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Test 3

STAT 021

Swarthmore College

Do not flip this page until instructed to do so.

Test organization: There are 12 questions in total on this test and they are organized into three subsections; the first 4 questions are matching or True/False with explanation questions, the next 5 questions are free response short answer and should not require more than a sentence or two to answer. The last section contains 3 long answer free response questions that require more than a couple of sentences to answer fully. There are a total of 60 points possible on this test. The last section explains an extra credit opportunity. If you need additional scratch paper you may come to the front of the class and pick some up.

Instructions: Answer each question to the best of your ability and raise your hand if you are confused by any of the wording in the questions or suspect a typo. For the short and long answer questions show all your work and provide enough justification and/or explanation in order to get full credit or to be considered for partial credit. You do not need a calculator to evaluate any expressions. For any calculation problems, simply writing out the formula to find the answer will suffice.

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Take a deep breath.

You have prepared for this test and with a clear and well-rested mind, you are ready to show me what you have learned this semester. The purpose of this test is to measure your understanding of the material we have covered this semester. This is nothing more than a metric for me to evaluate your preparedness to think statistically at this particular moment in time and in this particular setting. This is not a perfect measure of your knowledge and does not predict your future statistical skills.

Section 1: Matching and True/False problems

1. (5 points)

Suppose we are modeling the weight of birds (in kg) as a linear function of a categorical predictor variable for bird type (with levels pigeon, sparrow, and finch) and a numeric predictor for bird age. Given a "full" model

$B_4 x_1$ $B_5 x_2$

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_3 + \beta_5 x_2 x_3 + \epsilon,$$

where $x_1 = \begin{cases} 1, & \text{if sparrow} \\ 0, & \text{otherwise} \end{cases}$, $x_2 = \begin{cases} 1, & \text{if finch} \\ 0, & \text{otherwise} \end{cases}$ and x_3 is the age of the bird (in months), match the questions below to their corresponding null hypotheses.

a) For newly hatched birds (of age zero months), is there a statistically discernible difference in the weights of these three different bird types?

b) Does the effect of age on a bird's weight depend on what type of bird it is? interaction

c) Given we are only comparing birds of the same age, is there a statistically significant difference in the mean weight of sparrows and pigeons?

d) Given we are only comparing pigeons^{x_1}, is the effect of age on a bird's weight statistically significant?

e) Is there statistically discernible evidence of a linear relationship between bird age and type and bird weight?

1. E $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
2. C $H_0 : \beta_1 = 0$
3. D $H_0 : \beta_3 = 0$
4. B $H_0 : \beta_4 = \beta_5 = 0$
5. A $H_0 : \beta_1 = \beta_2 = 0$

b) $\cancel{B_0 + B_1 + B_4 x_3}$

Pigeons

$(B_0 + B_1) + B_3 x_3 +$

a) Age $= 0$

$B_0 + B_1 x_1 + B_2 x_2$

2. (5 points)

Determine which of the following statements about MLR models are true and false. For each statement that is false, provide a brief explanation as to why it is false.

(a) If predictors are collinear, then removing one variable will have no influence on the point estimate of another variable's coefficient.

False. The point estimate is based on the fact that the other predictor is included in the model.

False. This will only be true if x_1 is the only predictor in the model.

(b) Suppose a numerical variable x_1 has a coefficient of $\beta_1 = 2.5$ in the multiple regression model. Suppose also that the first observation has a value of $x_1 = 7.2$, the second observation has a value of $x_1 = 8.2$, and these two observations have the same values for all other predictors. The predicted value of the second observation will be 2.5 units higher than the prediction of the first observation based.

~~True. One unit increase in x_1~~ | x_1 7.2, 8.2, ; $\hat{y}$ 0 2.5

(c) As the total sample size increases, the degrees of freedom for the residuals increases as well.

≥ Mod ; ≤ ~~Mod~~ E ; = Tot

True. $DfE = n-k-1$

3. (5 points)

Determine which of the following statements about ANOVA models are true and false. For each statement that is false, provide a brief explanation as to why it is false.

If the null hypothesis that the means of four groups are all the same is rejected from an ANOVA model and overall F-test at a 5% significance level, then ...

(a) We can then conclude that all the means are different from one another.

False. We can most likely conclude that at least one group has a different mean.

(b) The standardized variability among the group averages is higher than the estimate of the variability of the data within each group. = group-group ≥ unit-unit

~~MSmod ≥~~ True.

(c) A post-hoc pairwise analysis will identify if there is at least one pair of means that are significantly different.

True

4. (5 points)

Determine if the following statements about statistical modeling are true or false, and explain your reasoning. If false, state how it could be corrected.

(a) If a given value (for example, the null hypothesized value of a parameter) is within a 95% confidence interval, it will also be within a 99% confidence interval.

True

(b) With large sample sizes, even small differences between the null value and the observed point estimate will be identified as statistically significant.

~~True~~ ~~False~~ True. n↑ df↑

(c) Correlation is a measure of the association between any two variables.

True

Section 2: Short answer questions

5. (4 points)

State two reasons why we might consider transforming the response variable to fit an appropriate multiple linear regression model to some data.

1. Non ~~confident~~ constent variance among residuals

2. Evidence that the relationship between the predicter and response is not linear, but shaped otherwise such as a $\sqrt{\ }$ or quadratic

6. (3 points)

If you could only use one measure (among the studentized residuals, leverage values, and Cook's distance values) to identify potentially influential data points, which would you choose and why?

Cooks distance because it takes into consideration both the studentized residuals and the leverage. Also, stdresid shows extreme values of the response, leverage shows extreme values in the predictor. Cooks D shows extremity in the combination of predicter and response.

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For questions 7-9 consider the following random sample of $n = 246$ online shoppers. We are going to model the average price (in US dollars) (`price`) as a linear function of the item's type (a categorical predictor with levels: trousers, skirts, blouses, on_sale). Below is the R summary output for this one-way ANOVA model.

4 levels

```
##
## Call:
## lm(formula = price ~ type, data = retail_dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -21.946  -8.946   0.893   6.054  35.054
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)     41.946      1.512  27.750  < 2e-16 ***
## typeon_sale     -5.438      2.128  -2.555  0.01123 *
## typeskirts       9.161      2.138   4.285 2.64e-05 ***
## typetrousers     5.937      1.987   2.988  0.00309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.31 on 242 degrees of freedom
## Multiple R-squared:  0.1913, Adjusted R-squared:  0.1813
## F-statistic: 19.09 on 3 and 242 DF,  p-value: 3.825e-11
```

$n = 246$

k 3

n-k-1 242

Tot 245

7. (3 points)

(a) What are the error degrees of freedom based on this model? 242

(b) What is the reference level?

blouses

8. (6 points)

price

Suppose the average ~~number of plate appearances~~ per game is 44.63 over all 246 data points. What is the estimated group effect for clothing type trousers?

5.937 ?

D = d_i

$B_0 + B_1 \text{ sale} + B_2 \text{ skirt} + B_3 \text{ trouser}$

$+ B_4 \text{ release} + B_5 \text{ produce}$

$+ B_6 \text{ release} \cdot \text{produc}$

9. (4 points)

Consider two additional numeric predictors: the amount of time the item has been available for purchase on this retailer's website, `release`, measured in weeks and the production cost associated with each item, `produce_cost`, measured in US dollars. If we were to fit a regression model including each of the three predictor variables (including `type`) and an interaction between the two numeric variables, explain the meaning of the coefficient for the interaction term within the context of this data. (You should be able to answer this in no more than two sentences.)

Regardless of the type of item, this coefficient estimates the effect of the combination of release and produce cost on weight.

Section 3: Long answer questions

10. (9 points)

x_1 age
x_2 tenure
x_3 sal
x_4 status

Suppose you have access to a data set on a random sample of Swarthmore faculty. The variables included in this data set are a numeric variable for each person's age, a binary categorical variable distinguishing faculty who are tenured from those who are not, a numeric variable for each faculty member's starting salary, and a categorical variable indicating if the faculty member attended a liberal arts college, or a university, or entered the work force after graduating high school.

State a research question that can be answered with the overall F-test for each of the following models. Also provide a mathematical representation of the model and state the null hypothesis based on the notation you define for each model.

(a) a simple linear regression model;

(b) an ANOVA model;

(c) a multiple linear regression model (not SLR or ANOVA).

A) Does ~~starting~~ starting salary depend on the persons age?

$H_0: B_1 = 0$

$Y = B_0 + B_1 x_1 + \mathcal{E}$

SLR, F Test = t Test

$\widehat{Salary} = \hat{B}_0 + \hat{B}_1 (Age)$

Cost = $B_0 + B_1$

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11. (8 points)

Consider the ANOVA model for the retail data you used in questions 7-8. Reference the R output on pg 5 and the plots on pg 10 to answer the following questions about this model.

(a) Check the conditions necessary for conducting a test to determine if the average cost of the purchased items are significantly different for different types of clothing type. (You do not need to check the zero mean or linearity conditions but you do need to describe what it means for the group effects to be constant in this context.)

(b) Write out in words and in symbols the hypotheses that would be tested in part (a). (Clearly define your notation.)

(c) What can you conclude about the test in part (b)? Write a paragraph discussing your conclusions and reference any relevant statistics and/or plots as part of your discussion.

A) Blouses shows a skewed distribution, so the assumption ~~is not met~~ of normality ~~is not met.~~ within this group is not met. The normal quantile plot is mostly normal with large tails, showing that overall, there is likely a constant effect across groups.

B) Ho:

$U_1 - U_2 = 0$

$U_1 - U_3 = 0$

$U_1 - U_4 = 0$

$U_2 - U_3 = 0$

$U_{2} - {}_{4} = 0$

$U_3 - U_4 = 0$

H_A: Atleast one ~~is~~ is not zero

Ho: ~~All differences~~ there is no diff. between the ~~effects of groups~~ mean price of groups

H_A: Atleast one group has a different ~~effect~~ ~~on price~~ average price

price = cost ✓

c) ~~ESBZ~~ Since the overall F test for the model has a small enough p-value we can compute LSD to determine which means are different

12. (8 points)

Suppose two people are studying the historic data set about the amount of arsenic (Arsenic) in local wells. This data contains $n = 70$ observations from a random selection of well water samples from across the state. In addition to the levels of arsenic, the data also records the year the data was collected (Year) and the distance from the well to the nearest mining site (Miles).

Person A fits the following MLR model to the data:

$$\frac{SS\,Mod}{SS\,TOT} = .26$$

$$Arsenic = \beta_0 + \beta_1 Year + \beta_2 Miles + \epsilon$$

and computes an adjusted R^2 value of 0.26.

Person B considers the following correlations:

$$Cor(Arsenic, Year) = \rho_1; \quad Cor(Arsenic, Miles) = \rho_2$$

and estimates each with their sample correlations $r_1 = 0.77$ and $r_2 = -0.34$. Are the two people's conclusions contradictory? Explain your answer.

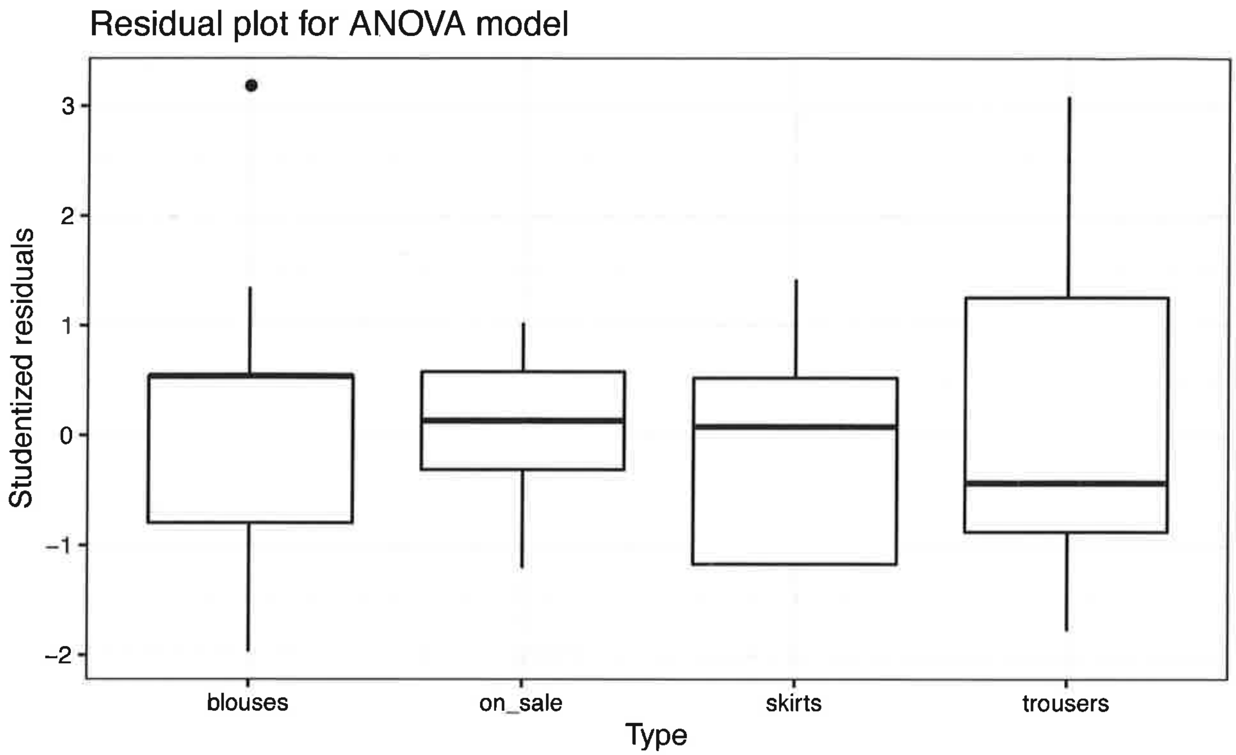
Not necessarily. The first person concludes that the predictors Year, and Miles account for approximately 26% of the variation in the response variable, arsenic. This person makes no reference to the direction or strength of either predictor. Person B identifies that Year has a strong positive assosuciation with Arsenic, and Miles has a moderate negative connection with Arsenic. Person B gives no info on the efficacy of the two predictors used together.

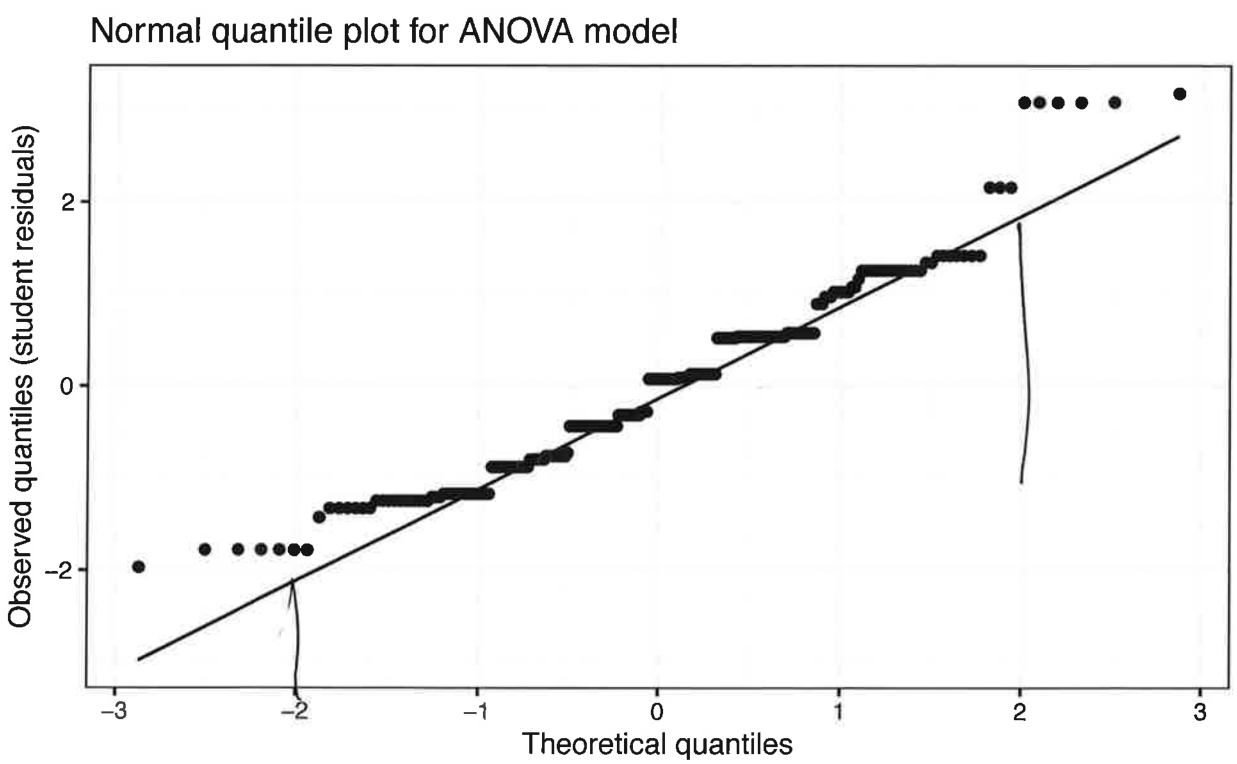
Section 4: Extra credit opportunity

If the response rate to my end of the semester evaluation form (on Moodle under Week 13 and 14) is at least 85% of our class size (over both sections), two percentage points will be added to everyone's Test 3 grade (up to 100 total possible points). **Hint:** You may not know how to or want to contact everyone in my class but you do know your group mates pretty well.

Ipontrc 2

Retail ANOVA Model





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10 B) Does ~~starting~~ the average age among professors who are tenured differ from the average age of professors who arent tenured?

$$Y = \beta_0 + \beta_1 x_1 + \varepsilon \quad \begin{cases} x_1 & 1 \text{ if tenured} \\ & 0 \text{ if not} \end{cases}$$

$$\hat{Age} = \hat{\beta}_0 + \hat{\beta}_1 (\text{tenured})$$

~~$H_0: \beta_0 = \beta_1 = 0$~~ $H_0: \beta_0 - \beta_1 = 0$

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c) In an MLR, an overall F-Test alone is unreliable. ~~However: Is there evidence to conclude that atleast one~~

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