Chapter 5 – The Standard Deviation as a Ruler and the Normal Model

Section 5.1

1. Stats test.

Gregor scored 65 points on the test: $z = \frac{y-\mu}{\sigma} \Rightarrow -2 = \frac{y-75}{5} \Rightarrow y = 65, \text{ or } 75 - 2(5) = 65.$

2. Mensa.

According to this scale, persons with an IQ of 137.5 or higher are considered geniuses:
$z = \frac{y-\mu}{\sigma} \Rightarrow 2.5 = \frac{y-100}{15} \Rightarrow y = 137.5, \text{ or } 100 + 2.5(15) = 137.5.$

3. Temperatures.

In January, with mean temperature 36° and standard deviation in temperature 10°, a high temperature of 55° is almost 2 standard deviations above the mean. In July, with mean temperature 74° and standard deviation 8°, a high temperature of 55° is more than two standard deviations below the mean. A high temperature of 55° is less likely to happen in July, when 55° is farther away from the mean.

4. Placement Exams.

On the French exam, the mean was 72 and the standard deviation was 8. The student's score of 82 was 10 points, or 1.25 standard deviations, above the mean. On the math exam, the mean was 68 and the standard deviation was 12. The student's score of 86 was 18 points or 1.5 standard deviations above the mean. The student did "better" on the math exam.

Section 5.2

5. Shipments.

a) Adding 4 ounces will affect only the median. The new median will be 68 + 4 = 72 ounces, and the IQR will remain at 40 ounces.

b) Changing the units will affect both the median and IQR. The median will be 72/16 = 4.5 pounds and the IQR will be 40/16 = 2.5 pounds.

6. Hotline.

a) Changing the units will affect both the median and IQR. The median will be 4.4(60) = 264 seconds and the IQR will be 2.3(60) = 138 seconds.

b) Subtracting 24 seconds will affect only the median. The new median will be 264 – 24 = 240 seconds and the new IQR will remain 138 seconds.

7. Men's shoe sizes.

a) The mean US shoe size will be affected by both the multiplication and subtraction. $\text{USsize} = \text{EuroSize} \times 0.7865 - 24 = 44.65 \times 0.7865 - 24 \approx 11.12$. The average US shoe size for the respondents is about 11.12.

b) The standard deviation of US shoe sizes will only be affected by the multiplication. Adding or subtracting a constant from each value doesn't affect the spread. $\sigma_{US} = \sigma_{Euro} \times 0.7865 = 2.03 \times 0.7865 \approx 1.597.$ The standard deviation of the respondents' shoe sizes, in US units, is about 1.597.

8. Women's shoe sizes.

a) The mean US shoe size will be affected by both the multiplication and subtraction. $\text{USsize} = \text{EuroSize} \times 0.7865 - 24 = 38.46 \times 0.7865 - 22.5 \approx 7.75$. The average US shoe size for the respondents is about 11.12.

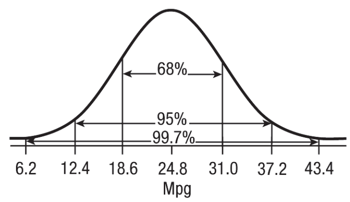
8. (continued)

b) The standard deviation of US shoe sizes will only be affected by the multiplication. Adding or subtracting a constant from each value doesn't affect the spread. $\sigma_{US} = \sigma_{Euro} \times 0.7865 = 1.84 \times 0.7865 \approx 1.447$. The standard deviation of the respondents' shoe sizes, in US units, is about 1.447.

Section 5.3

9. Guzzlers?

a) The Normal model for auto fuel economy is shown below.



b) Approximately 68% of the cars are expected to have highway fuel economy between 18.6 mpg and 31.0 mpg.

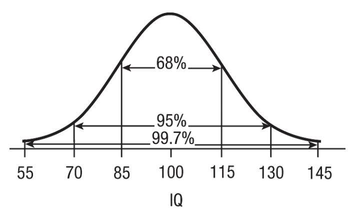
c) Approximately 16% of the cars are expected to have highway fuel economy above 31 mpg.

d) Approximately 13.5% of the cars are expected to have highway fuel economy between 31 mpg and 37 mpg.

e) The worst 2.5% of cars are expected to have fuel economy below approximately 12.4 mpg.

10. IQ.

a) The Normal model for IQ scores is shown below.



b) Approximately 95% of the IQ scores are expected to be within the interval 70 to 130 IQ points.

c) Approximately 16% of IQ scores are expected to be above 115 IQ points.

d) Approximately 13.5% of IQ scores are expected to be between 70 and 85 IQ points.

e) Approximately 2.5% of the IQ scores are expected to be above 130.

11. Checkup.

The boy's height is 1.88 standard deviations below the mean height of American children his age.

12. Stats test.

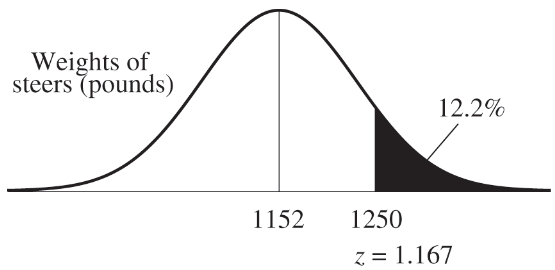
a) Your score was 2.2 standard deviations higher than the mean score in the class.

b) According to the Normal model, approximately 16% of scores are expected to be lower than a score that is 1 standard deviation below the mean.

Section 5.4

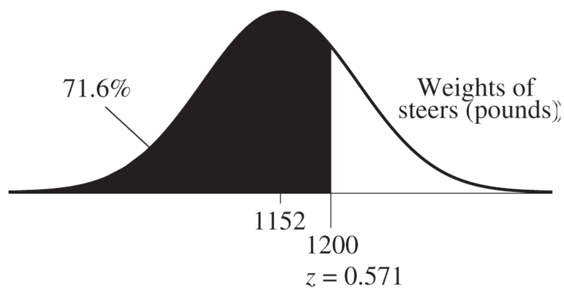
13. Normal cattle.

a) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{1250-1152}{84} \approx 1.167$



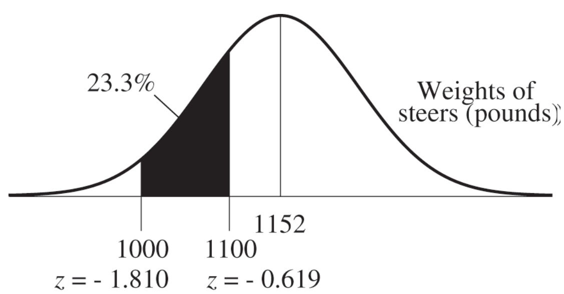
According to the Normal model, we expect 12.2% of steers to weigh over 1250 pounds.

b) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{1200-1152}{84} \approx 0.571$



According to the Normal model, 71.6% of steers are expected to weigh under 1200 pounds.

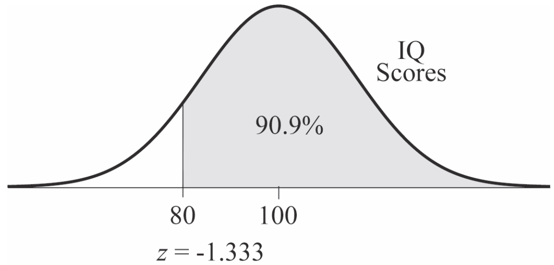
c) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{1000-1152}{84} \approx -1.810$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{1100-1152}{84} \approx -0.619$



According to the Normal model, 23.3% of steers are expected to weigh between 1000 and 1100 pounds.

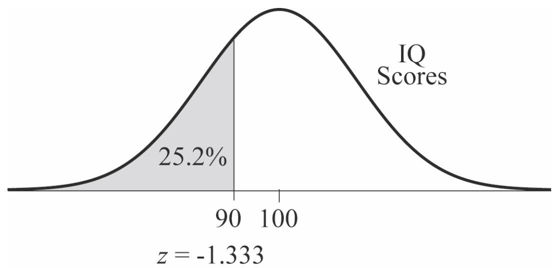
14. IQs revisited.

a) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{80-100}{15} = -1.333$



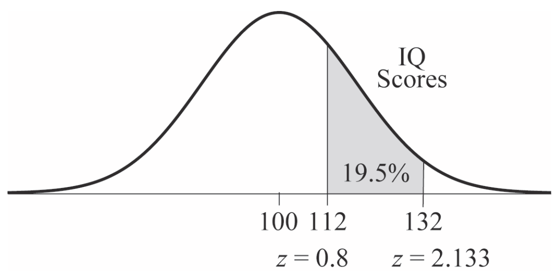
According to the Normal model, 90.9% of IQ scores are expected to be over 80.

b) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{90-100}{15} = -0.667$



According to the Normal model, 25.2% of IQ scores are expected to be under 90.

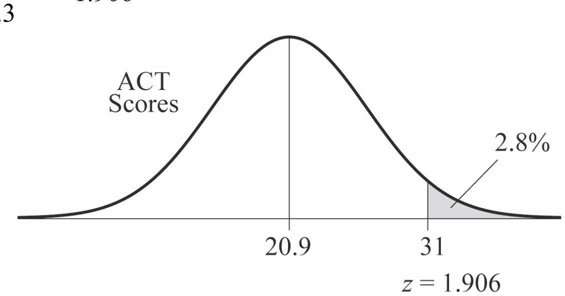
c) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{112-100}{15} = 0.8$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{132-100}{15} = 2.133$



According to the Normal model, about 19.5% of IQ scores are between 112 and 132.

15. ACT scores.

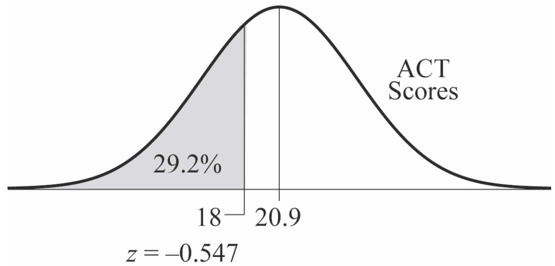
a) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{31-20.9}{5.3} = 1.906$



According to the Normal model, 2.8% of ACT scores are expected to be over 31.

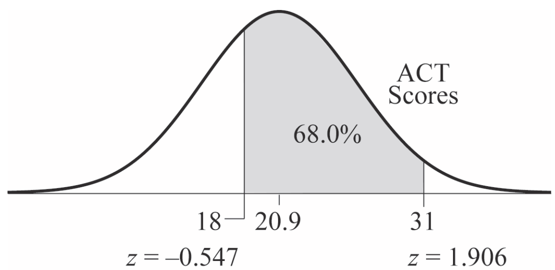
15. (continued)

b) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{18-20.9}{5.3} = -0.547$



According to the Normal model, 29.2% of ACT scores are expected to be under 18.

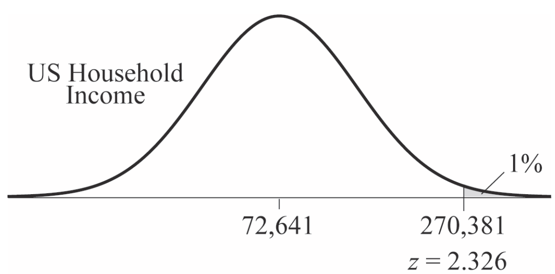
c) $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{18-20.9}{5.3} = -0.547$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{31-20.9}{5.3} = 1.906$



According to the Normal model, about 68.0% of ACT scores are between 18 and 31.

16. Incomes.

a) $z = \dfrac{y-\mu}{\sigma} \Rightarrow 2.32635 = \dfrac{y-72{,}641}{85{,}000} \Rightarrow y = 270{,}381$



According to the Normal model, the household income of the top 1% would be approximately $270,381.

b) We don't have very much faith in this estimate. The top 1% of incomes in 2014 were actually above $388,000.

c) The mean is much greater than the median, implying that the distribution of incomes is heavily skewed to the right and/or has high outliers. Additionally, even one standard deviation below the mean is a negative income. We shouldn't have used the Normal model to make an estimate about household income in the US.

Section 5.5

17. Music library.

a) The Normal probability plot is not straight, so there is evidence that the distribution of the lengths of songs in Corey's music library is not Normal.

17. (continued)

b) The distribution of the lengths of songs in Corey's music library appears to be skewed to the right. The Normal probability plot show that the longer songs in Corey's library are much longer than the lengths predicted by the Normal model. The song lengths are much longer than their quantile scores would predict for a Normal model.

18. Wisconsin ACT math 2015.

a) The distribution of mean ACT math scores is bimodal, so it is not approximately Normal. Also, 80.1% of scores are within one standard deviation of the mean. If a Normal model was useful, we would need approximately 68% of the scores within one standard deviation of the mean.

b) The Normal probability plot for the distribution of mean ACT scores of all schools is not straight, so the Normal model is not appropriate. With Milwaukee area schools removed, the distribution of mean ACT math scores is slightly skewed, but the Normal probability plot is reasonably straight, so the Normal model is appropriate.

Chapter Exercises.

19. Payroll.

a) The distribution of salaries in the company's weekly payroll is skewed to the right. The mean salary, \$700, is higher than the median, \$500.

b) The IQR, \$600, measures the spread of the middle 50% of the distribution of salaries. $Q3 - Q1 = IQR$
$\Rightarrow Q3 = Q1 + IQR = \$350 + \$600 = \$950,$ so 50% of the salaries are found between \$350 and \$950.

c) If a \$50 raise were given to each employee, all measures of center or position would increase by \$50. The minimum would change to \$350, the mean would change to \$750, the median would change to \$550, and the first quartile would change to \$400. Measures of spread would not change. The entire distribution is simply shifted up \$50. The range would remain at \$1200, the IQR would remain at \$600, and the standard deviation would remain at \$400.

d) If a 10% raise were given to each employee, all measures of center, position, and spread would increase by 10%. The minimum would change to \$330, the mean would change to \$770, the median would change to \$550, the first quartile would change to \$385, the IQR would change to \$660, the range would change to \$1320, and the standard deviation would change to \$440.

20. Hams.

a) Range = Max – Min = 7.45 – 4.15 = 3.30 pounds; IQR = Q3 – Q1 = 6.55 – 5.6 = 0.95 pounds

b) The distribution of weights of hams is slightly skewed to the left because the mean is lower than the median and the first quartile is farther from the median than the third quartile.

c) All of the statistics are multiplied by 16 in the conversion from pounds to ounces.

Mean = 96 oz.	St. Dev. = 10.4 oz.	First Quartile = 89.6 oz.
Third Quartile = 104.8 oz.	Median = 99.2 oz.	IQR = 15.2 oz.
Range = 52.8 oz.		

d) Measures of position increase by 30 ounces. Measures of spread remain the same.

Mean = 126 oz.	St. Dev. = 10.4 oz.	First Quartile = 119.6 oz.
Third Quartile = 134.8 oz.	Median = 129.2 oz.	IQR = 15.2 oz.
Range = 52.8 oz.		

e) If a 10-pound ham were added to the distribution, the mean would change, since the total weight of all the hams would increase. The standard deviation would also increase, since 10 pounds is far away from the mean. The overall spread of the distribution would increase. The range would increase, since 10 pounds would be the new maximum. The median, quartiles, and IQR may not change. These measures are summaries of the middle 50% of the distribution, and are resistant to the presence of outliers, like the 10-pound ham.

21. SAT or ACT?

Measures of center and position (lowest score, top 25% above, mean, and median) will be multiplied by 40 and increased by 150 in the conversion from ACT to SAT by the rule of thumb. Measures of spread (standard deviation and IQR) will only be affected by the multiplication.

Lowest score = 910	Mean = 1230	Standard deviation = 120
Third Quartile = 1350	Median = 1270	IQR = 240

22. Cold U?

Measures of center and position (maximum, median, and mean) will be multiplied by $9/5$ and increased by 32 in the conversion from Fahrenheit to Celsius. Measures of spread (range, standard deviation, IQR) will only be affected by the multiplication.

Maximum temperature = 51.8°F	Range = 59.4°F
Mean = 33.8°F	Standard deviation = 12.6°F
Median = 35.6°F	IQR = 28.8°F

23. Music library again.

On the Nickel, by Tom Waits has a *z*-score of 1.202: $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{380-242.4}{114.51} = 1.202.$

24. Windy.

a) February: $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{6.73-2.324}{1.577} = 2.79$

June: $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{3.93-0.857}{0.795} = 3.865$

August: $z = \dfrac{y-\mu}{\sigma} \Rightarrow z = \dfrac{2.53-0.63}{0.597} = 3.18$

b) The June event was the most extraordinary since it has the largest *z*-score.

25. Combining test scores.

The *z*-scores, which account for the difference in the distributions of the two tests, are 1.5 and 0 for Derrick and 0.5 and 2 for Julie. Derrick's total is 1.5 which is less than Julie's 2.5.

26. Combining scores again.

The *z*-scores, which account for the difference in the distributions of the two tests, are 0 and 1 for Reginald, for a total of 1. For Sara, they are 2.0 and –0.33 for a total of 1.67. While her raw score is lower, her *z*-score is higher.

27. Final Exams.

a) Anna's average is $\dfrac{83+83}{2} = 83$ and Megan's average is $\dfrac{77+95}{2} = 86.$

Only Megan qualifies for language honors, with an average higher than 85.

b) On the French exam, the mean was 81 and the standard deviation was 5. Anna's score of 83 was 2 points, or 0.4 standard deviations, above the mean. Megan's score of 77 was 4 points, or 0.8 standard deviations below the mean.

On the Spanish exam, the mean was 74 and the standard deviation was 15. Anna's score of 83 was 9 points, or 0.6 standard deviations, above the mean. Megan's score of 95 was 21 points, or 1.4 standard deviations, above the mean.

Measuring their performance in standard deviations is the only fair way in which to compare the performance of the two women on the test.

27. (continued)

Anna scored 0.4 standard deviations above the mean in French and 0.6 standard deviations above the mean in Spanish, for a total of 1.0 standard deviation above the mean.

Megan scored 0.8 standard deviations below the mean in French and 1.4 standard deviations above the mean in Spanish, for a total of only 0.6 standard deviations above the mean.

Anna did better overall, but Megan had the higher average. This is because Megan did very well on the test with the higher standard deviation, where it was comparatively easy to do well.

28. MP3s.

a) Standard deviation measures variability, which translates to consistency in everyday use. Batteries with a small standard deviation would be more likely to have lifespans close to their mean lifespan than batteries with a larger standard deviation.

b) RockReady batteries have a higher mean lifespan and smaller standard deviation, so they are the better battery. 8 hours is 2.67 standard deviations below the mean lifespan of RockReady and 1.5 standard deviations below the mean lifespan of DuraTunes. DuraTunes batteries are more likely to *fail* before the 8 hours have passed.

c) 16 hours is 2.5 standard deviations higher than the mean lifespan of DuraTunes, and 2.67 standard deviations higher than the mean lifespan of RockReady. Neither battery has a good chance of lasting 16 hours, but DuraTunes batteries have a greater chance than RockReady batteries.

29. Cattle.

a) Since $z = \dfrac{y-\mu}{\sigma} = \dfrac{1000-1152}{84} \approx -1.81,$ a steer weighing 1000 pounds would be about 1.81 standard deviations below the mean weight.

b) A steer weighing 1000 pounds is more unusual. Its z-score of –1.81 is further from 0 than the 1250 pound steer’s z-score of $z = \dfrac{1250-1152}{84} \approx 1.17.$

30. Car speeds 100.

a) Since $z = \dfrac{y-\mu}{\sigma} = \dfrac{20-23.84}{3.56} \approx -1.08,$ a car going the speed limit of 20 mph would be about 1.08 standard deviations below the mean speed.

b) A car going 10 mph would be more unusual. Its z-score of –3.89 is further from 0 than the 34 mph car’s z-score of $\dfrac{34-23.84}{3.56} \approx 2.85.$

31. More cattle.

a) The new mean would be 1152 – 1000 = 152 pounds. The standard deviation would not be affected by subtracting 1000 pounds from each weight. It would still be 84 pounds.

b) The mean selling price of the cattle would be 0.40(1152) = \$460.80. The standard deviation of the selling prices would be 0.40(84) = \$33.60.

32. Car speeds 100 again.

a) The new mean would be 23.84 – 20 = 3.84 mph over the speed limit. The standard deviation would not be affected by subtracting 20 mph from each speed. It would still be 3.56 miles per hour.

b) The mean speed would be 23.84(1.609) = 38.359 kph. The speed limit would convert to 1.609(20) = 32.18 kph. The standard deviation would be 1.609(3.56) = 5.728 kph.

33. Cattle, part III.

Generally, the minimum and the median would be affected by the multiplication and subtraction. The standard deviation and the IQR would only be affected by the multiplication.

Minimum = 0.40(980) – 20 = \$372.00
Median = 0.40(1140) – 20 = \$436
Standard deviation = 0.40(84) = \$33.60
IQR = 0.40(102) = \$40.80

34. Caught speeding.

Generally, the mean and the maximum would be affected by the multiplication and addition. The standard deviation and the IQR would only be affected by the multiplication.

Mean = 100 + 10(28 – 20) = \$180
Maximum = 100 +10(33 – 20) = \$230
Standard deviation = 10(2.4) = \$24
IQR = 10(3.2) = \$32

35. Professors.

The standard deviation of the distribution of years of teaching experience for college professors must be 6 years. College professors can have between 0 and 40 (or possibly 50) years of experience. A workable standard deviation would cover most of that range of values with ± 3 standard deviations around the mean. If the standard deviation were 6 months $\left(\frac{1}{2}\text{ year}\right)$, , some professors would have years of experience 10 or 20 standard deviations away from the mean, whatever it is. That isn't possible. If the standard deviation were 16 years, ± 2 standard deviations would be a range of 64 years. That's way too high. The only reasonable choice is a standard deviation of 6 years in the distribution of years of experience.

36. Rock concerts.

The standard deviation of the distribution of the number of fans at the rock concerts would most likely be 2000. A standard deviation of 200 fans seems much too consistent. With this standard deviation, the band would be very unlikely to draw more than 1000 fans (5 standard deviations!) above or below the mean of 21,359 fans. It seems like rock concert attendance could vary by much more than that. If a standard deviation of 200 fans is too small, then so is a standard deviation of 20 fans. 20,000 fans is too large for a likely standard deviation in attendance, unless they played several huge venues. Zero attendance is only a bit more than 1 standard deviation below the mean, although it seems very unlikely. 2000 fans is the most reasonable standard deviation in the distribution of number of fans at the concerts.

37. Small steer.

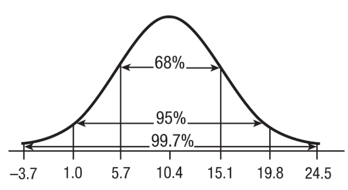
Any weight more than 2 standard deviations below the mean, or less than 1152 – 2(84) = 984 pounds might be considered unusually low. We would expect to see a steer below 1152 – 3(84) = 900 very rarely.

38. High IQ.

Any IQ more than 2 standard deviations above the mean, or more than 100 + 2 (15) = 130 might be considered unusually high. We would expect to find someone with an IQ over 100 + 3(15) = 145 very rarely.

39. Trees.

a) The Normal model for the distribution of tree diameters is at the right.



b) Approximately 95% of the trees are expected to have diameters between 1.0 inch and 19.8 inches.

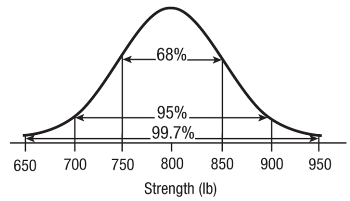
c) Approximately 2.5% of the trees are expected to have diameters less than an inch.

d) Approximately 34% of the trees are expected to have diameters between 5.7 inches and 10.4 inches.

e) Approximately 16% of the trees are expected to have diameters over 15 inches.

40. Rivets.

a) The Normal model for the distribution of shear strength of rivets is at the right.



b) 750 pounds is 1 standard deviation below the mean, meaning that the Normal model predicts that approximately 16% of the rivets are expected to have a shear strength of less than 750 pounds. These rivets are a poor choice for a situation that requires a shear strength of 750 pounds, because 16% of the rivets would be expected to fail. That's too high a percentage.

c) Approximately 97.5% of the rivets are expected to have shear strengths below 900 pounds.

d) In order to make the probability of failure very small, these rivets should only be used for applications that require shear strength several standard deviations below the mean, probably farther than 3 standard deviations. (The chance of failure for a required shear strength 3 standard deviations below the mean is still approximately 3 in 2000.) For example, if the required shear strength is 500 pounds (6 standard deviations below the mean), the chance of one of these bolts failing is approximately 1 in 1,000,000.

41. Trees, part II.

The use of the Normal model requires a distribution that is unimodal and symmetric. The distribution of tree diameters is neither unimodal nor symmetric, so use of the Normal model is not appropriate.

42. Car speeds 100, the picture.

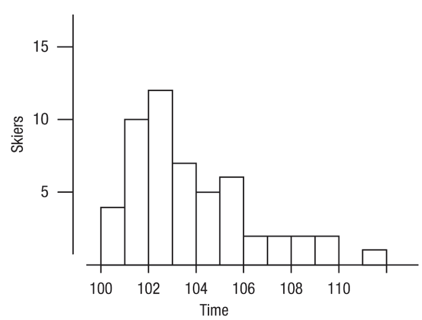
The distribution of cars speeds shown in the histogram is unimodal and roughly symmetric, and the normal probability plot looks quite straight, so a Normal model is appropriate.

43. Winter Olympics 2018.

a) The 2018 Winter Olympics downhill times have mean of 103.833 seconds and standard deviation 2.645 seconds. 101.233 seconds is approximately 1 standard deviation below the mean. If the Normal model is appropriate, 16% of the times should be below 101.233 seconds.

b) 7 out of 53 times (13.2%) are below 101.233 seconds.

c) The percentages in parts (a) and (b) do not agree because the Normal model is not appropriate in this situation.



d) The histogram of 2018 Winter Olympic Downhill times is skewed to the right, with a cluster of high times as well. The Normal model is not appropriate for the distribution of times, because the distribution is not symmetric.

44. Check the model.

a) We know that 95% of the observations from a Normal model fall within 2 standard deviations of the mean. That corresponds to 23.84 – 2(3.56) = 16.72 mph and 23.84 + 2(3.56) = 30.96 mph. These are the 2.5 percentile and 97.5 percentile, respectively. According to the Normal model, we expect only 2.5% of the speeds to be below 16.72 mph, and 97.5% of the speeds to be below 30.96 mph.

b) The actual 2.5 percentile and 97.5 percentile are 16.638 and 30.976 mph, respectively. These are very close to the predicted values from the Normal model. The histogram from Exercise 20 is unimodal and roughly symmetric. It is very slightly skewed to the right and there is one outlier, but the Normal probability plot is quite straight. We should not be surprised that the approximation from the Normal model is a good one.

45. Receivers 2015.

a) Approximately 2.5% of the receivers are expected to gain more yards than 2 standard deviations above the mean number of yards gained.

b) The distribution of the number of yards gained has mean 274.73 yards and standard deviation 327.32 yards. According to the Normal model, we expect 2.5% of the receivers, or about 12 of them, to gain more than 2 standard deviations above the mean number of yards. This means more than 274.73 + 2(327.32) = 929.37 yards. In 2015, 31 receivers ran for more than 1122 yards.

c) The distribution of the number of yards run by wide receivers is skewed heavily to the right. Use of the Normal model is not appropriate for this distribution, since it is not symmetric.

46. Customer database.

a) The median of 93% is the better measure of center for the distribution of the percentage of white residents in the neighborhoods, since the distribution is skewed to the left. The median is a better summary for skewed distributions since the median is resistant to effects of the skewness, while the mean is pulled toward the tail.

b) The IQR of 17% is the better measure of spread for the distribution of the percentage of white residents in the neighborhoods, since the distribution is skewed to the left. IQR is a better summary for skewed distributions since the IQR is resistant to effects of the skewness, and the standard deviation is not.

c) According to the Normal model, approximately 68% of neighborhoods are expected to have a percentage of whites within 1 standard deviation of the mean.

d) The mean percentage of whites in a neighborhood is 83.59%, and the standard deviation is 22.26%. 83.59% $\pm$ 22.26% = 61.33% to 105.85%. Estimating from the graph, more than 75% of the neighborhoods have a percentage of whites greater than 61.33%.

e) The distribution of the percentage of whites in the neighborhoods is strongly skewed to the left. The Normal model is not appropriate for this distribution. There is a discrepancy between part (c) and part (d) because part (c) is wrong!

47. CEO compensation 2014 sampled.

a) According to the Normal model, about 2.5% of CEOs would be expected to earn more than 2 standard deviation above the mean compensation.

b) The Normal model is not appropriate, since the distribution of CEO compensation is skewed to the right, not symmetric.

c) The Normal model is not appropriate, since the distribution of the sample means taken from samples of 30 CEO compensations is still skewed to the right, not symmetric.

d) The distribution of the sample means taken from samples of 100 CEO compensations is more symmetric, so the 68-95-99.7 Rule should work reasonably well.

e) The distribution of the sample means taken from samples of 200 CEO compensations is nearly Normal, so the 68-95-99.7 Rule should work quite well.

48. CEO compensation logged and sampled.

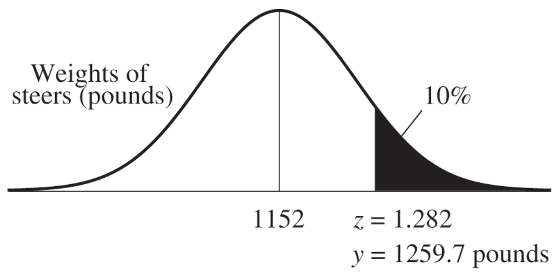
a) According to the Normal model, about 2.5% of CEOs would be expected to earn more than 2 standard deviation above the mean compensation.

b) The distribution of log compensation is unimodal and symmetric, so the Normal model is appropriate.

c) The distribution of the sample means take from samples of 30 log compensations is unimodal and symmetric, so the 68-95-99.7 Rule applies.

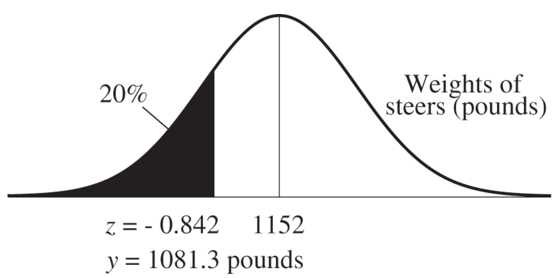
49. More cattle.

a) $z = \dfrac{y-\mu}{\sigma} \Rightarrow 1.282 = \dfrac{y-1152}{84} \Rightarrow y \approx 1259.7$



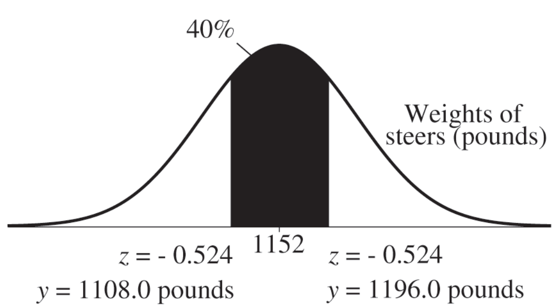
According to the Normal model, the highest 10% of steer weights are expected to be above approximately 1259.7 pounds.

b) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -0.842 = \dfrac{y-1152}{84} \Rightarrow y \approx 1081.3$



According to the Normal model, the lowest 20% of weights of steers are expected to be below approximately 1081.3 pounds.

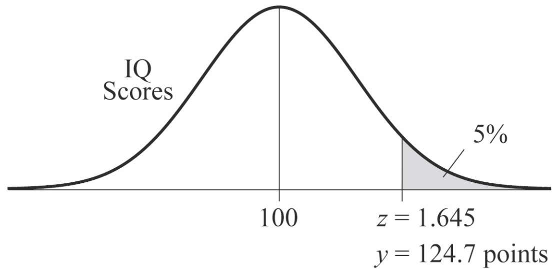
c) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -0.524 = \dfrac{y-1152}{84} \Rightarrow y \approx 1108.0$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow 0.524 = \dfrac{y-1152}{84} \Rightarrow y \approx 1196.0$



According to the Normal model, the middle 40% of steer weights is expected to be between about 1108 pounds and 1196 pounds.

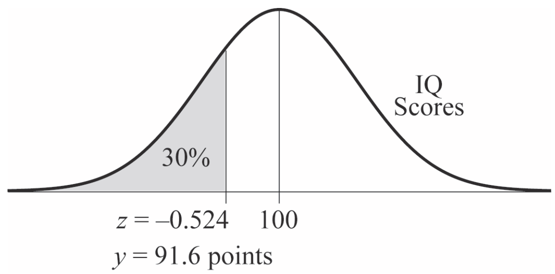
50. More IQs.

a) $z = \dfrac{y-\mu}{\sigma} \Rightarrow 1.645 = \dfrac{y-100}{15} \Rightarrow y \approx 124.7$



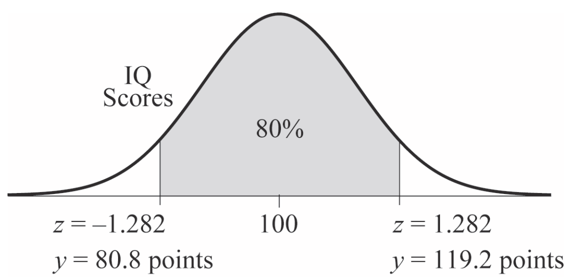
According to the Normal model, the highest 5% of IQ scores are above about 124.7 points.

b) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -0.524 = \dfrac{y-100}{15} \Rightarrow y \approx 92.1$



According to the Normal model, the lowest 30% of IQ scores are expected to be below about 92.1 points.

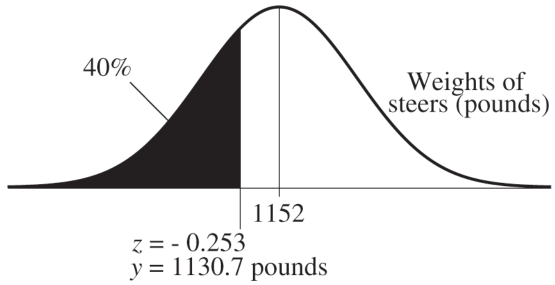
c) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -1.282 = \dfrac{y-100}{15} \Rightarrow y \approx 80.8$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow 1.282 = \dfrac{y-100}{15} \Rightarrow y \approx 119.2$



According to the Normal model, the middle 80% of IQ scores is expected to be between 80.8 points and 119.2 points.

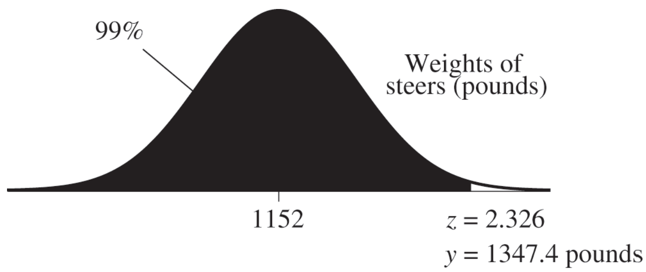
51. Cattle, finis.

a) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -0.253 = \dfrac{y-1152}{84} \Rightarrow y \approx 1130.7$



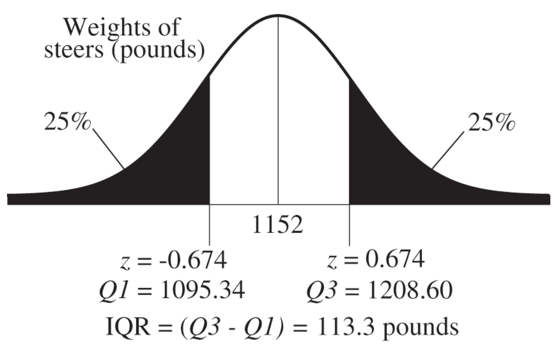
According to the Normal model, the weight at the 40th percentile is 1130.7 pounds. This means that 40% of steers are expected to weigh less than 1130.7 pounds.

b) $z = \dfrac{y-\mu}{\sigma} \Rightarrow 2.326 = \dfrac{y-1152}{84} \Rightarrow y \approx 1347.4$



According to the Normal model, the weight at the 99th percentile is 1347.4 pounds. This means that 99% of steers are expected to weigh less than 1347.4 pounds.

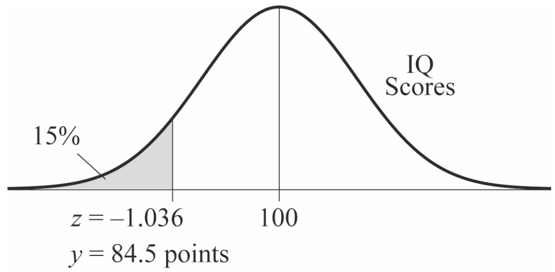
c) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -0.674 = \dfrac{\text{Q1}-1152}{84} \Rightarrow \text{Q1} \approx 1095.34$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow 0.674 = \dfrac{\text{Q3}-1152}{84} \Rightarrow \text{Q3} \approx 1208.60$



According to the Normal model, the IQR of the distribution of weights of Angus steers is about 113.3 pounds.

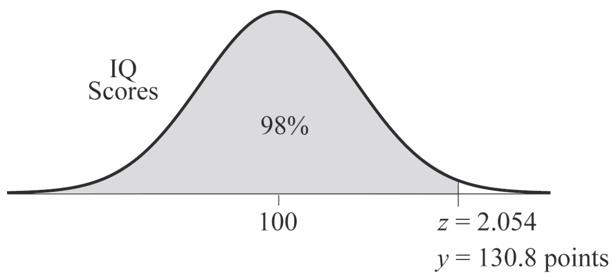
52. IQ, finis.

a) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -1.036 = \dfrac{y-100}{15} \Rightarrow y \approx 84.5$



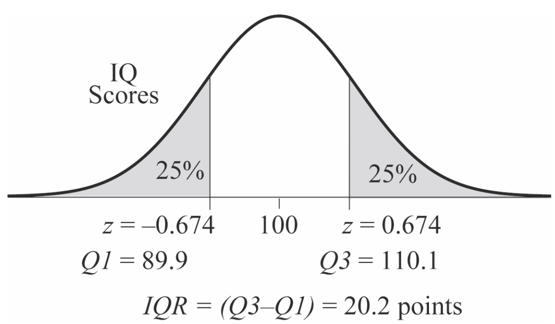
According to the Normal model, the 15th percentile of IQ scores is about 84.5 points. This means that we expect 15% of IQ scores to be lower than 84.5 points.

b) $z = \dfrac{y-\mu}{\sigma} \Rightarrow 2.054 = \dfrac{y-100}{15} \Rightarrow y \approx 130.8$



According to the Normal model, the 98th percentile of IQ scores is about 130.8 points. This means that we expect 98% of IQ scores to be lower than 130.8 points.

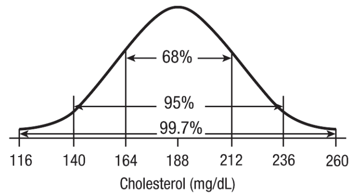
c) $z = \dfrac{y-\mu}{\sigma} \Rightarrow 0.674 = \dfrac{Q3-100}{15} \Rightarrow Q3 \approx 110.1$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow -0.674 = \dfrac{Q1-100}{15} \Rightarrow Q1 \approx 89.9$



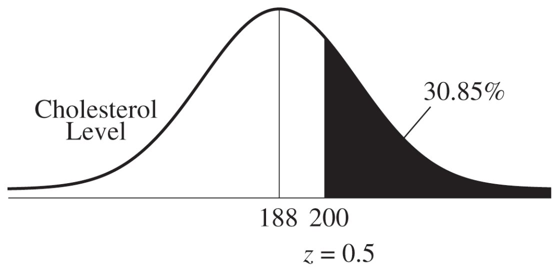
According to the Normal model, the IQR of the distribution of IQ scores is 20.2 points.

53. Cholesterol.

a) The Normal model for cholesterol levels of adult American women is shown below.

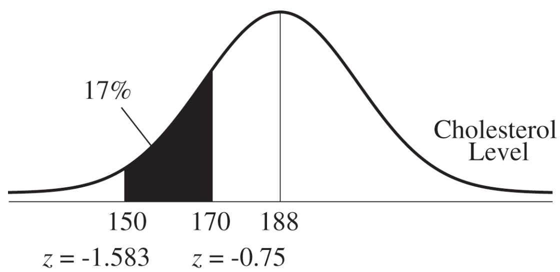


b) $z = \dfrac{y-\mu}{\sigma} = \dfrac{200-188}{24} = 0.5$

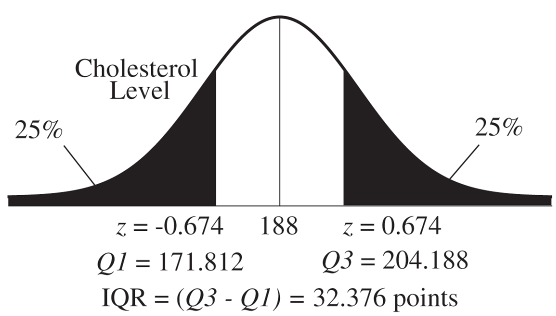


According to the Normal model, 30.85% of American women are expected to have cholesterol levels over 200 mg/dL.

c) According to the Normal model, 17.00% of American women are expected to have cholesterol levels between 150 and 170 mg/dL.

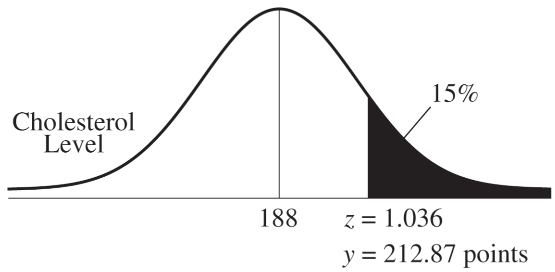


d) According to the Normal model**,** the interquartile range of the distribution of cholesterol levels of American women is approximately 32.4 mg/dL.



53. (continued)

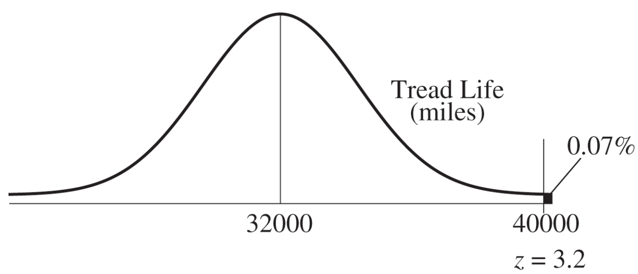
e) $z = \dfrac{y-\mu}{\sigma} \Rightarrow 1.036 = \dfrac{y-188}{24} \Rightarrow y \approx 212.9$



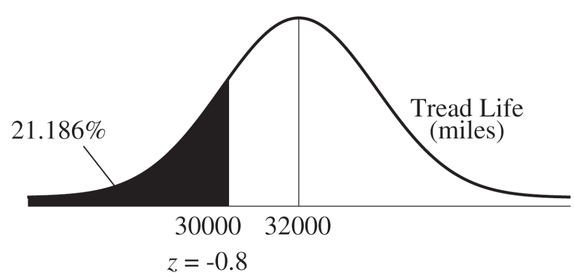
According to the Normal model, the highest 15% of women's cholesterol levels are above approximately 212.9 mg/dL.

54. Tires.

a) A tread life of 40,000 miles is 3.2 standard deviations above the mean tread life of 32,000. According to the Normal model, only approximately 0.07% of tires are expected to have a tread life greater than 40,000 miles. It would not be reasonable to hope that your tires lasted this long.



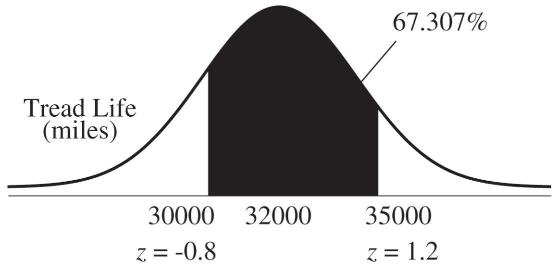
b) $z = \dfrac{y-\mu}{\sigma} = \dfrac{30,000-32,000}{2500} = -0.8$



According to the Normal model, approximately 21.2% of tires are expected to have a tread life less than 30,000 miles.

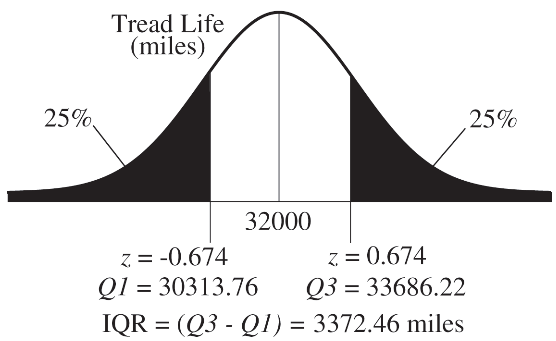
54. (continued)

c) $z = \dfrac{y-\mu}{\sigma} = \dfrac{35,000-32,000}{2500} = 1.2$



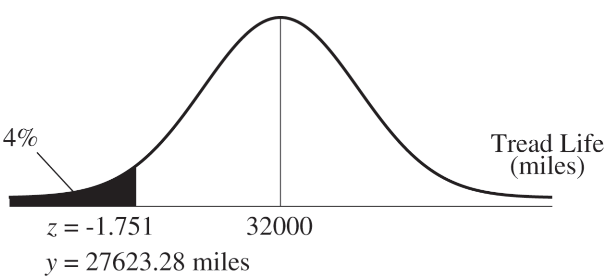
According to the Normal model, approximately 67.3% of tires are expected to last between 30,000 and 35,000 miles.

d) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -0.674 = \dfrac{\text{Q1}-32,000}{2500} \Rightarrow \text{Q1} = 30,314$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow 0.674 = \dfrac{\text{Q3}-32,000}{2500} \Rightarrow \text{Q3} = 33,686$



According to the Normal model, the interquartile range of the distribution of tire tread life is expected to be 33,686 – 30,314 = 3372.46 miles.

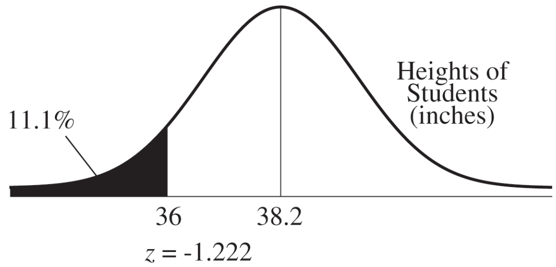
e) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -1.751 = \dfrac{y-32,000}{2500} \Rightarrow y = 27,623.28$



According to the Normal model, 1 of every 25 tires is expected to last less than 27,623 miles. If the dealer is looking for a round number for the guarantee, 27,000 miles would be a good tread life to choose.

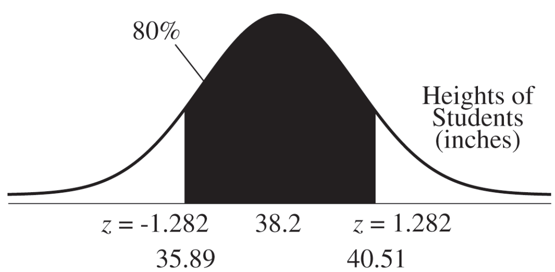
55. Kindergarten.

a) $z = \dfrac{y-\mu}{\sigma} = \dfrac{36-38.2}{1.8} = -1.222$



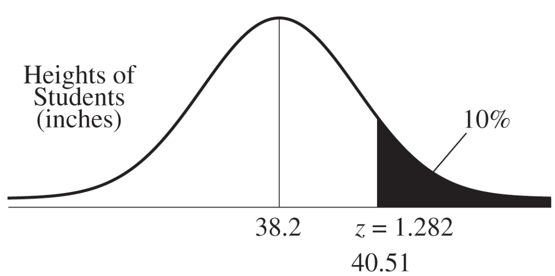
According to the Normal model, approximately 11.1% of kindergarten kids are expected to be less than three feet (36 inches) tall.

b) $z = \dfrac{y-\mu}{\sigma} \Rightarrow -1.282 = \dfrac{y_1-38.2}{1.8} \Rightarrow y_1 = 35.89$ and $z = \dfrac{y-\mu}{\sigma} \Rightarrow 1.282 = \dfrac{y_2-38.2}{1.8} \Rightarrow y_2 = 40.51$



According to the Normal model, the middle 80% of kindergarten kids are expected to be between 35.9 and 40.5 inches tall. (The appropriate values of $z = \pm 1.282$ are found by using right and left tail percentages of 10% of the Normal model.)

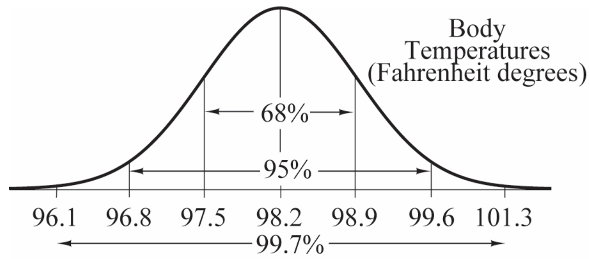
c) $z = \dfrac{y-\mu}{\sigma} \Rightarrow 1.282 = \dfrac{y_2-38.2}{1.8} \Rightarrow y_2 = 40.5$



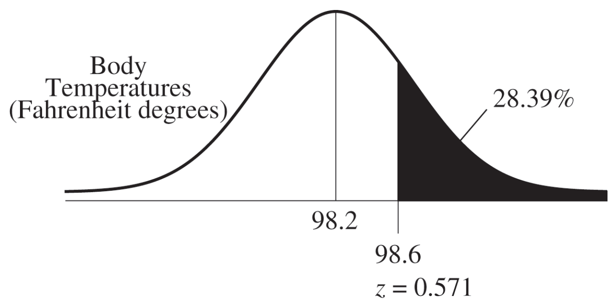
According to the Normal model, the tallest 10% of kindergarteners are expected to be at least 40.5 inches tall.

56. Body temperatures.

a) According to the Normal model (and based upon the 68-95-99.7 rule), 95% of people's body temperatures are expected to be between 96.8° and 99.6°. Virtually all people (99.7%) are expected to have body temperatures between 96.1° and 101.3°.

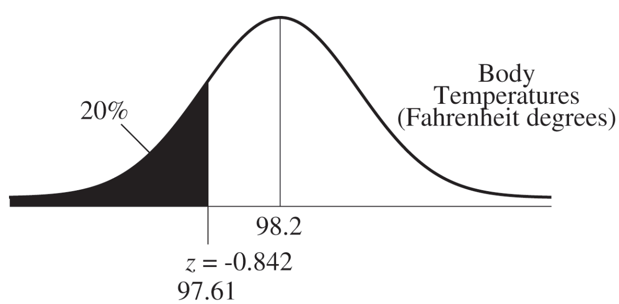


b) $z = \frac{y-\mu}{\sigma} = \frac{98.6-98.2}{0.7} = 0.571$



According to the Normal model, approximately 28.4% of people are expected to have body temperatures above 98.6°.

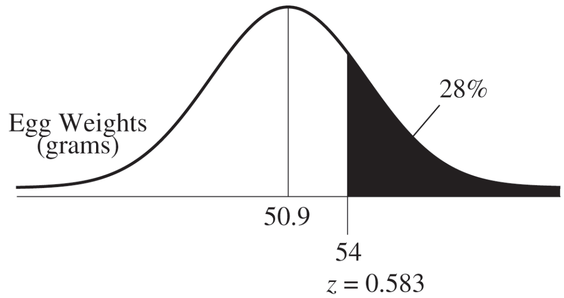
c) $z = \frac{y-\mu}{\sigma} \Rightarrow -0.842 = \frac{y-98.2}{0.7} \Rightarrow y = 97.61$



According to the Normal model, the coolest 20% of all people are expected to have body temperatures below 97.6°.

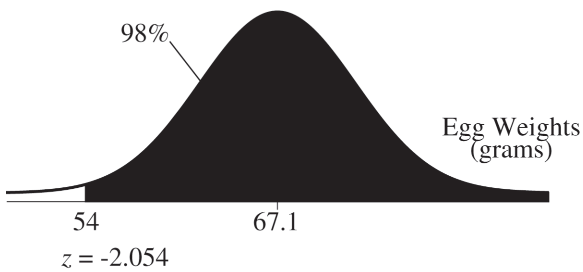
57. Eggs.

a) $z = \frac{y - \mu}{\sigma} \Rightarrow 0.583 = \frac{54 - 50.9}{\sigma} \Rightarrow 0.583\sigma = 3.1 \Rightarrow \sigma = 5.317$



According to the Normal model, the standard deviation of the egg weights for young hens is expected to be 5.3 grams.

b) $z = \frac{y - \mu}{\sigma} \Rightarrow -2.054 = \frac{54 - 67.1}{\sigma} \Rightarrow -2.054\sigma = -13.1 \Rightarrow \sigma = 6.377$

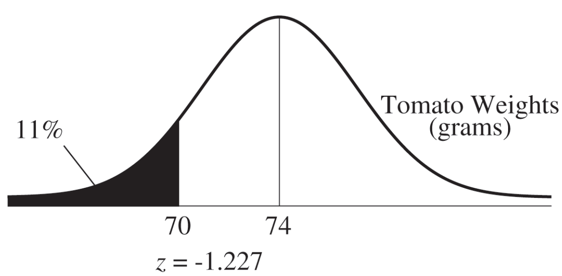


According to the Normal model, the standard deviation of the egg weights for older hens is expected to be 6.4 grams.

c) The younger hens lay eggs that have more consistent weights than the eggs laid by the older hens. The standard deviation of the weights of eggs laid by the younger hens is lower than the standard deviation of the weights of eggs laid by the older hens.

58. Tomatoes.

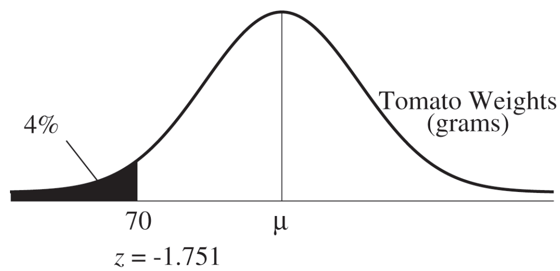
a) $z = \frac{y - \mu}{\sigma} \Rightarrow -1.227 = \frac{70 - 74}{\sigma} \Rightarrow -1.227\sigma = -4 \Rightarrow \sigma = 3.26$



According to the Normal model, the standard deviation of the weights of Roma tomatoes now being grown is 3.26 grams.

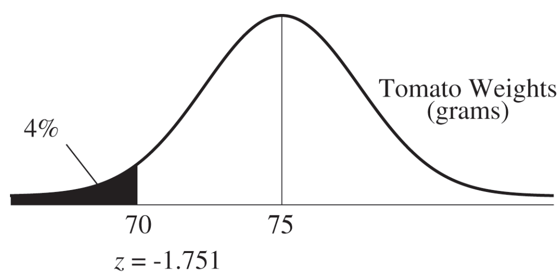
58. (continued)

b) $z = \frac{y-\mu}{\sigma} \Rightarrow -1.751 = \frac{70-\mu}{3.260} \Rightarrow -5.708 = 70-\mu \Rightarrow \mu = 75.70$



According to the Normal model, the target mean weight for the tomatoes should be 75.70 grams.

c) $z = \frac{y-\mu}{\sigma} \Rightarrow -1.751 = \frac{70-75}{\sigma} \Rightarrow \sigma = 2.856$



According to the Normal model, the standard deviation of these new Roma tomatoes is expected to be 2.86 grams.

d) The weights of the new tomatoes have a lower standard deviation than the weights of the current variety. The new tomatoes have more consistent weights.