Test 3

STAT 021

Swarthmore College

Do not flip this page until instructed to do so.

Test organization: There are 12 questions in total on this test and they are organized into three subsections: the first 4 questions are matching or True/False with explanation questions, the next 5 questions are free response short answer and should not require more than a sentence or two to answer. The last section contains 3 long answer free response questions that require more than a couple of sentences to answer fully. There are a total of 60 points possible on this test. The last section explains an extra credit opportunity. If you need additional scratch paper you may come to the front of the class and pick some up.

Instructions: Answer each question to the best of your ability and raise your hand if you are confused by any of the wording in the questions or suspect a typo. For the short and long answer questions show all your work and provide enough justification and/or explanation in order to get full credit or to be considered for partial credit. You do not need a calculator to evaluate any expressions. For any calculation problems, simply writing out the formula to find the answer will suffice.

First and Last Name: _____

Swarthmore Username: _____

Take a deep breath.

You have prepared for this test and with a clear and well-rested mind, you are ready to show me what you have learned this semester. The purpose of this test is to measure your understanding of the material we have covered this semester. This is nothing more than a metric for me to evaluate your preparedness to think statistically at this particular moment in time and in this particular setting. This is not a perfect measure of your knowledge and does not predict your future statistical skills.

Section 1: Matching and True/False problems

1. (5 points)

Suppose we are modeling the weight of birds (in kg) as a linear function of a categorical predictor variable for bird type (with levels pigeon, sparrow, and finch) and a numeric predictor for bird age. Given a "full" model

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_3 + \beta_5 x_2 x_3 + \epsilon,$$

where $x_1 = \begin{cases} 1, & \text{if sparrow} \\ 0, & \text{otherwise} \end{cases}$, $x_2 = \begin{cases} 1, & \text{if finch} \\ 0, & \text{otherwise} \end{cases}$ and x_3 is the age of the bird (in months), match the questions below to their corresponding null hypotheses.

a) For newly hatched birds (of age zero months), is there a statistically discernible difference in the weights of these three different bird types?

b) Does the effect of age on a bird's weight depend on what type of bird it is?

c) Given we are only comparing birds of the same age, is there a statistically significant difference in the mean weight of sparrows and pigeons?

d) Given we are only comparing pigeons, is the effect of age on a bird's weight statistically significant?

e) Is there statistically discernible evidence of a linear relationship between bird age and type and bird weight?

1.**C** $H_0 : \beta_1 = 0$

2. **D** $H_0 : \beta_3 = 0$
3. **E** $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
4. **B** $H_0 : \beta_4 = \beta_5 = 0$
5. **A** $H_0 : \beta_1 = \beta_2 = 0$

2. (5 points)

Determine which of the following statements are true and false. For each statement that is false, provide a brief explanation as to why it is false.

(a) If predictors are collinear, then removing one variable will have no influence on the point estimate of another variable's coefficient.

False with valid justification (2 pts), with invalid justification (1pt)

(b) Suppose one numeric predictor variable x_1 has an estimated MLR coefficient of $\hat{\beta}_1 = 0.5$, another numeric predictor x_2 has an estimated coefficient $\hat{\beta}_2 = 5$. If we consider a simultaneous one-unit change in both x_1 and x_2 (while holding any other predictor terms constant), this corresponds to an average change in $\hat{y}$ of at least 5 units.

True (2 pts); **False** if stated direction of unit changes matter (2 pts)

(c) As the total sample size increases, the degrees of freedom for the residuals increases as well.

True (1 pt)

3. (5 points)

Determine which of the following continuations of the statement below are true and which are false. For each statement that is false, provide a brief explanation as to why it is false.

If the null hypothesis that the means of four groups are all the same is rejected from an ANOVA model and overall F-test at a 5% significance level, then...

(a) We can then conclude that all the means are different from one another.

False with valid explanation (2 pts); with invalid explanation (1 pt)

(b) The standardized variability among the group averages is higher than the estimate of the variability of the data within each group.

True (2 pts); **False** but explanation demonstrates some understanding of concepts (1 pt)

(c) A post-hoc pairwise analysis will identify that there is at least one pair of means that are significantly different.

True (1 pt)

4. (5 points)

Determine if the following statements are true or false. For each statement that is false, provide a brief explanation as to why it is false.

(a) If a given value (for example, the null hypothesized value of a parameter) is within a 95% confidence interval, it will also be within a 99% confidence interval.

False with valid explanation (2 pts)

(b) Decreasing the significance level (α) will increase the probability of making a Type 1 Error.

False with valid explanation (2 pts); with invalid explanation (1 pt)

(c) Suppose the null hypothesis is $\beta_1 = 0$ and we fail to reject H_0. Under this scenario, the true slope of x_1 is zero.

False with valid explanation (1 pt)

Section 2: Short answer questions

5. (4 points)

When computing a confidence interval for the slope, β_j, of a numeric predictor variable, x_j, in a multiple linear regression model, provide a reason why someone might prefer to use a bootstrap procedure rather than rely on the formula $\widehat{\beta}_j \pm t^*_{(n-k-1),\alpha/2} \times SE(\widehat{\beta}_j)$?

Non-normality of random error

6. (3 points)

If you could only use one measure (among the studentized residuals, leverage values, and Cook's distance values) to identify potentially influential data points, which would you choose and why?

Cook'd D because it integrates both studentized residuals and leverage information

For questions 7-9 consider the following random sample of $n = 250$ Minor League baseball hitters who were drafted for the Major League between the years 1992 and 2006. We are going to consider modeling the average number of times each hitter appeared at the plate per game (`ave_plate_counts`) as the response variable and player position as a categorical predictor (with levels: Catcher, First Base, Outfield, Second Base, Shortstop, Third Base). Below is the R summary output for this one-way ANOVA model.

```
##
## Call:
## lm(formula = ave_plate_counts ~ position, data = baseball_dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.97080 -0.16440  0.06234  0.30217  0.87561
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)          3.48288    0.07475  46.592  < 2e-16 ***
## positionFirst Base   0.42739    0.12242   3.491  0.00057 ***
## positionOutfield     0.19625    0.08726   2.249  0.02541 *
## positionSecond Base  0.28036    0.12074   2.322  0.02105 *
## positionShortstop    0.30913    0.10311   2.998  0.00300 **
## positionThird Base   0.38201    0.11772   3.245  0.00134 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4547 on 244 degrees of freedom
## Multiple R-squared:  0.07043,    Adjusted R-squared:  0.05138
## F-statistic: 3.697 on 5 and 244 DF,  p-value: 0.003039
```

7. (3 points)

(a) What are the error degrees of freedom based on this model? **244** (1.5 pts)

(b) What is the reference level? **catcher** (1.5 pts); if gave mean rather than level name (1 pt)

8. (6 points)

Suppose the average number of plate appearances per game is 3.72 over all 250 data points. What is the estimated group effect for Shortstop players?

Solution:

$$\hat{\mu}_{sh} = 3.482 + 0.309 \text{ (4 pts)}$$

$$\hat{\alpha}_{sh} = 3.482 + 0.309 - 3.72 \text{ (6pts)}$$

9. (4 points)

Consider two additional numeric predictors: the age each player was drafted (`age_drafted`) and their overall pick order when they were drafted (`overall_pick`). If we were to fit a regression model including each of the three predictor variables and an interaction between the two numeric variables, explain the meaning of the coefficient for the interaction term within the context of this data. (You should be able to answer this in no more than two sentences.)

Solution:

The relationship between drafting age and number of plate appearances depends on the player's draft pick number.

OR

The relationship between draft pick number and number of plate appearances depends on a player's age.

Section 3: Long answer questions

10. (9 points)

Suppose you have access to a data set on a random sample of Swarthmore students. The variables included in this data set are a numeric variable for the average amount of time (in hr) spent studying each week school is in session, a binary categorical variable distinguishing students who are double majors from those who aren't double majors, a numeric variable for each student's current, cumulative GPA, and a categorical variable indicating if a student has ever worked more than two student jobs in a semester or if the most number of student jobs they've worked in a semester is one or if they have not worked any student jobs for any semesters.

State a research question that can be answered with the overall F-test for each of the following models. Also provide a mathematical representation of the model and state the null hypothesis based on the notation you define for each model.

(a) a simple linear regression model;

(b) an ANOVA model;

(c) a multiple linear regression model (not SLR or ANOVA).

Rubric

For each part:

- 1 pt for valid model
- 1 pt for valid question
- 1 pt for clear/correct mathematical notation in H_0

11. (8 points)

Consider the ANOVA model for the baseball data you used in questions 7-8. Reference the R output on pg 5 and the plots on pg 10 to answer the following questions about this model.

(a) Check the conditions necessary for conducting a test to determine if the average plate appearance (per game) is significantly different for players of these six different positions.

(b) Write out in words and in symbols the hypotheses that would be tested in part (a). (Clearly define your notation.)

(c) What can you conclude about the test in part (b)? Write a paragraph discussing your conclusions and reference any relevant statistics and/or plots as part of your discussion.

Solution/Rubric:

(a) 3 pts

- group effects constant means that the relationship between players' positions and their number of plate appearances doesn't depend on any other variables
- must mention independence
- reference box plot of residuals to comment on constant variance
- reference normal quantile plot to comment on normality

(b) 3 pts

Must use clear, consistent notation but can be in MLR format, treatment means, or treatment effects format

(c) 2 pts

Conclusion should not indicate causality but may be generalized to population of all minor league players. P-value interpretation should be consistent with their assessment of the model assumptions.

12. (8 points)

Suppose two people are studying the historic data set about the amount of arsenic (Arsenic) in local wells. This data contains $n = 70$ observations from a random selection of well water samples from across the state. In addition to the levels of arsenic, the data also records the year the data was collected (Year) and the distance from the well to the nearest mining site (Miles).

Person A fits the following MLR model to the data:

$$Arsenic = \beta_0 + \beta_1 Year + \beta_2 Miles + \epsilon$$

and computes an adjusted R^2 value of 0.26.

Person B considers the following correlations:

$$Cor(Arsenic, Year) = \rho_1; \quad Cor(Arsenic, Miles) = \rho_2$$

and estimates each with their sample correlations $r_1 = 0.77$ and $r_2 = -0.34$. Are the two people's conclusions contradictory? Explain your answer.

Solution:

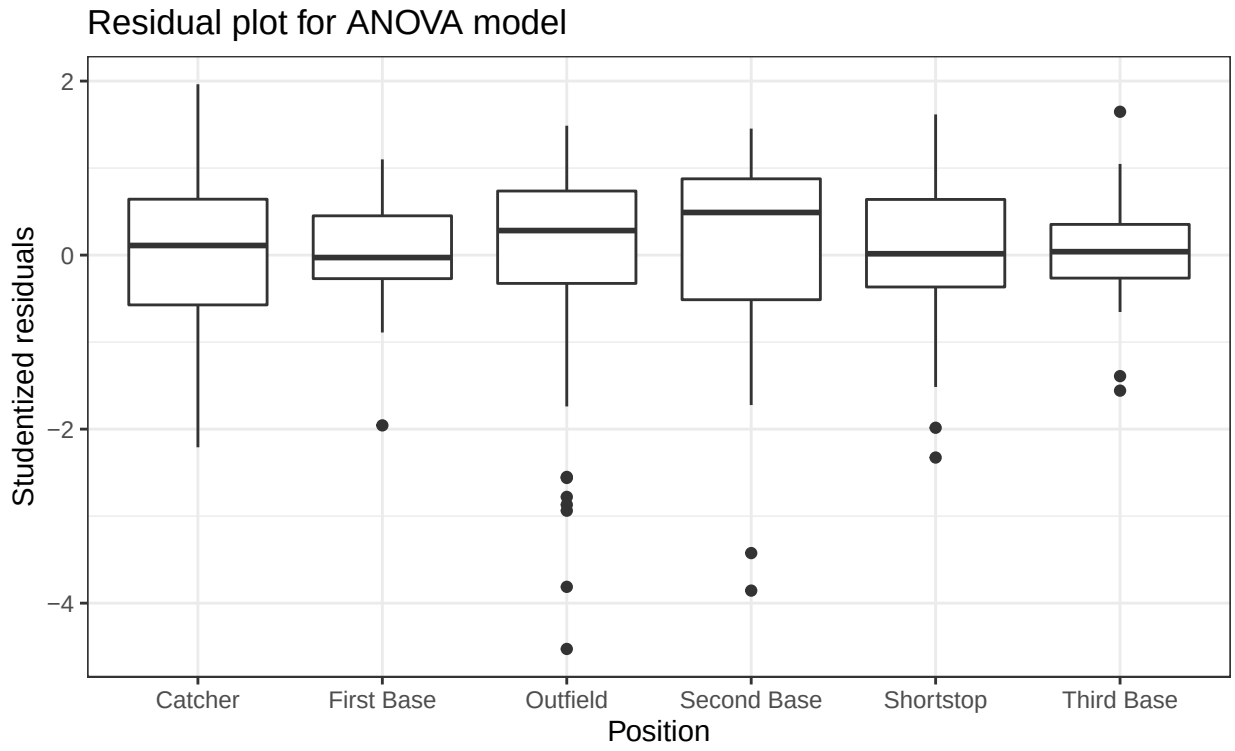
Responses should demonstrate understanding of

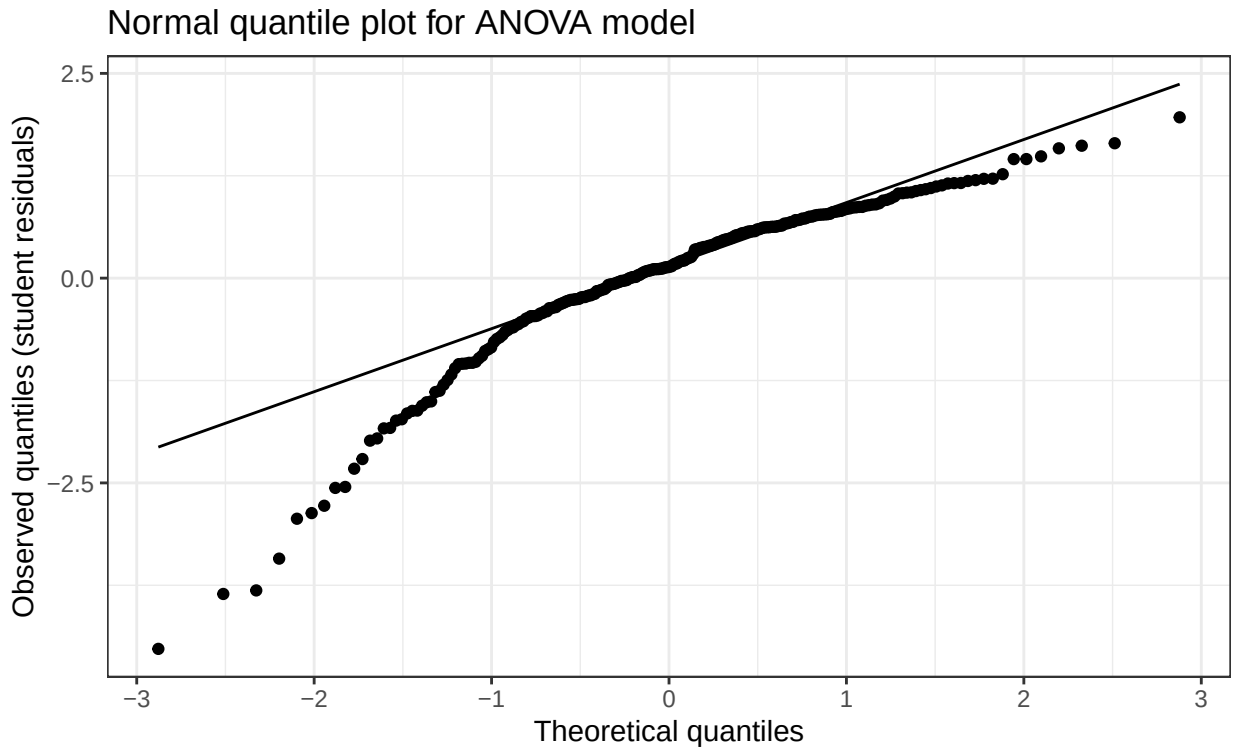
- demonstrate understanding of the differences between R^2 and correlation
- consider ways in which these results can happen

Section 4: Extra credit opportunity

If the response rate to my end of the semester evaluation form (on Moodle under Week 13 and 14) is at least 85% of our class size (over both sections), two percentage points will be added to everyone's Test 3 grade (up to 100 total possible points). **Hint:** You may not know how to or want to contact everyone in my class but you do know your group mates pretty well.

Baseball ANOVA Model





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