

Test 3

STAT 021

Swarthmore College

Do not flip this page until instructed to do so.

Test organization: There are 12 questions in total on this test and they are organized into three subsections: the first 4 questions are matching or True/False with explanation questions, the next 5 questions are free response short answer and should not require more than a sentence or two to answer. The last section contains 3 long answer free response questions that require more than a couple of sentences to answer fully. There are a total of 60 points possible on this test. The last section explains an extra credit opportunity. If you need additional scratch paper you may come to the front of the class and pick some up.

Instructions: Answer each question to the best of your ability and raise your hand if you are confused by any of the wording in the questions or suspect a typo. For the short and long answer questions show all your work and provide enough justification and/or explanation in order to get full credit or to be considered for partial credit. You do not need a calculator to evaluate any expressions. For any calculation problems, simply writing out the formula to find the answer will suffice.

First and Last Name: Abraham Porsch

Swarthmore Username: aporsch1

Take a deep breath.

You have prepared for this test and with a clear and well-rested mind, you are ready to show me what you have learned this semester. The purpose of this test is to measure your understanding of the material we have covered this semester. This is nothing more than a metric for me to evaluate your preparedness to think statistically at this particular moment in time and in this particular setting. This is not a perfect measure of your knowledge and does not predict your future statistical skills.

Section 1: Matching and True/False problems

1. (5 points)

Suppose we are modeling the weight of birds (in kg) as a linear function of a categorical predictor variable for bird type (with levels pigeon, sparrow, and finch) and a numeric predictor for bird age. Given a "full" model

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_3 + \beta_5 x_2 x_3 + \epsilon,$$

where $x_1 = \begin{cases} 1, & \text{if sparrow} \\ 0, & \text{otherwise} \end{cases}$, $x_2 = \begin{cases} 1, & \text{if finch} \\ 0, & \text{otherwise} \end{cases}$ and x_3 is the age of the bird (in months), match the questions below to their corresponding null hypotheses.

- ☒ a) For newly hatched birds (of age zero months), is there a statistically discernible difference in the weights of these three different bird types?
- ☒ b) Does the effect of age on a bird's weight depend on what type of bird it is?
- ☒ c) Given we are only comparing birds of the same age, is there a statistically significant difference in the mean weight of sparrows and pigeons?
- ☒ d) Given we are only comparing pigeons, is the effect of age on a bird's weight statistically significant?
- ☒ e) Is there statistically discernible evidence of a linear relationship between bird age and type and bird weight?

- 1. e $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
- 2. c $H_0 : \beta_1 = 0$
- 3. d $H_0 : \beta_3 = 0$
- 4. b $H_0 : \beta_4 = \beta_5 = 0$
- 5. a $H_0 : \beta_1 = \beta_2 = 0$

2. (5 points)

Determine which of the following statements about MLR models are true and false. For each statement that is false, provide a brief explanation as to why it is false.

- (a) If predictors are collinear, then removing one variable will have no influence on the point estimate of another variable's coefficient.

False, if predictors are collinear and one is removed the other variables will change to fill the effect of the removed variable since they are connected.

- (b) Suppose a numerical variable x_1 has a coefficient of $\beta_1 = 2.5$ in the multiple regression model. Suppose also that the first observation has a value of $x_1 = 7.2$, the second observation has a value of $x_1 = 8.2$, and these two observations have the same values for all other predictors. The predicted value of the second observation will be 2.5 units higher than the prediction of the first observation based.

TRUE, when all other predictors are held constant the slope is 2.5.

- (c) As the total sample size increases, the degrees of freedom for the residuals increases as well.

True, degrees of freedom increase directly with the number of data points.

3. (5 points)

Determine which of the following statements about ANOVA models are true and false. For each statement that is false, provide a brief explanation as to why it is false.

If the null hypothesis that the means of four groups are all the same is rejected from an ANOVA model and overall F-test at a 5% significance level, then ...

- (a) We can then conclude that all the means are different from one another.

FALSE, we can conclude that at least one mean is different.

- (b) The standardized variability among the group averages is higher than the estimate of the variability of the data within each group.

True, since there is some difference among the groups at least greater than data values the std. variance.

- (c) A post-hoc pairwise analysis will identify if there is at least one pair of means that are significantly different.

True, this is exactly what would answer question (a).

4. (5 points)

Determine if the following statements about statistical modeling are true or false, and explain your reasoning. If false, state how it could be corrected.

- (a) If a given value (for example, the null hypothesized value of a parameter) is within a 95% confidence interval, it will also be within a 99% confidence interval.

True, the 99% interval contains the entire 95% interval but is just expanded at the ends.

- (b) With large sample sizes, even small differences between the null value and the observed point estimate will be identified as statistically significant.

True, As sample size increases standard error decreases making it easier to get a low p-value for a statistical test. It does not guarantee a significant outcome though.

- (c) Correlation is a measure of the association between any two variables.

True, it measures how well one variable can predict another

Section 2: Short answer questions

5. (4 points)

State two reasons why we might consider transforming the response variable to fit an appropriate multiple linear regression model to some data.

If the data does not satisfy linearity and you can transform it to be linear. You can also transform to fit the zero-mean assumption.

6. (3 points)

If you could only use one measure (among the studentized residuals, leverage values, and Cook's distance values) to identify potentially influential data points, which would you choose and why?

I would use Cook's distance because I think that it is a pretty sophisticated measure that takes many of the same contributing variables that are used to calculate studentized residuals and leverage and combines them into one statistic.

For questions 7-9 consider the following random sample of $n = 246$ online shoppers. We are going to model the average price (in US dollars) (price) as a linear function of the item's type (a categorical predictor with levels: trousers, skirts, blouses, on_sale). Below is the R summary output for this one-way ANOVA model.

```
##
## Call:
## lm(formula = price ~ type, data = retail_dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -21.946  -8.946   0.893   6.054  35.054
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    41.946      1.512  27.750 < 2e-16 ***
## typeon_sale    -5.438      2.128  -2.555  0.01123 *
## typeskirts      9.161      2.138   4.285 2.64e-05 ***
## typetrousers    5.937      1.987   2.988  0.00309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.31 on 242 degrees of freedom
## Multiple R-squared:  0.1913, Adjusted R-squared:  0.1813
## F-statistic: 19.09 on 3 and 242 DF,  p-value: 3.825e-11
```

7. (3 points)

- (a) What are the error degrees of freedom based on this model?
 (b) What is the reference level?

$$246 - 3 - 1 = 242$$

blouses, since $\beta_{blouses} = 0$.

8. (6 points)

Suppose the average ^{price for each item} ~~number of plate appearances per game~~ is 44.63 over all 246 data points. What is the estimated group effect for clothing type trousers?

$$44.63 + 5.937$$

9. (4 points)

Consider two additional numeric predictors: the amount of time the item has been available for purchase on this retailer's website, release, measured in weeks and the production cost associated with each item, produce_cost, measured in US dollars. If we were to fit a regression model including each of the three predictor variables (including type) and an interaction between the two numeric variables, explain the meaning of the coefficient for the interaction term within the context of this data. (You should be able to answer this in no more than two sentences.)

It would show that with clothing type and one of the numerical predictors held constant, how much the numerical predictor held constant still affects the total price of the object with a one unit increase in the other numerical predictor.

Section 3: Long answer questions

10. (9 points)

Suppose you have access to a data set on a random sample of Swarthmore faculty. The variables included in this data set are a numeric variable for each person's age, a binary categorical variable distinguishing faculty who are tenured from those who are not, a numeric variable for each faculty member's starting salary, and a categorical variable indicating if the faculty member attended a liberal arts college, or a university, or entered the work force after graduating high school.

age, tenure, starting salary, liberal arts/univ/work
State a research question that can be answered with the overall F-test for each of the following models. Also provide a mathematical representation of the model and state the null hypothesis based on the notation you define for each model.

- (a) a simple linear regression model;
- (b) an ANOVA model;
- (c) a multiple linear regression model (not SLR or ANOVA).

a) $\text{starting salary} = \beta_0 + \beta_1(\text{age}) + \epsilon$, does age impact starting salary?

b) $\text{starting salary} = \beta_0 + \beta_1(\text{liberal arts}) + \beta_2(\text{university}) + \epsilon$, does faculty college/work impact starting salary?

c) $\text{starting salary} = \beta_0 + \beta_1(\text{age}) + \beta_2(\text{liberal arts}) + \beta_3(\text{university}) + \epsilon$

do age and college type / job out of high school significantly impact starting salary?

11. (8 points)

Consider the ANOVA model for the retail data you used in questions 7-8. Reference the R output on pg 5 and the plots on pg 10 to answer the following questions about this model.

- Check the conditions necessary for conducting a test to determine if the average cost of the purchased items are significantly different for different types of clothing type. (You do not need to check the zero mean or linearity conditions but you do need to describe what it means for the group effects to be constant in this context.)
- Write out in words and in symbols the hypotheses that would be tested in part (a). (Clearly define your notation.)
- What can you conclude about the test in part (b)? Write a paragraph discussing your conclusions and reference any relevant statistics and/or plots as part of your discussion.

a) The q-q plot has most of the points above the line signaling that it is probably not very close to normality and the variances are also not very constant across the different levels. For most of the levels the effect does seem to be additive but for blouses the data is so skewed it doesn't seem that way.

b) $H_0: \alpha_1 = \alpha_2 = \alpha_3$, this means that we are testing to see if there are any differences among any of the effects from each other.

c) All of the individual levels are significant as seen from their p-values and the model is also significant seen from its p-value of 3.073×10^{-11} . On sale has an estimated β value of ≈ -5.44 and a std error of 2.12, the closest β to -5.44 is 5.97 which is more than 4 times the SE of type III sum of squares which means there are at least one β significantly different from the others.

12. (8 points)

Suppose two people are studying the historic data set about the amount of arsenic (Arsenic) in local wells. This data contains $n = 70$ observations from a random selection of well water samples from across the state. In addition to the levels of arsenic, the data also records the year the data was collected (Year) and the distance from the well to the nearest mining site (Miles).

Person A fits the following MLR model to the data:

$$\text{Arsenic} = \beta_0 + \beta_1 \text{Year} + \beta_2 \text{Miles} + \epsilon$$

and computes an adjusted R^2 value of 0.26.

Person B considers the following correlations:

$$\text{Cor}(\text{Arsenic}, \text{Year}) = \rho_1; \quad \text{Cor}(\text{Arsenic}, \text{Miles}) = \rho_2$$

and estimates each with their sample correlations $r_1 = 0.77$ and $r_2 = -0.34$. Are the two people's conclusions contradictory? Explain your answer.

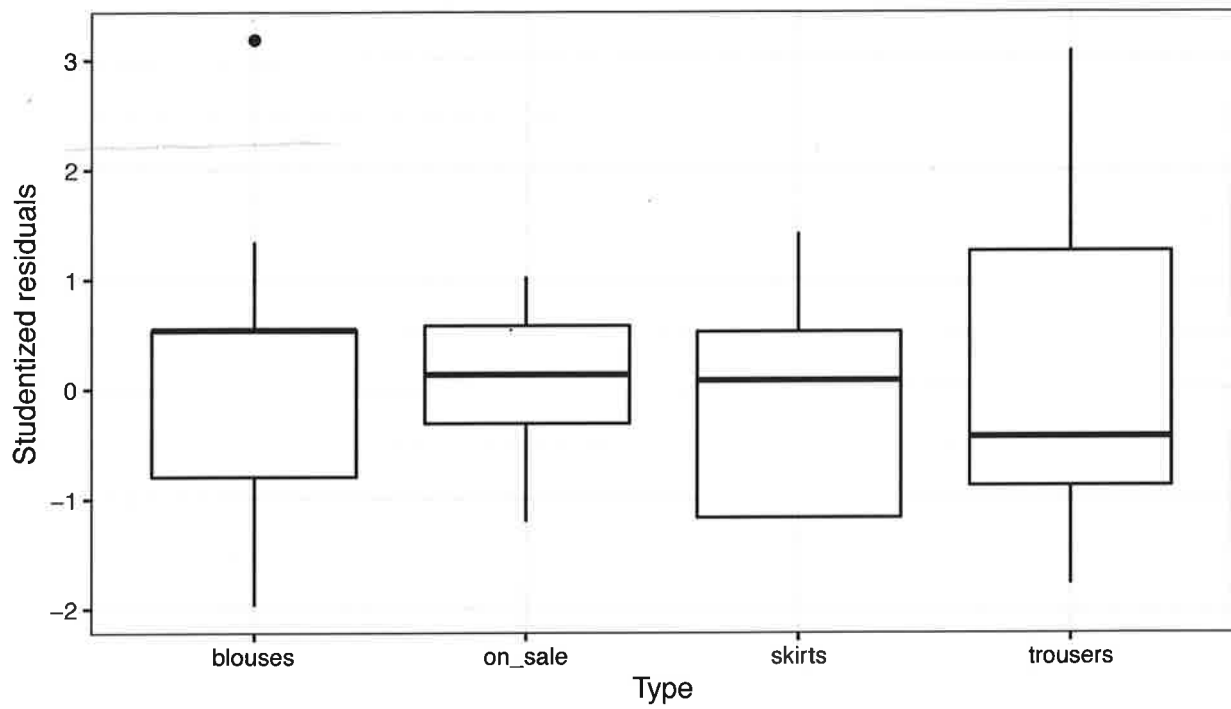
Not contradictory at all, you are allowed to fit a model with values that have positive and negative correlations, those signs just correspond to the slopes. Although the β_1 may be a slightly more significant value than β_2 because the correlation is higher. And I think the negative value is expected considering that as distance to a mine increases there would probably be more toxic chemicals such as arsenic. So not only is it not contradictory I think it is to be expected.

Section 4: Extra credit opportunity

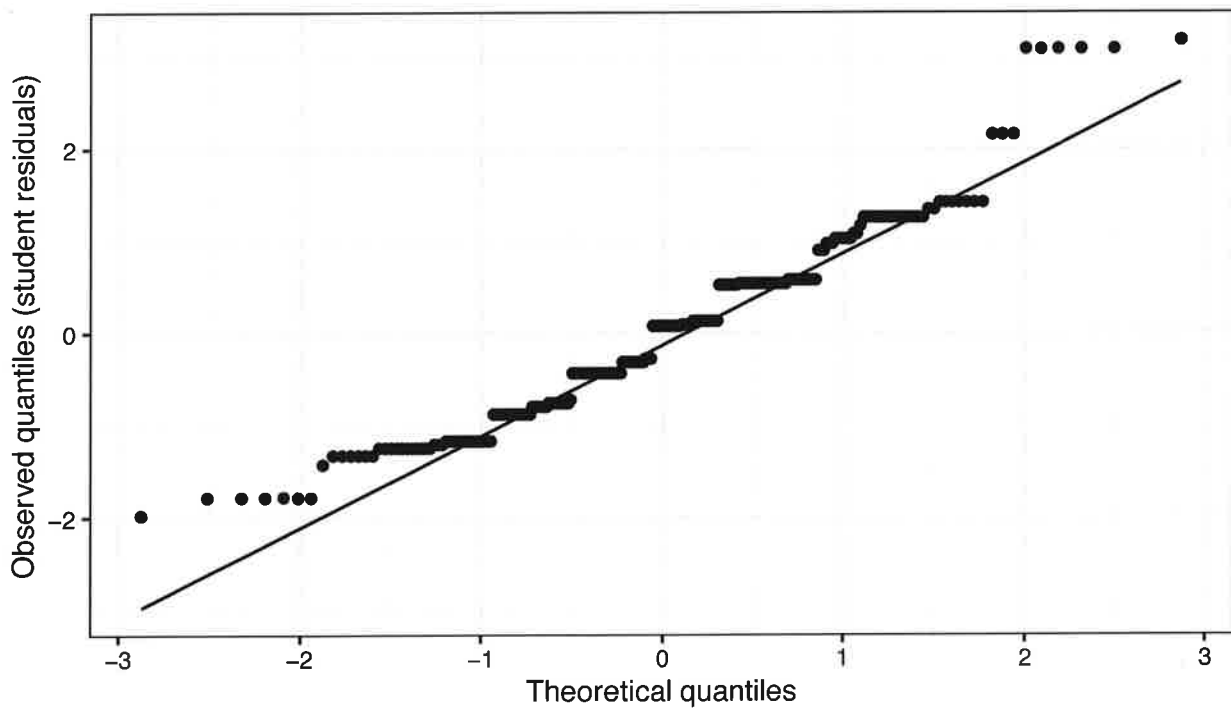
If the response rate to my end of the semester evaluation form (on Moodle under Week 13 and 14) is at least 85% of our class size (over both sections), two percentage points will be added to everyone's Test 3 grade (up to 100 total possible points). **Hint:** You may not know how to or want to contact everyone in my class but you do know your group mates pretty well.

Retail ANOVA Model

Residual plot for ANOVA model



Normal quantile plot for ANOVA model



“

