

# SYSTEM MODELING AND ANALYSIS

## CHAPTER 3 System Representation

# Content

3.1

- Important definitions (1 hour)

3.2

- Techniques of simplifying block diagrams. (2 hours)

3.3

- Signal flow graphs (0.5 hours)

3.4

- Changing block diagrams to signal flow graphs and vice versa (0.5 hours)

3.5

- Mason's Rule and example questions (1 hour)

# Introduction

- A control system consists of the inter-connection of subsystems.
- A more complicated system will have many inter-connected subsystems.
- For the purpose of analysis, we want to represent the multiple subsystems as a single transfer function.
- A system with multiple subsystems can be represented in two ways:
  - Block diagrams
  - Signal flow graphs

# 3.1

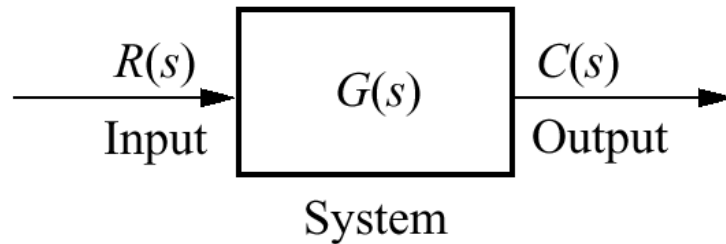
## IMPORTANT DEFINITIONS

# BLOCK DIAGRAMS

- The basic components in a block diagram are:
  - Signals

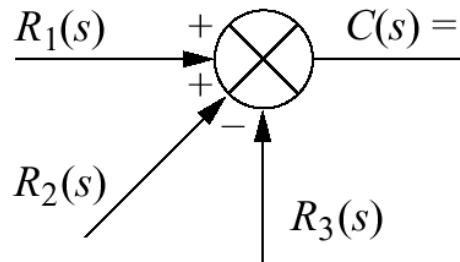


The direction of signal flow is shown by the arrow.

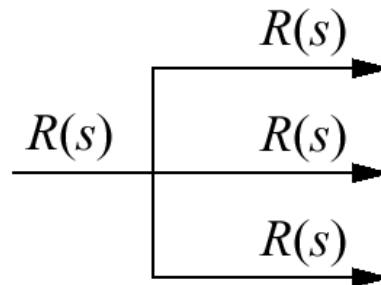


The system block represented by a transfer function.

## – Summing junctions



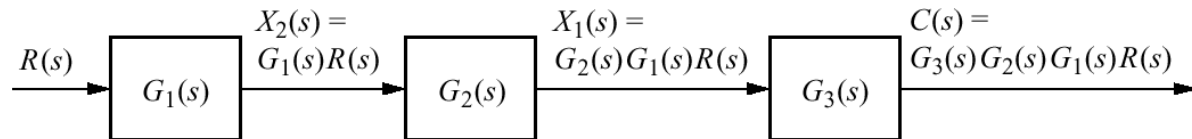
The signals are added/subtracted algebraically  
Pickoff points



The same signal is distributed to other subsystems

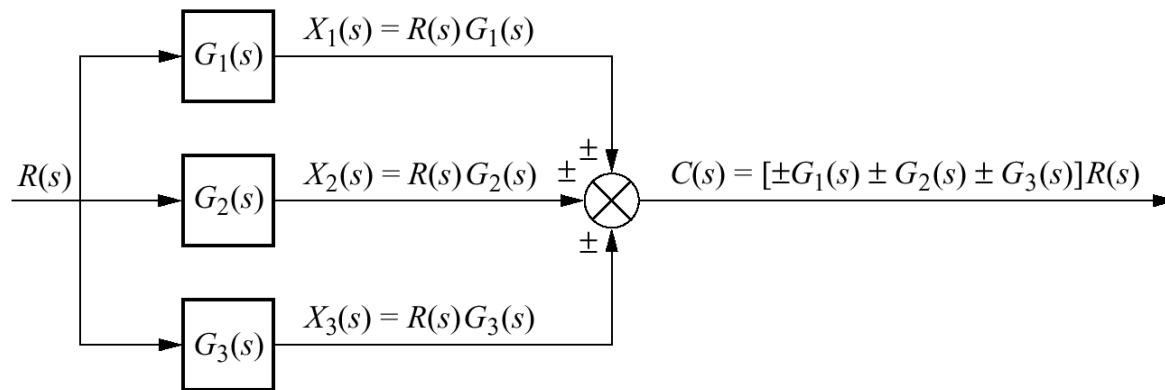
- The subsystems in a block diagram are normally connected in three forms:

- Cascade form



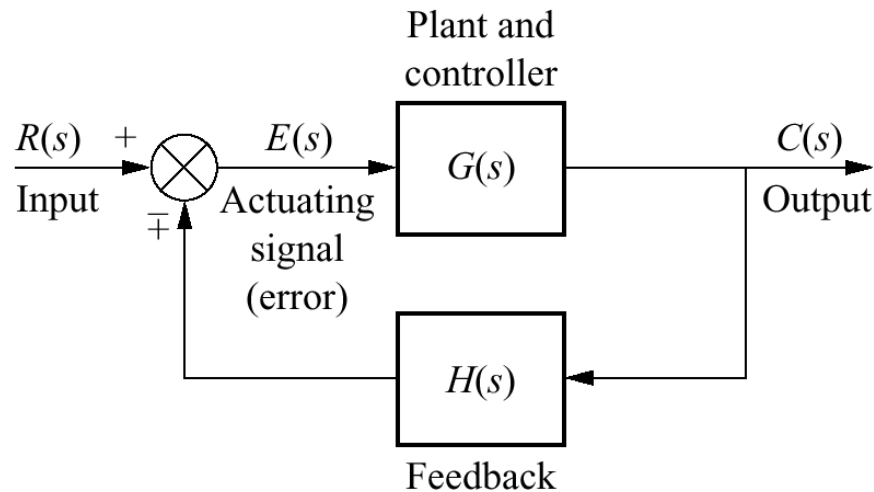
- The block diagram can be reduced into a single block by multiplying every block to give:

- Parallel form



- The block diagram can be reduced into a single block by summing every block to give:

### - Feedback form



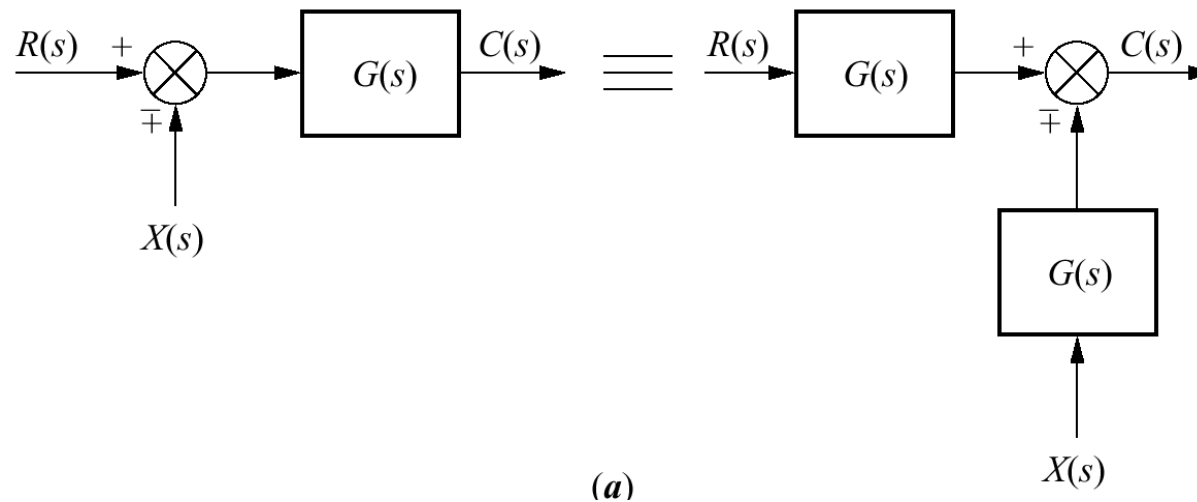
- closed-loop transfer function:
- open-loop transfer function:



## 3.2

# TECHNIQUES OF SIMPLIFYING BLOCK DIAGRAMS

- It is not always apparent to get block diagrams in the familiar forms.
  - We have to move blocks to get the familiar forms in order to be able to reduce the block diagram into single transfer function
1. Moving the summing junction to the front of a block



# Primary Aim:

- It is not always apparent to get block diagrams in the familiar forms.
- We have to move blocks to get the familiar forms in order to be able to reduce the block diagram into single transfer function

1

- Moving the summing junction to the front of a block

2

- Moving the summing junction to the back of a block

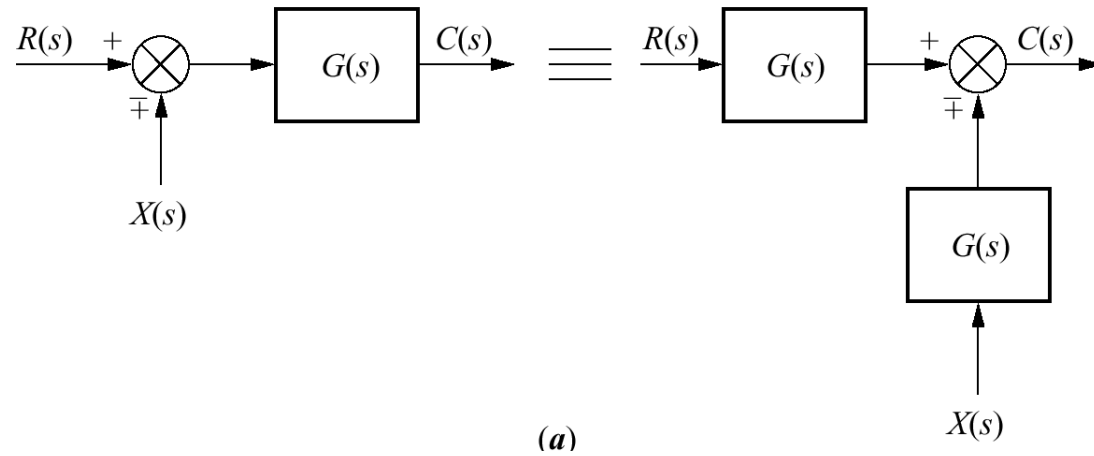
3

- Moving pick-off point to the front of a block

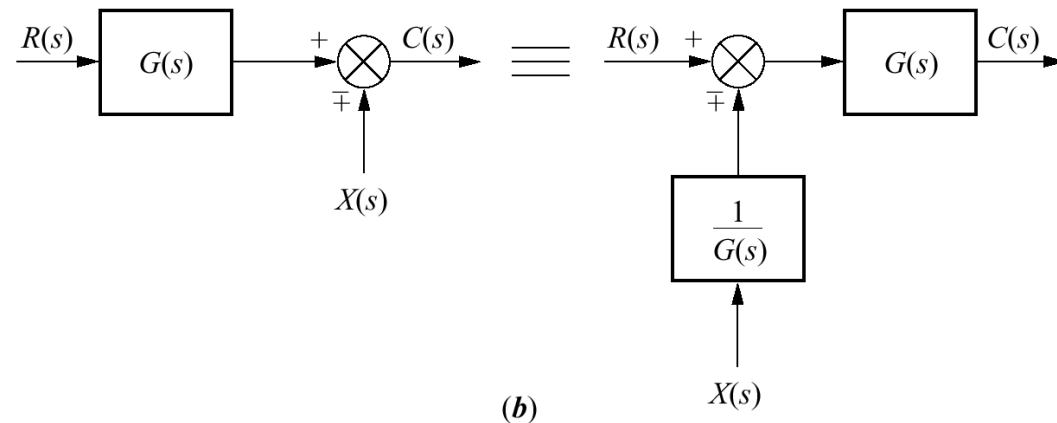
4

- Moving pick-off point to the back of a block

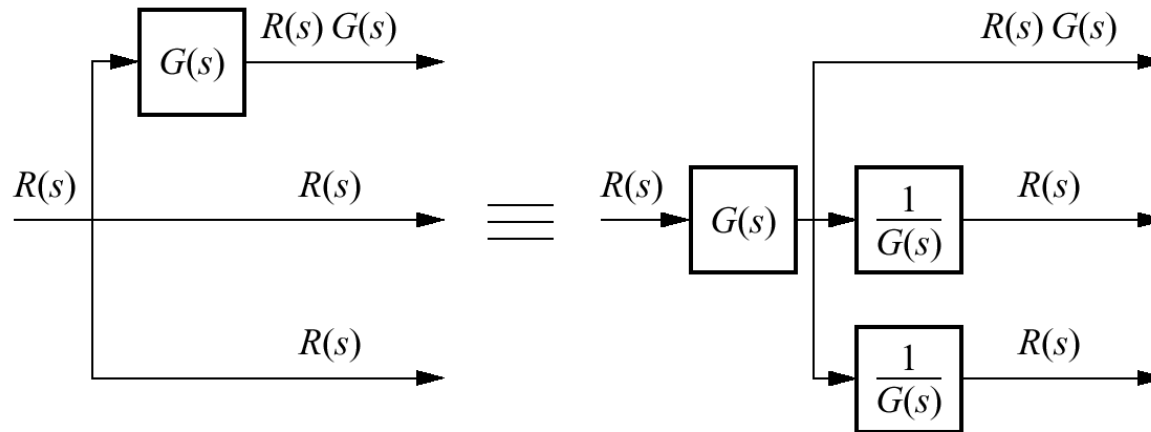
- Moving the summing junction to the front of a block



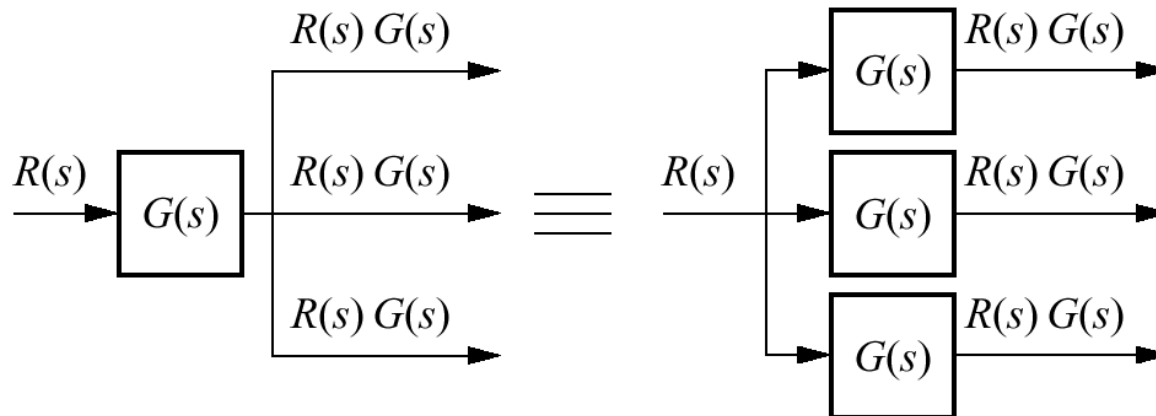
- Moving the summing junction to the back of a block



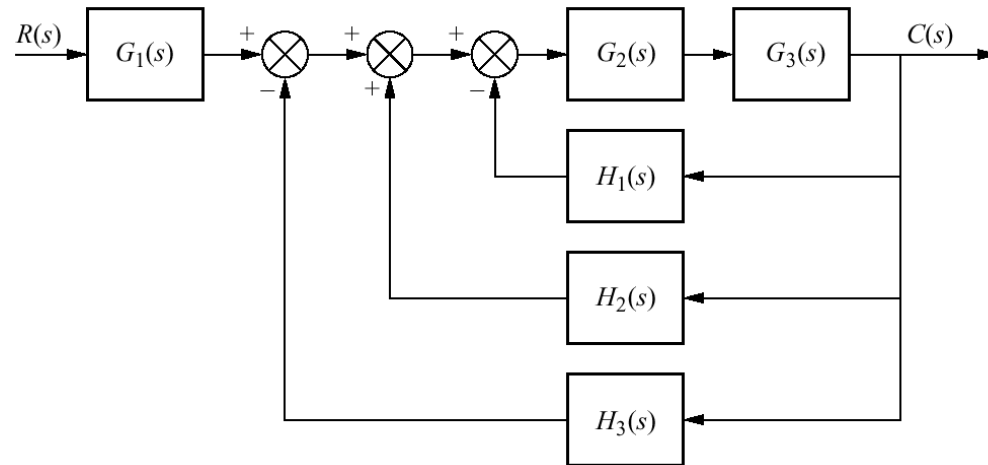
- Moving pick-off point to the front of a block



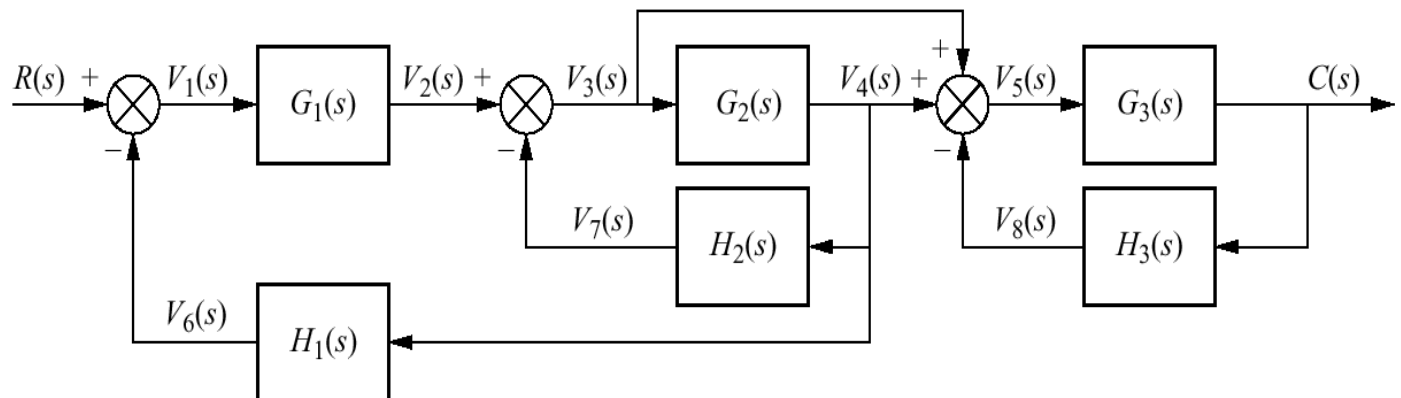
- Moving pick-off point to the back of a block



- Example (5.1): Reduce the following block diagram into a single transfer function



- Example (5.2): Reduce the following block diagram into a single transfer function.



## 3.3

# SIGNAL FLOW GRAPHS



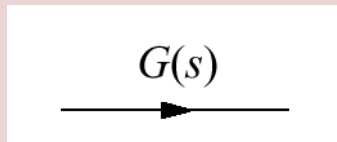
# Main Purpose

- An alternative to block diagrams.
- It consists only of branches to represent systems and nodes to represents signals.

# 2 Types of Signal Flow Graph

## 1. Branches

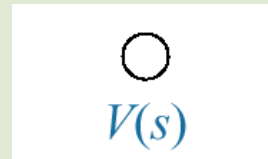
- Represented by a line with arrow showing the direction of signal flow through the system



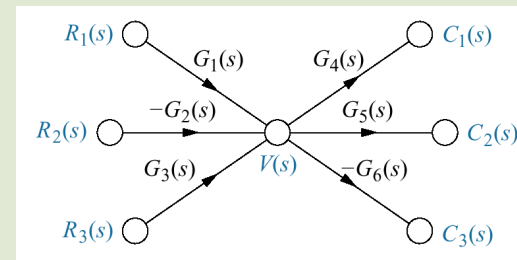
- The transfer function is written close to the line and arrow

## 2. Nodes

- Represented by a small circle with the signal's name is written adjacent to the node



- Example:



# 3.4

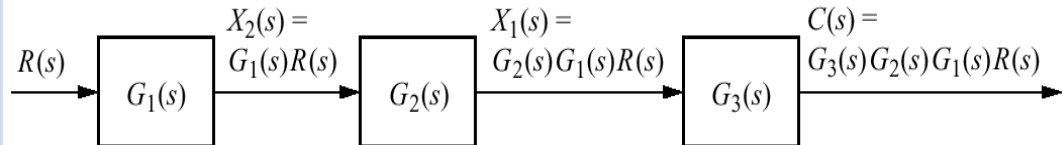
## CHANGING BLOCK DIAGRAMS TO SIGNAL FLOW GRAPHS AND VICE VERSA

## Converting block diagrams into signal-flow graphs

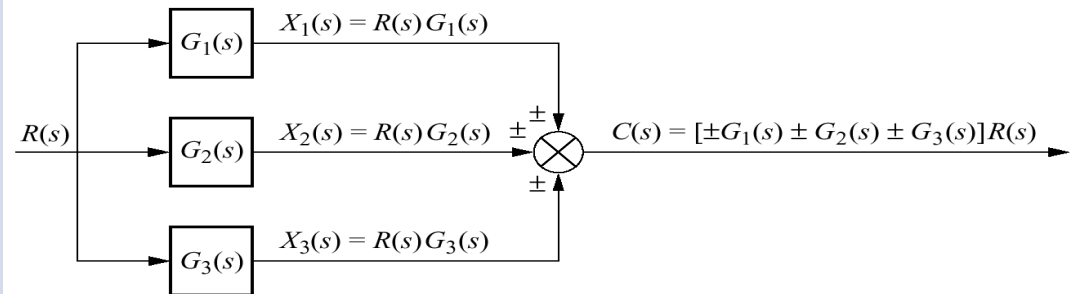
- The block diagrams in cascade, parallel and feedback forms can be converted into signal-flow diagrams.
- We can start with drawing the signal nodes, and then interconnect the signal nodes with system branches.

# Type of Changing Form

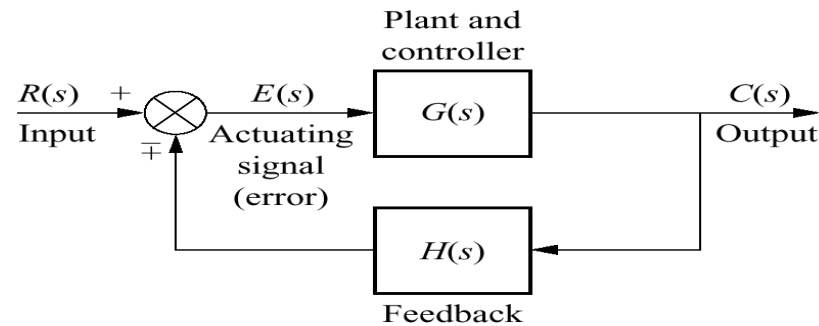
Cascade form



Parallel form

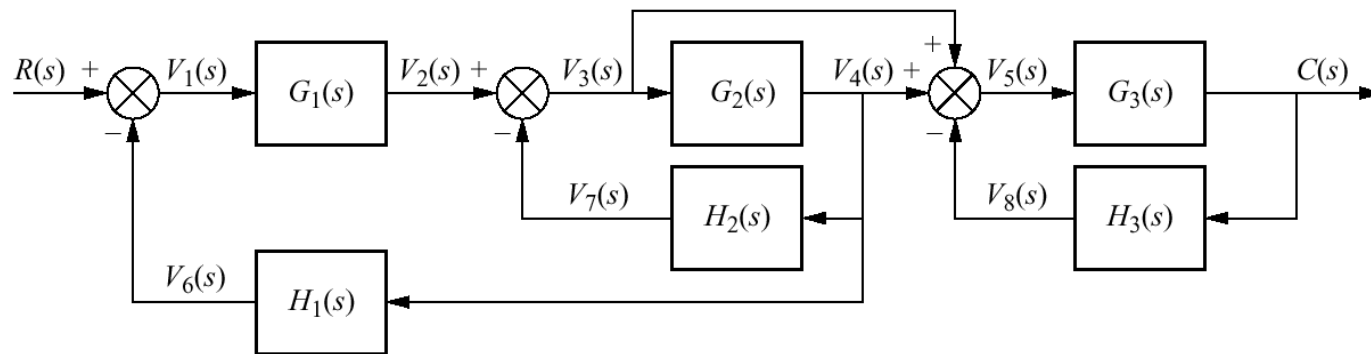


Feedback form



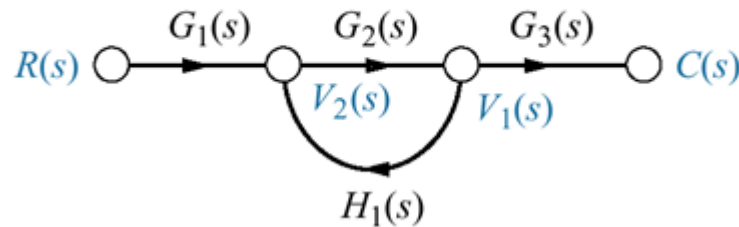
# Example

- Convert the following block diagram



# 3.5 MASON'S RULE

- Mason's rule is used to get the single transfer function in the signal-flow graph.
- We need to know some important definitions:



- **Loop** – starts and ends at the same node.
- **Loop gain** – the product of branch gains found by traversing a loop.
- **Forward-path** – a path from the input node to the output node of the signal-flow graph in the direction of signal flow.
- **Forward-path gain** – the product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.



- Non-touching loops – loops that do not have any nodes in common.
- Non-touching loops gain – the product of loop gains from non-touching loops taken two, three, four, or more at a time.
- Mason Rule's

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

$k$  = number of forward path

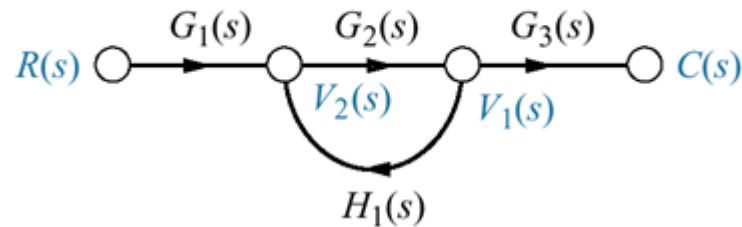
$T_k$  = the  $k$ -th forward-path gain

$\Delta = 1 - \text{S loop gains} + \text{S non-touching loops gain (taken 2 at a time)} - \text{S non-touching loops gain (taken 3 at a time)} + \text{S non-touching loops gain (taken 4 at a time)} - \dots$

$\Delta_k = \Delta - \text{S loop gain terms in } \Delta \text{ that touch the } k\text{-th forward path.}$   
In other words,  $\Delta_k$  is formed by eliminating from  $\Delta$  those loop gains that touch the  $k$ -th forward path.

# Example

- Find the transfer function,  $C(s)/R(s)$  of the following signal-flow graph,



END OF CHAPTER 3