10:15

Exam 1

STAT 021

Swarthmore College

2019/10/4

Name: Victor L Dos Anjos

Instructions:

There are seven questions on this exam. The points allotted for each question are given at the end of the problem. Please don't write an entire page response for any of the answers. Rather, answer these questions to the best of your ability with succinct, informative statements or observations. You may or may not use the following formulas and definitions.

Formulas and Definitions Linear model: $Y = \beta_0 + \beta_1 x + \epsilon$ or, equivalently, $E[Y] = \beta_0 + \beta_1 x$.

In the model(s) above, if we assume that the mean of ϵ is 0 and the variance of ϵ is some unknown number, σ^2, then the mean of the random variable Y is $\beta_0 + \beta_1 x$ and the variance of Y is σ^2.

Fitted/estimated model: $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

In the fitted model above, we solve for the least squares estimates of the parameters using these equations:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Definition of residuals: $\hat{y}_i - y_i = e_i$

Regression model sums of squares: $\sum_{i=1}^n (\hat{y}_i - \bar{y})^2$

Residual sums of squares: $\sum_{i=1}^n (y_i - \hat{y}_i)^2$

Total sums of squares: $\sum_{i=1}^n (y_i - \bar{y})^2$

Relationship among the sums of squares terms: $SS_{tot} = SS_{reg} + SS_{res}$

The sums of squares terms are used to calculate the following statistics:

$$\hat{\sigma} = \sqrt{\frac{SS_{res}}{n-2}}$$

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} = \frac{SS_{reg}}{SS_{tot}}$$

Categorical = Qualitative
Numerical = Quantitative

Problem 1 Suppose that the observational units in a study are patients who entered the emergency room at French Hospital in the previous week. For each of the following, indicate whether it is a categorical variable, a numerical variable, **or** not a variable with regard to these observational units. (10 points)

a) How long the patient waits to be seen by a medical professional

Categorical

No natural order between patients.

b) Day of the week on which the patient arrives

Numerical

Day of the week has a natural order.

c) Average wait time before the patient is seen by a medical professional

Categorical

No natural order for avg. wait time for patients seen.

d) Whether or not wait times tend to be longer on weekends than weekdays

Numerical

Weekends & weekdays have natural order.

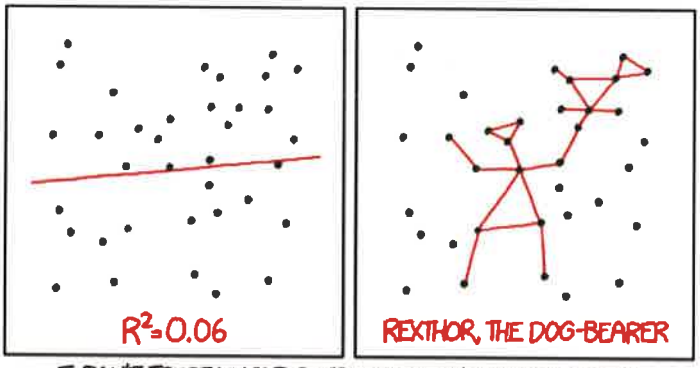
Problem 2 Consider the transactions at the Science Center coffee bar to be the observational units in a statistical study. In a paragraph of less, state a research question that involves two quantitative variables for these observational units. Also clearly identify what roles the two variables would play in the study and why. (10 points)

As students spend more money on coffee does the average price of their coffee decrease? The predictive variable would be money spent on coffee and the response variable would be the average price of their coffee. Both variables are quantitative.

Problem 3 Suppose a professor has a paper titled: *Estimation and hypothesis testing in regression in the presence of nonhomogeneous error variances* lying out on her desk.[1] In 1-2 sentences, explain what you think this paper is about? (10 points)

The paper is likely on heteroscedasticity and the effects that non-constant $Var[\varepsilon]$ has on estimation and hypothesis testing. The paper possibly has suggestions for correcting data so that the assumption $Var[\varepsilon] = \sigma^2 < \infty$ is met (one example may be transformations).

Problem 4 Based on the data shown in the scatter plot of this comic[2], what can you tell me about the relationship between the SS_{reg} and SS_{res} terms? (10 points)



I DON'T TRUST LINEAR REGRESSIONS WHEN IT'S HARDER TO GUESS THE DIRECTION OF THE CORRELATION FROM THE SCATTER PLOT THAN TO FIND NEW CONSTELLATIONS ON IT.

Assuming this is the scatterplot.

Because $R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$ or $\frac{SS_{reg}}{SS_{tot}}$ and the $R^2 = 0.06$ then that means that the SS_{res} must have been high (random error high) and SS_{reg} (variation between samples given X) low. In other words, given the spread of the data, the variation in Y that can be explained by the variation in X (non random variation) is low, while the variation that is random is high.

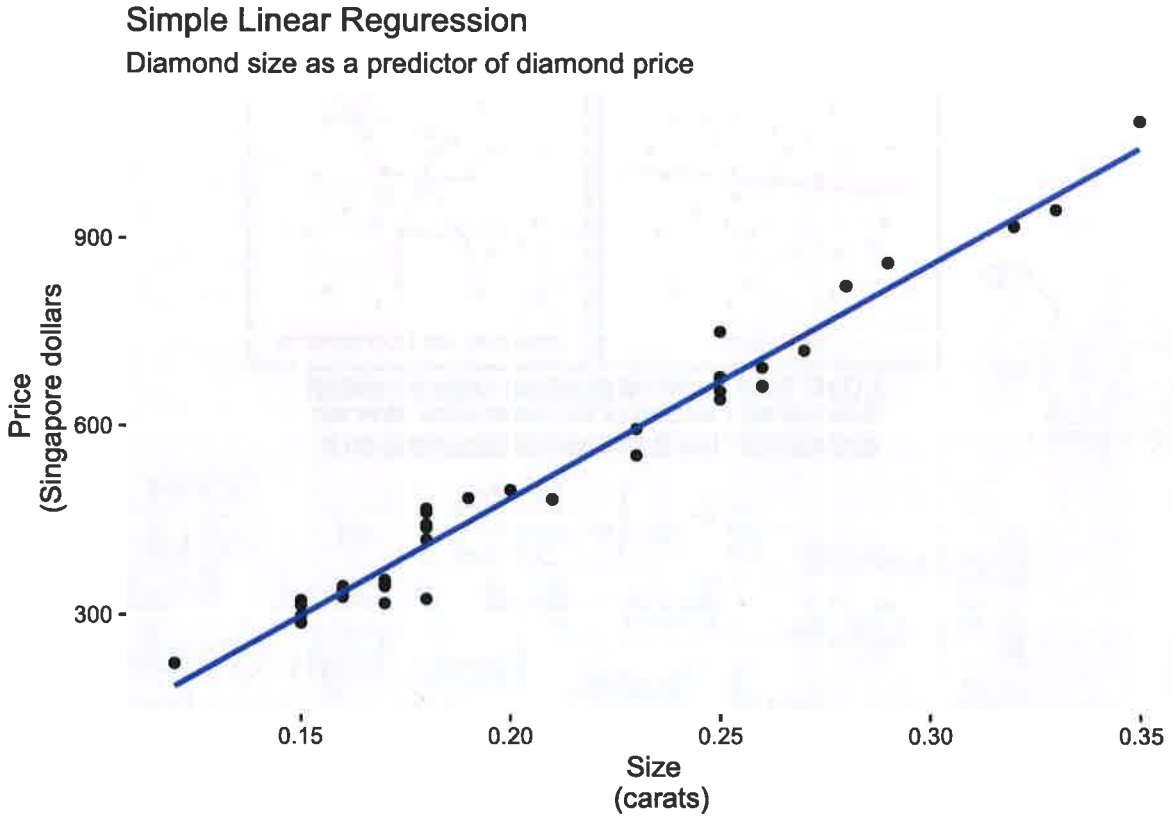
Problem 5 Suppose we have observed a small data set (say $n = 5$) without any significant measurement error (e.g. we are collecting data on vapor pressure and temperature but our instruments to measure each are exact). How do we find the line of best fit? (10 points)

You can put your data in R and use R or you can use the $Y = \beta_0 + \beta_1 x + \varepsilon$ model. Use your data to get estimated model $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$; use: $\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})}$ & $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ to get your estimates from the model.

Problem 6 Recall the diamond data that we discussed in class. For this data, we have a simple random sample of Singaporean diamonds and are interested in how the size of the diamond (in carats) can predict (or explain) what the cost of the diamond will be. Below is the R code for fitting this simple linear regression model. (25 points)

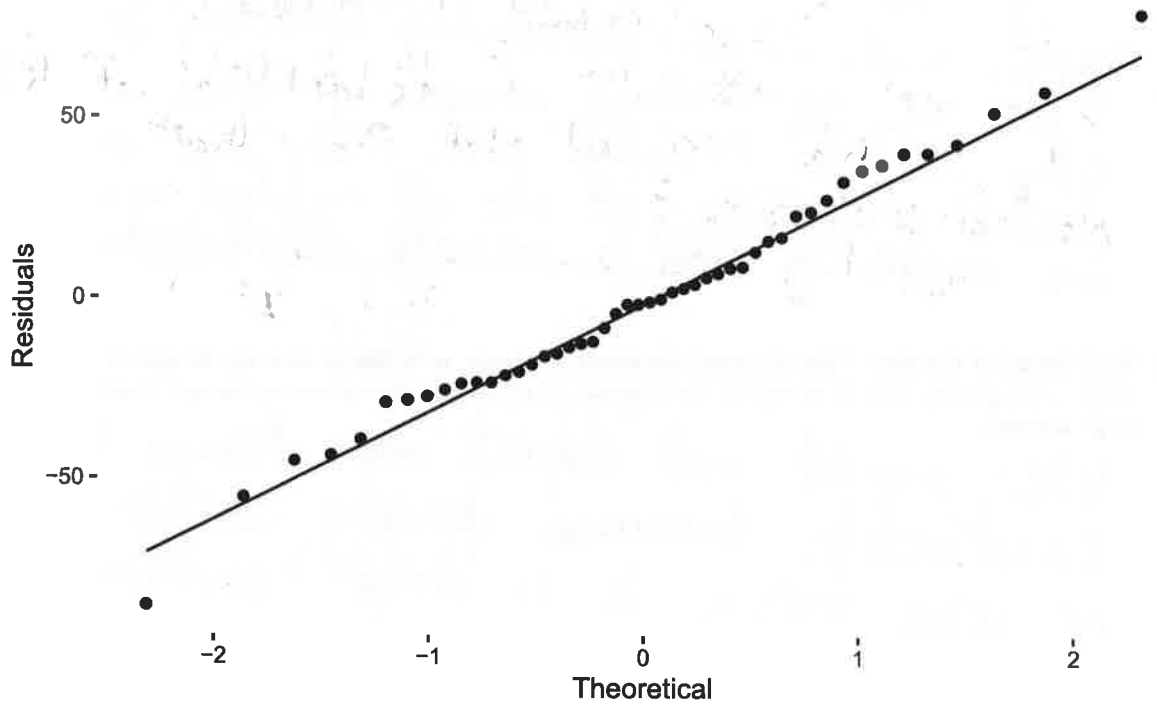
```
diamond_mod <- lm(price~size, data=diamond_dat)
diamond_mod_summary <- summary(diamond_mod)
```

Analyse the following three plots based on this regression model to answer the next two questions.



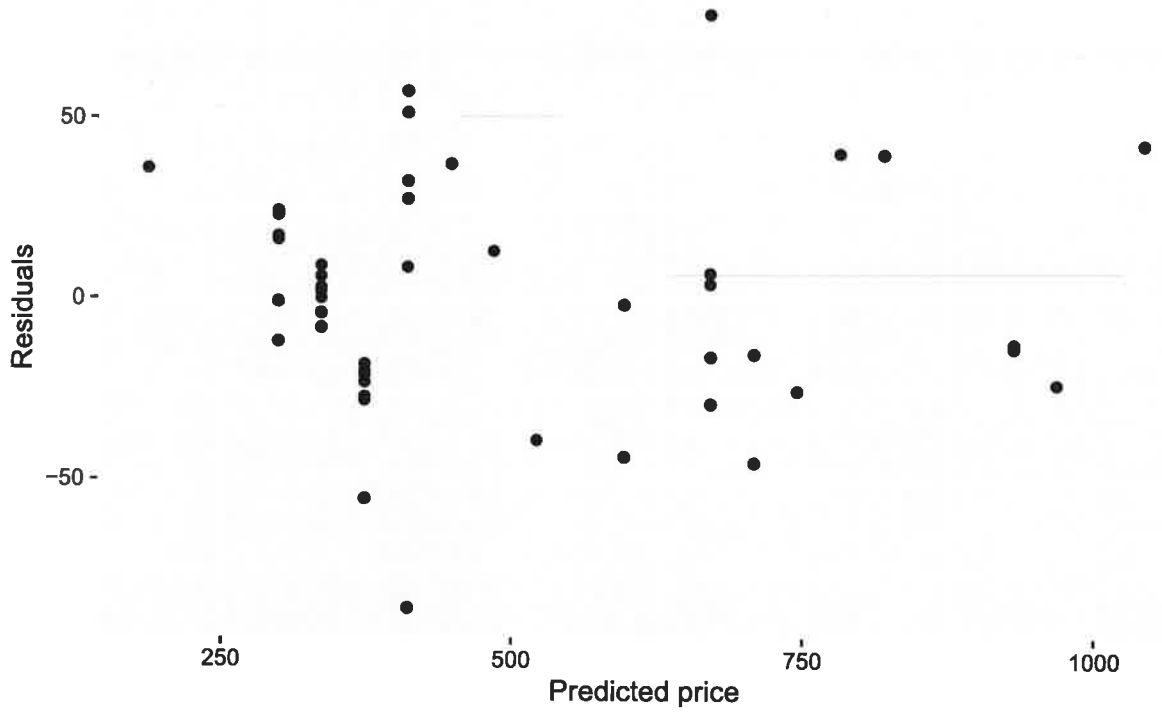
~~Residual plot~~ Normal Probability Plot

Fitting diamond prices as a linear function of size



Residual plot

Fitting diamond prices as a linear function of size



ξ = and

a) Based on these plots, what conclusions can we make about the presence of a linear relationship, if the random errors are constant, and if the random errors are Normally distributed?

Using the scatterplot, it seems very likely that there is a linear relationship between price & size. There are no obvious shapes or signs of heteroscedasticity in the residual plot. Finally, the values in the normal probability plot are all clusted near the $x=y$ line suggesting data is normally distributed.

Suggesting $Var[\varepsilon]$ are constant.

b) Say instead of the size of the diamond measured in carats, we'd like to look at the size in grams (1 carat = 0.2 grams). Would we expect the behavior of any of the plots above to change? Briefly explain your answers.

We would not expect any change in the relationships because changing carats to grams or vice versa is a linear transformation.

Problem 7 The data that appear in the data set "Four-Mile-Run-data.txt" were collected by a GPS watch worn by the runner of a four-mile course. Using heart rate measurements after each run, an analysis of the runner's post-exercise heart rate recovery provides an indication of cardiovascular fitness. We are interested in answering the question: is the speed of the run (in mph) related to the number of calories burned. Below is the R code and output for fitting such a linear model to this data.[3] (25 points)

```
run_dat <- read_table2("~/Google Drive Swat/Swat docs/Stat 21/Data/Four-Mile-Run-data.txt")
summary(lm(calories~aveSpeed, run_dat))
```

```
##
## Call:
## lm(formula = calories ~ aveSpeed, data = run_dat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -55.542 -18.918   2.212  16.376  56.130
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -208.21     161.63  -1.288  0.21495
## aveSpeed       80.82      22.51   3.590  0.00225 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 30.84 on 17 degrees of freedom
## Multiple R-squared:  0.4313, Adjusted R-squared:  0.3978
## F-statistic: 12.89 on 1 and 17 DF,  p-value: 0.002255
```

a) What is the estimate for the standard deviation of the number of calories burned based on this linear model?

$SE(\hat{\beta}_1) = 22.51$ calories

Assuming "number of calories burnt" refers to the slope of calories burnt as a function of average speed based on the model estimate. If the question refers to the random variation of calories burnt in a sample for a single mph value then RSE = 30.84 calories may be better.

b) On average, how many more (or fewer) calories can our runner expect to burn for each mph increase in average running speed?

80.82 calories per 1mph increase in average running speed.

$\hat{\beta}_1 = 80.82$

c) Suppose, on average, for any person within the same age group as our runner, every mph increase in running speed corresponds to 100 additional calories burnt. How can we determine if our runner's rate of burning calories is different from this average for all people in the age group?

You can run a t-test to test the null and alternative hypothesis. If the runners $\hat{\beta}_1 = 80.82$ calories lost per +1mph then we want to know if $\beta_1 = 100$ calories lost per +1mph is inside a 95% CI of the estimated β_1. The test could be:

$H_0: \beta_1 = 100$

$H_1: \beta_1 \neq 100$

d) What numbers in the R output above can help us determine if this model is a good fit for the data? Explain briefly. (There are at least two.)

R^2 is good because it tells us the amount of variation in y can be explained by variation in x given our model. The p-value from the t-test of F-test is also good because it tells us if we have enough information to reject the null hypothesis that there is no relationship b/w y and x (t & F test pvalues should be the same in SLR).

[1] Michael L. Deaton, Mation R. Reynolds Jr. & Raymond H. Myers (1983) Estimation and hypothesis testing in regression in the presence of nonhomogeneous error variances, Communications in Statistics - Simulation and Computation, 12:1, 45-66, DOI: 10.1080/03610918308812299

[2] https://xkcd.com/1725

[3] Paul J. Laumakis & Kevin McCormack (2014) Analyzing Exercise Training Effect and Its Impact on Cardiorespiratory and Cardiovascular Fitness, Journal of Statistics Education, 22:2, , DOI: 10.1080/10691898.2014.11889702]