First, lets define a few basic variables based on the madgraph model. In particular, we can define the mixing parameter η and ϵ (following https://github.com/davidrcurtin/HAHM/blob/m) as

$$\chi \to \frac{\epsilon}{\cos\theta_w}$$

$$\eta = \frac{\epsilon}{\cos\theta_w \sqrt{1 - \epsilon^2/\cos\theta_w^2}}$$

where $\eta = \frac{\chi}{\sqrt{1-\chi^2}}$ is the actual mixing parameter with the bosons written as

$$\begin{pmatrix} X_\mu \\ Y_\mu \end{pmatrix} = \begin{pmatrix} \sqrt{1-\chi^2} & 0 \\ -\chi & 1 \end{pmatrix} \begin{pmatrix} \hat{X}_\mu \\ \hat{Y}_\mu \end{pmatrix}$$

switching into a more normal configuration we have

$$\begin{pmatrix} Z \\ Z_D \end{pmatrix} = \begin{pmatrix} \cos\theta_a & \sin\theta_a \\ -\sin\theta_a & \cos\theta_a \end{pmatrix} \begin{pmatrix} Z_0 \\ X \end{pmatrix}$$

which we can write as (taking from the document) with $\Delta_z = \left(\frac{M_{z'}}{M_z}\right)^2$

$$\tan\theta_a \approx \sin\theta_w \frac{\eta}{\Delta_z - 1}$$

$$\tan\theta_a \approx \epsilon \frac{\tan\theta_w}{\Delta_z - 1}$$

From the model, the axial vector coupling to quarks can be written equivalently for the Dark photon model as

$$g_A = e\left(\frac{\cos\theta_a \eta}{6\cos\theta_w} + \frac{\sin\theta_a \sin\theta_w}{6\cos\theta_w}\right)$$

$$g_V = e\sin\theta_a \frac{\cos\theta_w}{2\sin\theta_w}$$

Now putting in all the approximations from above, we can take the above equation and replace $\sin\theta_a \approx \epsilon\frac{\sin\theta_w}{\Delta_z - 1}$

$$g_V \approx e\epsilon \frac{1}{(\Delta_z - 1)} \frac{\cos\theta_w}{2}$$

which solving for ϵ gives us the relation

$$\epsilon = g_V \frac{2(\Delta_z - 1)}{e\cos\theta_w}$$

in the low dark photon mass limit we ahve

$$\epsilon = \frac{2g_v}{e\cos\theta_w} \approx \frac{2g_v}{0.27}$$

Finally for reference, we have

$$\frac{e^2}{4\pi} = \frac{1}{128} \to e = \sqrt{\frac{4\pi}{128}}$$