Chapter 15 – Probability Models

Section 15.1

1. **Bernoulli.**

   a) These are not Bernoulli trials. The possible outcomes are 1, 2, 3, 4, 5, and 6. There are more than two possible outcomes.

   b) These may be considered Bernoulli trials. There are only two possible outcomes, Type A and not Type A. Assuming the 120 donors are representative of the population, the probability of having Type A blood is 43%. The trials are not independent, because the population is finite, but the 120 donors represent less than 10% of all possible donors.

   c) These are not Bernoulli trials. The probability of getting a heart changes as cards are dealt without replacement.

   d) These are not Bernoulli trials. We are sampling without replacement, so the trials are not independent. Samples without replacement may be considered Bernoulli trials if the sample size is less than 10% of the population, but 500 is more than 10% of 3000.

   e) These may be considered Bernoulli trials. There are only two possible outcomes, sealed properly and not sealed properly. The probability that a package is unsealed is constant, at about 10%, as long as the packages checked are a representative sample of all packages. Finally, the trials are not independent, since the total number of packages is finite, but the 24 packages checked probably represent less than 10% of the packages.

2. **Bernoulli 2.**

   a) These may be considered Bernoulli trials. There are only two possible outcomes, getting a 6 and not getting a 6. The probability of getting a 6 is constant at 1/6. The rolls are independent of one another, since the outcome of one die roll doesn't affect the other rolls.

   b) These are not Bernoulli trials. There are more than two possible outcomes for eye color.

   c) These can be considered Bernoulli trials. There are only two possible outcomes, properly attached buttons and improperly attached buttons. As long as the button problem occurs randomly, the probability of a doll having improperly attached buttons is constant at about 3%. The trails are not independent, since the total number of dolls is finite, but 37 dolls is probably less than 10% of all dolls.

   d) These are not Bernoulli trials. The trials are not independent, since the probability of picking a council member with a particular political affiliation changes depending on who has already been picked. The 10% condition is not met, since the sample of size 4 is more than 10% of the population of 19 people.

   e) These may be considered Bernoulli trials. There are only two possible outcomes, cheating and not cheating. Assuming that cheating patterns in this school are similar to the patterns in the nation, the probability that a student has cheated is constant, at 74%. The trials are not independent, since the population of all students is finite, but 481 is less than 10% of all students.

Section 15.2

3. **Toasters.**

   Let $X =$ the number of toasters that need repair.

   The condition of the toasters can be considered Bernoulli trials. There are only two possible outcomes, needing to be sent back for repairs and not needing to be sent back for repairs. The probability that a toaster needs repair is constant, given as $p = 0.05$. The trials are not independent, since there are a finite number of toasters, but 20 toasters in each carton is less than 10% of all toasters produced.

   The distribution of the number of repairs required follows Binom(20, 0.05).

   $P(\text{exactly 3 toasters need repair}) = P(X = 3) = {}_{20}C_3\,(0.05)^3\,(0.95)^{17} \approx 0.0596$; According to the Binomial model, the probability that exactly three toasters out of 20 require repair is 0.0596.

4. Soccer.

Let $X =$ the number of goals scored on corner kicks.

The outcome of the corner kicks can be considered Bernoulli trials. There are only two possible outcomes, scoring a goal or not scoring a goal. The probability of scoring a goal is constant at $p = 0.08$. We will assume that the corner kicks are taken independently from each other, and the outcome of one kick will not affect another kick.

The distribution of the number of goals scored follows Binom(15, 0.08).

$P(\text{exactly 2 goals scored}) = P(X = 2) = {}_{15}C_2\,(0.08)^2\,(0.92)^{13} \approx 0.227$; According to the Binomial model, the probability that exactly two corner kicks are made out of 15 attempts is 0.227.

Section 15.3

5. Toasters again.

A Binomial model and a Normal model are both appropriate for modeling the number of toasters that need to be sent back for minor repair.

Let $X =$ the number of toasters that need to be sent back. The condition of the toasters can be considered Bernoulli trials, as verified in Exercise 3.
The distribution of the number of repairs required follows Binom(10000, 0.05).

$E(X) = np = 10{,}000(0.05) = 500$ toasters; $SD(X) = \sqrt{npq} = \sqrt{10{,}000(0.05)(0.95)} \approx 21.79$ toasters

Since $np = 500$ and $nq = 9500$ are both greater than 10, Binom(10000, 0.05) may be approximated by the Normal model, $N(500, 21.79)$.

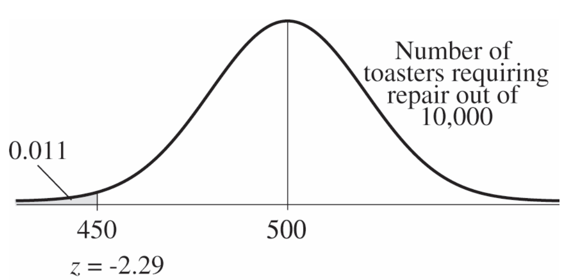
Using Binom(10000, 0.05):

$$\begin{aligned} P(\text{fewer than 450 toasters}) &= P(X < 450) \\ &= {}_{10000}C_0\,(0.05)^0\,(0.95)^{10000} + \ldots + {}_{10000}C_{449}\,(0.05)^{449}\,(0.95)^{9551} \\ &\approx 0.009 \end{aligned}$$

According to the Binomial model, the probability that fewer than 450 of 10,000 toasters need repair is approximately 0.009.

Using *N*(500, 21.79):

$$z = \frac{x-\mu}{\sigma} = \frac{450-500}{21.79} = -2.29$$



$P(X < 450) \approx P(z < -2.29) \approx 0.011$; According to the Normal model, the probability that fewer than 450 of 10,000 toasters need repair is approximately 0.011.

6. Soccer again.

A Binomial model and a Normal model are both appropriate for modeling the number of goals made from corner kicks in a season.

Let $X =$ the number of goals resulting from corner kicks. Attempting goals from corner kicks can be considered Bernoulli trials, as verified in Exercise 4.
The distribution of the number of goals made follows Binom(200, 0.08).

$E(X) = np = 200(0.08) = 16$ goals; $SD(X) = \sqrt{npq} = \sqrt{200(0.08)(0.92)} \approx 3.837$ goals

Since $np = 16$ and $nq = 184$ are both greater than 10, $\text{Binom}(200, 0.08)$ may be approximated by the Normal model, $N(16, 3.837)$.

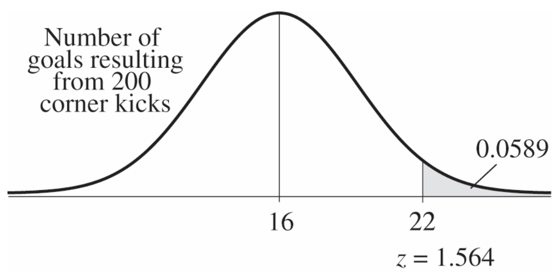
Using Binom(200, 0.08):

$$
\begin{aligned}
P(\text{more than 22 goals}) &= P(X > 22) \\
&= {}_{200}C_{23}(0.08)^{23}(0.92)^{177} + \ldots + {}_{200}C_{200}(0.08)^{200}(0.92)^{0} \\
&\approx 0.0507
\end{aligned}
$$

According to the Binomial model, the probability that more than 22 of 200 corner kicks result in goals is approximately 0.0507.

Using *N*(16, 3.837):

$$z = \frac{x-\mu}{\sigma} = \frac{22-16}{3.837} = 1.564$$



$P(X > 22) \approx P(z > 1.564) \approx 0.0589$; According to the Normal model, the probability that more than 22 of 200 corner kicks will result in goals is approximately 0.0589.

Section 15.4

7. Sell!

$$E(X) = \lambda = 5$$

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P(X = 3) = \frac{e^{-5}5^{3}}{3!} \approx 0.140$$

According to the Poisson model, the probability of the dealer selling 3 cars is 0.140.

8. Passing on.

$$E(X) = \lambda = 7$$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(X = 10) = \frac{e^{-7}7^{10}}{10!} \approx 0.071$$

According to the Poisson model, the probability that the hospital has 10 fatalities is 0.071.

Section 15.7

9. Telephone numbers.

a) The telephone numbers are distributed uniformly, since all numbers are equally likely to be selected.

b) According to the uniform distribution, the probability of choosing one of the numbers assigned to the incubator is 200/10,000 = 0.02.

c) According to the uniform distribution, the probability of choosing a number above 9000 is 1000/10,000 = 0.10

10. Serial numbers.

a) The serial numbers are distributed uniformly, since all numbers are equally likely to be selected.

b) According to the uniform distribution, the probability of choosing one of the last 100 phones to be produced is 100/1000 = 0.10.

c) According to the uniform distribution, the probability of choosing a phone from among the last 200 or first 50 to be produced is (200 + 50)/10000 = 0.25.

11. Component lifetimes.

a) The mean of the exponential model is $\mu = \frac{1}{\lambda}$, so $\lambda = \frac{1}{\mu} = \frac{1}{3}$.

b) Let $X =$ the number of years in the hard drive lifetime.

$P(X \le 5) = 1 - e^{-\lambda t} = 1 - e^{-(1/3)(5)} \approx 0.811$; According to the exponential model, the probability that hard drive lasts 5 years or less is approximately 0.811.

12. Website sales.

a) Since 5 sales are expected per hour, we would expect to wait 1/5 of an hour, or 12 minutes.

b) Let $X =$ the number of hours until the next sale. We want to know the probability that the next sale will occur within 6 minutes, or 0.1 hours.

$P(X \le 0.1) = 1 - e^{-\lambda t} = 1 - e^{-(5)(0.1)} \approx 0.393$; According to the exponential model, the probability that the next sale will occur within 6 minutes is 0.393.

Chapter Exercises

13. Simulating the model.

a) Answers will vary. A component is the simulation of the picture in one box of cereal. One possible way to model this component is to generate random digits 0–9. Let 0 and 1 represent Hope Solo and 2–9 a picture of another sports star. Each run will consist of generating random numbers until a 0 or 1 is generated. The response variable will be the number of digits generated until the first 0 or 1.

b) Answers will vary.

13. (continued)

c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you get a picture of Hope Solo in the first box. This is the number of trials in which a 0 or 1 was generated first, divided by the total number of trials. Perform similar calculations for the simulated probability that you have to wait until the second box, the third box, etc.

d) Let $X =$ the number of boxes opened until the first Hope Solo picture is found.

X	$P(X)$
1	0.20
2	$(0.80)^1(0.20) = 0.16$
3	$(0.80)^2(0.20) = 0.128$
4	$(0.80)^3(0.20) = 0.1024$
5	$(0.80)^4(0.20) \approx 0.0819$

X	$P(X)$
6	$(0.80)^5(0.20) \approx 0.0655$
7	$(0.80)^6(0.20) \approx 0.0524$
8	$(0.80)^7(0.20) \approx 0.0419$
≥ 9	0.1679

e) Answers will vary.

14. Simulation II.

a) Answers will vary. A component is the simulation of one die roll. One possible way to model this component is to generate random digits 1–6. Let 1 represent getting 1 (the roll you need) and let 2–6 represent not getting the roll you need. Each run will consist of generating random numbers until 1 is generated. The response variable will be the number of digits generated until the first 1.

b) Answers will vary.

c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you roll a 1 on the first roll. This is the number of trials in which a 1 was generated first divided by the total number of trials. Perform similar calculations for the simulated probability that you have to wait until the second roll, the third roll, etc.

d) Let $X =$ the number of rolls until the first 1 is rolled.

X	$P(X)$
1	$\left(\frac{1}{6}\right) \approx 0.1667$
2	$\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) \approx 0.1389$
3	$\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) \approx 0.1157$
4	$\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) \approx 0.0964$
5	$\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) \approx 0.0804$

X	$P(X)$
6	$\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) \approx 0.0670$
7	$\left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right) \approx 0.0558$
8	$\left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right) \approx 0.0465$
≥ 9	0.2326

e) Answers will vary.

15. Hope, again.

a) Answers will vary. A component is the simulation of the picture in one box of cereal. One possible way to model this component is to generate random digits 0–9. Let 0 and 1 represent Hope Solo and 2–9 a picture of another sports star. Each run will consist of generating five random numbers. The response variable will be the number of 0s and 1s in the five random numbers.

b) Answers will vary.

c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you get no pictures of Hope Solo in the five boxes. This is the number of trials in which neither 0 nor 1 were generated divided by the total number of trials. Perform similar calculations for the simulated probability that you would get 1 picture, 2 pictures, etc.

d) Let $X =$ the number of Hope Solo pictures in 5 boxes.

X	$P(X)$	X	$P(X)$
0	${}_5C_0(0.20)^0(0.80)^5 \approx 0.3277$	3	${}_5C_3(0.20)^3(0.80)^2 \approx 0.0512$
1	${}_5C_1(0.20)^1(0.80)^4 \approx 0.4096$	4	${}_5C_4(0.20)^4(0.80)^1 = 0.0064$
2	${}_5C_2(0.20)^2(0.80)^3 \approx 0.2048$	5	${}_5C_5(0.20)^5(0.80)^0 \approx 0.0$

e) Answers will vary.

16. Seatbelts.

a) Answers will vary. A component is the simulation of one driver in a car. One possible way to model this component is to generate pairs of random digits 00–99. Let 01–75 represent a driver wearing his or her seatbelt and let 76–99 and 00 represent a driver not wearing his or her seatbelt. Each run will consist of generating five pairs of random digits. The response variable will be the number of pairs of digits that are 00–75.

b) Answers will vary.

c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that none of the five drivers are wearing seatbelts. This is the number of trials in which no pairs of digits were 00–75, divided by the total number of trials. Perform similar calculations for the simulated probability that one driver is wearing his or her seatbelt, two drivers, etc.

d) Let $X =$ the number of drivers wearing seatbelts in 5 cars.

X	$P(X)$	X	$P(X)$
0	${}_5C_0(0.75)^0(0.25)^5 \approx 0.0$	3	${}_5C_3(0.75)^3(0.25)^2 \approx 0.2637$
1	${}_5C_1(0.75)^1(0.25)^4 \approx 0.015$	4	${}_5C_4(0.75)^4(0.25)^1 \approx 0.3955$
2	${}_5C_2(0.75)^2(0.25)^3 \approx 0.0879$	5	${}_5C_5(0.75)^5(0.25)^0 \approx 0.2373$

e) Answers will vary.

17. On time.

These departures cannot be considered Bernoulli trials. Departures from the same airport during a 2-hour period may not be independent. They all might be affected by weather and delays.

18. Lost luggage.

The fate of these bags cannot be considered Bernoulli trials. The fate of 22 pieces of luggage, all checked on the same flight, probably aren't independent.

19. Hoops.

The player's shots may be considered Bernoulli trials. There are only two possible outcomes (make or miss), the probability of making a shot is constant (80%), and the shots are independent of one another (making, or missing, a shot does not affect the probability of making the next).

Let $X =$ the number of shots until the first missed shot.
Let $Y =$ the number of shots until the first made shot.

Since these problems deal with shooting until the first miss (or until the first made shot), a geometric model, either $\text{Geom}(0.8)$ or $\text{Geom}(0.2)$, is appropriate.

a) Use $\text{Geom}(0.2)$. $P(\text{X} = 5) = (0.8)^4(0.2) = 0.0819$ (Four shots made, followed by a miss.)

b) Use $\text{Geom}(0.8)$. $P(Y = 4) = (0.2)^3(0.8) = 0.0064$ (Three misses, then a made shot.)

c) Use $\text{Geom}(0.8)$. $P(Y = 1) + P(Y = 2) + P(Y = 3) = (0.8) + (0.2)(0.8) + (0.2)^2(0.8) = 0.992$

20. Chips.

The selection of chips may be considered Bernoulli trials. There are only two possible outcomes (fail testing and pass testing). Provided that the chips selected are a representative sample of all chips, the probability that a chip fails testing is constant at 2%. The trials are not independent, since the population of chips is finite, but we won't need to sample more than 10% of all chips.

Let $X =$ the number of chips required until the first bad chip. The appropriate model is $\text{Geom}(0.02)$.

a) $P(X = 5) = (0.98)^4(0.02) \approx 0.0184$ (Four good chips, then a bad one.)

b) $P(1 \le X \le 10) = (0.02) + (0.98)(0.02) + (0.98)^2(0.02) + \ldots + (0.98)^9(0.02) \approx 0.183$
(Use the geometric model on a calculator or computer for this one!)

21. More hoops.

As determined in Exercise 19, the shots can be considered Bernoulli trials, and since the player is shooting until the first miss, $\text{Geom}(0.02)$ is the appropriate model.

$E(X) = \frac{1}{p} = \frac{1}{0.2} = 5$ shots; The player is expected to take 5 shots until the first miss.

22. Chips ahoy.

As determined in Exercise 20, the selection of chips can be considered Bernoulli trials, and since the company is selecting until the first bad chip, $\text{Geom}(0.02)$ is the appropriate model.

$E(X) = \frac{1}{p} = \frac{1}{0.02} = 50$ chips; The first bad chip is expected to be the 50th chip selected.

23. Customer center operator.

The calls can be considered Bernoulli trials. There are only two possible outcomes, taking the promotion, and not taking the promotion. The probability of success is constant at 5% (50% of the 10% Platinum cardholders). The trials are not independent, since there are a finite number of cardholders, but this is a major credit card company, so we can assume we are selecting fewer than 10% of all cardholders. Since we are calling people until the first success, the model $\text{Geom}(0.05)$ may be used.

$E(\text{calls}) = \frac{1}{p} = \frac{1}{0.05} = 20$ calls; We expect it to take 20 calls to find the first cardholder to take the double miles promotion.

24. Cold calls.

The donor contacts can be considered Bernoulli trials. There are only two possible outcomes, giving \$100 or more, and not giving \$100 or more. The probability of success is constant at 1% (5% of the 20% of donors who will make a donation). The trials are not independent, since there are a finite number of potential donors, but we will assume that she is contacting less than 10% of all possible donors. Since we are contacting people until the first success, the model $\text{Geom}(0.01)$ may be used.

$E(\text{contacts}) = \frac{1}{p} = \frac{1}{0.01} = 100$ contacts; We expect that Justine will have to contact 100 potential donors to find a \$100 donor.

25. Blood.

These may be considered Bernoulli trials. There are only two possible outcomes, Type AB and not Type AB. Provided that the donors are representative of the population, the probability of having Type AB blood is constant at 4%. The trials are not independent, since the population is finite, but we are selecting fewer than 10% of all potential donors. Since we are selecting people until the first success, the model $\text{Geom}(0.04)$ may be used.

Let $X =$ the number of donors until the first Type AB donor is found.

a) $E(X) = \frac{1}{p} = \frac{1}{0.04} = 25$ people; We expect the 25th person to be the first Type AB donor.

b) $P(\text{a Type AB donor among the first 5 people checked})$

$$= P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$$
$$= (0.04)+(0.96)(0.04)+(0.96)^2(0.04)+(0.96)^3(0.04)+(0.96)^4(0.04) \approx 0.185$$

c) $P(\text{a Type AB donor among the first 6 people checked})$

$$= P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)+P(X=6)$$
$$= (0.04)+(0.96)(0.04)+(0.96)^2(0.04)$$
$$+(0.96)^3(0.04)+(0.96)^4(0.04)+(0.96)^5(0.04) \approx 0.217$$

d) $P(\text{no Type AB donor before the tenth person checked}) = P(X > 9) = (0.96)^9 \approx 0.693$; This one is a bit tricky. There is no implication that we actually find a donor on the tenth trial. We only care that nine trials passed with no Type AB donor.

26. Colorblindness.

These may be considered Bernoulli trials. There are only two possible outcomes, colorblind and not colorblind. As long as the men selected are representative of the population of all men, the probability of being colorblind is constant at about 8%. Trials are not independent, since the population is finite, but we won't be sampling more than 10% of the population.

Let $X =$ the number of people checked until the first colorblind man is found.
Since we are selecting people until the first success, the model $\text{Geom}(0.08)$, may be used.

a) $E(X) = \frac{1}{p} = \frac{1}{0.08} = 12.5$ people; We expect to examine 12.5 people until finding the first colorblind person.

b) $P(\text{no colorblind men among the first 4}) = P(X > 4) = (0.92)^4 \approx 0.716$

c) $P(\text{first colorblind is the sixth man checked}) = P(X = 6) = (0.92)^5(0.08) \approx 0.0527$

26. (continued)

d) $P(\text{she finds a colorblind man before the tenth man}) = P(1 \le X \le 9)$

$= (0.08) + (0.92)(0.08) + (0.92)^2(0.08) + \ldots + (0.92)^8(0.08) \approx 0.528$

(Use the geometric model on a calculator or computer for this one!)

27. Coins and intuition.

a) Intuitively, we expect 50 heads.

b) $E(\text{heads}) = np = 100(0.5) = 50$ heads

28. Roulette and intuition.

a) Intuitively, we expect 2 balls to wind up in a green slot.

b) $E(\text{green}) = np = 38\left(\dfrac{2}{38}\right) = 2$ green

29. Lefties.

These may be considered Bernoulli trials. There are only two possible outcomes, left-handed and not left-handed. Since people are selected at random, the probability of being left-handed is constant at about 13%. The trials are not independent, since the population is finite, but a sample of 5 people is certainly fewer than 10% of all people.

Let $X =$ the number of people checked until the first lefty is discovered.
Let $Y =$ the number of lefties among $n = 5$.

a) Use $\text{Geom}(0.13)$.

$P(\text{first lefty is the fifth person}) = P(X = 5) = (0.87)^4(0.13) \approx 0.0745$

b) Use $\text{Binom}(5, 0.13)$.

$$\begin{aligned} P(\text{some lefties among the 5 people}) &= 1 - P(\text{no lefties among the first 5 people}) \\ &= 1 - P(Y = 0) \\ &= 1 - {}_5C_0(0.13)^0(0.87)^5 \\ &\approx 0.502 \end{aligned}$$

c) Use $\text{Geom}(0.13)$.

$$\begin{aligned} P(\text{first lefty is second or third person}) &= P(X = 2) + P(X = 3) \\ &= (0.87)(0.13) + (0.87)^2(0.13) \approx 0.211 \end{aligned}$$

d) Use $\text{Binom}(5, 0.13)$.

$P(\text{exactly 3 lefties in the group}) = P(Y = 3) = {}_5C_3(0.13)^3(0.87)^2 \approx 0.0166$

e) Use $\text{Binom}(5, 0.13)$.

$$\begin{aligned} P(\text{at least 3 lefties in the group}) &= P(Y = 3) + P(Y = 4) + P(Y = 5) \\ &= {}_5C_3(0.13)^3(0.87)^2 + {}_5C_4(0.13)^4(0.87)^1 + {}_5C_5(0.13)^5(0.87)^0 \\ &\approx 0.0179 \end{aligned}$$

29. (continued)

f) Use $\text{Binom}(5, 0.13)$.

$$\begin{aligned} P(\text{at most 3 lefties in the group}) &= P(Y=0)+P(Y=1)+P(Y=2)+P(Y=3) \\ &= {}_5C_0(0.13)^0(0.87)^5 + {}_5C_1(0.13)^1(0.87)^4 \\ &\quad + {}_5C_2(0.13)^2(0.87)^3 + {}_5C_3(0.13)^3(0.87)^2 \\ &\approx 0.9987 \end{aligned}$$

30. Arrows.

These may be considered Bernoulli trials. There are only two possible outcomes, hitting the bull's-eye and not hitting the bull's-eye. The probability of hitting the bull's-eye is given, $p = 0.80$. The shots are assumed to be independent.

Let $X =$ the number of shots until the first bull's-eye.
Let $Y =$ the number of bull's-eyes in $n = 6$ shots.

a) Use $\text{Geom}(0.80)$.

$$P\text{ (first bull's-eye is on the third shot)} = P(X=3) = (0.20)^2(0.80) \approx 0.032$$

b) Use $\text{Binom}(6, 0.80)$.

$$\begin{aligned} P(\text{at least one miss out of 6 shots}) &= 1 - P(\text{6 out of 6 hits}) \\ &= 1 - P(Y=6) \\ &= 1 - {}_6C_6(0.80)^6(0.20)^0 \\ &\approx 0.738 \end{aligned}$$

c) Use $\text{Geom}(0.80)$.

$$\begin{aligned} P(\text{first hit on fourth or fifth shot}) &= P(X=4)+P(X=5) \\ &= (0.20)^3(0.80)+(0.20)^4(0.80) = 0.00768 \end{aligned}$$

d) Use $\text{Binom}(6, 0.80)$.

$$\begin{aligned} P(\text{exactly four hits}) &= P(Y=4) \\ &= {}_6C_4(0.80)^4(0.20)^2 \\ &\approx 0.246 \end{aligned}$$

e) Use $\text{Binom}(6, 0.80)$.

$$\begin{aligned} P(\text{at least four hits}) &= P(Y=4)+P(Y=5)+P(Y=6) \\ &= {}_6C_4(0.80)^4(0.20)^2 + {}_6C_5(0.80)^5(0.20)^1 + {}_6C_6(0.80)^6(0.20)^0 \\ &\approx 0.901 \end{aligned}$$

f) Use $\text{Binom}(6, 0.80)$.

$$\begin{aligned} P(\text{at most four hits}) &= P(Y=0)+P(Y=1)+P(Y=2)+P(Y=3)+P(Y=4) \\ &= {}_6C_0(0.80)^0(0.20)^6 + {}_6C_1(0.80)^1(0.20)^5 + {}_6C_2(0.80)^2(0.20)^4 \\ &\quad + {}_6C_3(0.80)^3(0.20)^3 + {}_6C_4(0.80)^4(0.20)^2 \\ &\approx 0.345 \end{aligned}$$

31. Lefties, redux.

a) In Exercise 29, we determined that the selection of lefties could be considered Bernoulli trials. Since our group consists of 5 people, use Binom(5,0.13).

Let $Y =$ the number of lefties among $n = 5$.

$E(Y) = np = 5(0.13) = 0.65$ lefties

b) $SD(Y) = \sqrt{npq} = \sqrt{5(0.13)(0.87)} \approx 0.75$ lefties

c) Use Geom(0.13). Let $X =$ the number of people checked until the first lefty is discovered.

$$E(X) = \frac{1}{p} = \frac{1}{0.13} \approx 7.69 \text{ people}$$

32. More arrows.

a) In Exercise 30, we determined that the shots could be considered Bernoulli trials. Since the archer is shooting 6 arrows, use Binom(6, 0.80).

Let $Y =$ the number of bull's-eyes in $n = 6$ shots.

$E(Y) = np = 6(0.80) = 4.8$ bull's-eyes.

b) $SD(Y) = \sqrt{npq} = \sqrt{6(0.80)(0.20)} \approx 0.98$ bull's-eyes

c) Use Geom(0.80). Let $X =$ the number of arrows shot until the first bull's-eye.

$$E(X) = \frac{1}{p} = \frac{1}{0.80} = 1.25 \text{ shots}$$

33. Still more lefties.

a) In Exercise 29, we determined that the selection of lefties (and also righties) could be considered Bernoulli trials. Since our group consists of 12 people, and now we are considering the righties, use Binom(12, 0.87).

Let $Y =$ the number of righties among $n = 12$.

$E(Y) = np = 12(0.87) = 10.44$ righties; $SD(Y) = \sqrt{npq} = \sqrt{12(0.87)(0.13)} \approx 1.16$ righties

b)
$$\begin{aligned} P(\text{not all righties}) &= 1 - P(\text{all righties}) \\ &= 1 - P(Y = 12) \\ &= 1 - {}_{12}C_{12}(0.87)^{12}(0.13)^{0} \\ &\approx 0.812 \end{aligned}$$

c)
$$\begin{aligned} P(\text{no more than 10 righties}) &= P(Y \le 10) \\ &= P(Y=0) + P(Y=1) + P(Y=2) + \ldots + P(Y=10) \\ &= {}_{12}C_0(0.87)^0(0.13)^{12} + \ldots + {}_{12}C_{10}(0.87)^{10}(0.13)^2 \\ &\approx 0.475 \end{aligned}$$

d)
$$\begin{aligned} P(\text{exactly six of each}) &= P(Y = 6) \\ &= {}_{12}C_6(0.87)^6(0.13)^6 \\ &\approx 0.00193 \end{aligned}$$

33. (continued)

e)
$$\begin{aligned}P(\text{majority righties}) &= P(Y \geq 7)\\ &= P(Y=7)+P(Y=8)+P(Y=9)+\ldots+P(Y=12)\\ &= {}_{12}C_7(0.87)^7(0.13)^5+\ldots+{}_{12}C_{12}(0.87)^{12}(0.13)^0\\ &\approx 0.998\end{aligned}$$

34. Still more arrows.

a) In Exercise 30, we determined that the archer's shots could be considered Bernoulli trials. Since our archer is now shooting 10 arrows, use $\text{Binom}(10, 0.80)$.

Let $Y =$ the number of bull's-eyes hit from $n = 10$ shots.

$E(Y) = np = 10(0.80) = 8$ bull's-eyes hit; $SD(Y) = \sqrt{npq} = \sqrt{10(0.80)(0.20)} \approx 1.26$ bull's-eyes hit

b)
$$\begin{aligned}P(\text{no misses out of 10 shots}) &= P(\text{all hits out of 10 shots})\\ &= P(Y=10)\\ &= {}_{10}C_{10}(0.80)^{10}(0.20)^0\\ &\approx 0.107\end{aligned}$$

c)
$$\begin{aligned}P(\text{no more than 8 hits}) &= P(Y \leq 8)\\ &= P(Y=0)+P(Y=1)+P(Y=2)+\ldots+P(Y=8)\\ &= {}_{10}C_0(0.80)^0(0.20)^{10}+\ldots+{}_{10}C_8(0.80)^8(0.20)^2\\ &\approx 0.624\end{aligned}$$

d)
$$\begin{aligned}P(\text{exactly 8 out of 10 shots}) &= P(Y=8)\\ &= {}_{10}C_8(0.80)^8(0.20)^2\\ &\approx 0.302\end{aligned}$$

e)
$$\begin{aligned}P(\text{more hits than misses}) &= P(Y \geq 6)\\ &= P(Y=6)+P(Y=7)+\ldots+P(Y=10)\\ &= {}_{10}C_6(0.80)^6(0.20)^4+\ldots+{}_{10}C_{10}(0.80)^{10}(0.20)^0\\ &\approx 0.967\end{aligned}$$

35. Vision.

The vision tests can be considered Bernoulli trials. There are only two possible outcomes, nearsighted or not. The probability of any child being nearsighted is given as $p = 0.12$. Finally, since the population of children is finite, the trials are not independent. However, 169 is certainly less than 10% of all children, and we will assume that the children in this district are representative of all children in relation to nearsightedness. Use $\text{Binom}(169, 0.12)$.

$\mu = E(\text{nearsighted}) = np = 169(0.12) = 20.28$ children

$\sigma = SD(\text{nearsighted}) = \sqrt{npq} = \sqrt{169(0.12)(0.88)} \approx 4.22$ children

36. International students.

The students can be considered Bernoulli trials. There are only two possible outcomes, international or not. The probability of any freshmen being an international student is given as $p = 0.06$. Finally, since the population of freshmen is finite, the trials are not independent. However, 40 is likely to be less than 10% of all students, and we are told that the freshmen in this college are randomly assigned to housing. Use $\text{Binom}(40, 0.06)$.

$\mu = E(\text{international}) = np = 40(0.06) = 2.4$ students

$\sigma = SD(\text{international}) = \sqrt{npq} = \sqrt{40(0.06)(0.94)} \approx 1.5$ students

37. Tennis, anyone?

The first serves can be considered Bernoulli trials. There are only two possible outcomes, successful and unsuccessful. The probability of any first serve being good is given as $p = 0.70$. Finally, we are assuming that each serve is independent of the others. Since she is serving 6 times, use $\text{Binom}(6, 0.70)$.

Let $X =$ the number of successful serves in 6 first serves.

a) $P(\text{six serves in }) = P(X = 6) = {}_6C_6(0.70)^6(0.30)^0 \approx 0.118$

b) $P(\text{exactly four serves in}) = P(X = 4) = {}_6C_4(0.70)^4(0.30)^2 \approx 0.324$

c) $P(\text{at least four serves in}) = P(X = 4) + P(X = 5) + P(X = 6)$

$$= {}_6C_4(0.70)^4(0.30)^2 + {}_6C_5(0.70)^5(0.30)^1 + {}_6C_6(0.70)^6(0.30)^0$$
$$\approx 0.744$$

d) $P(\text{no more than four serves in})$

$$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$
$$= {}_6C_0(0.70)^0(0.30)^6 + {}_6C_1(0.70)^1(0.30)^5 + {}_6C_2(0.70)^2(0.30)^4 + {}_6C_3(0.70)^3(0.30)^3 + {}_6C_4(0.70)^4(0.30)^2$$
$$\approx 0.580$$

38. Frogs.

The frog examinations can be considered Bernoulli trials. There are only two possible outcomes, having the trait and not having the trait. If the frequency of the trait has not changed, and the biologist collects a representative sample of frogs, then the probability of a frog having the trait is constant, at $p = 0.125$. The trials are not independent since the population of frogs is finite, but 12 frogs is fewer than 10% of all frogs. Since the biologist is collecting 12 frogs, use $\text{Binom}(12, 0.125)$.

Let $X =$ the number of frogs with the trait out of 12 frogs.

a) $P(\text{no frogs have the trait}) = P(X = 0)$

$$= {}_{12}C_0(0.125)^0(0.875)^{12}$$
$$\approx 0.201$$

b) $P(\text{at least two frogs}) = P(X \geq 2)$

$$= P(X = 2) + P(X = 3) + \ldots + P(X = 12)$$
$$= {}_{12}C_2(0.125)^2(0.875)^{10} + \ldots + {}_{12}C_{12}(0.125)^{12}(0.875)^0$$
$$\approx 0.453$$

c) $P(\text{three or four frogs have trait}) = P(X = 3) + P(X = 4)$

$$= {}_{12}C_3(0.125)^3(0.875)^9 + {}_{12}C_4(0.125)^4(0.875)^8$$
$$\approx 0.171$$

38. (continued)

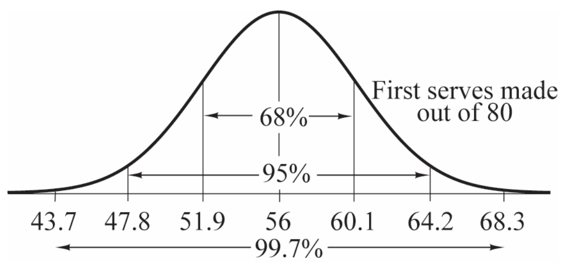
d)
$$\begin{aligned} P(\text{no more than four}) &= P(X \le 4) \\ &= P(X=0)+P(X=1)+\ldots+P(X=4) \\ &= {}_{12}C_0(0.125)^0(0.875)^{12}+\ldots+{}_{12}C_4(0.125)^4(0.875)^8 \\ &\approx 0.989 \end{aligned}$$

39. And more tennis.

The first serves can be considered Bernoulli trials. There are only two possible outcomes, successful and unsuccessful. The probability of any first serve being good is given as $p = 0.70$. Finally, we are assuming that each serve is independent of the others. Since she is serving 80 times, use Binom(80, 0.70).

Let $X =$ the number of successful serves in $n = 80$ first serves.

a) $E(X) = np = 80(0.70) = 56$ first serves in; $SD(X) = \sqrt{npq} = \sqrt{80(0.70)(0.30)} \approx 4.10$ first serves in



b) Since $np = 56$ and $nq = 24$ are both greater than 10, Binom(80, 0.70) may be approximated by the Normal model, N(56, 4.10).

c) According to the Normal model, in matches with 80 serves, she is expected to make between 51.9 and 60.1 first serves approximately 68% of the time, between 47.8 and 64.2 first serves approximately 95% of the time, and between 43.7 and 68.3 first serves approximately 99.7% of the time.

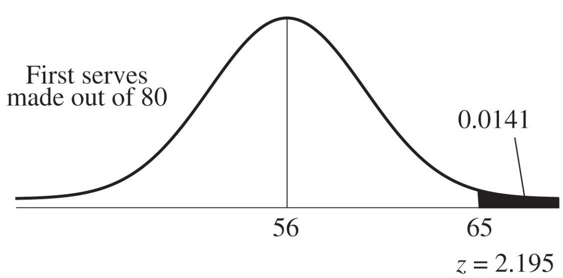
d) **Using Binom(80, 0.70):**

$$\begin{aligned} P(\text{at least 65 first serves}) &= P(X \ge 65) \\ &= P(X=65)+P(X=66)+\ldots+P(X=80) \\ &= {}_{80}C_{65}(0.70)^{65}(0.30)^{15}+\ldots+{}_{80}C_{80}(0.70)^{80}(0.30)^0 \\ &\approx 0.0161 \end{aligned}$$

According to the Binomial model, the probability that she makes at least 65 first serves out of 80 is approximately 0.0161.

Using *N*(56, 4.10):

$$z = \frac{x-\mu}{\sigma} = \frac{65-56}{4.10} \approx 2.195$$



$P(X \ge 65) \approx P(z > 2.195) \approx 0.0141$; According to the Normal model, the probability that she makes at least 65 first serves out of 80 is approximately 0.0141.

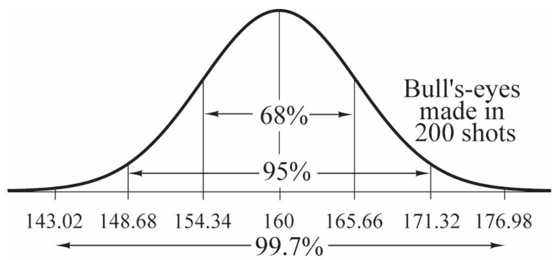
40. More arrows.

These may be considered Bernoulli trials. There are only two possible outcomes, hitting the bull's-eye and not hitting the bull's-eye. The probability of hitting the bull's-eye is given, $p = 0.80$. The shots are assumed to be independent. Since she will be shooting 200 arrows, use Binom(200, 0.80).

Let $Y =$ the number of bull's-eyes in $n = 200$ shots.

a) $E(Y) = np = 200(0.80) = 160$ bull's-eyes; $SD(Y) = \sqrt{npq} = \sqrt{200(0.80)(0.20)} \approx 5.66$ bull's-eyes

b) Since $np = 160$ and $nq = 40$ are both greater than 10, $\text{Binom}(200, 0.80)$ may be approximated by the Normal model, $N(160, 5.66)$.



c) According to the Normal model, in matches with 200 arrows, she is expected to get between 154.34 and 165.66 bull's-eyes approximately 68% of the time, between 148.68 and 171.32 bull's-eyes approximately 95% of the time, and between 143.02 and 176.98 bull's-eyes approximately 99.7% of the time.

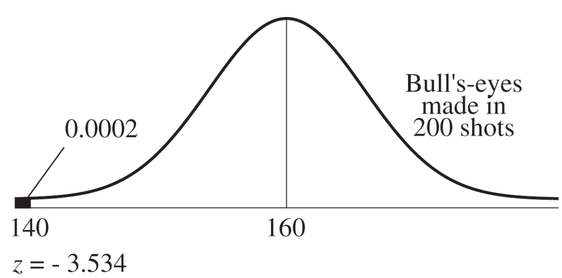
d) **Using Binom(**200, 0.80):

$$\begin{aligned} P(\text{at most 140 hits}) &= P(Y \le 140) \\ &= P(Y = 0) + P(Y = 1) + \ldots + P(Y = 140) \\ &= {}_{200}C_0 (0.80)^0 (0.20)^{200} + \ldots + {}_{200}C_{140} (0.80)^{140} (0.70)^{60} \\ &\approx 0.0005 \end{aligned}$$

According to the Binomial model, the probability that she makes at most 140 bull's-eyes out of 200 is approximately 0.0005.

Using *N*(160, 5.66):

$$z = \frac{y - \mu}{\sigma} = \frac{140 - 160}{5.66} \approx -3.534$$



$P(Y \le 200) \approx P(z < -3.534) \approx 0.0002$; According to the Normal model, the probability that she hits at most 140 bull's-eyes out of 200 is approximately 0.0002. Using either model, it is apparent that it is very unlikely that the archer would hit only 140 bull's-eyes out of 200.

41. Apples.

a) A Binomial model and a Normal model are both appropriate for modeling the number of cider apples that may come from the tree.

Let $X =$ the number of cider apples found in the $n = 300$ apples from the tree.

The quality of the apples may be considered Bernoulli trials. There are only two possible outcomes, cider apple or not a cider apple. The probability that an apple must be used for a cider apple is constant, given as $p = 0.06$. The trials are not independent, since the population of apples is finite, but the apples on the tree are undoubtedly less than 10% of all the apples that the farmer has ever produced, so model with Binom(300, 0.06).

$E(X) = np = 300(0.06) = 18$ cider apples; $SD(X) = \sqrt{npq} = \sqrt{300(0.06)(0.94)} \approx 4.11$ cider apples

Since $np = 18$ and $nq = 282$ are both greater than 10, $\text{Binom}(300, 0.06)$ may be approximated by the Normal model, $N(18, 4.11)$.

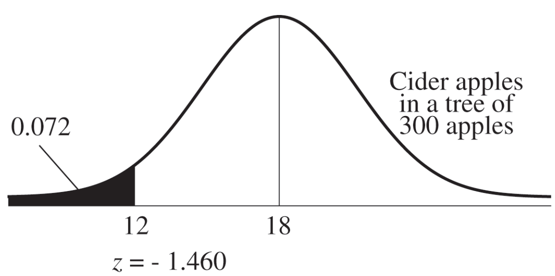
b) Using Binom(300, 0.06):

$$\begin{aligned} P(\text{at most 12 cider apples}) &= P(X \le 12) \\ &= P(X = 0) + \ldots + P(X = 12) \\ &= {}_{300}C_0(0.06)^0(0.94)^{300} + \ldots + {}_{300}C_{12}(0.06)^{12}(0.94)^{282} \\ &\approx 0.085 \end{aligned}$$

According to the Binomial model, the probability that no more than 12 cider apples come from the tree is approximately 0.085.

Using *N*(18, 4.11):

$$z = \frac{x - \mu}{\sigma} = \frac{12 - 18}{4.11} = -1.460$$



$P(X \le 12) \approx P(z < -1.460) \approx 0.072$; According to the Normal model, the probability that no more than 12 apples out of 300 are cider apples is approximately 0.072.

c) It is extremely unlikely that the tree will bear more than 50 cider apples. Using the Normal model, $N(18, 4.11)$, 50 cider apples is approximately 7.8 standard deviations above the mean.

42. Frogs, part II.

The frog examinations can be considered Bernoulli trials. There are only two possible outcomes, having the trait and not having the trait. If the frequency of the trait has not changed, and the biologist collects a representative sample of frogs, then the probability of a frog having the trait is constant, at $p = 0.125$. The trials are not independent since the population of frogs is finite, but 150 frogs is fewer than 10% of all frogs. Since the biologist is collecting 150 frogs, use $\text{Binom}(150, 0.125)$.

42. (continued)

Let $X =$ the number of frogs with the trait, from $n = 150$ frogs.

a) $E(X) = np = 150(0.125) = 18.75$ frogs; $SD(X) = \sqrt{npq} = \sqrt{150(0.125)(0.875)} \approx 4.05$ frogs

b) Since $np = 18.75$ and $nq = 131.25$ are both greater than 10, Binom(200, 0.125) may be approximated by the Normal model, $N(18.75, 4.05)$.

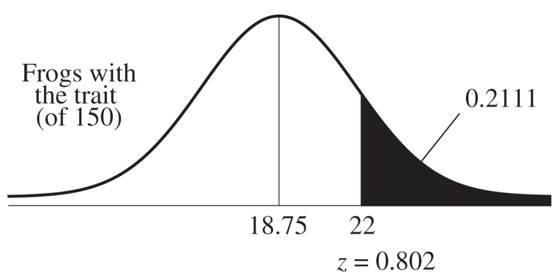
c) **Using Binom(150, 0.125):**

$$\begin{aligned} P(\text{at least 22 frogs}) &= P(X \geq 22) \\ &= P(X = 22) + \ldots + P(X = 150) \\ &= {}_{150}C_{22}(0.125)^{22}(0.875)^{128} + \ldots + {}_{150}C_{150}(0.125)^{150}(0.875)^{0} \\ &\approx 0.2433 \end{aligned}$$

According to the Binomial model, the probability that at least 22 frogs out of 150 have the trait is approximately 0.2433.

Using *N*(18.75, 4.05):

$$z = \frac{x - \mu}{\sigma} = \frac{22 - 18.75}{4.05} \approx 0.802$$



$P(X \geq 22) \approx P(z > 0.802) \approx 0.2111$; According to the Normal model, the probability that at least 22 frogs out of 150 have the trait is approximately 0.2111. Using either model, the probability that the biologist discovers 22 of 150 frogs with the trait simply as a result of natural variability is quite high. This doesn't prove that the trait has become more common.

43. Lefties, again.

Let $X =$ the number of righties among a class of $n = 188$ students.

Using Binom(188, 0.87):

These may be considered Bernoulli trials. There are only two possible outcomes, right-handed and not right-handed. The probability of being right-handed is assumed to be constant at about 87%. The trials are not independent, since the population is finite, but a sample of 188 students is certainly fewer than 10% of all people. Therefore, the number of righties in a class of 188 students may be modeled by Binom(188, 0.87).

If there are 171 or more righties in the class, some righties have to use a left-handed desk.

$$\begin{aligned} P(\text{at least 171 righties}) &= P(X \geq 171) \\ &= P(X = 171) + \ldots + P(X = 188) \\ &= {}_{188}C_{171}(0.87)^{171}(0.13)^{17} + \ldots + {}_{188}C_{188}(0.87)^{188}(0.13)^{0} \\ &\approx 0.061 \end{aligned}$$

According to the Binomial model, the probability that a right-handed student has to use a left-handed desk is approximately 0.061.

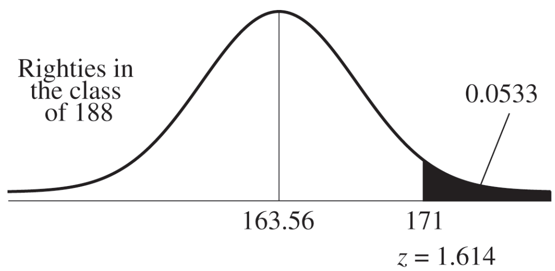
43. (continued)

Using *N*(163.56, 4.61):

$E(X) = np = 188(0.87) = 163.56$ righties; $SD(X) = \sqrt{npq} = \sqrt{188(0.87)(0.13)} \approx 4.61$ righties

Since $np = 163.56$ and $nq = 24.44$ are both greater than 10, $\text{Binom}(188, 0.87)$ may be approximated by the Normal model, *N*(163.56, 4.61).

$$z = \frac{x-\mu}{\sigma} = \frac{171-163.56}{4.61} \approx 1.614$$



$P(X \geq 171) \approx P(z > 1.614) \approx 0.053$; According to the Normal model, the probability that there are at least 171 righties in the class of 188 is approximately 0.0533.

44. No-shows.

Let $X =$ the number of passengers that show up for the flight of $n = 275$ passengers.

Using Binom(275, 0.95):
These may be considered Bernoulli trials. There are only two possible outcomes, showing up and not showing up. The airlines believe the probability of showing up is constant at about 95%. The trials are not independent, since the population is finite, but a sample of 275 passengers is certainly fewer than 10% of all passengers. Therefore, the number of passengers who show up for a flight of 275 may be modeled by Binom(275, 0.95).

If 266 or more passengers show up, someone has to get bumped off the flight.

$$\begin{aligned} P(\text{at least 266 passengers}) &= P(X \geq 266) \\ &= P(X = 266) + \ldots + P(X = 275) \\ &= {}_{275}C_{266}(0.95)^{266}(0.05)^{9} + \ldots + {}_{275}C_{275}(0.95)^{275}(0.05)^{0} \\ &\approx 0.116 \end{aligned}$$

According to the Binomial model, the probability someone on the flight must be bumped is approximately 0.116.

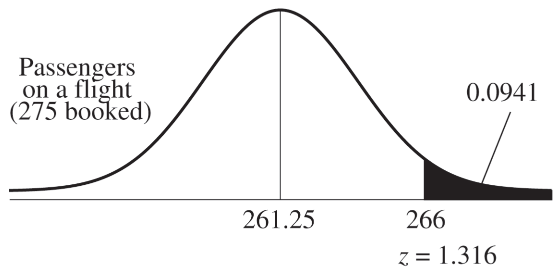
44. (continued)

Using *N*(261.25, 3.61):

$E(X) = np = 275(0.95) = 261.25$ passengers; $SD(X) = \sqrt{npq} = \sqrt{275(0.95)(0.05)} \approx 3.61$ passengers

Since $np = 261.25$ and $nq = 13.75$ are both greater than 10, $\text{Binom}(275, 0.95)$ may be approximated by the Normal model, $N(261.25, 3.61)$.

$$z = \frac{x - \mu}{\sigma} = \frac{266 - 261.25}{3.61} \approx 1.316$$



$P(X \geq 266) \approx P(z > 1.316) \approx 0.0941$; According to the Normal model, the probability that at least 266 passengers show up is approximately 0.0941.

45. Annoying phone calls.

Let $X =$ the number of sales made after making $n = 200$ calls.

Using Binom(200, 0.12):
These may be considered Bernoulli trials. There are only two possible outcomes, making a sale and not making a sale. The telemarketer was told that the probability of making a sale is constant at about $p = 0.12$. The trials are not independent, since the population is finite, but 200 calls is fewer than 10% of all calls. Therefore, the number of sales made after making 200 calls may be modeled by Binom(200, 0.12).

$$\begin{aligned} P(\text{at most } 10) &= P(X \leq 10) \\ &= P(X = 0) + \ldots + P(X = 10) \\ &= {}_{200}C_0 (0.12)^0 (0.88)^{200} + \ldots + {}_{200}C_{1900} (0.12)^{10} (0.88)^{190} \\ &\approx 0.0006 \end{aligned}$$

According to the Binomial model, the probability that the telemarketer would make at most 10 sales is approximately 0.0006.

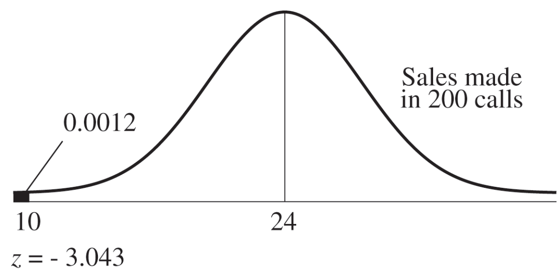
Using *N*(24, 4.60):

$E(X) = np = 200(0.12) = 24$ sales; $SD(X) = \sqrt{npq} = \sqrt{200(0.12)(0.88)} \approx 4.60$ sales

Since $np = 24$ and $nq = 176$ are both greater than 10, $\text{Binom}(200, 0.12)$ may be approximated by the Normal model, $N(24, 4.60)$.

45. (continued)

$$z = \frac{x-\mu}{\sigma} = \frac{10-24}{4.60} \approx -3.043$$



$P(X \le 10) \approx P(z < -3.043) \approx 0.0012$; According to the Normal model, the probability that the telemarketer would make at most 10 sales is approximately 0.0012.

Since the probability that the telemarketer made 10 sales, given that the 12% of calls result in sales is so low, it is likely that he was misled about the true success rate.

46. The euro.

Let $X =$ the number of heads after spinning a Belgian euro $n = 250$ times.

Using Binom(250, 0.5):
These may be considered Bernoulli trials. There are only two possible outcomes, heads and tails. The probability that a fair Belgian euro lands heads is $p = 0.5$. The trials are independent, since the outcome of a spin does not affect other spins. Therefore, Binom(250, 0.5) may be used to model the number of heads after spinning a Belgian euro 250 times.

$$\begin{aligned} P(\text{at least } 140) &= P(X \ge 140) \\ &= P(X = 140) + \ldots + P(X = 250) \\ &= {}_{250}C_{140}(0.5)^{140}(0.5)^{110} + \ldots + {}_{250}C_{250}(0.5)^{250}(0.5)^{0} \\ &\approx 0.0332 \end{aligned}$$

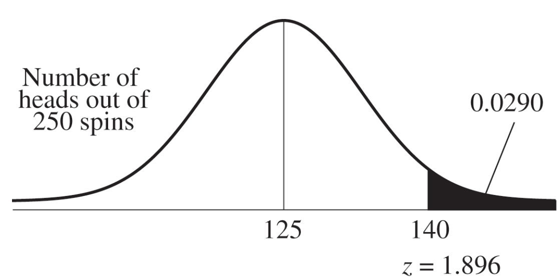
According to the Binomial model, the probability that a fair Belgian euro comes up heads at least 140 times is 0.0332.

Using *N*(125, 7.91):

$E(X) = np = 250(0.05) = 125$ heads; $SD(X) = \sqrt{npq} = \sqrt{250(0.5)(0.5)} \approx 7.91$ heads

Since $np = 125$ and $nq = 125$ are both greater than 10, Binom$(250, 0.5)$ may be approximated by the Normal model, *N*(125, 7.91).

$$z = \frac{x-\mu}{\sigma} = \frac{140-125}{7.91} \approx 1.896$$



46. (continued)

$P(X \geq 140) \approx P(z > 1.896) \approx 0.0290$; According to the Normal model, the probability that a fair Belgian euro lands heads at least 140 out of 250 spins is approximately 0.0290. Since the probability that a fair Belgian euro lands heads at least 140 out of 250 spins is low, it is unlikely that the euro spins fairly. However, the probability is not extremely low, and we aren't sure of the source of the data, so it might be a good idea to spin it some more.

47. Hurricanes, redux.

a) $E(X) = \lambda = 2.45$

$$P(\text{no hurricanes next year}) = \frac{e^{-2.45}(2.45)^0}{0!} \approx 0.0863$$

b) $P(\text{exactly one hurricane in next 2 years})$

$= P(\text{hurr. first yr})P(\text{no hurr. second yr}) + P(\text{no hurr. first yr})P(\text{hurr. second yr})$

$$= \left(\frac{e^{-2.45}(2.45)^1}{1!}\right)\left(\frac{e^{-2.45}(2.45)^0}{0!}\right) + \left(\frac{e^{-2.45}(2.45)^0}{0!}\right)\left(\frac{e^{-2.45}(2.45)^1}{1!}\right)$$

≈ 0.0365

48. Bank tellers.

a) Because the Poisson model scales according to the sample size, we can calculate the mean for 10 minutes. A Poisson model with mean 2 customers per hour (60 minutes) is equivalent to a Poisson model with mean $\frac{2}{6} = \frac{1}{3}$ customers per $\frac{60}{6} = 10$ minutes.

Let $X =$ the number of customers arriving in 10 minutes.

$$P(X = 0) = \frac{e^{-\frac{1}{3}}\left(\frac{1}{3}\right)^0}{0!} \approx 0.7165$$

b) $$P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - \left[\frac{e^{-\frac{1}{3}}\left(\frac{1}{3}\right)^0}{0!} + \frac{e^{-\frac{1}{3}}\left(\frac{1}{3}\right)^1}{1!}\right] \approx 0.0446$$

c) No. The probabilities do not change based on what has just happened. Even though two customers just came in, the probability of no customers in the next 10 minutes will not change. This is neither a better nor a worse time.

49. TB.

a) $E(X) = \lambda = np = 8000(0.0005) = 4$ cases

b) $$P(\text{at least one new case}) = 1 - P(\text{no new cases}) = 1 - \frac{e^{-4}(4)^0}{0!} \approx 0.9817$$

50. Earthquakes.

a) $E(X) = \lambda = np = 1000\left(\frac{1}{10{,}000}\right) = \frac{1}{10}$

b) $$P(\text{at least one earthquake in next 100 days}) = 1 - P(\text{no earthquakes}) = 1 - \frac{e^{-\frac{1}{10}}\left(\frac{1}{10}\right)^0}{0!} \approx 0.0952$$

51. Seatbelts II.

These stops may be considered Bernoulli trials. There are only two possible outcomes, belted or not belted. Police estimate that the probability that a driver is buckled is 80%. (The probability of not being buckled is therefore 20%.) Provided the drivers stopped are representative of all drivers, we can consider the probability constant. The trials are not independent, since the population of drivers is finite, but the police will not stop more than 10% of all drivers.

a) Let $X =$ the number of cars stopped before finding a driver whose seat belt is not buckled. Use Geom(0.2) to model the situation.

$$E(X) = \frac{1}{p} = \frac{1}{0.2} = 5 \text{ cars}$$

b) $P(\text{First unbelted driver is in the sixth car}) = P(X = 6) = (0.8)^5(0.2) \approx 0.066$

c) $P(\text{The first ten drivers are wearing seatbelts}) = (0.8)^{10} \approx 0.107$

d) Let $Y =$ the number of drivers wearing their seatbelts in 30 cars. Use Binom(30, 0.8).

$$E(Y) = np = 30(0.8) = 24 \text{ drivers; } SD(Y) = \sqrt{npq} = \sqrt{30(0.8)(0.2)} \approx 2.19 \text{ drivers}$$

e) Let $W =$ the number of drivers not wearing their seatbelts in 120 cars.

Using Binom(120, 0.2):

$$\begin{aligned} P(\text{at least } 20) = P(W \geq 20) &\\ &= P(W = 20) + \ldots + P(W = 120) \\ &= {}_{120}C_{20}(0.2)^{20}(0.8)^{100} + \ldots + {}_{120}C1_{20}(0.2)^{120}(0.8)^{0} \\ &\approx 0.848 \end{aligned}$$

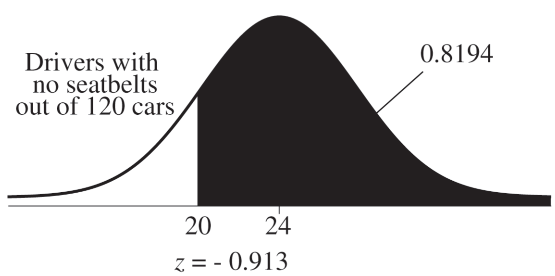
According to the Binomial model, the probability that at least 20 out of 120 drivers are not wearing their seatbelts is approximately 0.848.

Using *N*(24, 4.38):

$$E(W) = np = 120(0.2) = 24 \text{ drivers; } SD(W) = \sqrt{npq} = \sqrt{120(0.2)(0.8)} \approx 4.38 \text{ drivers}$$

Since $np = 24$ and $nq = 96$ are both greater than 10, Binom(120, 0.2) may be approximated by the Normal model, *N*(24, 4.38).

$$z = \frac{w - \mu}{\sigma} = \frac{20 - 24}{4.38} \approx -0.913$$



$P(W \geq 120) \approx P(z > -0.913) \approx 0.8194$; According to the Normal model, the probability that at least 20 out of 120 drivers stopped are not wearing their seatbelts is approximately 0.8194.

52. Rickets.

The selection of these children may be considered Bernoulli trials. There are only two possible outcomes, vitamin D deficient or not vitamin D deficient. Recent research indicates that 20% of British children are vitamin D deficient. (The probability of not being vitamin D deficient is therefore 80%.) Provided the students at this school are representative of all British children, we can consider the probability constant. The trials are not independent, since the population of British children is finite, but the children at this school represent fewer than 10% of all British children.

a) Let $X =$ the number of students tested before finding a student who is vitamin D deficient. Use Geom(0.2) to model the situation.

$$P(\text{First vit. D def. child is the eighth one tested}) = P(X = 8) = (0.8)^7(0.2) \approx 0.042$$

b) $P(\text{The first ten children tested are okay}) = (0.8)^{10} \approx 0.107$

c) $E(X) = \dfrac{1}{p} = \dfrac{1}{0.2} = 5$ kids

d) Let $Y =$ the number of children who are vitamin D deficient out of 50 children.
Use Binom(50, 0.2).

$$E(Y) = np = 50(0.2) = 10 \text{ kids}; \; SD(Y) = \sqrt{npq} = \sqrt{50(0.2)(0.8)} \approx 2.83 \text{ kids}$$

e) Using Binom(320, 0.2):

$$\begin{aligned} P(\text{no more than 50 children have the deficiency}) &= P(X \le 50) \\ &= P(X = 0) + \ldots + P(X = 50) \\ &= {}_{320}C_0(0.2)^0(0.8)^{320} + \ldots + {}_{320}C_0(0.2)^{50}(0.8)^{270} \\ &\approx 0.027 \end{aligned}$$

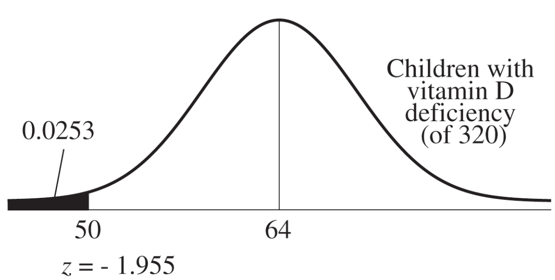
According to the Binomial model, the probability that no more than 50 of the 320 children have the vitamin D deficiency is approximately 0.027.

Using *N*(64, 7.16):

$$E(Y) = np = 320(0.2) = 64 \text{ kids}; \; SD(Y) = \sqrt{npq} = \sqrt{320(0.2)(0.8)} \approx 7.16 \text{ kids}$$

Since $np = 64$ and $nq = 256$ are both greater than 10, Binom(320, 0.2) may be approximated by the Normal model, $N(64, 7.16)$.

$$z = \frac{y - \mu}{\sigma} = \frac{50 - 64}{7.16} \approx -1.955$$



$P(Y \le 50) \approx P(z < -1.955) \approx 0.0253$; According to the Normal model, the probability that no more than 50 out of 320 children have the vitamin D deficiency is approximately 0.0253.

53. ESP.

Choosing symbols may be considered Bernoulli trials. There are only two possible outcomes, correct or incorrect. Assuming that ESP does not exist, the probability of a correct identification from a randomized deck is constant, at $p = 0.20$. The trials are independent, as long as the deck is shuffled after each attempt. Since 100 trials will be performed, use Binom(100, 0.2).

Let $X =$ the number of symbols identified correctly out of 100 cards.

$E(X) = np = 100(0.2) = 20$ correct identifications; $SD(X) = \sqrt{npq} = \sqrt{100(0.2)(0.8)} = 4$ correct identifications

Answers may vary. In order be convincing, the "mind reader" would have to identify at least 32 out of 100 cards correctly, since 32 is 3 standard deviations above the mean. Identifying fewer cards than 32 could happen too often, simply due to chance.

54. True–false.

Guessing at answers may be considered Bernoulli trials. There are only two possible outcomes, correct or incorrect. If the student was guessing, the probability of a correct response is constant, at $p = 0.50$. The trials are independent, since the answer to one question should not have any bearing on the answer to the next. Since 50 questions are on the test use Binom(500, 0.5).

Let $X =$ the number of questions answered correctly out of 50 questions.

$E(X) = np = 50(0.5) = 25$ correct answers; $SD(X) = \sqrt{npq} = \sqrt{50(0.5)(0.5)} \approx 3.54$ correct answers

Answers may vary. In order be convincing, the student would have to answer at least 36 out of 50 questions correctly, since 36 is approximately 3 standard deviations above the mean. Answering fewer than 36 questions correctly could happen too often, simply due to chance.

55. Hot hand.

A streak like this is not unusual. The probability that he makes 4 in a row with a 55% free throw percentage is $(0.55)(0.55)(0.55)(0.55) \approx 0.09$. We can expect this to happen nearly 1 in 10 times for every set of 4 shots that he makes. One out of 10 times is not that unusual.

56. New bow.

A streak like this is not unusual. The probability that she makes 6 consecutive bull's-eyes with an 80% bull's-eye percentage is $(0.8)(0.8)(0.8)(0.8)(0.8)(0.8) \approx 0.26$. If she were to shoot several flights of 6 arrows, she is expected to get 6 bull's-eyes about 26% of the time. An event that happens due to chance about one out of four times is not that unusual.

57. Hotter hand.

The shots may be considered Bernoulli trials. There are only two possible outcomes, make or miss. The probability of success is constant at 55%, and the shots are independent of one another. Therefore, we can model this situation with Binom(32, 0.55).

Let $X =$ the number of free throws made out of 40.

$E(X) = np = 40(0.55) = 22$ free throws made; $SD(X) = \sqrt{npq} = \sqrt{40(0.55)(0.45)} \approx 3.15$ free throws

Answers may vary. The player's performance seems to have increased. 32 made free throws is $(32 - 22)/3.15 \approx 3.17$ standard deviations above the mean, an extraordinary feat, unless his free throw percentage has increased. This does NOT mean that the sneakers are responsible for the increase in free throw percentage. Some other variable may account for the increase. The player would need to set up a controlled experiment in order to determine what effect, if any, the sneakers had on his free throw percentage.

58. New bow, again.

The shots may be considered Bernoulli trials. There are only two possible outcomes, hit or miss the bull's-eye. The probability of success is constant at 80%, and the shots are independent of one another. Therefore, we can model this situation with Binom(50, 0.8).

Let $X =$ the number of bull's-eyes hit out of 50.

$E(X) = np = 50(0.8) = 40$ bull's-eyes hit; $SD(X) = \sqrt{npq} = \sqrt{50(0.8)(0.2)} \approx 2.83$ bull's-eyes

Answers may vary. The archer's performance doesn't seem to have increased. Forty-five bull's-eyes is $(45-40)/2.83 \approx 1.77$ standard deviations above the mean. This isn't unusual for an archer of her skill level.

59. Web visitors.

a) The Poisson model because it is a good model to use when the data consists of counts of occurrences. The events must be independent and the mean number of occurrences stays constant.

b) The probability that in any 1 minute at least one purchase is made is $P(X=1)+P(X=2)+...$

$$= \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} + ... = 0.9502.$$

c) The probability that no one makes a purchase in the next 2 minutes is $P(X=0) = \frac{e^{-6}6^0}{0!} = 0.0025.$

60. Quality control.

a) The Poisson model because it is a good model to use when the data consists of counts of occurrences. The events must be independent and the mean number of occurrences stays constant.

b) The probability that no faulty cell phones will be produced tomorrow is $P(X=0) = \frac{e^{-2}2^0}{0!} = 0.1353.$

c) The probability that 3 or more faulty cell phones were produced today is $P(X=3)+P(X=4)+...$

$$= \frac{e^{-2}2^3}{3!} + \frac{e^{-2}2^4}{4!} + \frac{e^{-2}3^5}{5!} + ... = 0.3233.$$

61. Web visitors, part 2.

a) The exponential model can be used to model the time between events especially when the number of arrivals of those events can be modeled by a Poisson model.

b) The mean time between purchases: 3 purchases are made per minute so each purchase is made every 1/3 minute.

c) The probability that the time to the next purchase will be between 1 and 2 minutes is $e^{-3(1)} - e^{-3(2)} = 0.0473.$

62. Quality control, part 2.

a) The exponential model can be used to model the time between events especially when the number of arrivals of those events can be modeled by a Poisson model.

b) The probability that the time to the next failure is 1 day or less is $e^{-2(0)} - e^{-2(1)} = 0.8647.$

c) The mean time between failures: the mean number of defective cell phones is 2 per day so a defective cell phone occurs every $\frac{1}{2}$ day.