m_e}{2\pi kT}\right)^{3/2} dv \qquad \int \frac{kT}{m} dy\, e^{-y} = \frac{kT}{m} e^{-y_{min}} = \frac{kT}{m} e^{-h\nu/kT}$$

$$= n_i n_e \frac{128\pi^3 Z^2 q^6}{3\sqrt{3}\, m_e^2 c^3} g_{ff} \left(\frac{m_e}{2\pi kT}\right)^{3/2} \frac{kT}{m_e} e^{-h\nu/kT}$$

$$\boxed{4\pi\varepsilon_\nu = \frac{32\sqrt{2}\,\pi^{3/2} Z^2 q^6}{3\sqrt{3}\, m_e^2 c^3}\, n_i n_e\, g_{ff} \left(\frac{m_e}{kT}\right)^{1/2} e^{-h\nu/kT}}$$

$$\varepsilon_\nu = 5.44\times 10^{-39}\, Z^2 n_i n_e\, T^{-1/2} e^{-h\nu/kT} g_{ff} \quad \text{erg cm}^{-3}\ \text{s}^{-1}\ \text{Hz}^{-1}\ \text{ster.}^{-1}$$

$$\Lambda = 4\pi\int_0^\infty \varepsilon_\nu\, d\nu = \left(\frac{2\pi k}{3 m_e}\right)^{1/2} \frac{32\pi q^6}{3h\, m_e^2 c^3} Z^2 T^{1/2} n_i n_e\, \bar{g}_{ff} \quad \text{erg cm}^{-3}\ \text{s}^{-1}$$

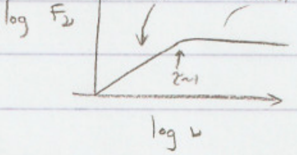
$$\frac{\varepsilon_\nu}{\kappa_\nu} = B_\nu(T) \simeq \frac{2\nu^2}{c^2} kT \quad \text{low freq. approx. (radio)}$$

$$\kappa_\nu \simeq \frac{c^2}{2\nu^2}\frac{\varepsilon_\nu}{kT} \propto \underline{\nu^{-2} n_i n_e T^{-3/2}} \qquad h\nu \ll kT$$

$$I_\nu = S_\nu\left(1-e^{-\tau_\nu}\right) \begin{cases} S_\nu \tau_\nu & \tau_\nu \ll 1 \\ S_\nu & \tau_\nu \gg 1 \end{cases} \qquad \tau_\nu = \kappa_\nu L$$

At low freq $\kappa_\nu \uparrow \Rightarrow \tau_\nu \gg 1$ $\qquad F_\nu = \Omega I_\nu \qquad I_\nu = \varepsilon_\nu L$

$h\nu + H \rightleftarrows H^+ + e^-$



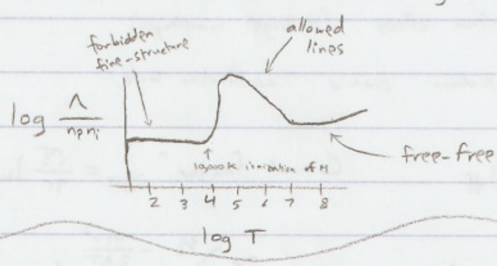
$$\tau \sim \int \frac{n_e^2}{T^{3/2}} ds \qquad T = 10{,}000\,K \quad \text{hot ionized gas}$$

can get hotter with additional sources of heat (ex. shockwaves)

$$\int n_e^2 ds = \text{emission measure } [cm^{-6}\,pc]$$

You get free-free emission from ionized gas

Solar Abundances



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Solid grains

(a) $\qquad x = \frac{2\pi a}{\lambda} \qquad$ Mie theory describes scattering, etc. using EnM

$x \ll 1 \qquad Q = \frac{\text{true crossection}}{\text{geometric crossection}} = \frac{\sigma_{eff}}{\pi a^2}$

$$Q_{abs} = -4x\,\mathrm{Im}\left\{\frac{m^2-1}{m^2+2}\right\} \qquad Q_{scat} = \frac{8}{3}x^4\left|\frac{m^2-1}{m^2+2}\right|^2 \qquad m = n - ik \quad \text{index of refraction}$$

different from $\vec{k}$

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \qquad \vec{k}_\nu = m\vec{k}_0$$

$x \gg 1 \qquad Q_{abs} \simeq 1 \qquad Q_{scat} \simeq 1 \qquad$ scattering $\propto \lambda^{-4}$

$$\sigma_{abs} = Q_{abs}\pi a^2 = -\frac{8\pi a^3}{\lambda}\mathrm{Im}\left\{\frac{m^2-1}{m^2+2}\right\} \qquad M_{gr} = \frac{4\pi}{3}a^3\rho_s$$

solid density

$$\sigma_{abs} = -\frac{6M_{gr}}{\lambda\rho_s}\mathrm{Im}\left\{\frac{m^2-1}{m^2+2}\right\} \qquad \chi = \frac{\sigma_{abs}}{M_{gr}}$$

$m^2 = n^2 - k^2 - 2ikn$ $\qquad \frac{m^2-1}{m^2+2} = \frac{n^2-k^2-2ikn-1}{n^2-k^2-2ikn+2}$

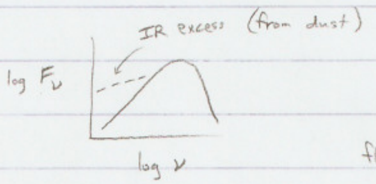
$= \frac{n^2-k^2-2ikn-1 \quad (n^2-k^2+2+2ikn)}{((n^2-k^2+2)-2ikn)\ ((n^2-k^2+2)+2ikn)}$

$\mathrm{Im}\{\ \} = \frac{-6nk}{(n^2-k^2+2)^2 + 4n^2k^2}$ $\qquad$ Silicate feature around $\sim 10\mu m$

$n \approx k \approx \left(\frac{\lambda \sigma_0}{c}\right)^{1/2}$ $\qquad \sigma_0$: DC conductivity $\rightarrow$ for conductors

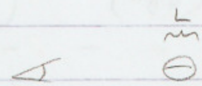
for long wavelengths $\quad \sigma_{abs} \propto \lambda^{-2}$ $\qquad \mathrm{Im}() \sim \frac{1}{nk} \sim \frac{1}{\lambda} \qquad \frac{1}{\lambda}\mathrm{Im}() \sim \frac{1}{\lambda^2}$

PAH features $\quad$ ⬡ absorbs photon, reemitts after distributing E through structure $\rightarrow$ vibration modes



$F_\nu = \varepsilon_\nu L \Omega$

flux $\quad$ emissivity $\quad$ path length through source $\quad$ solid angle



$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad I_\nu = S_\nu(1-e^{-\tau_\nu}) \qquad \tau_\nu \ll 1 \quad I_\nu = S_\nu \tau_\nu = \frac{\varepsilon_\nu}{\kappa_\nu}\kappa_\nu L = \varepsilon_\nu L$

$F_\nu = I_\nu \Omega$

thermal emission $\quad S_\nu = B_\nu(T)$

$\varepsilon_\nu = \kappa_\nu B_\nu(T) = \chi_\nu \rho B_\nu(T) \qquad \chi_\nu$ = opacity $[cm^2\, g^{-1}]$ $\quad \rho$ = mass density $[g\, cm^{-3}]$

D = distance to source $\qquad A = \Omega D^2 \qquad$ A = projected area on sky

$F_\nu = \chi_\nu \rho B_\nu(T) L \frac{A}{D^2} = \frac{\chi_\nu B_\nu(T)}{D^2}(\rho L A) = \frac{\chi_\nu B_\nu(T)}{D^2} M$

$M = \frac{D^2 F_\nu}{B_\nu(T)\chi_\nu}$ $\qquad$ Rayleigh-Jeans limit $\qquad M \cong \frac{D^2 F_\nu \lambda^2}{2\chi_\nu kT}$ $\qquad$ for optically thin

---

E, V, M, *Sun $\qquad \Sigma$ = surface density $\qquad$ Pulverize Earth

$\Sigma = \frac{M_E}{\pi R^2} \sim 10\ g/cm^2 \qquad \chi = \frac{\sigma}{m} = \frac{Q\pi a^2}{\frac{4\pi}{3}\rho_s a^3} = \frac{3}{4}\frac{Q}{a\rho_s}$

if $Q \simeq 1 \quad \chi = \frac{3}{4}\frac{1}{a\rho_s}$ $\qquad$ Silicates $\rho_s = 3\,g\,cm^{-3}$, $a = 10^{-3}\,cm$ $\qquad \chi = 25000\ cm^2 g^{-1} \qquad \tau = \Sigma\chi = 2.5\times10^5 \gg 1$

Earth atmosphere $\quad \Sigma \sim 10^4\ g/cm^2 \quad$ add some smoke (solid particles) $\rightarrow$ opaque $\quad \chi \sim 25{,}000$

Star $*$ Grain $\cdot$ → what T?

Steady state Energy Balance not necessarily true for smallest particles

$$\int_0^\infty Q_\nu(abs)\, \pi a^2\, 4\pi J_\nu\, d\nu = \int_0^\infty Q_\nu(abs)\, \pi a^2\, 4\pi B_\nu(T)\, d\nu$$

Energy In Energy out

Simple case $Q_\nu = 1$

$$\int_0^\infty J_\nu\, d\nu = \int_0^\infty B_\nu(T)\, d\nu \qquad J_\nu = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} I_\nu \sin\theta\, d\theta\, d\varphi$$

$$F_\nu = \int I_\nu \cos\theta\, d\Omega$$

far from star $\cos\theta \simeq 1$

$$J_\nu = \frac{F_\nu}{4\pi} = \frac{L_\nu}{4\pi D^2\, 4\pi}$$

→ OoMA Question

T of star

$$\int_0^\infty J_\nu\, d\nu = \int \frac{L_\nu}{(4\pi)^2 D^2}\, d\nu = \frac{4\pi R_*^2\, \sigma_{SB} T_*^4}{(4\pi)^2 D^2} = \frac{\sigma_{SB} T^4}{\pi}$$

$$T = T_*\left(\frac{R_*^2}{4D^2}\right)^{1/4} \qquad \boxed{T = \frac{T_*}{\sqrt{2}}\left(\frac{R_*}{D}\right)^{1/2}}$$

Approx: Q goes as power law

$$Q_\nu = Q_0\left(\frac{\nu}{\nu_0}\right)^p$$

$$\to \int_0^\infty \nu^p J_\nu\, d\nu = \int_0^\infty \nu^p B_\nu(T)\, d\nu \qquad J_\nu = \frac{F_\nu}{4\pi} = \frac{\pi B_\nu(T_*)\, 4\pi R_*^2}{4\pi D^2\, 4\pi} = \frac{1}{4}\frac{R_*^2}{D^2} B_\nu(T_*)$$

$$\frac{1}{4}\left(\frac{R_*}{D}\right)^2 \int_0^\infty \nu^p \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT_*}-1}\, d\nu = \int_0^\infty \nu^p \frac{2h\nu^3}{c^2}\frac{d\nu}{e^{h\nu/kT}-1}$$

$$\to \int_0^\infty \frac{\nu^{3+p}\, d\nu}{e^{h\nu/kT}-1} \qquad \frac{x = h\nu/kT}{dx = \frac{h}{kT}d\nu} \qquad \propto T^{4+p}$$

$$\frac{1}{4}\left(\frac{R_*}{D}\right)^2 T_*^{4+p} = T^{4+p} \qquad T = T_*\left(\frac{R_*}{2D}\right)^{\frac{2}{4+p}} = \boxed{T_*\left(\frac{1}{4}\right)^{\frac{1}{4+p}}\left(\frac{R_*}{D}\right)^{\frac{2}{4+p}} = T}$$

Dust emits inefficiently so are warmer than blackbody

Atomic Spectroscopy

2/5/2007

Saha Eq. local thermodynamic equilibrium (LTE)

V: volume N: total # of atoms $n_T = N/V$

$$\sum_j n(x^{+j}) = n_T(x) \qquad \frac{N(x^{+j+1})\,N_e}{N(x^{+j})} = \frac{f_{j+1}\, f_e}{f_j}\, e^{-I_0/kT}$$

→ ionization potential

j=0 neutral
j=1 singly ionized
X: species (H, O, ...)

partition function $\quad f_e = \frac{2}{h^3}(2\pi m_e kT)^{3/2}\, V$

depends on number density ... not ∞

H-atoms $\quad f_+ = \frac{2}{h^3}(2\pi m_H kT)^{3/2}\, V \qquad f_0 = 4\,\frac{(2\pi m_H kT)^{3/2}}{h^3}\, V\left(\sum_{j=1}^{j_{max}} j^2 e^{-E_j/kT}\right)$

(neutral H)

Pure H $\quad n_e = n(H^+)$

$$\frac{(N(H^+))^2}{N(H)} = \frac{(2\pi m_e kT)^{3/2}}{h^3}\, e^{-I_0/kT}$$

Saha Eq. for H

complication: need to consider H^-, H_2

Photoionization

$h\nu + H \rightleftharpoons H^+ + e^-$ Detailed Balance

rate of photo-ionization

$$\underset{\text{rate/atom}}{\Gamma} = \int_{\nu_0}^{\infty} 4\pi \frac{J_\nu}{h\nu}\,\sigma\, d\nu \qquad [\mathrm{erg\ cm^{-2}\ s^{-1}\ H^{-1}\ ster^{-1}}] = [J_\nu] \quad \therefore\ \Gamma \to [s^{-1}]$$

$h\nu_0$ = ionization potential (I_0)

$\Gamma\, n(H)$ # of ionizations per s per cm³

rate of recombination $\alpha(T)$ [cm³ s⁻¹] $\qquad \alpha(T) = \langle \sigma v\rangle \qquad n_e\, n(H^+)\,\alpha(T) = \Gamma\, n(H)$

$$\alpha(T) = \frac{n(H)\,\Gamma}{n_e\, n(H^+)}$$

LTE (using Saha Eq and $J_\nu = B(T)$)

$$\alpha(T) = \frac{h^3 e^{I_0/kT}}{(2\pi m_e kT)^{3/2}} \int_{\nu_0}^{\infty} 4\pi \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\,\frac{\sigma}{h\nu}\, d\nu$$

Approx:

$\sigma = \sigma_0\left(\frac{\nu_0}{\nu}\right)^2$ (not exact) $\qquad e^{h\nu/kT} \gg 1$

$$\alpha(T) = \frac{h^3 e^{I_0/kT}}{(2\pi m_e kT)^{3/2}}\,\frac{8\pi}{c^2}\,\sigma_0\nu_0^2 \int_{\nu_0}^{\infty} e^{-h\nu/kT}\, d\nu$$

$$\alpha(T) = \frac{8\pi\sigma_0 h^2 \nu_0^2}{(2\pi m_e)^{3/2} c^2}\left(\frac{1}{kT}\right)^{1/2}$$

$t = \frac{b}{v} \qquad b \sim 10^{-8}$ cm $\qquad v \sim 10^{8}$ cm/s $\qquad t \sim 10^{-16}$ s

$\frac{10^{-16}\,\mathrm{s}}{10^{-9}\,\mathrm{s}} \sim 10^{-7}$ chance of recombination

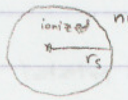
(time to emit photon)

$e^- + XY^+ \to X + Y$ Dissociative recombination
— can be faster than radiative recomb.

$$\frac{dn_e}{dt} = -n_e\, n(x^+)\,\alpha(T)$$

characteristic time $\quad t \sim \frac{1}{n(x^+)\,\alpha(T)} \sim \frac{1}{10^6\, 10^{-12}} \sim 10^6$ s

Strongren Sphere



$\therefore$ l mean free path of ionizing photon $l = \frac{1}{n\sigma}$

$\therefore$ $l \ll r_s$

I_* rate of emission of ionizing photons

$$\frac{4\pi}{3} r_S^3 n_e^2 \alpha(T) = I_* \qquad T \sim 10{,}000\,K$$

Forbidden lines

O III 5007 Å, 4959 Å

$\hookrightarrow O^{+2} \rightarrow 6e^-$ $\quad 1s^2 2s^2 2p^2$

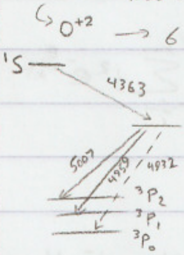
$l=1 \quad L = 2, 1, 0$

ground state $\left\{ {}^3P_2, {}^3P_1, {}^3P_0 \right.$

$3 = 2S+1 \quad \therefore S = 1$ total spin is 1

(spin of 2 outer e^- are aligned) Gp

$^{2S+1}L_J$ — orbital, J total angular momentum



3 : 1

(5007) (4959)

$e^- + O^{+2} \rightarrow O^{+2\,*} \rightarrow O^{+2} + h\nu$

'D need $T \sim 10{,}000$

'S need $T \sim 50{,}000$

ratio of 'S to 'D can give info on T

(Maxwell-Boltzmann distn. $\sim e^{-\frac{1}{2}\frac{mv^2}{kT}}$) $\rightarrow$ 'S stronger than 'D

Detailed Balance with Thermal

$$\Lambda = \text{heating rate / ionization} = \int_{\nu_0}^{\infty} 4\pi \frac{J_\nu \sigma_\nu}{h\nu}\, d\nu\, (h\nu - h\nu_0)$$

$$\Lambda\, n(H) = \sum_x n_e\, n(x^+) \langle \sigma v \rangle \langle \Delta E \rangle \qquad [\text{erg cm}^{-3}\,\text{s}^{-1}]$$

$$\Gamma\, n(H) = n_e^2\, \alpha(T) \quad \text{photoionization}$$

$$\frac{\Lambda\, \cancel{n_e^2}\, \alpha(T)}{\Gamma} = \cancel{n_e^2}\, f(T, \text{compos})$$

$\rightarrow$ common to have $T \sim 10{,}000\,K$ in ionized regions

lower T $\rightarrow$ slow down particles $\rightarrow$ heat up $\rightarrow$? cool down

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2/7/2007

Molecular Spectroscopy

Diatomic ⚬–⚬ $\quad E = B\,J(J+1) \quad B = \frac{\hbar^2}{2I} \quad g_J = 2J+1$ statistical weight

$I = \mu r^2 \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$ $\rightarrow$ leads to differences in isotopes ex: ^{13}CO, ^{12}CO

J=2

J=1

J=0

$\Delta J = 1$ allowed transitions

ortho	para
half-integer total nuclear spin	integer total nuclear spin
for H_2: 0	for H_2: 1

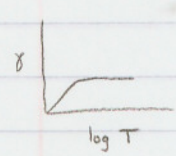
H_2 J	g (total)
0	1
1	9
2	5
3	21

$\rightarrow (2J+1)(3)$

$$f = \sum_i g_i e^{-E_i/kT}$$

$\therefore$ partition function more complicated, not so for HD

Avg. Energy $\quad \bar{E} = -\frac{\partial}{\partial \beta} \ln f \quad \beta = \frac{1}{kT} \quad C_V = \frac{d\bar{E}}{dt}$ specific heat $\quad \gamma = \frac{C_P}{C_V}$



if $\gamma < 4/3 \rightarrow$ self gravitating system is unstable and will collapse

grav self energy $\propto \frac{GM^2}{R} = \frac{GM^2}{V^{1/3}}$ thermal energy $\propto MT$ $P \propto \rho^\gamma$ $T \propto \left(\frac{M}{V}\right)^\gamma$

$\gamma = 1$ isothermal $\rightarrow T$ is constant during collapse

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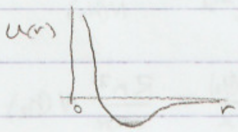
Raman scattering : photon scatters off molecule and excites rotational states
$\rightarrow$ not that important to astrophysics

Vibrational modes

$$E_n = \left(n + \tfrac{1}{2}\right) h\nu$$



$\Delta E = h\nu$ always? $\rightarrow$ no: $U \sim \frac{1}{2}kx^2$ is only approx to ⤴ $\therefore$ not all photons won't have same E

more complicated molecules (ex. PAHs) have more complicated modes

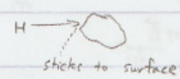
electronic : $\Sigma = S$ $\Lambda = P$ $\Delta = D$
new notation for molecules — total angular momentum

	ΔJ
O	-2
P	-1
Q	0
R	1
S	2

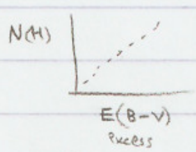
Franck – Condon Principle : nuclei stay where they are during electronic transition but can get diff. vibrational states

Molecules usually don't break apart with high electronic transitions (but atoms dissociate)
$\rightarrow e^-$ fall back quickly

H_2 how to form? $H + H \rightarrow H_2 + h\nu$ ... highly forbidden ... very slow
$\rightarrow$ forms catalytically on surface of grains
can form layer of H, which combine to form
H_2 and escape grain : $H + H + \text{grain} \rightarrow H_2 + \text{grain}$



$n(H)\, n_{gr} \langle \sigma_{gr} v \rangle \epsilon$ (efficiency) $\frac{n_{gr}\sigma_{gr}}{n} = \text{const}$ Column density $N = N(H) + 2N(H_2)$



$n(H)\, n R$ = formation rate $[cm^{-3}\, s^{-1}]$ $R = n_{gr} \langle \sigma_{gr} v \rangle \epsilon$ formation rate coefficient

I_d = destruction

$$\underbrace{I_d}_{\text{destruction rate}} n(H_2) = R\, n(H)\, n$$

(photodissociation)

$N(H) = \int n\, ds$ $\tau_v = \int n_{gr}\, \sigma_{gr}(v)\, ds$

$A_v = 1.09\, \tau_v$ $e^{-\tau_v} = 10^{-0.4 A_v}$ grains $\sim 0.1\, \mu m$ with wide range

$h\nu + H_2 \to H_2^*$ $\quad h\nu + H_2^* \to H_2^{**}$ can have enough vibration to break apart

$$I_d = \sum_i p_i \int_{-\infty}^{\infty} 4\pi J_\nu(\Delta\nu) \frac{\pi e^2}{mc} f_i \, \phi(\Delta\nu) \, d(\Delta\nu)$$

optically thin $J_\nu(\Delta\nu)$ is constant $\qquad I_d \, n(H_2) = n(H)\, n R$

$$\frac{n(H_2)}{n(H)} = \frac{Rn}{I_d} \sim \text{small number like } 10^{-5}$$

as it gets optically thick $\quad I_d \sim \frac{\text{const}}{N(H_2)} \qquad n(H_2) = \frac{d\,N(H_2)}{dx}$

$$\frac{d\,N(H_2)}{dx} I_d = Rn^2 \qquad \frac{d\,N(H_2)}{dx} = \frac{Rn^2}{\text{const}} N(H_2)$$

exponential growth

self-shielding $\quad \to H_2$ grows fast, high column densities

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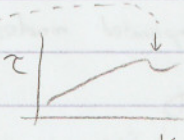
2200 Å



← absorption by interstellar medium

probably caused by Carbon in ring structure

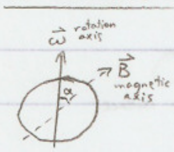
Diffuse Interstellar Bands (absorptions)

→ correlated to dust, more dust → stronger bands

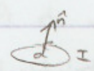
origin: unknown

2/12/2007

Pulsars



$\vec{m}$ magnetic dipole $\quad \vec{m} = \frac{Ia}{c}\hat{n}$



$$\vec{B} = \frac{3\hat{r}(\hat{r}\cdot\vec{m}) - \vec{m}}{|\vec{r}|^3}$$

$\vec{m} = m_0 \hat{z} \qquad \hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta \qquad \vec{B} = \frac{m_0}{|\vec{r}|^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$

At surface of star, max value $B_0 = \frac{2m_0}{R_*^3}$

power radiated $\quad P = \frac{2}{3}\frac{|\ddot{\vec{m}}|^2}{c^3}$

time varying $\quad \vec{m} = m_0 \sin\alpha\,(\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}) + m_0 \cos\alpha\,\hat{z}$

$$\ddot{\vec{m}} = \omega^2 m_0 \sin\alpha(\ldots) \qquad |\ddot{\vec{m}}|^2 = \omega^4 m_0^2 \sin^2\alpha$$

$$P = \frac{2}{3}\frac{1}{c^3}\omega^4 \sin^2\alpha \frac{R_*^6 B_0^2}{4}$$

$E_{rot} = \frac{1}{2} I \omega^2$ $\quad I = \frac{2}{5} M_* R_*^2$ $\quad E_{rot} = \frac{1}{5} M_* R_*^2 \omega^2$

$P = -\frac{dE_{rot}}{dt}$ $\quad T$ = period $\quad \omega = \frac{2\pi}{T}$ $\quad E_{rot} = \frac{1}{5} M_* R_*^2 \frac{4\pi^2}{T^2}$

$$-\frac{dE_{rot}}{dt} = + \frac{8\pi^2}{5} \frac{M_* R_*^2}{T^3}\frac{dT}{dt} = \frac{8\pi^4}{3c^3 T^4} \sin^2\alpha\, R_*^6 B_0^2$$

$\frac{dT}{dt} = \frac{1}{T} \frac{5\pi^2 B_0^2 R_*^4 \sin^2\alpha}{3c^3 M_*}$ $\quad B_0 \simeq 10^{12}$ Gauss

$\frac{dT}{dt} = 2\pi \frac{d\omega^{-1}}{dt} = \sim \frac{1}{T} \sim \omega$ $\quad$ If you take Br^2 = const for star, then can cause ~1 Gauss initial fields can be increased as star shrinks

$\frac{d\omega}{dt} = -k\omega^3$ $\quad \frac{d^2\omega}{dt^2} = 3\frac{1}{\omega}\frac{d\omega}{dt}$

n "braking index" $\quad \ddot{\omega} = n\frac{\dot{\omega}^2}{\omega}$

radio pulsars powered by spin-down

Anomalous X-ray pulsars
- pulsate in X-rays (not radio) — more energy radiated than available from spin-down

magnetar model: E radiated comes from dissipating $\vec{B}$ fields

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$\nabla\cdot\vec{E} = 4\pi\rho$ $\quad \nabla\cdot\vec{B} = 0$ $\quad \nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$ $\quad \nabla\times\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$

$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ $\quad \vec{E}_0 = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$

$i\vec{k}\cdot\vec{E} = 4\pi\rho$ $\quad i\vec{k}\cdot\vec{B} = 0$ $\quad i\vec{k}\times\vec{E} = \frac{i\omega}{c}\vec{B}$ $\quad i\vec{k}\times\vec{B} = -\frac{i\omega}{c}\vec{E} + \frac{4\pi}{c}\vec{J}$

n_e - density of e^- $\quad m_e\dot{\vec{v}} = -q\vec{E} = -i\omega m_e\vec{v}$ $\quad \vec{v} = \frac{q\vec{E}}{i\omega m_e}$

$\vec{J} = -q n_e \vec{v} = \frac{-q^2 n_e}{i\omega m_e}\vec{E} = \sigma\vec{E}$

$\sigma = \frac{i n_e q^2}{\omega m_e}$ $\quad \frac{\partial\rho}{\partial t} + \nabla\cdot\vec{J} = 0$ $\quad -i\omega\rho + i\vec{k}\cdot\vec{J} = 0$

$\rho = \frac{1}{\omega}\vec{k}\cdot\vec{J} = \frac{\sigma}{\omega}\vec{k}\cdot\vec{E}$

$i\vec{k}\cdot\vec{E} = 4\pi\frac{\sigma}{\omega}\vec{k}\cdot\vec{E}$ $\quad i(\vec{k}\cdot\vec{E})\left(1 - \frac{4\pi\sigma}{i\omega}\right) = 0$

$i(\vec{k}\cdot\vec{E})\underbrace{\left(1 - \frac{4\pi n_e q^2}{\omega^2 m_e}\right)}_{\epsilon} = 0$ $\quad i\vec{k}\times\vec{B} = -i\frac{\omega}{c}\vec{E} + \frac{4\pi}{c}\frac{i n_e q^2}{\omega m_e}\vec{E} = -i\frac{\omega}{c}\vec{E}\left(1 - \frac{4\pi n_e q^2}{\omega^2 m_e}\right)$

$\epsilon = 1 - \frac{\omega_p^2}{\omega^2}$ $\quad \omega_p^2 = \frac{4\pi n_e q^2}{m_e}$ plasma freq.

$c^2k^2 = \epsilon\omega^2 = \omega^2 - \omega_p^2$ $\quad$ $\omega \gg \omega_p$ vacuum like; $\omega < \omega_p$ no propagation $\quad \omega = \sqrt{c^2k^2 + \omega_p^2}$

$\sim \int n_e\, ds$ ← dispersion measure

$v_g = \frac{\partial\omega}{\partial k} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$ $\quad t_p = \int\frac{ds}{v_g} = \int\frac{ds}{c\sqrt{1-\frac{\omega_p^2}{\omega^2}}} \simeq \frac{s}{c} + \frac{1}{c\omega^2}\int\omega_p^2\, ds$

$\int B_\parallel n_e\, ds$ rotation measure if you have Faraday Rotation

Synchrotron Radiation

2/14/2007

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \qquad \frac{d}{dt}(\gamma m \vec{v}) = q\frac{\vec{v}}{c}\times\vec{B} \qquad \gamma m \frac{d\vec{v}}{dt} = q\frac{\vec{v}}{c}\times\vec{B}$$

no $\vec{E}$ field
Energy is constants ($\vec{B}$ due no work) $\therefore\ \gamma = \text{constant}$

$\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$ $\quad \vec{v}_{\parallel}$ — along $\hat{z}$ $\quad \vec{v}_{\perp} = v_x\hat{x} + v_y\hat{y}$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_{\parallel} \\ 0 & 0 & B \end{vmatrix} = \hat{x}v_yB - \hat{y}v_xB \qquad \gamma m_e \frac{dv_x}{dt} = \frac{q}{c}v_yB \quad \text{in } \hat{x}$$

$$\gamma m_e \frac{dv_y}{dt} = -\frac{q}{c}v_xB$$

$$\gamma m_e \frac{d^2v_x}{dt^2} = \frac{q}{c}B\frac{dv_y}{dt} = -\frac{q^2}{c^2}B^2\frac{1}{\gamma m_e}v_x \qquad \frac{d^2v_x}{dt^2} = -\frac{q^2B^2}{\gamma^2m_e^2c^2}v_x \qquad v_x = v_0\cos(\omega_B t)$$

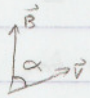
$$\omega_B = \frac{qB}{\gamma m_e c} \quad \text{relativistic cyclotron freq.}$$

Guiding-Center Approx. if $\vec{B}$ is not uniform

$$P = \frac{2}{3}\frac{q^2}{c^3}\gamma^4\left(a_\perp^2 + \gamma^2 a_\parallel^2\right) \qquad a - \text{acceleration}$$

$a_\perp = v_\perp\omega_B$ (circular motion) $\qquad a_\parallel = 0$

$$P = \frac{2}{3}\frac{q^2}{c^3}\gamma^4 v_\perp^2\omega_B^2 = \frac{2}{3}\gamma^2\frac{q^4B_0^2v_\perp^2}{m_e^2c^5}$$



$\alpha \leftrightarrow \theta$

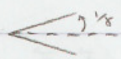
$$\langle v_\perp^2\rangle = v_0^2\langle\sin^2\alpha\rangle = v_0^2\frac{1}{4\pi}\underbrace{\int\sin^2\alpha\, d\Omega}_{\frac{8\pi}{3}} = \frac{2}{3}v_0^2$$

$$P \simeq \frac{4}{9}\frac{q^4\gamma^2B_0^2v_0^2}{m_e^2c^5} \simeq \frac{4}{9}\gamma^2\frac{q^4B_0^2}{m_e^2c^3} \quad \ldots v_0\simeq c \ldots \text{ this is power for } 1e^-$$

$$E = \gamma mc^2 \qquad t_{syn} = \frac{E}{P} = \frac{\gamma m_ec^2\ 9m_e^2c^3}{4\gamma^2q^4B_0^2} = \frac{1}{\gamma}\frac{1}{B_0^2}\frac{9}{4}\frac{m_e^3c^5}{q^4}$$

$L = n_e P$ — # of relativistic e^-

Relativistic beaming



... fraction that beam sweeps → not 2π but $\frac{2}{\gamma}$

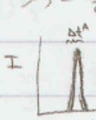
Δt — effective time of pulse emission $\sim \frac{1}{\omega_B}\frac{2}{\gamma}$

arrival time: Δt^A ← less than Δt since it moves closer to us

$$\Delta t^A = \Delta t(1 - v/c) \qquad 1 - \frac{v}{c} \simeq \frac{1}{2\gamma^2} \text{ for } v\approx c \qquad v/c = \beta \qquad 1-\beta = \varepsilon$$

$$\frac{1}{2\gamma^2} = \frac{1}{2}(1-\beta^2) = \frac{1}{2}(1-\beta)(1+\beta) = \frac{1}{2}(\varepsilon)(2-\varepsilon) = \varepsilon - \frac{\varepsilon^2}{2} \simeq \varepsilon = 1-\beta$$

$$\therefore\ \Delta t^A = \frac{1}{\gamma^3\omega_B}$$



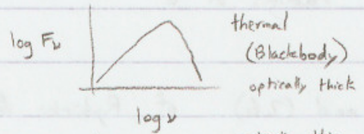
Fourier analyze to get spectrum

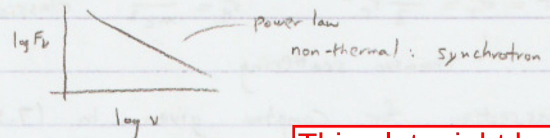
narrow peak → inverse of time will be characteristic freq. $\qquad \omega_c \simeq \frac{1}{\Delta t^A} \simeq \omega_B\gamma^3$

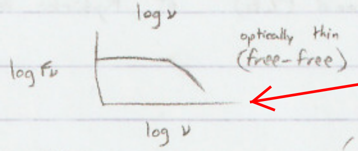
$$\nu_c = \frac{1}{2\pi}\omega_B\gamma^3 \qquad \nu_c \propto \gamma^2$$

How to know if you're seeing synchrotron radiation:

1) Spectrum







This plot might be backward.
Refer to Free-free emission.

2) Surface Brightness flux $F_\nu = I_\nu \Omega$ (surface brightness; solid angle) $I_\nu = S_\nu(1-e^{-\tau_\nu})$ $I_\nu = S_\nu$ $\tau_\nu \gg 1$; $I_\nu = S_\nu \tau_\nu$ $\tau_\nu \ll 1$; $S_\nu \geq I_\nu$

blackbody $S_\nu = B_\nu(T)$ radio $S_\nu = \frac{2\nu^2}{c^2} kT$ $F_\nu = \frac{2\nu^2}{c^2} kT\Omega$

can measure Ω, F_ν to determine T if T is very high $\rightarrow$ synchrotron

ex. $T \sim 10^{12}$ K $\rightarrow$ relativistic

3) Polarization due to $\vec{B}$, α, etc.

4) Time Variation relativistic particles loose E by radiation

$N(E)\,dE \sim N_0 E^{-p}\,dE$ # of relativistic particles

$$P_\nu(tot) = \int_{E_{min}}^{E_{max}} N_0 E^{-p} G\left(\frac{\nu}{\nu_c}\right) dE$$

$G\left(\frac{\nu}{\nu_c}\right)$ = Fourier analysis of synchrotron peak function of spectrum

$\nu_c \propto \omega_B \gamma^3 \alpha \gamma^2$
$E \propto \gamma$

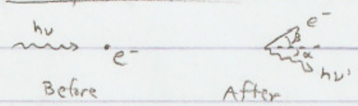
$$\nu_c = kE^2 \qquad x = \frac{\nu}{\nu_c} = \frac{\nu}{kE^2} \qquad E = \left(\frac{\nu}{k}\frac{1}{x}\right)^{1/2} \qquad dE = \left(\frac{\nu}{k}\right)^{1/2}\left(-\frac{1}{2}\right) x^{-3/2}\,dx$$

$$P_\nu(tot) = \nu^{1/2} N_0 \nu^{-p/2} k^{p/2} \int_{x_{min}}^{x_{max}} \frac{x^{p/2} G(x)}{2} k^{-1/2} x^{-3/2}\,dx$$

$$P_\nu \propto \nu^{-\frac{p}{2}+\frac{1}{2}} \qquad P_\nu \propto \nu^{-s} \qquad s = \frac{p-1}{2} \text{ spectral index}$$

2/21/2007

Compton Effect



$h\nu + mc^2 = h\nu' + \gamma mc^2$ Energy correction

$\hat{x}$: $\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\alpha + m\gamma v\cos\beta$

$\hat{y}$: $0 = \frac{h\nu'}{c}\sin\alpha - m\gamma v\sin\beta$

Momentum conservation

$$\cos\beta = \left(\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\alpha\right)\left(\frac{1}{\gamma m v}\right) \qquad \sin\beta = \left(\frac{h\nu'}{c}\sin\alpha\right)\left(\frac{1}{\gamma m v}\right)$$

$$\cos^2\beta = \left(\frac{1}{\gamma m v}\right)^2\left(\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\alpha\right)^2 \qquad \sin^2\beta = \left(\frac{1}{\gamma m v}\right)^2\left(\frac{h\nu'}{c}\right)^2\sin^2\alpha \longrightarrow \text{add together and simplify}$$

$\gamma^2 m^2 v^2 c^2 = (h\nu)^2 + (h\nu')^2 - 2h^2\nu\nu'\cos\alpha$

energy: $\gamma^2 m^2 c^4 = (h\nu + mc^2 - h\nu')^2$

subtract $(c^2 - v^2) = \left(1 - \frac{v^2}{c^2}\right)c^2 = \frac{1}{\gamma^2}c^2$

$$(1-\cos\alpha) = \frac{mc^2}{h}\frac{\nu - \nu'}{\nu\nu'} = \frac{mc^2}{h}\left(\frac{\lambda'}{c} - \frac{\lambda}{c}\right) \qquad 1-\cos\alpha = \frac{mc}{h}\Delta\lambda$$

$$\boxed{\Delta\lambda = \lambda_0(1-\cos\alpha)} \qquad \lambda_0 = \frac{h}{mc} \text{ Compton wavelength} \qquad 1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$$

$\sigma = \sigma_T = \frac{8\pi}{3} r_e^2 \qquad r_e = \frac{e^2}{mc^2}$ classical radius of e^-

└ Thomson scattering

Crosssection for Compton given in (7.5) and (7.6) of Rybicki & Lightman

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Before $\rightsquigarrow \quad \overleftarrow{e^-}$ After $\leftsquigarrow \quad \overleftarrow{e^-}$ $\quad \alpha = 180°$

Lab $\quad E_B = h\nu$

e^- frame $\quad E_B' = h\nu\,\gamma(1+\beta) = h\nu' \qquad E_A' = h\nu'\left(1 - \frac{2h\nu'}{mc^2}\right) = h\nu_A'$

$\nu = \frac{c}{\lambda} = \frac{c}{\lambda_0 + \Delta\lambda} \simeq \frac{c}{\lambda_0}\left(1 - \frac{\Delta\lambda}{\lambda}\right)$

$\nu = \nu_0\left(1 - \frac{\Delta\nu}{\nu}\right)$

$\Delta\lambda = 2\lambda_0$

Lab $\quad E_A = \gamma h\nu_A'(1+\beta) = h\nu\,\gamma^2(1+\beta)^2\left(1 - \frac{2\gamma(1+\beta)h\nu}{mc^2}\right)$ Compton correction $\quad \alpha = 180$

$h\nu \ll mc^2 \qquad v/c \ll 1 \qquad E_A = h\nu(1+2\beta)$ Doppler shift for radar

$\beta \sim 1 \quad E_A = 4\gamma^2 E_B$ Inverse Compton Effect photon gets energy from relativistic e^-

$E_B\,\gamma^2(2)^2\left(1 - \underbrace{2\gamma}_{\text{small for } \gamma < 10^4}\right)$ for $\gamma = 10^3 \quad E_A = 4\times10^6 E_B$

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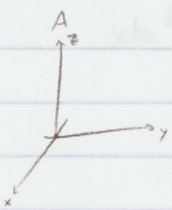
now: an ensemble



e^- at origin with $\vec{v} = v_0(\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z})$

field of photons $\quad \hat{n} = \sin\theta'\cos\phi'\,\hat{x} + \sin\theta'\sin\phi'\,\hat{y} + \cos\theta'\,\hat{z}$

$(-\hat{n})$

electron frame: $\frac{\Delta\nu'}{\nu'} = \left(1 + \frac{\vec{v}\cdot\hat{n}}{c}\right)\gamma \qquad \frac{\Delta\nu''}{\nu''} = \left(1 + \frac{\vec{v}\cdot\hat{z}}{c}\right)\gamma$: photon knocked back to observer (going to lab)

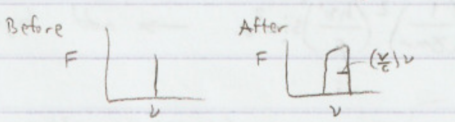
total doppler shift: $\frac{\Delta\nu}{\nu} = \gamma^2\left(1 + \frac{\vec{v}\cdot\hat{n}}{c}\right)\left(1 + \frac{\vec{v}\cdot\hat{z}}{c}\right)$ (have ignored Compton effect)

what is average doppler shift? Ignoring aberrations

$$\left\langle\frac{\Delta\nu}{\nu}\right\rangle = \frac{1}{(4\pi)^2}\int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi}\int_0^{\pi}\gamma^2\Big(1 + \frac{v_0}{c}\underbrace{(\sin\theta\sin\theta'\cos\phi\cos\phi' + \sin\theta\sin\theta'\sin\phi\sin\phi'}_{\text{will drop out } \int_0^{2\pi}\cos\phi\, d\phi = 0 \text{ or } \sin\phi} + \cos\theta\cos\theta')\Big)\left(1 + \frac{v_0}{c}\cos\theta\right)\sin\theta\sin\theta'\,d\theta\,d\theta'\,d\phi\,d\phi'$$

$$\left\langle\frac{\Delta\nu}{\nu}\right\rangle = \frac{1}{(4\pi)^2}(2\pi)^2\int \dots = \frac{1}{4}\int_0^{\pi}\int_0^{\pi}\gamma^2\left(1 + \frac{v_0}{c}\cos\theta\cos\theta'\right)\left(1 + \frac{v_0}{c}\cos\theta\right)\sin\theta\sin\theta'\,d\theta\,d\theta' = \gamma^2$$

off by factor of $4/3$ (?)

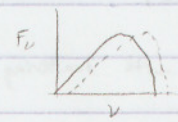


energy is flowing

$\gamma^2 = \frac{1}{1-\beta^2} \simeq 1 + \beta^2$

Zeldovich – Sunyaev Effect

Cluster of galaxies → CMB photons get doppler shifted



CMB Planck function is shifted

can be used to measure Hubble constant (with X-ray and Radio)

$F \propto n_e^2 V T^{1/2}$ (free-free emission)

brightness temperature $T \sim 10^{12}$ K maximum: if higher → synchrotron losses reduce it
or other(?)

2/26/2007

Scattering

isotropic scattering n - density σ_{scat} – crossection

$$d\tau_s = n\sigma_s\,dx \qquad d\tau = d\tau_s + d\tau_A \qquad d\tau = n(\sigma_s+\sigma_A)\,dx$$

absorption

$= n\sigma_s J$

production rate — mean intensity $J = \frac{1}{4\pi}\int I\,d\Omega$

equivalent to emissivity [erg cm^{-3} s^{-1} Hz^{-1} ster^{-1}]

both s+A emissivity

$$\frac{dI}{dx} = -\kappa_T I + \varepsilon + n\sigma_s J \qquad \frac{dI}{d\tau} = -I + \frac{\varepsilon}{\kappa_T} + \frac{n\sigma_s}{\kappa_T}J$$

$\kappa_T = n(\sigma_s+\sigma_A)$

$$\frac{dI}{d\tau} = -I + \frac{\varepsilon}{\kappa_A}\frac{\kappa_A}{\kappa_T} + \frac{\kappa_{scat}J}{\kappa_T}$$

thermal source function $\frac{\varepsilon}{\kappa_A}$ Planck function at LTE

albedo $a = \frac{\kappa_s}{\kappa_s+\kappa_A} = \frac{\sigma_s}{\sigma_s+\sigma_A}$

$$\frac{dI}{d\tau} = -I + B(1-a) + aJ = -I + S$$

$S = (1-a)B + aJ$

if S = constant $I = S(1-e^{-\tau}) \simeq S\tau$ if $\tau \ll 1$

Ex: Sky (dustless, cloudless) $F_{Sun} = \frac{L}{4\pi D^2}$ $F = \int I\cos\theta\,d\Omega$ $J = \frac{1}{4\pi}\int I\,d\Omega$

$J \simeq \frac{F}{4\pi}$

$\cos\theta \simeq 1$

$\therefore\ J = \frac{1}{(4\pi)^2}\frac{L}{D^2}$

$I = S\tau$ $S = J$ $(a=1)$ (for optical λ)

$I = \frac{1}{(4\pi)^2}\frac{L}{D^2}\tau = \tau\frac{F}{4\pi}$ $I = J\tau$

Now more precisely:

$F_{Sun}(\text{directly}) = \frac{L}{4\pi D^2}e^{-\tau/\mu_*}$ vertical optical depth

zenith angle $\mu_* = \cos\theta_*$

τ θ $\tau/\cos\theta$

$$I = \frac{L}{16\pi^2 D^2}\left(1-e^{-\tau/\mu}\right)$$

$$F_{sky} = \int_0^{\pi/2}\int_0^{2\pi}\left(1-e^{-\tau/\mu}\right)\frac{L}{16\pi^2D^2}\mu\sin\theta\,d\theta\,d\varphi = 2\pi\underbrace{\frac{L}{(4\pi)^2D^2}}_{J}\int_0^{\pi/2}\left(1-e^{-\tau/\mu}\right)\mu\sin\theta\,d\theta$$

$$= \pi J - 2\pi J\int_0^{\pi/2}e^{-\tau/\mu}\mu\sin\theta\,d\theta$$

$x = \frac{1}{\cos\theta}$ $dx = +\frac{\sin\theta\,d\theta}{\cos^2\theta}$ $\cos^2\theta\,dx = \sin\theta\,d\theta = \frac{dx}{x^2}$

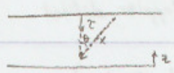
$$\therefore \int_1^\infty e^{-\tau x}\frac{dx}{x^3} = E_3(\tau) \qquad E_n(\tau) = \int_1^\infty \frac{e^{-\tau x}}{x^n}dx$$

$F_{sky} = \pi J - 2\pi J \, E_3(\tau)$ for $\tau = 0.1$ $\theta_* = 45°$ $\frac{F_{sky}}{F_{sun}} = 0.048$

plane parallel atmosphere with scattering

$$\frac{dI}{d\tau} = -I + (1-a)B + aJ$$

(generalized τ)



$dx = -\frac{dz}{\cos\theta} = -\frac{dz}{\mu}$ (Stellar Atmosphere)

Now plane-parallel τ: (vertical τ)

$$\mu \frac{dI}{d\tau} = I - (1-a)B - aJ$$

Two Stream Approximation

I_+ intensity in up direction $-\tau, +z$ ($\mu = +1$) I_- intensity down $-z, +\tau$ ($\mu = -1$)

$$\frac{dI_+}{d\tau} = I_+ - (1-a)B - aJ \qquad \frac{dI_-}{d\tau} = -I_- + (1-a)B + aJ \qquad J = \tfrac{1}{2}(I_+ + I_-)$$

$H = \text{flux} = \frac{1}{2}(I_+ - I_-)$

add: $\frac{d}{d\tau}(I_+ + I_-) = 2\frac{dJ}{d\tau} = 2H$ $\qquad \frac{dJ}{d\tau} = H$

subtract: $\frac{d}{d\tau}(2H) = 2J - 2(1-a)B - 2aJ$ $\qquad \frac{dH}{d\tau} = (1-a)J - (1-a)B$

Say: $B = \text{const.}$ $\qquad \frac{d^2J}{d\tau^2} = \frac{dH}{d\tau} = J(1-a) - B(1-a)$

$$J = C_1 e^{\sqrt{1-a}\,\tau} + C_2 e^{-\sqrt{1-a}\,\tau} + B$$

$C_1 = 0$ to keep J bounded $\qquad$ top of atmosphere $I_- = 0$, $\tau = 0$ no incoming atmosphere (this is stellar atm)

$H = J$ at $\tau = 0$

$$\frac{dJ}{d\tau} = -C_2\sqrt{1-a}\,e^{-\sqrt{1-a}\,\tau} \qquad J(0) = C_2 + B \qquad H(0) = -C_2\sqrt{1-a}$$

$\therefore C_2 + B = -C_2\sqrt{1-a}$ $\qquad C_2(1+\sqrt{1-a}) = -B$ $\qquad C_2 = \frac{-B}{1+\sqrt{1-a}}$

$$J = B\left(1 - \frac{e^{-\sqrt{1-a}\,\tau}}{1+\sqrt{1-a}}\right) \qquad H = B\frac{\sqrt{1-a}}{1+\sqrt{1-a}}e^{-\sqrt{1-a}\,\tau} \qquad I_+ = B\left(1 - e^{-\sqrt{1-a}\,\tau}\right)$$

sum of $J+H$

$I_+(0) \simeq 0$ if $a \simeq 1$

scattering removes light $\rightarrow$ strong calcium lines

$\tau \to \infty$ (deep in atm) $\quad I_+ \to B$, $H \to 0$ as expected

Non LTE Line Emission

2/28/2007

$S_\nu = \frac{\varepsilon_\nu}{\kappa_\nu} \neq B_\nu(T)$ ex: $S_\nu = (1-a)B_\nu + aJ_\nu$ ... scattering

$\Delta E \{$ — u, — L $\quad$ n – density of colliders

n_L – density in L level

n_u – " " u "

$n n_L C_{Lu} = n n_u C_{uL}$ in steady state

C_{Lu} – collisional excitation coefficient

C_{uL} – " deexcitation "

$C_{Lu} = \langle \sigma_{Lu} v \rangle$ [$cm^3 s^{-1}$]

$C_{uL} = \langle \sigma_{uL} v \rangle$

$$\frac{n_u}{n_L} = \frac{C_{Lu}}{C_{uL}} \left(\frac{n_u}{n_L} = \frac{g_u}{g_L} e^{-\frac{\Delta E}{kT}} \text{ in LTE} \right)$$ g – statistical weight in level

$$C_{Lu} = C_{uL} \frac{g_u}{g_L} e^{-\frac{\Delta E}{kT}}$$

$\sigma_{uL} \simeq 10^{-16}$ cm^2 $\qquad \bar{v} = \left(\frac{8}{\pi}\frac{kT}{m}\right)^{1/2}$

$$n_L(\underbrace{nC_{Lu}}_{\text{collisional excitation}} + \underbrace{B_{Lu}\bar{J}}_{\text{radiative emission}}) = n_u(\underbrace{nC_{uL}}_{\text{collisions}} + \underbrace{B_{uL}\bar{J}}_{\text{stimulated emission}} + \underbrace{A_{uL}}_{\text{spontaneous emission}})$$

$$\bar{J} = 4\pi \int_{-\infty}^{\infty} J_\nu(\Delta\nu)\,\phi(\Delta\nu)\,d(\Delta\nu)$$

mean intensity $= \frac{1}{4\pi}\int I_\nu d\Omega$

line profile

Ex: formation of Calcium lines in Sun

Hydrostatic Equil. $\frac{dP}{dz} = -\rho g$ z: upwards $\qquad \frac{dP}{\rho dz} = -g = \frac{\chi dP}{\chi \rho dz}$ $\quad \chi$ – opacity [$cm^2 g^{-1}$]

$d\tau = -\chi \rho dz \qquad \chi \frac{dP}{d\tau} = g \qquad \frac{dP}{d\tau} = \frac{g}{\chi} \qquad P \simeq \frac{g}{\chi}\tau$ (if χ = const. which it isn't)

photosphere $\tau = \frac{2}{3} \qquad P \simeq \frac{2}{3}\frac{g}{\chi} \qquad P = nkT \quad \therefore n \approx \frac{g}{\chi kT}$ characteristic density in photosphere

$g = 2.7 \times 10^4 \frac{cm}{s^2} \qquad T \approx 6000-7000$ K

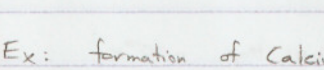
if fully ionized: $\chi = 0.4$ cm^{2}g^{-1}

$h\nu + H^- \rightarrow H + e^-$: source of opacity $\qquad \chi = \frac{\sigma}{m} = \frac{4\times10^{-17}}{1.7\times10^{-24}} = 2\times10^7$ cm^{2}g^{-1} but not all H is in H^-

another source is excited H $\quad H^* + h\nu \rightarrow H^+ + e^-$ but not that significant for Sun

take $\chi \sim 1$ cm^{2}g^{-1} $\Rightarrow n \simeq 2\times10^{16}$ cm^{-3} lower density than in Earth's atmosphere

$C_{uL} \sim 10^{-10}$ cm^3 s^{-1} $\qquad A_{uL}$ higher for Calcium so use scattering and not collisional deexcitation



Calcium K line (3933.4 Å) $\qquad Ca^+$

$$H = \frac{\sqrt{1-a}}{1+\sqrt{1-a}} e^{-\sqrt{1-a}\,z} B \qquad z=0 \quad H = \frac{\sqrt{1-a}}{1+\sqrt{1-a}} B$$

$a=1 \quad H=0$

$a=0.5 \quad H \sim 0.5B$

$\kappa_{scat} = \kappa_{abs} \qquad \kappa = \rho\chi$

scattering = absorption $\rightarrow$ opacity in line = opacity in continuum (say H^-) dominant

$\underset{\text{(continuum)}}{\rho_H \chi_H} = \underset{\text{(line)}}{\rho_{Ca}\chi_{Ca}} \qquad n(H)m_H\chi_H = n(Ca^+)m_{Ca}\chi_{Ca^+}$

assume all Ca is Ca^+

$$\chi(Ca^+)[\Delta\nu] = \underbrace{\frac{\pi e^2 f}{m_e c^2}\frac{\delta}{\pi(\Delta\nu)^2}}_{\text{crosssection}}\frac{1}{m_{Ca}}$$

$$\chi(Ca^+)[\Delta\nu] = 1100 \text{ cm}^2 g^{-1}$$

$\delta = \frac{A_{uL}}{4\pi}$

$$\frac{n(Ca^+)}{n(H)} = \frac{m_H}{m_{Ca}} \frac{\chi_H}{\chi_{Ca^+}}$$ take $\chi_H \sim 0.1\ cm^2 g^{-1}$

$\simeq \frac{1}{40} \frac{10^{-1}}{10^3}$ → $\frac{n(Ca^+)}{n(H)} = 2\times10^{-6}$

$$\kappa_\nu = \frac{\pi e^2}{m_e c} f\left(1-e^{-h\nu/kT}\right)\phi(\Delta\nu)\, n_L$$

$$\sigma_\nu = \frac{\pi e^2}{mc} f\left(1-e^{-h\nu/kT}\right)\phi(\Delta\nu) \qquad \chi_\nu = \frac{\kappa_\nu}{m}$$

Thermal: $\phi(\Delta\nu) = \frac{1}{\sqrt{\pi}}\frac{1}{(\Delta\nu_0)} e^{-\left(\frac{\Delta\nu}{\Delta\nu_0}\right)^2}$

Masers

$$I_{obs} = I_{back}\, e^{-\tau} + S\left(1-e^{-\tau}\right)$$

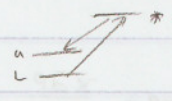
$\tau_\nu = \kappa_\nu L \qquad S_\nu = \frac{\varepsilon_\nu}{\kappa_\nu}$ if κ_ν is negative: $I_{obs} \simeq -S e^{-\tau}$

$$\kappa_\nu = \left(n_L B_{Lu} - \underbrace{n_u B_{uL}}_{\sim\text{stimulated emission}}\right)\frac{\phi(\Delta\nu)}{4\pi}$$ more stimulated emission than absorption

$$n_L\left(n C_{Lu} + B_{Lu}\bar{J}\right) = n_u\left(n C_{uL} + \bar{J} B_{uL} + A_{uL}\right) \qquad \frac{n_u}{n_L} = \frac{n C_{Lu} + B_{Lu}\bar{J}}{n C_{uL} + \bar{J} B_{uL} + A_{uL}}$$

$$\frac{C_{Lu}}{C_{uL}} \le \frac{g_u}{g_L} \qquad \frac{B_{Lu}}{B_{uL}} = \frac{g_u}{g_L} \qquad \therefore\ \frac{n_u}{n_L} \le \frac{g_u}{g_L}$$ cant have maser with just 2 levels

need:



$$I_{obs} = \frac{F\ (\text{flux})}{\Omega\ (\text{solid angle})} = \frac{2kT_b}{\lambda^2}$$ brightness temp. $\sim 10^{10}$ K in radio (see pg. 4)

OH, H_2O, SiO Masers

in ISM or outer envelopes of stars

Masers vary quickly (small changes can reduce amplification)

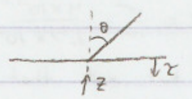
can be highly polarized

Masers have narrow lines: $e^{\tau} \qquad \tau_\nu \sim \phi(\Delta\nu)$

3/5/2007

Stellar Atmospheres

plane parallel



$\mu = \cos\theta$

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$ flow of radiation $\quad I_\nu = I_\nu(\theta, \tau)$

Constant flux through atm (no creation/destruction of E)

$$F = \int_0^\infty \int_0^\pi \int_0^{2\pi} I_\nu \cos\theta \sin\theta\, d\theta\, d\varphi\, d\nu \qquad F = 2\pi \int_0^\infty \int_{-1}^{1} I_\nu\, \mu\, d\mu\, d\nu$$

Integrate over all freq: grey approx $\tau_\nu = \tau$

$$\mu \frac{dI}{d\tau} = I - S \qquad S = \int_0^\infty S_\nu\, d\nu \qquad \text{in LTE } S_\nu = B_\nu(T)$$

$$S = \int_0^\infty B_\nu\, d\nu = \frac{\sigma_{SB}}{\pi} T^4$$

Integrate over sky

$$\frac{d}{d\tau}\int \mu I \, d\Omega = \int I \, d\Omega - \int S \, d\Omega$$

$$\frac{dF}{d\tau} = 4\pi J - 4\pi S$$

if flux is constant: $J = S$

$$\mu^2 \frac{dI}{d\tau} = \mu I - \mu S \qquad \frac{d}{d\tau}\int \mu^2 I \, d\Omega = \int \mu I \, d\Omega - \int \mu S \, d\Omega$$

$0 : \int_0^\pi \sin\theta \cos\theta \, d\theta = 0$

Eddington Approx (or Diffusion Approx) $\quad I \cong J \quad$ mostly isotropic

$$\frac{dJ}{d\tau}\int \mu^2 d\Omega = 2\pi \frac{dJ}{d\tau}\int_0^\pi \cos^2\theta \sin\theta \, d\theta = 2\pi \frac{dJ}{d\tau}\int_{-1}^{1} \mu^2 d\mu = \frac{4\pi}{3}\frac{dJ}{d\tau}$$

$$\frac{4\pi}{3}\frac{dJ}{d\tau} = F \qquad J = \frac{3}{4\pi}F\tau + C$$

at $\tau = 0$ no incoming radiation: $J = \frac{I}{2}$ also $F = \pi I$ $\therefore$ $J = \frac{F}{2\pi}$

$$\therefore C = \frac{F}{2\pi} \qquad J = \frac{\sigma T^4}{\pi} \qquad F = \sigma T_e^4$$

T_e: effective temp.

$$\sigma T^4 = \frac{3}{4\pi}\sigma T_e^4 \tau + \frac{1}{2\pi}\sigma T_e^4$$

$$\boxed{T^4 = T_e^4\left(\frac{1}{2} + \frac{3}{4}\tau\right)} \qquad T^4 = \frac{3}{4}T_e^4\left(\tau + \frac{2}{3}\right)$$

Kramer's

$$\mu \frac{dI_\nu}{dz} = -\chi_\nu \rho I_\nu + \varepsilon_\nu \qquad \frac{\mu}{\chi_\nu \rho}\frac{dI_\nu}{dz} = -I_\nu + \frac{\varepsilon_\nu}{\chi_\nu \rho} = -I_\nu + B_\nu$$

(opacity)

multiply by μ $\therefore$ integrate over sky + freq.

$$\frac{1}{\rho}\int_0^\infty \int \frac{\mu^2}{\chi_\nu}\frac{dI_\nu}{dz} \, d\Omega \, d\nu = -F \qquad \frac{1}{3\rho}\int_0^\infty \frac{1}{\chi_\nu}\frac{dB_\nu}{dz} d\nu = -F$$

$4\pi I \to J \quad \int \mu^2 d\Omega = \frac{4\pi}{3} \quad J \to B$

$$\frac{1}{\bar\chi} = \frac{\int_0^\infty \frac{1}{\chi_\nu}\frac{dB_\nu}{dz} d\nu}{\int_0^\infty \frac{dB_\nu}{dz} d\nu} \qquad \text{Rosseland Mean Opacity}$$

$$\frac{dB_\nu}{dz} = \frac{\partial B_\nu}{\partial T}\frac{\partial T}{\partial z} \quad \therefore \quad \frac{1}{\bar\chi} = \frac{\int_0^\infty \frac{1}{\chi_\nu}\frac{\partial B_\nu}{\partial T} d\nu}{\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu} = \frac{\int_0^\infty \frac{1}{\chi_\nu}\frac{\partial B_\nu}{\partial T} d\nu}{\frac{\partial}{\partial T}(B(T))}$$

$$B = \frac{\sigma}{\pi}T^4 \qquad \frac{\partial B}{\partial T} = \frac{4\sigma}{\pi}T^3$$

$$\therefore \boxed{\frac{1}{\bar\chi} = \frac{\pi}{4\sigma T^3}\int_0^\infty \frac{1}{\chi_\nu}\frac{\partial B_\nu}{\partial T} d\nu}$$

$$\bar\chi = \frac{\int_0^\infty \chi_\nu B_\nu d\nu}{\int_0^\infty B_\nu d\nu}$$

Planck mean

Weak Spectral Line in atm.

$$T^4 = T_e^4\left(\frac{1}{2} + \frac{3}{4}\tau\right) \qquad S_\nu = a_\nu + b_\nu \tau_\nu \qquad F_\nu = \left(a_\nu + \frac{2}{3} b_\nu\right)\pi$$

$$b_\nu = \frac{\partial S_\nu}{\partial \tau_\nu}(\tau_\nu = 0) \qquad F_\nu = \pi\left(a_\nu + \frac{2}{3}\kappa_\nu^{-1}\frac{\partial S_\nu}{\partial z}\right)$$

$$\underset{\text{line}}{\kappa} = \kappa_L + \underset{\text{continuum}}{\kappa_c} \qquad F_c = \pi\left(a + \frac{2}{3}\frac{1}{\kappa_c}\frac{\partial S_\nu}{\partial z}\right) \qquad F_L = \pi\left(a + \frac{2}{3}\frac{1}{\kappa_c + \kappa_L}\frac{\partial S_\nu}{\partial z}\right)$$

$$\kappa_L \ll \kappa_c \qquad F_L = \pi\left(a + \frac{2}{3}\frac{1}{\kappa_c}\left(1 - \frac{\kappa_L}{\kappa_c}\right)\frac{\partial S_\nu}{\partial z}\right)$$

F_c F_L residual intensity $$r_\nu = \frac{F_c - F_L}{F_c} = \frac{\pi\left(\frac{2}{3}\frac{1}{\kappa_c}\frac{\kappa_L}{\kappa_c}\frac{\partial S_\nu}{\partial z}\right)}{F_c}$$

need gradient in z to get absorption

$$\kappa_L = n_L \frac{\pi e^2}{mc} f\, \phi(\Delta\nu)\left(1 - e^{-h\nu/kT}\right) \qquad \kappa_c = n_c \sigma_c \left(1 - e^{-h\nu/kT}\right)$$

equivalent width $$W_\nu = \int_{-\infty}^{\infty} r_\nu\, d(\Delta\nu) \qquad \int_{-\infty}^{\infty}\phi(\Delta\nu)\, d(\Delta\nu) = 1$$

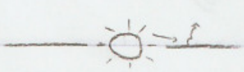
$$W_\nu = \frac{n_L \frac{\pi e^2}{mc} f}{n_c \sigma_c}\,\frac{2}{3}\pi\,\frac{\frac{\partial S_\nu}{\partial z}}{\kappa_c F_c} \qquad \frac{1}{\kappa_c}\frac{\partial S_\nu}{\partial z} = \frac{\partial S_\nu}{\partial \tau} \qquad F_\nu \simeq \pi S_\nu$$

$$\frac{1}{S}\frac{\partial S}{\partial \tau} = \frac{\partial(\ln S)}{\partial \tau}$$

$$\boxed{W_\nu = \frac{n_L}{n_c \sigma_c}\frac{\pi e^2}{mc} f\, \frac{2}{3}\, \frac{\partial(\ln S_\nu)}{\partial \tau}}$$

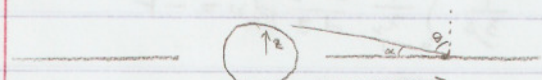
3/7/2007

Passive Disks

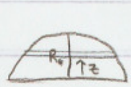


energy is from star reradiated by disk

Flat Disk, locally isothermal $T(D)$

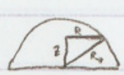


Flux-in

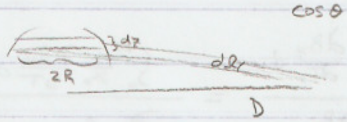


from star $I_* = \frac{\sigma_{SB} T_*^4}{\pi}$ $\qquad F = \int \cos\theta\, I\, d\Omega$

$\cos\theta = \frac{z}{D} = \sin\alpha$



$z^2 + R^2 = R_*^2 \qquad d\Omega = \frac{dz}{D}2\frac{R}{D}$



area: $dz\, 2R$
distance: D
$\therefore\ d\Omega = \frac{dz\, 2R}{D^2}$ $\qquad d\Omega = \frac{2\,dz}{D^2}\sqrt{R_*^2 - z^2}$

$$F(\text{onto disk}) = \int_0^{R_*} \frac{\sigma_{SB} T_*^4}{\pi}\,\frac{z}{D}\,\frac{2\sqrt{R_*^2 - z^2}}{D^2}\, dz$$

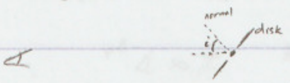
incident $F = \frac{\sigma_{SB} T_*^4}{\pi} \frac{2}{D^3} \int_0^{R_*} z\sqrt{R_*^2 - z^2}\, dz$ $\quad u = R_*^2 - z^2$, $du = -2z\,dz$ $\quad \int_0^{R_*^2} u^{1/2} \frac{du}{2}$

$$\therefore\ F = \frac{2}{3} \frac{\sigma_{SB} T_*^4}{\pi} \frac{R_*^3}{D^3}$$

outgoing flux $F = \sigma_{SB} T^4$ $\quad \therefore\ \cancel{\sigma_{SB}} T^4 = \frac{2}{3} \frac{\cancel{\sigma_{SB}} T_*^4}{\pi} \frac{R_*^3}{D^3}$

$$\boxed{T = \left(\frac{2}{3\pi}\right)^{1/4} T_* \left(\frac{R_*}{D}\right)^{3/4}}$$ flat disk

what we observe is flux



$I_\nu(\text{disk}) = B_\nu(T)$

$i = 0$: face-on

$$F_\nu = 2\pi \cos i \int_{D_{in}}^{D_{out}} B_\nu(T)\, \frac{D\, dD}{D_*^2}$$

--- area: $2\pi D\, dD$

D_* — distance to star

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \qquad F_\nu = \frac{2\pi \cos i}{D_*^2} \frac{2h\nu^3}{c^2} \int \frac{D\, dD}{e^{h\nu/kT(D)} - 1}$$

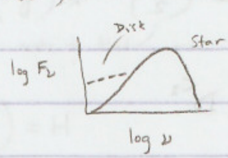
$$x = \frac{h\nu}{kT} = \frac{h\nu}{kT_*} \left(\frac{D}{R_*}\right)^{3/4} \left(\frac{3\pi}{2}\right)^{1/4} \qquad D = x^{4/3} \left(\frac{kT_*}{h\nu}\right)^{4/3} \left(\frac{2}{3\pi}\right)^{1/3} R_*$$

$$dD = \frac{4}{3} x^{1/3} R_* \left(\frac{kT_*}{h\nu}\right)^{4/3} \left(\frac{2}{3\pi}\right)^{1/3} dx$$

$$F_\nu = \frac{2\pi \cos i}{D_*^2} \frac{2h\nu^3}{c^2} \frac{4}{3} \left(\frac{2}{3\pi}\right)^{2/3} \left(\frac{kT_*}{h\nu}\right)^{8/3} R_*^2 \int_{x_{in}}^{x_{out}} \frac{x^{5/3}\, dx}{e^x - 1}$$

$$F_\nu = 12\pi^{1/3} \left(\frac{R_*}{D_*}\right)^2 \frac{h\nu^3}{c^2} \left(\frac{2\, kT_*}{3\, h\nu}\right)^{8/3} \cos i \underbrace{\int_{x_{in}}^{x_{out}} \frac{x^{5/3}\, dx}{e^x - 1}}_{\sim 1.9 \ \left(\text{for } x_{in} = 0,\ x_{out} = \infty\right)} \qquad F_\nu \propto \nu^{1/3}$$

Star $F_{\nu *} = \pi \frac{R_*^2}{D_*^2} \frac{2\nu^2}{c^2} kT_*$ (Rayleigh-Jeans tail)



Good for WD disk, not great for pre-MS stars

└ can have gas and be flared

$\uparrow z$ —— ○ —— $\frac{dP}{dz} = -\rho g_z$ grav force component downward

if isothermal $P = \frac{\rho}{\mu} kT$ mean molecular weight $\frac{kT}{\mu}\frac{d\rho}{dz} = -\rho g$

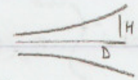
$g_{total} = \frac{GM_*}{D^2}$ $g_z = \frac{GM_*}{D^2}\frac{z}{D}$ $\frac{1}{\rho}\frac{d\rho}{dz} = \frac{d}{dz}(\ln\rho) = -\frac{\mu}{kT}\frac{GM_*}{D^3} z$

$\rho = \rho_0 e^{-\left(z^2/H^2\right)}$ $H^2 = \frac{2kTD^3}{\mu GM_*}$ At $z=0$ $\rho=\rho_0$

Σ – surface density of disk $\int_{-\infty}^{\infty}\rho\, dz$ $\Sigma = \int_{-\infty}^{\infty}\rho_0 e^{-z^2/H^2} = \rho_0 H\sqrt{\pi}$

$\boxed{H = \left(\frac{2kTD^3}{\mu GM_*}\right)^{1/2}}$ characteristic height for flat disk $T \propto D^{-3/4}$ $\therefore H \propto D^{9/8}$

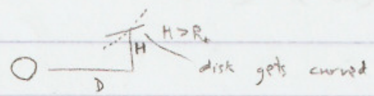
$\frac{H}{D} \propto D^{1/8}$ H increases outward → flaring disk



flat disk approx will break down when $H > R_*$

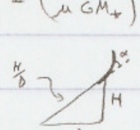
top of disk $\tau = 1$ (depends on grain size) $\tau_\perp = \chi\Sigma$ through whole disk we'll take $z=H$ as top of disk | $M = 6\times10^{27}$ g, $R = 1.5\times10^{13}$ cm $\Sigma = \frac{M}{\pi R^2} \sim 10$ g cm^{-2} $\chi = \frac{\pi a^2}{\frac{4\pi}{3}\rho a^3} = \frac{3}{4\rho a} \sim 10^4$ cm^2 g^{-1} (grain radius)

Notes: critical D at which $H = R_*$



Taylor expand $\frac{H}{D}$ versus D $\alpha = D\frac{d}{dD}\left(\frac{H}{D}\right)$

$H = \left(\frac{2k}{\mu GM_*}\right)^{1/2} D^{3/2} T_*^{1/2}\left(\frac{2}{3\pi}\right)^{1/8}\left(\frac{R_*}{D}\right)^{3/8}$ $H = R_*$ when $D_{crit} = \left(\frac{3\pi}{32}\right)^{1/9}\left(\frac{GM_*\mu}{kT_* R_*}\right)^{4/9} R_*$



α is measure angle between local portion of disk and illumination of star

$T = \left(\frac{\alpha}{2}\right)^{1/4} T_*\left(\frac{R_*}{D}\right)^{1/2}$ ← from flux in = flux out (half of star)

$H^2 = \frac{2kTD^3}{GM_*\mu}$

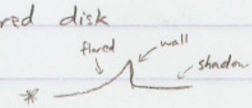
$T = C_1 D^{C_2}$ $H = \left(\frac{2kC_1}{GM_*\mu}\right)^{1/2} D^{\frac{3+C_2}{2}}$ $\frac{H}{D} = \left(\frac{2kC_1}{GM_*\mu}\right)^{1/2} D^{\frac{1+C_2}{2}}$

$\alpha = D\frac{d}{dD}\left(\frac{H}{D}\right) = \left(\frac{2kC_1}{GM_*\mu}\right)^{1/2}\left(\frac{1+C_2}{2}\right) D^{\frac{1+C_2}{2}}$ $T = \left(\frac{\alpha}{2}\right)^{1/4} T_* R_*^{1/2} D^{-1/2} = C_1 D^{C_2}$

$\left(\left(\frac{2kC_1}{GM_*\mu}\right)^{1/2}\left(\frac{1+C_2}{4}\right)\right)^{1/4} T_* R_*^{1/2} D^{\frac{1+C_2}{8}} D^{-1/2} = C_1 D^{C_2}$ $C_2 = \frac{1}{8} + \frac{C_2}{8} - \frac{1}{2}$ $\therefore C_2 = -3/7$

$T \propto D^{-1/2}$ optically thin $T \propto D^{-3/4}$ flat disk $T \propto D^{-3/7}$ flared disk

$\boxed{T = \left(\frac{1}{7}\right)^{2/7}\left(\frac{R_*}{D}\right)^{3/7}\left(\frac{2kT_* R_*}{GM_*\mu}\right)^{1/7} T_*}$ $F_\nu \propto \nu^{-5/3}$



dust emission lines → imply $T = T(z)$

Accretion and Active Disks

3/12/2007

Disk —•— spherical

$$L = \frac{G M_* \dot{M}}{R_*}$$ (mass of star; R_* — radius of star)

Black hole $R_s = \frac{2GM_*}{c^2}$ $R_{*,eff} \simeq 3R_s$ $L = \frac{GM_* \dot{M}}{\frac{6GM_*}{c^2}} \simeq \frac{1}{6}\dot{M}c^2$

$L \sim 0.1 \dot{M} c^2$ can be low for BH → no surface so doesn't have to slow down

Accretion Disk

Approx circular orbits $v^2 = \frac{GM_*}{D}$

$$E = m\left(\frac{1}{2}v^2 - \frac{GM_*}{D}\right) = m\left(\frac{1}{2}\frac{GM_*}{D} - \frac{GM_*}{D}\right) = -\frac{1}{2}\frac{GM_*}{D}m$$

viscosity causes matter to fall inward, angular momentum decreases and matter speeds up

matter → angular momentum

Annulus → ring of thickness ΔD

ΔE_{orb} — orbital energy lost moving from D to ΔD

$$\Delta E_{orb} = \frac{1}{2}\frac{GM_* m}{D^2}\Delta D \longleftarrow \frac{\partial}{\partial D} \text{ of } E$$

$$\frac{\Delta E_{orb}}{\Delta t} = \frac{1}{2}\frac{GM_*}{D^2}\Delta D \frac{\Delta m}{\Delta t} = \frac{1}{2}\frac{GM_*}{D^2}\dot{m}\Delta D$$

Power $(F \cdot v)$ for rotational motion: $N\Omega$ (torque, angular velocity) $v = \Omega D$

$$\Omega = \left(\frac{GM_*}{D}\right)^{1/2}\frac{1}{D} \qquad \Omega = \left(\frac{GM_*}{D^3}\right)^{1/2} \qquad N = -D^2\Omega\dot{M}$$ generic torque source unknown

$$\frac{dE_{torque}}{dt} = \frac{d}{dD}(N\Omega)\Delta D = -\frac{d}{dD}(D^2\Omega^2\dot{M})\Delta D = -\dot{M}\Delta D \frac{d}{dD}\left(\frac{D^2 GM_*}{D^3}\right)$$

$$\frac{dE_{torque}}{dt} = \frac{GM_*}{D^2}\dot{M}\Delta D \leftarrow \text{twice of } \frac{dE_{orb}}{dt}$$

$$\frac{dE_{total}}{dt} = \frac{dE_{orb}}{dt} + \frac{dE_{torque}}{dt} = \frac{3}{2}\frac{GM_*}{D^2}\Delta D$$

Dissipated locally in ring by radiation

$$\underbrace{2(2\pi D\Delta D)}_{\text{area (2 sides)}}\sigma T^4 = 4\pi D\Delta D\,\sigma T^4 = \frac{3}{2}\frac{GM_*}{D^2}\Delta D \qquad \sigma T^4 = \frac{3}{8\pi}\frac{GM_*}{D^3}\dot{M}$$

$T \propto D^{-3/4}$ just like passive flat disk

Boundary Layer

Shocked-Heated gas $\qquad v_{rot} \propto \left(\frac{GM_*}{R_*}\right)^{1/2} \longrightarrow$ to zero

$\gamma = 5/3 \qquad kT_{shock} = \frac{3}{16}\mu v_s^2$

Pre-Shock $\qquad$ Post-Shock

$\rho_B v_B = \rho_A v_A \qquad$ Mass, Momentum, Energy Conservation

Rankine-Hugoniot Relations

$$kT_{shock} = \frac{3}{16}\mu \frac{GM_*}{R_*} \qquad \frac{kT_s}{\mu} = \frac{3}{16}\frac{GM_*}{R_*} = \frac{3}{32}v_{esc}^2 \qquad v_{esc}^2 = \frac{2GM_*}{R_*}$$

very hot gas in shock (neutron star $v_{esc} \sim 0.1c$ so very high T_s)

X-rays

from Maxwellian dist $\quad v^2 = \frac{8}{\pi}\frac{kT}{\mu} \qquad \frac{kT}{\mu} = \frac{\pi}{8}v^2 \qquad \therefore \frac{\pi}{8}v^2 = \frac{3}{32}v_{esc}^2 \qquad v \sim v_{esc}$

$\therefore$ tend to form spherical accretion near star: accretion halo

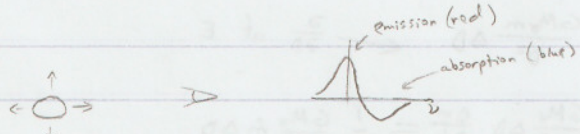
in BH $v_{esc} \sim c$ so very high E, if you add $\vec{B}$ fields can produce relativistic jets (photons can Compton scatter with e^- in jets)

$\therefore$ total flux from active disks can be different from passive disks

3/14/2007

Winds

P Cygni Profile



outflow speed $\sim$ constant $= v \qquad \dot{M}$ = mass outflow rate constant $\qquad 4\pi R^2 v \rho = \dot{M} \qquad \rho = \frac{\dot{M}}{4\pi R^2 v}$

$$n = \frac{\rho}{\mu} = \frac{\dot{M}}{4\pi R^2 v \mu} \qquad \tau = \int_{R_*}^{\infty} n\sigma \, dR = \frac{\dot{M}\sigma}{4\pi\mu R_* v}$$ ← can use this to estimate outflow

ionized gas if star is hot

$T = 10{,}000$ K $\qquad$ free-free absorption $\quad \alpha_\nu \propto \frac{n_e^2}{\nu^2} \qquad \tau \propto \int \frac{n_e^2 \, dz}{\nu^2} \propto \frac{1}{\nu^2}\int n_p^2 \, dz$

optically thick inner region, optically thin outer region

b = impact parameter

within b: $\tau > 1$ $\quad$ further $\tau < 1$

z

$$F_\nu = \underbrace{\frac{2\nu^2}{c^2}kT}_{\substack{\text{intensity} \\ \text{(Rayleigh-Jeans)}}} \underbrace{\frac{\pi b^2}{D^2}}_{\Omega}$$ within b

Beyond b: $\tau < 1 \qquad I_\nu \propto (I_\nu)_b \, \phi^{-3} \qquad \phi = \frac{b}{D}$

ϕ = offset angle

$R^2 = b^2 + z^2$

$n_e \propto \frac{1}{R^2} \qquad \tau \propto \int n_e^2 \, dz$

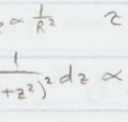
$$I_\nu \simeq S_\nu \tau_\nu$$

$$\tau \propto \int_{-\infty}^{\infty} \frac{1}{(b^2+z^2)^2} dz \propto \frac{1}{b^3}$$

$\tau_\nu = 1 \, \phi_0^{-3}$

$$F_\nu = \int_{\phi_0}^{\pi/2}\int_0^{2\pi} S_\nu \tau_\nu \, d\Omega \qquad F_\nu = 2\frac{\nu^2}{c^2}kT \int_b^{\infty} \frac{2\pi R'}{D}\frac{dR'}{D}\left(\frac{b}{R'}\right)^3$$

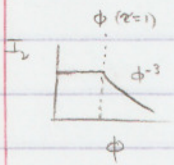
$\uparrow \tau$ so that $\tau = 1$ at $R' = b$



$d\Omega = \frac{2\pi R' \, dR'}{D^2}$

$$F_\nu = 4\pi\frac{\nu^2}{c^2}kT\frac{b^3}{D^2}\int_b^{\infty}\frac{dR'}{R'^2} = 4\pi\frac{b^2}{D^2}\frac{\nu^2}{c^2}kT$$

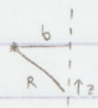
(b is now fixed so $b \to R'$)



inner $F_\nu = 2\pi \frac{b^2}{D^2}\frac{\nu^2}{c^2} kT$ outer $F_\nu = 4\pi \frac{b^2}{D^2}\frac{\nu^2}{c^2} kT$

$F_\nu = (F_\nu)_{inner} + (F_\nu)_{outer}$ $(F_\nu)_{outer} = 2(F_\nu)_{inner}$

$F_\nu = 3(F_\nu)_{inner}$ $\boxed{F_\nu = 6\pi \frac{b^2}{D^2}\frac{\nu^2}{c^2} kT}$



$\tau = \int_{-\infty}^{\infty} \alpha_\nu \, dz$ $\alpha_\nu = \frac{\kappa_{FF} n^2}{\nu^2 T^{3/2}}$ $\tau = \frac{\kappa_{FF}}{T^{3/2}\nu^2}\int_{-\infty}^{\infty} n^2 \, dz$ $n = \frac{\dot{M}}{4\pi R^2 v \mu}$ $R^2 = b^2 + z^2$

$\tau = \frac{\alpha_{FF}}{T^{3/2}\nu^2}\frac{\dot{M}^2}{(4\pi v \mu)^2}\int_{-\infty}^{\infty}\frac{dz}{(b^2+z^2)^2}$ $\boxed{b = \left(\frac{\kappa_{FF}\dot{M}^2}{32\pi \nu^2 T^{3/2}\mu^2 v^2}\right)^{1/3}}$

$F_\nu = 6\pi \frac{\nu^2}{c^2} kT \frac{1}{D^2}\left(\sim\right)^{2/3}$ $\boxed{4\left(\frac{1}{3}\right)^{3/4}\frac{F_\nu^{3/4} c^{3/2} D^{3/2} \mu v}{\pi^{1/2}\nu^{1/2} k_B^{3/4} \kappa_{FF}^{1/2}} = \dot{M}}$ $F_\nu \propto \nu^{2/3}$