Stat 61 In-Class Worksheet

Original group members:

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Suppose we observe $X_1, \ldots, X_n$ IID data points from a $N(\mu, 0.4^2)$ distribution where $n = 16$ and we wish to test $H_0 : \mu = 37$.

1. For a simple alternative, $H_1 : \mu = 36.8$ with $\alpha = 0.025$, what is the rejection region A_α? What is the power of this test?

2. How does the rejection region change if we increase n to $n = 64$ but keep everything else the same?

3. How would the power change if we decreased α but kept everything else the same?

4. How would A_α change if we instead tested against $H_1 : \mu = 36$?

The Neyman-Pearson lemma implies that, for testing $H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1$ with $\mu_1 < \mu_0$, the test that rejects for $\bar{X} < c_\alpha$ is most powerful of all tests with comparable α.

5. Is the test in (1) above uniformly most powerful for any pair of hypotheses? If so which ones and why?

6. For the test in (1) above, suppose you observe data where $\bar{x}_{obs} = 36.85$. What is the p-value for this one-sided test? (I.e. what is the smallest α level that would lead to rejecting H_0?)

7. For testing $H_0 : \mu = \mu_0$ vs $H_1 : \mu < \mu_0$ at an $\alpha = 0.025$ level, if we observe $\bar{x}_{obs} = 36.85$ ($n = 16$), what are the range of μ_0 values that would NOT be rejected? (I.e. find a one-sided confidence interval for μ.)