Chapter 12 – From Randomness to Probability

Section 12.1

1. Flipping a coin.

In the long run, a fair coin will generate 50% heads and 50% tails, approximately. But for each flip we cannot predict the outcome.

2. Dice.

In the long run, a fair die will produce roughly equal amounts of the numbers 1 through 6 when rolled. But each roll is unpredictable.

3. Flipping a coin II.

There is no law of averages for the short run. The first five flips do not affect the sixth flip.

4. Dice II.

The dice have no memory and in the short run there is no guarantee about what will happen next.

Section 12.2

5. Wardrobe.

a) There are a total of 10 shirts, and 3 of them are red. The probability of randomly selecting a red shirt is 3/10 = 0.30.

b) There are a total of 10 shirts, and 8 of them are not black. The probability of randomly selecting a shirt that is not black is 8/10 = 0.80.

6. Playlists.

a) There are a total of 20 songs, and 7 of them are rap songs. The probability of randomly selecting a rap song is 7/20 = 0.35.

b) There are a total of 20 songs, and 17 of them are not country songs. The probability of randomly selecting noncountry song is 17/20 = 0.85.

Section 12.3

7. Cell phones and surveys.

a) If 25% of homes don't have a landline, then 75% of them do have a landline. The probability that all 5 houses have a landline is $(0.75)^5 \approx 0.237$.

b) $P(\text{at least one without landline}) = 1 - P(\text{all landlines}) = 1 - (0.75)^5 \approx 0.763$

c) $P(\text{at least one with landline}) = 1 - P(\text{no landlines}) = 1 - (0.25)^5 \approx 0.999$

8. Cell phones and surveys II.

a) The probability that all 4 adults have only a cell phone is $(0.49)^4 \approx 0.0576$.

b) If 49% have only a cell phone and no landline, then 51% don't have this combination of phones. $P(\text{no one with only a cell phone}) = (0.51)^4 \approx 0.0677$.

c) $P(\text{at least one with only cell phone}) = 1 - P(\text{cellphone and/or landline}) = 1 - (0.51)^4 \approx 0.9323$

Chapter Exercises.

9. Sample spaces.

a) S = {HH, HT, TH, TT}; All of the outcomes are equally likely to occur.

b) S = {0, 1, 2, 3}; All outcomes are not equally likely. A family of 3 is more likely to have, for example, 2 boys than 3 boys. There are three equally likely outcomes that result in 2 boys (BBG, BGB, and GBB), and only one that results in 3 boys (BBB).

c) S = {H, TH, TTH, TTT}; All outcomes are not equally likely. For example, the probability of getting heads on the first try is $\frac{1}{2}$. The probability of getting 3 tails is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

d) S = {1, 2, 3, 4, 5, 6}; All outcomes are not equally likely. Since you are recording only the larger number (or the number if there is a tie) of two dice, 6 will be the larger when the other die reads 1, 2, 3, 4, or 5. The outcome 2 will only occur when the other die shows 1 or 2.

10. Sample spaces.

a) S = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}; All outcomes are not equally likely. For example, there are four equally likely outcomes that result in a sum of 5 (1 + 4, 4 + 1, 2 + 3, and 3 + 2), and only one outcome that results in a sum of 2 (1 + 1).

b) S = {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}; All outcomes are equally likely.

c) S = {0, 1, 2, 3, 4}; All outcomes are not equally likely. For example, there are 4 equally likely outcomes that produce 1 tail (HHHT, HHTH, HTHH, and THHH), but only one outcome that produces 4 tails (TTTT).

d) S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}; All outcomes are not equally likely. A string of 3 heads is much more likely to occur than a string of 10 heads in a row.

11. Roulette.

If a roulette wheel is to be considered truly random, then each outcome is equally likely to occur, and knowing one outcome will not affect the probability of the next. Additionally, there is an implication that the outcome is not determined through the use of an electronic random number generator.

12. Rain.

When a weather forecaster makes a prediction such as a 25% chance of rain, this means that when weather conditions are like they are now, rain happens 25% of the time in the long run.

13. Winter.

Although acknowledging that there is no law of averages, Knox attempts to use the law of averages to predict the severity of the winter. Some winters are harsh and some are mild over the long run, and knowledge of this can help us to develop a long-term probability of having a harsh winter. However, probability does not compensate for odd occurrences in the short term. Suppose that the probability of having a harsh winter is 30%. Even if there are several mild winters in a row, the probability of having a harsh winter is still 30%.

14. Snow.

The radio announcer is referring to the “law of averages,” which is not true. Probability does not compensate for deviations from the expected outcome in the recent past. The weather is not more likely to be bad later on in the winter because of a few sunny days in autumn. The weather makes no conscious effort to even things out, which is what the announcer’s statement implies.

15. Cold streak.

There is no such thing as being “due to make a shot.” This statement is based on the so-called law of averages, which is a mistaken belief that probability will compensate in the short term for odd occurrences in the past. The player’s chance of making a shot does not change based on recent successes or failures.

16. Crash.

- **a)** There is no such thing as the "law of averages." The overall probability of an airplane crash does not change due to recent crashes.
- **b)** Again, there is no such thing as the "law of averages." The overall probability of an airplane crash does not change due to a period in which there were no crashes. It makes no sense to say a crash is "due." If you say this, you are expecting probability to compensate for strange events in the past.

17. Auto insurance.

- **a)** It would be foolish to insure your neighbor against automobile accidents for \$1500. Although you might simply collect \$1500, there is a good chance you could end up paying much more than \$1500. That risk is not worth the \$1500.
- **b)** The insurance company insures many people. The overwhelming majority of customers pay the insurance and never have a claim, or have claims that are lower than the cost of their payments. The few customers who do have a claim are offset by the many who simply send their premiums without a claim. The relative risk to the insurance company is low.

18. Jackpot.

- **a)** The Excalibur can afford to give away millions of dollars on a \$3 bet because almost all of the people who bet do not win the jackpot.
- **b)** The press release generates publicity, which entices more people to come and gamble. Of course, the casino wants people to play, because the overall odds are always in favor of the casino. The more people who gamble, the more the casino makes in the long run. Even if that particular slot machine has paid out more than it ever took in, the publicity it gives to the casino more than makes up for it. If the casino is successful, then they will buy more slot machines from the slot machine maker.

19. Spinner.

- **a)** This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1.
- **b)** This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1.
- **c)** This is not a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, but the sum of the probabilities is greater than 1.
- **d)** This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1. However, this game is not very exciting!
- **e)** This probability assignment is not legitimate. The sum of the probabilities is 0, and there is one probability, –1.5, that is not between 0 and 1, inclusive.

20. Scratch off.

- **a)** This is not a legitimate probability assignment. Although each outcome has probability between 0 and 1, inclusive, the sum of the probabilities is less than 1.
- **b)** This is not a legitimate probability assignment. Although each outcome has probability between 0 and 1, inclusive, the sum of the probabilities is greater than 1.
- **c)** This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1.
- **d)** This probability assignment is not legitimate. Although the sum of the probabilities is 1, there is one probability, –0.25, that is not between 0 and 1, inclusive.
- **e)** This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1. This is also known as a 10% off sale!

21. Electronics.

A family may have both a computer and an HD TV. The events are not disjoint, so the Addition Rule does not apply.

22. Homes.

A home may have both a garage and a pool. The events are not disjoint, so the Addition Rule does not apply.

23. Speeders.

When cars are traveling close together, their speeds are not independent. For example, a car following directly behind another can't be going faster than the car ahead. Since the speeds are not independent, the Multiplication Rule does not apply.

24. Lefties.

There may be a genetic factor making handedness of siblings not independent. The Multiplication Rule does not apply.

25. College admissions.

a) Jorge had multiplied the probabilities.

b) Jorge assumes that being accepted to the colleges are independent events.

c) No. Colleges use similar criteria for acceptance, so the decisions are not independent. Students that meet these criteria are more likely to be accepted at all of the colleges. Since the decisions are not independent, the probabilities cannot be multiplied together.

26. College admissions II.

a) Jorge has added the probabilities.

b) Jorge is assuming that getting accepted to the colleges are disjoint events.

c) No. Students can get accepted to more than one of the three colleges. The events are not disjoint, so the probabilities cannot simply be added together.

27. Car repairs.

Since all of the events listed are disjoint, the addition rule can be used.

a) $P(\text{no repairs}) = 1 - P(\text{some repairs}) = 1 - (0.17 + 0.07 + 0.04) = 1 - (0.28) = 0.72$

b) $P(\text{no more than one repair}) = P(\text{no repairs or one repair}) = 0.72 + 0.17 = 0.89$

c) $P(\text{some repairs}) = P(\text{one or two or three or more repairs}) = 0.17 + 0.07 + 0.04 = 0.28$

28. Stats projects.

Since all of the events listed are disjoint, the addition rule can be used.

a) $P(\text{two or more semesters of Calculus}) = 1 - (0.55 + 0.32) = 0.13$

b) $P(\text{some Calculus}) = P(\text{one semester or two or more semesters}) = 0.32 + 0.13 = 0.45$

c) $P(\text{no more than one semester}) = P(\text{no Calculus or one semester}) = 0.55 + 0.32 = 0.87$

29. More repairs.

Assuming that repairs on the two cars are independent from one another, the multiplication rule can be used. Use the probabilities of events from Exercise 27 in the calculations.

a) $P(\text{neither will need repair}) = (0.72)(0.72) = 0.5184$

b) $P(\text{both will need repair}) = (0.28)(0.28) = 0.0784$

c) $P(\text{at least one will need repair}) = 1 - P(\text{neither will need repair}) = 1 - (0.72)(0.72) = 0.4816$

30. Another project.

Since students with Calculus backgrounds are independent from one another, use the multiplication rule. Use the probabilities of events from Exercise 28 in the calculations.

a) $P(\text{neither has studied Calculus}) = (0.55)(0.55) = 0.3025$

b) $P(\text{both have studied at least one semester of Calculus}) = (0.45)(0.45) = 0.2025$

c) $P(\text{at least one has studied more than one semester of Calculus})$
$= 1 - P(\text{neither has studied more than one semester of Calculus})$
$= 1 - (0.87)(0.87) = 0.2431$

31. Repairs, again.

a) The repair needs for the two cars must be independent of one another.

b) This may not be reasonable. An owner may treat the two cars similarly, taking good (or poor) care of both. This may decrease (or increase) the likelihood that each needs to be repaired.

32. Final project.

a) The Calculus backgrounds of the students must be independent of one another.

b) Since the professor assigned the groups at random, the Calculus backgrounds are independent.

33. Energy 2013.

a) $P(\text{response is “Increase oil, gas, and coal”}) = 164/529 \approx 0.310$

b) $P(\text{“Equally important” or “No opinion”}) = 37/529 + 16/529 = 53/529 \approx 0.100$

34. Failing fathers?

a) $P(\text{response is “Harder”}) = 682/2005 \approx 0.340$

b) $P(\text{response is “Same” or “Easier”}) = 802/2005 + 501/2005 = 1303/2005 \approx 0.650$

35. More energy.

a) $P(\text{all three respond “Develop wind and solar”}) = \left(\frac{312}{529}\right)\left(\frac{312}{529}\right)\left(\frac{312}{529}\right) \approx 0.205$

b) $P(\text{none respond “Equally important”}) = \left(\frac{492}{529}\right)\left(\frac{492}{529}\right)\left(\frac{492}{529}\right) \approx 0.805$

c) In order to compute the probabilities, we must assume that responses are independent.

d) It is reasonable to assume that responses are independent, since the three people were chosen at random.

36. Fathers revisited.

a) $P(\text{both think being a father is easier}) = \left(\frac{501}{2005}\right)\left(\frac{501}{2005}\right) \approx 0.062$

b) $P(\text{neither thinks being a father is easier}) = \left(\frac{1504}{2005}\right)\left(\frac{1504}{2005}\right) \approx 0.563$

c) $P(\text{first thinks being a father is easier, the second doesn't}) = \left(\frac{501}{2005}\right)\left(\frac{1504}{2005}\right) \approx 0.187$

d) In order to compute the probabilities, we must assume that responses are independent.

e) It is reasonable to assume that responses are independent, since the two people were chosen at random.

37. Polling.

a)
$$\begin{aligned}&P(\text{household is contacted and household refuses to cooperate})\\&= P(\text{household is contacted}) \cdot P(\text{household refuses} \mid \text{contacted})\\&= (0.62)(1-0.14) = 0.5332\end{aligned}$$

b)
$$\begin{aligned}&P(\text{fail to contact household or contacting and not getting interview})\\&= P(\text{fail to contact}) + P(\text{contact household}) \cdot P(\text{not getting interview} \mid \text{contacted})\\&= (1-0.62) + (0.62)(1-0.14) = 0.9132\end{aligned}$$

c) The question in part b covers all possible occurrences *except* contacting the house and getting the interview.

$$\begin{aligned}&P(\text{failing to contact household or contacting and not getting the interview})\\&= 1 - P(\text{contacting the household and getting the interview})\\&= 1 - (0.62)(0.14) = 0.9132\end{aligned}$$

38. Polling, part II.

a)
$$\begin{aligned}&P(\text{2012 household is contacted and household cooperates})\\&= P(\text{household is contacted}) \cdot P(\text{household cooperates} \mid \text{contacted})\\&= (0.62)(0.14) = 0.0868\end{aligned}$$

b)
$$\begin{aligned}&P(\text{1997 household is contacted and cooperates})\\&= P(\text{household is contacted}) \cdot P(\text{household cooperates} \mid \text{contacted})\\&= (0.90)(0.43) = 0.387\end{aligned}$$

It was more likely for pollsters to obtain an interview at the next household in 1997 than in 2003.

39. M&M's.

a) Since all of the events are disjoint (an M&M can't be two colors at once!), use the addition rule where applicable.

1. $P(\text{brown}) = 1 - P(\text{not brown}) = 1 - P(\text{yellow or red or orange or blue or green})$
$$= 1 - (0.20 + 0.20 + 0.10 + 0.10 + 0.10) = 0.30$$

2. $P(\text{yellow or orange}) = 0.20 + 0.10 = 0.30$

3. $P(\text{not green}) = 1 - P(\text{green}) = 1 - 0.10 = 0.90$

4. $P(\text{striped}) = 0$

39. (continued)

b) Since the events are independent (picking out one M&M doesn't affect the outcome of the next pick), the multiplication rule may be used.

1. $P(\text{all three are brown}) = (0.30)(0.30)(0.30) = 0.027$

2. $P(\text{the third one is the first one that is red}) = P(\text{not red and not red and red})$
$= (0.80)(0.80)(0.20) = 0.128$

3. $P(\text{no yellow}) = P(\text{not yellow and not yellow and not yellow}) = (0.80)(0.80)(0.80) = 0.512$

4. $P(\text{at least one is green}) = 1 - P(\text{none are green}) = 1 - (0.90)(0.90)(0.90) = 0.271$

40. Blood.

a) Since all of the events are disjoint (a person cannot have more than one blood type!), use the addition rule where applicable.

1. $P(\text{Type AB}) = 1 - P(\text{not Type AB}) = 1 - P(\text{Type O or Type A or Type B})$
$= 1 - (0.45 + 0.40 + 0.11) = 0.04$

2. $P(\text{Type A or Type B}) = 0.40 + 0.11 = 0.51$

3. $P(\text{not Type O}) = 1 - P(\text{Type O}) = 1 - 0.45 = 0.55$

b) Since the events are independent (one person's blood type doesn't affect the blood type of the next), the multiplication rule may be used.

1. $P(\text{all four are Type O}) = (0.45)(0.45)(0.45)(0.45) \approx 0.041$

2. $P(\text{no one is Type AB}) = P(\text{not AB and not AB and not AB and not AB})$
$= (0.96)(0.96)(0.96)(0.96) \approx 0.849$

3. $P(\text{not all Type A}) = 1 - P(\text{all Type A}) = 1 - (0.40)(0.40)(0.40)(0.40) \approx 0.974$

4. $P(\text{at least one person is Type B}) = 1 - P(\text{no one is Type B}) = 1 - (0.89)(0.89)(0.89)(0.89) \approx 0.373$

41. Disjoint or independent?

a) For one draw, the events of getting a red M&M and getting an orange M&M are disjoint events. Your single draw cannot be both red and orange.

b) For two draws, the events of getting a red M&M on the first draw and a red M&M on the second draw are independent events. Knowing that the first draw is red does not influence the probability of getting a red M&M on the second draw.

c) Disjoint events can never be independent. Once you know that one of a pair of disjoint events has occurred, the other one cannot occur, so its probability has become zero. For example, consider drawing one M&M. If it is red, it cannot possible be orange. Knowing that the M&M is red influences the probability that the M&M is orange. It's zero. The events are not independent.

42. Disjoint or independent?

a) For one person, the events of having Type A blood and having Type B blood are disjoint events. One person cannot have both Type A and Type B blood.

b) For two people, the events of the first having Type A blood and the second having Type B blood are independent events. Knowing that the first person has Type A blood does not influence the probability of the second person having Type B blood.

c) Disjoint events can never be independent. Once you know that one of a pair of disjoint events has occurred, the other one cannot occur, so its probability has become zero. For example, consider selecting one person, and checking his or her blood type. If the person's blood type is Type A, it cannot possibly be Type B. Knowing that the person's blood type is Type A influences the probability that the person's blood type is Type B. It is zero. The events are not independent.

43. Dice.

a) $P(6) = \frac{1}{6}$, so $P(\text{all 6's}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) \approx 0.0046$

b) $P(\text{odd}) = P(1 \text{ or } 3 \text{ or } 5) = \frac{3}{6}$, so $P(\text{all odd}) = \left(\frac{3}{6}\right)\left(\frac{3}{6}\right)\left(\frac{3}{6}\right) = 0.125$

c) $P(\text{not divisible by } 3) = P(1 \text{ or } 2 \text{ or } 4 \text{ or } 5) = \frac{4}{6}$, so $P(\text{none divisible by } 3) = \left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\left(\frac{4}{6}\right) \approx 0.296$

d) $P(\text{at least one } 5) = 1 - P(\text{no 5's}) = 1 - \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) \approx 0.421$

e) $P(\text{not all 5's}) = 1 - P(\text{all 5's}) = 1 - \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) \approx 0.995$

44. Slot Machine.

Each wheel runs independently of the others, so the multiplication rule may be used.

a) $P(\text{lemon on 1 wheel}) = 0.30$, so $P(3 \text{ lemons}) = (0.30)(0.30)(0.30) = 0.027$

b) $P(\text{bar or bell on 1 wheel}) = 0.50$, so $P(\text{no fruit symbols}) = (0.50)(0.50)(0.50) = 0.125$

c) $P(\text{bell on 1 wheel}) = 0.10$, so $P(3 \text{ bells}) = (0.10)(0.10)(0.10) = 0.001$

d) $P(\text{no bell on 1 wheel}) = 0.90$, so $P(\text{no bells on 3 wheels}) = (0.90)(0.90)(0.90) = 0.729$

e) $P(\text{no bar on 1 wheel}) = 0.60$, so $P(\text{at least one bar on 3 wheels}) = 1 - P(\text{no bars}) = 1 - (0.60)(0.60)(0.60)$
$= 0.784$

45. Champion bowler.

Assuming each frame is independent of others, the multiplication rule may be used.

a) $P(\text{no strikes in 3 frames}) = (0.30)(0.30)(0.30) = 0.027$

b) $P(\text{makes first strike in the third frame}) = (0.30)(0.30)(0.70) = 0.063$

c) $P(\text{at least one strike in first three frames}) = 1 - P(\text{no strikes}) = 1 - (0.30)^3 = 0.973$

d) $P(\text{perfect game}) = (0.70)^{12} \approx 0.014$

46. The train.

Assuming the arrival time is independent from one day to the next, the multiplication rule may be used.

a) $P(\text{gets stopped on Monday and gets stopped on Tuesday}) = (0.15)(0.15) = 0.0225$

b) $P(\text{gets stopped for the first time on Thursday}) = (0.85)(0.85)(0.85)(0.15) \approx 0.092$

c) $P(\text{gets stopped every day}) = (0.15)^5 \approx 0.00008$

d) $P(\text{gets stopped at least once}) = 1 - P(\text{never gets stopped}) = 1 - (0.85)^5 \Rightarrow 0.556$

47. Voters.

Since you are calling at random, one person's political affiliation is independent of another's. The multiplication rule may be used.

a) $P(\text{all Republicans}) = (0.29)(0.29)(0.29) \approx 0.024$

b) $P(\text{no Democrats}) = (1-0.37)(1-0.37)(1-0.37) \approx 0.25$

c) $P(\text{at least one Independent}) = 1 - P(\text{no Independents}) = 1 - (0.77)(0.77)(0.77) \approx 0.543$

48. Religion.

Since you are calling at random, one person's religion is independent of another's. The multiplication rule may be used.

a) $P(\text{all Christian}) = (0.62)(0.62)(0.62)(0.62) \approx 0.148$

b) $P(\text{no Jews}) = (1-0.12)(1-0.12)(1-0.12)(1-0.12) \approx 0.600$

c)
$$\begin{aligned} P(\text{at least one person who is nonreligious}) &= 1 - P(\text{no nonreligious people}) \\ &= 1 - (0.90)(0.90)(0.90)(0.90) \approx 0.344 \end{aligned}$$

49. Lights.

Assume that the defective light bulbs are distributed randomly to all stores so that the events can be considered independent. The multiplication rule may be used.

$$\begin{aligned} P(\text{at least one of five bulbs is defective}) &= 1 - P(\text{none are defective}) \\ &= 1 - (0.94)(0.94)(0.94)(0.94)(0.94) \approx 0.266 \end{aligned}$$

50. Pepsi.

Assume that the winning caps are distributed randomly, so that the events can be considered independent. The multiplication rule may be used.

$P(\text{you win something}) = 1 - P(\text{you win nothing}) = 1 - (0.90)^6 \approx 0.469$

51. 9/11?

a) For any date with a valid three-digit date, the chance is 0.001, or 1 in 1000. For many dates in October through December, the probability is 0. For example, there is no way three digits will make 1015, to match October 15.

b) There are 65 days when the chance to match is 0. (October 10 through October 31, November 10 through November 30, and December 10 through December 31.) That leaves 300 days in a year (that is not a leap year) in which a match might occur, so $P(\text{no matches in 300 days}) = (0.999)^{300} \approx 0.741$.

c) $P(\text{at least one match in a year}) = 1 - P(\text{no matches in a year}) = 1 - 0.741 \approx 0.259$

d) $P(\text{at least one match on 9/11 in one of the 50 states}) = 1 - P(\text{no matches in 50 states})$

$$= 1 - (0.999)^{50} \approx 0.049$$

52. Red cards.

a) Your thinking is correct. There are 42 cards left in the deck, 26 black and only 16 red.

b) This is not an example of the Law of Large Numbers. There is no "long run." You'll see the entire deck after 52 cards, and you know there will be 26 of each color then.