Revisiting the Geodesic Distance-based Approach for Sentinel-1 GRD Product

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Abstract

Recently, the author of this note with others generalized the Geodesic Distance-based approach from quad polarimetric (QP) mode Synthetic Aperture Radar (SAR) data to other partial polarimetric mode data as well as ESA Sentinel-1 dual polarimetric (DP) Ground Range Detected (GRD) product [2]. In this note the author revisits the particular Section III.C. titled "*GRD Product: Only one GD-derived parameter*" which needs further clarification for the readers to better appreciate the work and use the open source codes associated with it.

1 Background

The geodesic distance (GD) between two covariance matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ defined in equation (6) in [2] is given as:

$$\mathrm{GD}(\mathbf{C_1}, \mathbf{C_2}) = \frac{2}{\pi} \cos^{-1} \left(\frac{\mathrm{Tr}(\mathbf{C_1}^{*T} \mathbf{C_2})}{\sqrt{\mathrm{Tr}(\mathbf{C_1}^{*T} \mathbf{C_1})} \sqrt{\mathrm{Tr}(\mathbf{C_2}^{*T} \mathbf{C_2})}} \right) \tag{1}$$

where superscript $(\cdot)^*$ is the complex conjugate operator, $(\cdot)^T$ is matrix transpose, and Tr is the trace operator.

For the Sentinel-1 dual poalrimetric GRD data a proxy for the covariance matrix is defined in equation (8) in [2]. We recall the same below:

$$\mathbf{C}^D := \begin{bmatrix} \sigma^0_{XX} & 0 \\ 0 & \sigma^0_{XY} \end{bmatrix} \tag{2}$$

where X and $Y(\neq X)$ can be H or V and σ^0_P stands for the SAR backscatter corresponding to polarimetric channel P.

The three roll-invariant parameters defined in case of quad polarimetric mode SAR data (QP) [3] using Kennaugh matrices have been expressed here in terms of polarimetric covariance matrices in Table 1 (as Table IV in [2]).

Table 1: GD-based roll-invariant parameters in QP

Parameter	Definition
α_{GD}	$90^\circ \times \mathrm{GD}(\mathbf{C}, \mathbf{C}_{th})$
τ_{GD}	$45^\circ \times (1 - \sqrt{\mathrm{GD}(\mathbf{C}, \mathbf{C}_{lh}) \times \mathrm{GD}(\mathbf{C}, \mathbf{C}_{rh})})$
P_{GD}	$\{\frac{3}{\pi}\cos^{-1}\left(0.5 \times \mathrm{Span}/\sqrt{\{Tr(\mathbf{C}^{*T}\mathbf{C})\}}\right)\}^2$

Later in Section III.C. expression for α_{GD} is presented as:

$$\alpha_{\mathrm{GD}} = 90^\circ \times \frac{2}{\pi}\cos^{-1}\left(\frac{\sigma^0_{XX}}{\sigma^0_{XX} + \sigma^0_{XY}}\right) \tag{3}$$

In retrospective, the expression in equation (10) in [2] also shown here in equation (3) *is not in accordance with the formulation in equation* (1) *and Table 1 where the denominator in the argument of* $\cos^{-1}(\cdot)$ *has been intentionally modified. This change also has effect in concluding that* τ_{GD}, P_{GD} *are constants. The author feels that this needs further clarification for the readers to better appreciate the work and use the open source codes associated with it.*

2 The Solution

To remedy this situation, we first derive the exact expression for GD for Sentinel 1A GRD data and the three parameters α_{GD}, τ_{GD} and P_{GD}. And later explain why a modification was justifiably carried out. Given two GRD observations, let the corresponding matrices be $\mathbf{C}_1^D$ and $\mathbf{C}_2^D$ respectively. We further define them as follows:

$$\mathbf{C}_1^D = \begin{bmatrix} \sigma^{0,1}_{XX} & 0 \\ 0 & \sigma^{0,1}_{XY} \end{bmatrix} \tag{4}$$

$$\mathbf{C}_2^D = \begin{bmatrix} \sigma^{0,2}_{XX} & 0 \\ 0 & \sigma^{0,2}_{XY} \end{bmatrix} \tag{5}$$

where the second superscript in the polarimetric channel backscatter term denotes the the nominal index. We substitute these matrices in equation (1) to obtain the GD expression for GRD data as follows:

$$\mathrm{GD}(\mathbf{C}_1^D, \mathbf{C}_2^D) = \frac{2}{\pi}\cos^{-1}\left(\frac{\mathrm{Tr}(\mathbf{C}_1^{D*T}\mathbf{C}_2^D)}{\sqrt{\mathrm{Tr}(\mathbf{C}_1^{D*T}\mathbf{C}_1^D)}\sqrt{\mathrm{Tr}(\mathbf{C}_2^{D*T}\mathbf{C}_2^D)}}\right) \tag{6}$$

$$= \frac{2}{\pi}\cos^{-1}\left(\frac{\sigma^{0,1}_{XX}\sigma^{0,2}_{XX} + \sigma^{0,1}_{XY}\sigma^{0,2}_{XY}}{\sqrt{(\sigma^{0,1}_{XX})^2 + (\sigma^{0,1}_{XY})^2}\sqrt{(\sigma^{0,2}_{XX})^2 + (\sigma^{0,2}_{XY})^2}}\right) \tag{7}$$

Note σ_0 *values are real. For other polarimetric modes where GRD data is possible, the XX and XY may be two distinct polarimetric channels instead.*

Next we present the scattering vector k_L and their 1-look covariance matrices for canonical scatterers like trihedral and helix scatterers in QP mode data as shown in the Table 2. The relationship between scattering vector k_L corresponding covariance matrix $\mathbf{C}$ is given by

$$\mathbf{C} = \langle k_L {k_L}^{*T} \rangle \tag{8}$$

where $\langle \cdot \rangle$ denote the ensemble averaging [1]. However, there is no ensemble averaging in 1-look cases.

Table 2: Covariance Matrix for Canonical Scatterers[1]

	Trihedral ($\mathbf{C}_{th}$)	R/L Helix ($\mathbf{C}_{rh/lh}$)
QP	$k_L = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$	$k_L = \begin{pmatrix} 1 \\ \mp\sqrt{2}i \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & \mp\sqrt{2}i & -1 \\ \pm\sqrt{2}i & 2 & \mp\sqrt{2}i \\ -1 & \pm\sqrt{2}i & 1 \end{pmatrix}$
DPH	$k_L = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$k_L = \begin{pmatrix} 1 \\ \mp i \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & \mp 1i \\ \pm 1i & 1 \end{pmatrix}$
DPV	$k_L = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$k_L = \begin{pmatrix} -1 \\ \mp i \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & \pm 1i \\ \mp 1i & 1 \end{pmatrix}$

Now $\mathbf{C}^D_{th}$ or $\mathbf{C}^D_{lh/rh}$ for S1 GRD data can be obtained from the respective covariance matrices *by setting the off-diagonal entries to zero* shown in Table 3.

Table 3: Reference matrices in case of S1 GRD

	Trihedral ($\mathbf{C}^D_{th}$)	R/L Helix ($\mathbf{C}^D_{rh/lh}$)
DPH	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
DPV	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Now we derive the three GD-derived parameters for DP mode S1 GRD observation $\mathbf{C}^D$ (ref. equation (2)) using Table 1 and equation (6):

$$\alpha_{\text{GD}} = 90^\circ \times \frac{2}{\pi} \cos^{-1}\left(\frac{\sigma^0_{XX}}{\sqrt{(\sigma^0_{XX})^2 + (\sigma^0_{XY})^2}} \right)$$

$$\tau_{\text{GD}} = 45^\circ \times \left(1 - \underbrace{\frac{2}{\pi} \cos^{-1}\left(\frac{\sigma^0_{XX} + \sigma^0_{XY}}{\sqrt{2}\sqrt{(\sigma^0_{XX})^2 + (\sigma^0_{XY})^2}} \right)}_{\text{squareroot over squared}} \right)$$

$$P_{\text{GD}} = \{\frac{3}{\pi} \times \cos^{-1}\left(\frac{0.5 \times \text{Span}}{\sqrt{(\sigma^0_{XX})^2 + (\sigma^0_{XY})^2}} \right)\}^2$$

[2]where Span $= \sigma^0_{XX} + \sigma^0_{XY}$.

3 Discussion on GRD Product

The Figure 1 shows reference RGB, Span in decibel scale (dB), and the three GD-derived parameters computed on a real Sentinel-1 Extra Wide mode GRD format data containing primarily sea-ice and open water used in [2]. The ranges of the GD-derived parameters are clipped between 2nd and 98th percentile for better contrast within the scene.

Additionally, it may be noted that the effect of HH channel is dominant in comparison to HV channel in sea ice/open water studies. Hence, the τ_{GD} appears to be predominantly similar in spread of values on the same color scale but in respective ranges. However, in general, it principally different from α_{GD}.

The information conveyed by τ_{GD} and P_{GD} is also similar because the expressions containing σ^0_P values in both of them differ by a constant. So it captures the same variability over the GRD data.

The P_{GD} for GRD data attributes the open water correctly towards higher value i.e., close to pure scattering in comparison to sea ice which has relatively lower values. It may be noted that this preservation of polarimetric property may be deceptive because the original definition of P_{GD} for Quad Polarimetric mode (QP) emerged from a necessary but insufficient condition for determining the coherent nature of polarimetric backscatter from a target [3]. This requires the off-diagonal elements of covariance matrix with their phases, which is absent in the GRD format case.

Hence, the GD-derived parameters in GRD case takes advantage mainly of the imbalance between real σ^0_{XX} and σ^0_{XY} values. Otherwise, the polarimetric content from QP to partial polarimetric mode to GRD format data is at a minimum in a relative comparison. Nonetheless, they can be explored for various Earth Observation applications.

4 Modification of Expression

Taking cue from the above discussion, it is enough to work with any one of the GD-derived parameters for identifying sea-ice type/open water. In the paricular case of study of sea ice using Sentinel-1 dual polarimetric GRD product data, the expression for α_{GD} has been modified to a computationally simpler form.

The trick used in [2] is to change the denominator of the argument for $\cos^{-1}(.)$ from $\sqrt{(\sigma^0_{XX})^2 + (\sigma^0_{XY})^2}$ to simply Span $= \sigma^0_{XX} + \sigma^0_{XY}$ for all the three GD-derived parameters. After this step, the *modified* $\alpha_{\mathbf{GD}}$ is the only varying parameter, while the *modified* $\tau_{\mathbf{GD}}$ and $P_{\mathbf{GD}}$ parameters become constants. *This is the missing piece in the discussion in [2] without which the readers could easily confuse.* To avoid this, we explicitly

[2]MATLAB codes are accessible from this link.

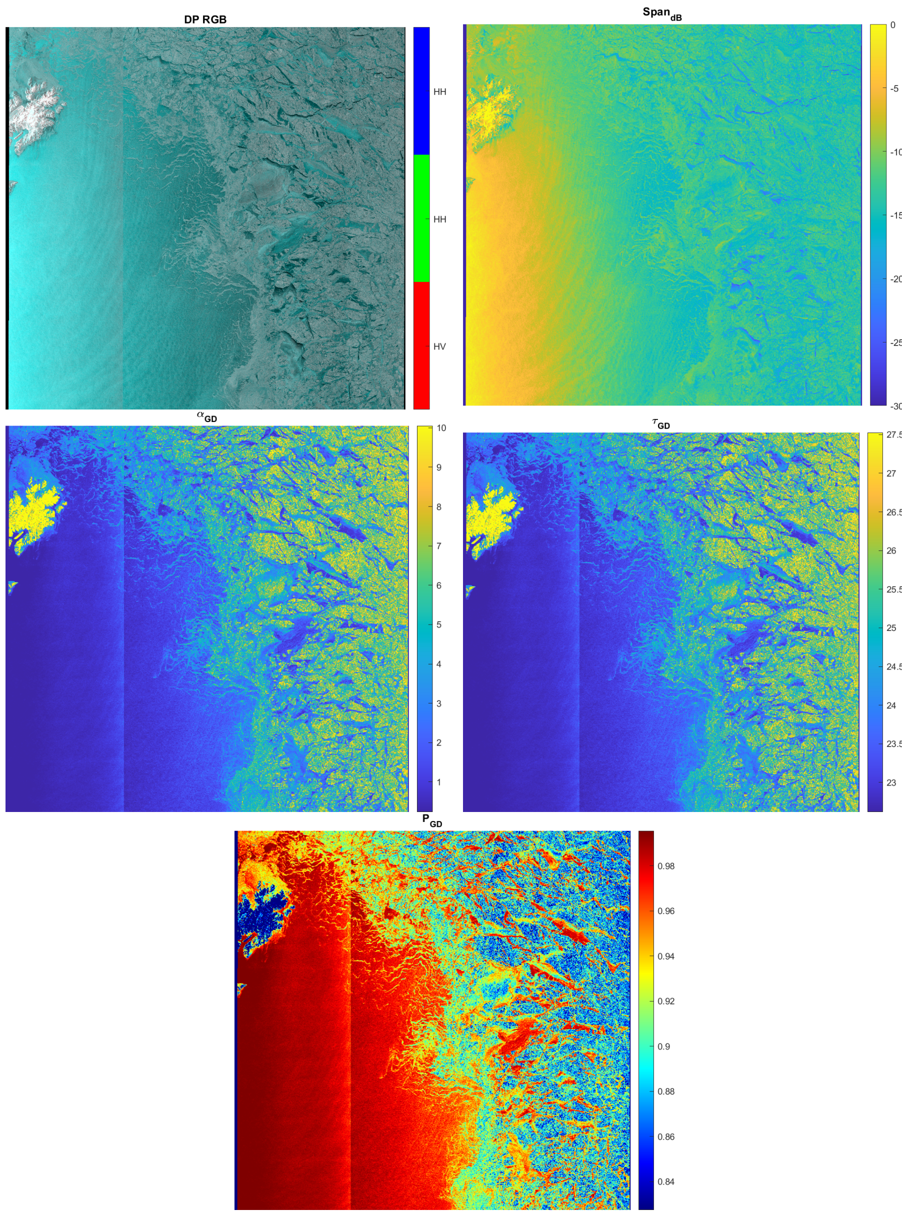

Figure 1: (From let to right): Reference RGB, Span in dB scale, α_{GD}, τ_{GD} and P_{GD} for a real Sentinel-1 EW GRD product

provide the final table of expressions for original and modified forms of the GD-derived parameters in Table 4. And we implore everyone (including ourselves) to use the word ***modified*** appropriately before α_{GD} to have a distinction between the original and modified expression for particular study of sea ice in future discussions using S1 GRD data.

Table 4: The exact and modified expressions for the GD-derived parameters for Sentinel-1 dual polarimetric GRD mode data.

S1 GRD	Original Expression	Modified (for sea ice)
α_{GD}	$90^\circ \times \frac{2}{\pi}\cos^{-1}\left(\frac{\sigma^0_{XX}}{\sqrt{(\sigma^0_{XX})^2+(\sigma^0_{XY})^2}}\right)$	$90^\circ \times \frac{2}{\pi}\cos^{-1}\left(\frac{\sigma^0_{XX}}{\sigma^0_{XX}+\sigma^0_{XY}}\right)$
τ_{GD}	$45^\circ \times \left(1-\frac{2}{\pi}\cos^{-1}\left(\frac{\frac{1}{\sqrt{2}}\times\mathrm{Span}}{\sqrt{(\sigma^0_{XX})^2+(\sigma^0_{XY})^2}}\right)\right)$	constant
P_{GD}	$\{\frac{3}{\pi}\times\cos^{-1}\left(\frac{0.5\times\mathrm{Span}}{\sqrt{(\sigma^0_{XX})^2+(\sigma^0_{XY})^2}}\right)\}^2$	constant

5 Conclusion

We have shown that in case of Sentinel-1 dual polarimetric GRD data, only one GD-derived parameter is sufficient for study of sea ice. We have provided the justification behind the modification of the expression for the α_{GD} completing the discussion in Sec III.C. in [2]. We emphasize for correctness of future discussions that the new expression for α_{GD} for studying sea ice using GRD product data be preceded by the keyword ***modified*** to make the necessary distinction from its original counterpart.

References

[1] Jong-Sen Lee and Eric Pottier. *Polarimetric radar imaging: from basics to applications*. CRC press, 2017.

[2] Debanshu Ratha, Andrea Marinoni, and Torbjørn Eltoft. A generalized geodesic distance-based approach for analysis of sar observations across polarimetric modes. *IEEE Transactions on Geoscience and Remote Sensing*, 61:1–16, 2023.

[3] Debanshu Ratha, Eric Pottier, Avik Bhattacharya, and Alejandro C. Frery. A polsar scattering power factorization framework and novel roll-invariant parameter-based unsupervised classification scheme using a geodesic distance. *IEEE Transactions on Geoscience and Remote Sensing*, 58(5):3509–3525, 2020.