Estimating Seats–Votes Partisan Advantage

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Abstract

A new classification of metrics is introduced: Seats–Votes Partisan Advantage ($PA|SV$). Metrics in this class measure whether redistricting plans favor one political party or the other. Ten prominent measures of partisan bias are computed for a cross-section of past, present, and hypothetical congressional plans and are classified with respect to this new taxonomy. Of these, ten, only (dis)proportionality and the efficiency gap are found to be measures of $PA|SV$. One newer metric measures $PA|SV$ as well.

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Introduction

With *Rucho v. Common Cause, 139 S. Ct. 2484* (2019), the Supreme Court finally and definitively ruled that partisan gerrymandering claims were not justiciable in federal court (Khokher 2021). This refocused attention on pursuing claims of partisan gerrymandering in state courts (Wang et al. 2019a) or by prohibiting it altogether with new federal legislation such as the Senate's Freedom to Vote Act (S2747).

Thirteen states have constitutional provisions or laws that prohibit drawing congressional or state legislative districts that unduly favor or disfavor a political party (NCSL).[1]

Florida's constitutional amendment is representative:

> No apportionment plan or individual district shall be drawn with the intent to **favor or disfavor a political party** or an incumbent; and districts shall not be drawn with the intent or result of denying or abridging the equal opportunity of racial or language minorities to participate in the political process or to diminish their ability to elect

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[1] See Appendix A for specific state-by-state language.

> representatives of their choice; and districts shall consist of contiguous territory. [emphasis added] (Florida Constitution)

Many more state constitutions also have "free" elections clauses which offer another, if more complex, avenue for pursuing claims of partisan gerrymandering (Wang et al. 2019a).[2]

Similarly, the Senate's recent Freedom to Vote Act (S2747) would prohibit states from drawing redistricting plans that have "the effect of materially **favoring or disfavoring any political party**" [emphasis added] (US Senate, S2747, 376).[3]

Both routes beg the question, "How do you know whether a plan favors one party or the other?"[4]

The Senate bill begins to answer that question, saying that to assess whether plans favor or disfavor political parties, the court shall:

> Determine … whether the number of districts that would have been carried by any party's candidate … results in partisan advantage or disadvantage … as determined by standard quantitative **measures of partisan fairness that relate a party's share of the statewide vote to that party's share of seats**. [emphasis added] (US Senate, S2747, 380)

In other words, the bill defined the effect of "favor" as a difference of seats won (or lost).

This notion agrees with how partisan map drawers evaluate maps in real life. Our political system is dominated by two parties. The number of seats that candidates of the two parties win determines the control of Congress and state legislatures: seats won are political currency. Outside courtrooms where arguments fall back on legal interpretations, the more a redistricting plan will likely translate statewide votes into seats in a way that significantly deviates from some expected seats–votes relationship, the more the plan favors one party or the other; the more a plan will likely translate votes into a number of seats that closely matches that ideal, the more the plan is politically neutral.

This definition of what it means for a redistricting plan to "favor" one party or the other – a difference of seats (or seat shares) – suggests an answer to that same question for the state provisions. But it begs a further question: "What is the ideal seat–votes relationship to which redistricting plans should be compared?"

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[2] They are beyond the scope of this paper.

[3] While the concept of Seats–Votes Partisan Advantage ($PA|SV$) in this paper was developed independently and from first principles, it is a fully elaborated version of the measure introduced in S2747 which uses the same nomenclature.

[4] You have the same issue with automated redistricting programs using techniques such as Markov Chain Monte Carlo (MCMC) simulation.

This paper proceeds as follows. Section 1 sketches the breadth of criteria for evaluating redistricting plans and enumerates the confusing depth of metrics in the "bias" subcategory. Section 2 explains the relatively new concept of Partisan Advantage and extends it to use a seats–votes benchmark ($PA|SV$) instead of Eguia's jurisdictional benchmark ($PA|J$).[5] Several properties of valid metrics are proposed, and some specific examples are considered.

Section 3 introduces three sets of plans: the congressional plans drawn in 2011 for 11 states using a composite of 2012 elections, the corresponding 2020 plans using a composite of 2016–2020 election results, and a dozen carefully curated hypothetical plans. For each, ten prominent metrics are computed: declination (δ), lopsided outcomes (LO), mean–median (MM), seat bias (α_S), vote bias (α_V), geometric seat bias (β), global symmetry (GS), proportional (PR), efficiency gap (EG), and gamma (γ). Some apparent contradictions between the measures are discussed.

Finally, section 4 classifies these measures according to whether they satisfy the properties of $PA|SV$ metrics. Of these prominent metrics, only (dis)proportionality and the efficiency gap are found to reliably measure $PA|SV$ across a wide range of statewide vote shares.

A $PA|SV$ metric bound to a specific seats–votes standard could be incorporated into congressional legislation or codified in state constitutional or statutory law.

1. Redistricting in 2020

Redistricting technology was substantially democratized in the 2020 census cycle. Anyone with access to a web browser could draw block-level maps suitable for submission to redistricting officials and could quickly evaluate plans they and others drew,[6] all free of charge (Dave's Redistricting, District Builder, PlanScore).

One may consider many factors when evaluating redistricting plans.[7] These range from basic requirements, like whether a plan is complete and contiguous,[8] to complex expert analyses to determine whether districts are VRA compliant.[9] In between are a large body of metrics. These includes straightforward formulas, like population deviation and measures of compactness,[10]

[5] I use initial caps to differentiate the formal concept of Partisan Advantage defined and discussed here from the generic phrasing "partisan advantage."
[6] While DRA uses one methodology for partisan metrics (see https://bit.ly/2ZlqVfl) and PlanScore uses another (see https://bit.ly/3or5IgK), they both use multiple elections and aggregate census block results up to district-by-district and statewide vote shares.
[7] DRA supports all of these, except the qualitative considerations, of course.
[8] Even something as conceptually simple as contiguity is sometimes non-trivial in real life. See the discussion of 'operational contiguity' in "Contiguity" (https://bit.ly/2ZtmB0Q).
[9] DRA incorporates some heuristics for evaluating the opportunity for minority representation from PGP (2018).
[10] DRA computes Reock, Polsby–Popper, and "know it when you see it" compactness (Kaufman et al. 2021).

much discussed partisan measures of bias and responsiveness,[11] and how much counties, cities, and communities are split by districts.[12] In addition, there are more qualitative considerations for which there are no established metrics, such as the effects on incumbents and preservation of district cores.

Even just under the “bias” subset of the partisan dimension above, there are ten prominent metrics that may yield seemingly conflicting results for any given map:[13] declination (δ), lopsided outcomes (LO), mean–median (MM), seat bias (α_S), vote bias (α_V), geometric seat bias (β), global symmetry (GS), proportional (PR), efficiency gap (EG),[14] and gamma (γ).

To complicate matters, different experts focus on different subsets of these metrics, and there is no consensus on which of them to use under what circumstances. Moreover, political analysts frequently report simple delegation splits in whole seats, for example, 10–3 Republicans / Democrats, instead of any of these formal metrics (Wasserman).

2. Seats–Votes Partisan Advantage

The problem sketched above stems from loose terminology and imprecise definitions. In their search for a standard for partisan gerrymandering, many authors used the terms “bias” and “fairness,” or “partisan bias” and “partisan fairness,” somewhat interchangeably, as though they measured the same thing.[15] This is despite “partisan bias” having a specific meaning in the literature.[16] Instead, some metrics measure partisan gerrymandering via packing & cracking,[17] some measure the bias inherent in a state’s political geography,[18] others measure aspects of

[11] For example: Tufte 1973; Grofman 1983; Wang 2016; McGhee 2017, McDonald et al. 2018; Stephanopoulos and McGhee 2018; Wang et al. 2019b; Warrington 2019; Katz et al. 2020; Nagle and Ramsay 2021.

[12] Appendix 6 of Duchin (2018) and Wang et al. (2021) are the bases for the county-splitting and community-splitting metrics in Dave’s Redistricting, respectively.

[13] I study these ten metrics, because Nagle and Ramsay (2021) analyzed them and DRA reports all of them. DRA also computes Eguia’s measure of geographic bias which requires precinct-by-precinct election data (Eguia 2021). PlanScore calculates the efficiency gap, partisan bias, mean–median, and declination. In this paper, I take an approach like McGhee (2017): where he evaluates metrics against the Efficiency Principle, I assess them with respect to $PA|SV$.

[14] I calculate the efficiency gap using the formula shown in Equation 16 and the actual statewide vote share. This is what DRA does as well.

[15] For example, see Nagle and Ramsay (2021), Wang et al. (2019a), and Katz et al. (2020).

[16] Geometric seats bias (β) measure in Katz et al. (2020).

[17] Declination (δ) does provably well on this (Warrington 2019).

[18] For example, Eguia (2021).

partisan symmetry,[19] and still others measure "fairness" relative to some normative standard such as proportionality.[20]

Here I define what it means for a redistricting plan to "favor" one political party or the other, by extending the relatively new concept of Partisan Advantage[21] to specify a class of measures which I call Seats–Votes Partisan Advantage.

2.1. Definition

To gauge whether a proposed or adopted redistricting plan will favor one party or the other, one simply needs to compare how votes will *likely* be translated into seats under the plan to some normative ideal of how they *should* be translated into.[22] Simply put:

Partisan Advantage is the difference between the ideal and actual seat shares.

Eguia (2021) used a jurisdictional benchmark to account for political geography when he introduced the concept of Partisan Advantage with his Jurisdictional Partisan Advantage that I refer to here as $PA|J$. I use a seats–votes benchmark here instead which I dub $PA|SV$.[23]

2.2. General Formula & Units

To formalize this concept, refer to the ideal seats–votes relationship as $S = s(V)$, where V and the resulting S are the two-party Democratic vote share and seat share, respectively, and s is an ideal seats–votes function.[24] As you will see below, there are many candidates for this expected seats–votes relationship, not just proportionality.

Similarly, refer to the actual statewide vote share and the resulting seat share as $\bar{V}$ and $\bar{S}$, respectively.

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[19] Partisan symmetry is the principle that a plan should treat the two parties equally.

[20] These apples and oranges categories cannot be meaningfully compared or combined.

[21] Eguia (2021) introduces this concept using a jurisdictional benchmark.

[22] One's notion of how votes *should* translate into seats won does not need to depend solely on statewide vote share or an inferred seats-votes curve. It could depend on other factors such as the geographic or racial distribution of votes shares or of turnout across the state. I restrict my focus here to those that be computed from these two inputs.

[23] While one might consider $PA|SV$ to be an (uppercase) Democratic notion of "fairness" – it is, after all, what Senate Democrats' put in the Freedom to Vote Act – and Eguia's version with a jurisdictional benchmark, $PA|J$, to be "fairness" as it would be defined by Republicans when it comes to what should be enshrined in laws, my view is that the partisans *of both parties* undoubtedly use $PA|SV$ implicitly when deciding whether a plan favors one party or the other *in practice*.

[24] I use two-party Democratic vote shares by convention. Two-party Replication vote shares are simply $1 - V$.

Seats–Votes Partisan Advantage is simply the difference between the expected and actual seat shares:[25]

$$PA|SV = s(\bar{V}) - \bar{S} \tag{1}$$

In other words, the difference between the share of seats that *should* be won given a statewide vote share and the share of seats *actually* won. The unit of measure is a difference in seat shares (ΔS).[26] Hence, metrics that do not compare the difference between actual (or likely) seat shares and some ideal measure some other aspect of a redistricting plan. Those other quantities are interesting, but they are not measures of $PA|SV$.

A composite of prior elections can be used as a proxy for a not-yet-held election to infer a *likely* seats–votes curve.[27]

2.3. Properties

I propose that seats–votes standards for $PA|SV$ should also satisfy the following criteria.

Property 1 (only statewide vote share). First, a seats–votes ideal should only depend on the statewide vote share.[28] All votes count equally, regardless of who casts them or where.

Votes are tallied by precincts, so given m precincts and two parties, the votes in an election may be represented a 2 by m matrix

$$\boldsymbol{V} \equiv \begin{bmatrix} V_1^1 & V_2^1 & \cdots & V_m^1 \\ V_1^2 & V_2^2 & \cdots & V_m^2 \end{bmatrix}$$

where each row represents a party, each column a precinct, and the intersection a vote total. Then, if $v(\boldsymbol{V})$ is the statewide vote share given vote matrix $\boldsymbol{V}$ and s is the ideal seats–votes function, this property requires that for any $\boldsymbol{V}$ and $\boldsymbol{V}'$ such that $v(\boldsymbol{V}) = v(\boldsymbol{V}')$, then $s(\boldsymbol{V}) = s(\boldsymbol{V}')$. Since my focus here is on advantage in the legislative body, the only thing that matters is how total votes should translate to seats.

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[25] This is essentially the same formula as Eguia (2021, 7) with different notation.

[26] In programming terms, we call this the return type of the function.

[27] DRA uses Nagle's fractional seat probabilities (2019a) and his method for inferring a seats–votes curve using proportional shift (2019b). PlanScore takes a directionally similar approach different in the details.

[28] One's notion of how votes *should* translate into seats won does not need to depend solely on a statewide vote share. It could depend on other factors such as the geographic or racial distribution of votes shares or of turnout across the state. Jon Eguia pointed out in a private conversation: "There are other notions of fairness leading to other 'ideals' about seat outcomes. For instance, under the VRA, or under the Northern Ireland Good Friday Accords, *who* casts each vote matters when determining whether an outcome is fair, since group representation of distinct ethnic groups is deemed essential to fairness. Similarly, under much of the US Democratic tradition – including most first State constitutions -- *where* a vote was cast also mattered, since representation to state assemblies was by county."

Property 2 (majoritarian). Second, seats–votes standards should be majoritarian.

This means that more than half the votes wins more than half the seats: if $\bar{V} > 0.5$, then $s(\bar{V}) > 0.5$.[29] Graphically, ideal seats–votes curves are only in quadrants 1 (upper right) and 3 (lower left) of Figure 1.

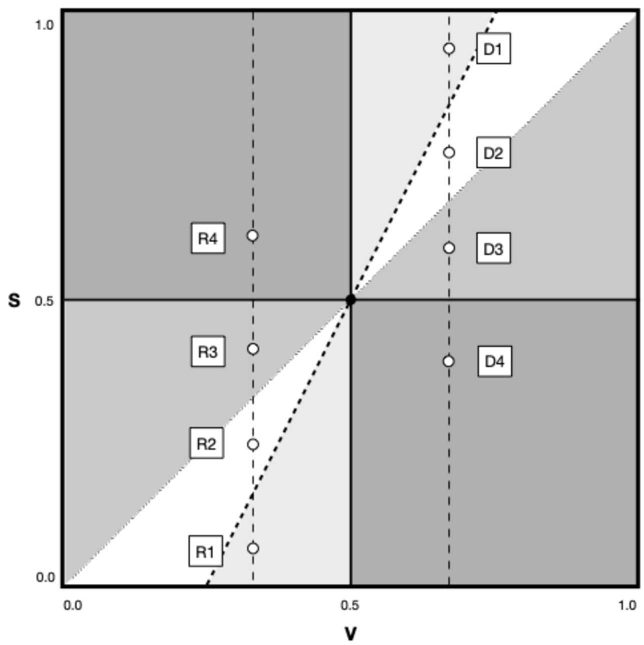

Fig. 1. $s(V)$ space with 2-proportionality ideal (dashed line)

Property 3 (monotonicity). Third, more votes should garner more seats. The requirement that if $v(\boldsymbol{V}') > v(\boldsymbol{V})$, then $s(\boldsymbol{V}') \geq s(\boldsymbol{V})$ precludes downward sloping ideal seats–votes curves.[30]

Property 4 (locality). Fourth, while a seats–votes curve shows the seat shares corresponding to all theoretically possible vote shares,[31] only a small range around the typical statewide vote share is likely in practice.[32] Hence, like the *principle of locality* in physics,[33] absent some theory and some empirical evidence to support it, there is no reason to believe that any metric that measures a seats–votes curve far away from the likely statewide vote share measures anything related to $PA|SV$ close to it.

[29] McGann et al. (2016, 204) show that political equality implies majority rule.
[30] I follow Katz et al. (2020, Online Appendices, 4) to ensure that seat shares never decline with increased vote shares.
[31] Because statewide vote shares tend to not fall much outside the range [0.4, 0.6], I only infer the points of the seats–votes curve for the range [0.25, 0.75].
[32] I consider the zone of uncertainty to be the 5% range that brackets the statewide vote share, because the average uncertainty for the seats–votes curves in (Nagle and Ramsay 2021) was roughly 2%.
[33] <https://en.wikipedia.org/wiki/Principle_of_locality>.

If d is a notion of distance that applies to pairs of vote results, $d((\boldsymbol{V}), (\boldsymbol{V}'))$, then $PA|SV$ should be estimated using only real, predicted, inferred, or counterfactual election results that are a small distance from an actual recent election result, according to d.

Property 5 (neutrality). Finally, if parties switch votes (and thus statewide vote shares), they should also switch seat shares, i.e., the label of the parties does not matter (McDonald 2009, 3).

Formally, using $\boldsymbol{V}$ above and given the mirror image vote matrix

$$\boldsymbol{V}' \equiv \begin{bmatrix} V_1^2 & V_2^2 & \cdots & V_m^2 \\ V_1^1 & V_2^1 & \cdots & V_m^1 \end{bmatrix}$$

then $s(\boldsymbol{V}') = s(\boldsymbol{V})$.

This symmetry also implies the fair ideal that half the votes wins half the seats (Nagle and Ramsay 2021, 117), i.e., ideal seats–votes curves pass through the $\big(V, s(V)\big)$ point of $(0.5, 0.5)$.

2.4. Examples

The concept of $PA|SV$ is agnostic to what you believe the ideal seats-votes relationship should be, and I will not advocate a specific one here. Nonetheless, to give you a sense of the breadth of possibilities, I enumerate some examples.

One candidate is simple proportionality, the $45°$ line on a graph of votes (x-axis) and seats (y-axis) in Fig. 1:

$$S = V \tag{2}$$

On purely little 'd' democratic principles one might say that this is the ideal seats–votes relationship.[34] Alternatively, in the most real-world terms, only "butts in seats" vote in legislatures! Hence, at some level, any deviation from proportionality is an advantage for one party and a disadvantage for the other.

A second example is the two-times winner bonus embedded in the formula for the efficiency gap (EG). This is the less familiar functional form:

$$S = 2V - 0.5 \tag{3}$$

Stephanopoulos and McGhee (2018) showed that this relationship comports well with how congressional and state legislative redistricting plans have performed historically.

These first two possibilities are among the metrics analyzed in section 4. They can be

[34] But I'm <u>not</u> making that argument here!

generalized to any k-prop line that passes through the $(0.5, 0.5)$ center point of symmetry,[35] where k is the constant responsiveness (r):[36]

$$S = k(V - 0.5) + 0.5 \tag{4}$$

Barton (2021) proposes a modified version of the EG specified in terms of *minority* vote share:

$$S = 2V^2 \tag{5}$$

Recast for the full range of vote shares, this becomes:

$$S = \begin{cases} 2V^2, & \text{if } 0.0 \le V \le 0.5 \\ 1 - 2(1 - V)^2, & \text{if } 0.5 < V \le 1.0 \end{cases} \tag{6}$$

Alternatively, though Tufte (1973) discredited it on both empirical and theoretical grounds, one might consider the non-linear "cube law" the ideal seats–votes relationship:[37]

$$\frac{S}{1-S} = \left(\frac{V}{1-V}\right)^3 \tag{7}$$

It reduces to:

$$S = V^3/(1 - 3V + 3V^2) \tag{8}$$

One might argue instead for a winner-take-all relationship in which the party that receives most of the votes wins *all* the seats; that is, $s(V) = 1$ for any $V > 0.5$.

Finally, one might advocate that how votes translate into seats should reflect the underlying political geography of and jurisdictional boundaries within a state (Eguia 2021). This metric requires more than the statewide vote share to compute though, and thus violates Property 1. Hence, while interesting and important, it is not a candidate for the $PA|SV$ benchmark.

As this brief survey illustrates, there are many possible ideal seats–votes relationships. Extending the $PA|SV$, $PA|J$ notation, one could, for example, use $PA|1v$, $PA|2v$, $PA|v^2$, and $PA|v^3$, to refer to their linear with slope 1, linear with slope 2, quadratic, and cubic fair seat benchmarks, respectively.

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[35] All symmetric seats–votes curves pass through this point.

[36] On purely logical grounds, if US elections conformed to the Seat Product Model, one would expect there to be a small winner's bonus in the US congressional system: with $M = 1$ and $S = 435$, the ratio of the Laws of Largest Party Seats (s_1) and Largest Party Votes (v_1) is ~10% or $k \approx 1.1$ (Shugart and Taagepera 2017). The US has fewer seat-winning and a lower effective number of parties than the model suggests though which muddies the waters. In any event, one could reasonably expect a winner's bonus to be somewhat more than the philosophical ideal of pure proportionality ($k = 1$) but substantially less than the observed two-times winner bonus ($k = 2$).

[37] Alternatively, the power of three can be replaced by any constant.

With $PA|SV$ specified, we can now sort through the various metrics enumerated in section 1 and decide whether they are members of this new taxonomic classification or not.

3. Sample Plans

This section presents three groups of redistricting plans for which I compute the ten metrics enumerated above.[38] The 34 plans in them represent a broad cross-section of partisan characteristics and thus provide a good test suite for the metrics.

3.1. Select 2012 Congressional Plans

The first set of maps is the congressional plans studied by Nagle and Ramsay (2021). In terms of their typical statewide two-party vote shares, four lean heavily Democratic (California, Illinois, Massachusetts, and Maryland), three lean heavily Republican (South Carolina, Tennessee, and Texas), and four are nearly politically balanced (Colorado, North Carolina, Ohio, and Pennsylvania).

The plans were drawn in 2011, and the profiles use a composite of 2012 election results.[39] The partisan profiles for these plans can be found in the Supplementary Materials.

	$\bar{V}$	$\bar{S}$	R	r	δ	LO	MM	α_S	α_V	β	GS	PR	EG	γ
CA	59.2%	72.9%	2.49	1.9	-3	9.2%	-1.0%	-1.8%	-0.6%	3.3%	-1.8%	-13.7%	-4.5%	-5.1%
IL	60.0%	74.7%	2.46	3.1	6	11.7%	2.2%	6.5%	1.8%	2.5%	2.1%	-14.6%	-4.6%	6.9%
MD	59.3%	85.0%	3.74	1.1	-33	4.1%	-1.1%	-5.5%	-1.0%	1.4%	-2.7%	-25.6%	-16.3%	-24.9%
MA	60.0%	95.8%	4.57	2.0	N/A	N/A	2.3%	6.7%	1.0%	-3.0%	2.0%	-35.8%	-25.8%	-26.0%
CO	50.6%	50.8%	1.38	3.8	0	0.3%	1.4%	1.4%	0.4%	1.4%	0.9%	-0.2%	0.4%	1.4%
NC	51.5%	32.3%	-11.81	4.4	37	11.1%	5.7%	21.7%	4.5%	21.0%	6.7%	19.3%	20.8%	24.3%
OH	51.3%	39.0%	-8.44	4.1	22	6.9%	4.2%	15.5%	3.3%	14.9%	4.4%	12.3%	13.6%	16.3%
PA	52.9%	41.7%	-2.83	3.4	24	8.7%	5.7%	16.9%	4.7%	15.5%	5.7%	11.3%	14.2%	18.4%
TN	41.6%	21.1%	3.44	0.5	35	0.6%	4.1%	18.4%	3.2%	-3.2%	6.5%	20.5%	12.1%	25.0%
TX	40.4%	28.9%	2.20	1.2	17	-3.0%	2.1%	5.5%	1.4%	-8.3%	5.4%	11.5%	1.9%	9.9%
SC	43.0%	15.8%	4.88	0.9	48	4.2%	4.7%	15.0%	2.4%	0.7%	5.1%	27.2%	20.2%	28.1%

Table 1 – Metrics for Select 2012 Congressional Plans

The measurements in Table 1 are grouped into five sets of columns:

- The 1st set shows the state, statewide vote share ($\bar{V}$), and seat share ($\bar{S}$).
- The 2nd set shows the overall responsiveness or winner bonus for the plan (R) and responsiveness at the typical statewide vote share (r).
- The 3rd set shows the measures of partisan gerrymandering via packing & cracking: declination (δ), lopsided outcomes (LO), and mean–median difference (MM).

[38] The plans are characterized by “profiles” consisting of a statewide vote share and district-by-district vote shares, using Democratic two-party votes by convention.
[39] I call them “2012” plans hereafter.

- The 4th set shows the measures of partisan symmetry: seat bias (α_S), vote bias (α_V), geometric seat bias (β), and global symmetry (GS).
- The last set shows the deviation from proportionality (PR), efficiency gap (EG), and gamma (γ).

All values are percentages, except δ which is an angle in degrees and the two measures of responsiveness, R and r. By convention, positive values indicate Republican advantage and negative values indicate Democratic advantage.

As Table 1 shows and will be discussed in section 4, the metrics seem to conflict for some of the plans. For example, for the IL plan, seven bias measures give one indication (Republican advantage) while PR, EG, and γ suggest the opposite (Democratic advantage).

The seat–vote curves for the IL and CO plans below are examples of the D1 and D2 outcomes noted in Fig. 1, respectively.[40]

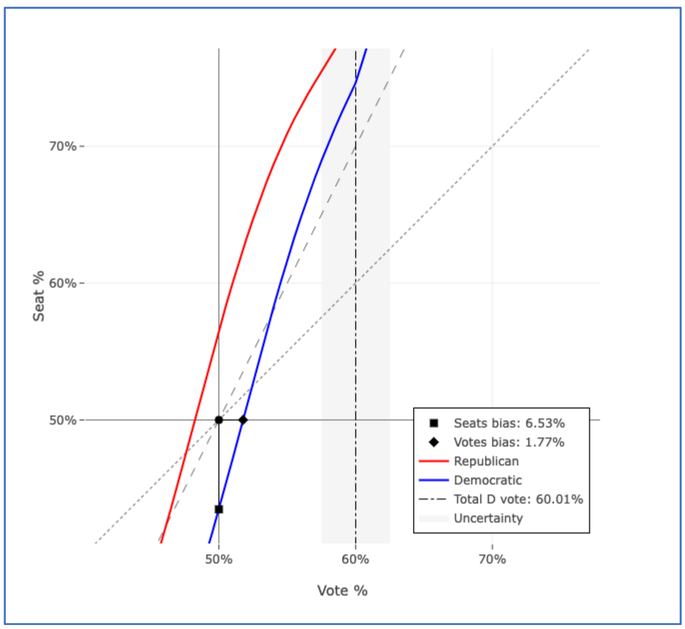

Fig. 2. IL 2012 Congressional

The NC, OH, and PA seat–vote curves (not shown) are all examples of D4 anti-majoritarian outcomes. The seats–votes curves for all the plans can be found in the Supplementary Materials.

[40] The seats–votes curves here are analogous to the folded seats–votes curves in Cervas and Grofman (2022).

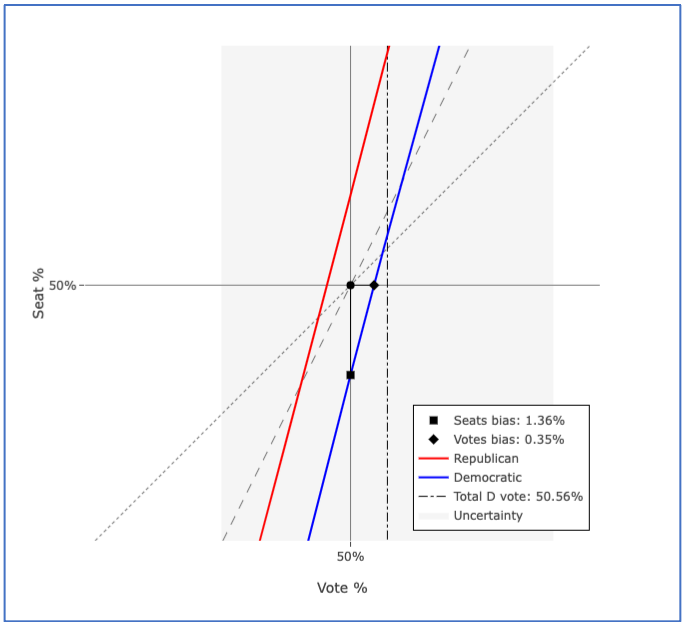

Fig. 3. CO 2012 Congressional

3.2. Corresponding 2020 Congressional Plans

The second set of plans are the corresponding 2020 maps for the same states but using a composite of 2016–2020 elections.[41] The plans are otherwise the same as the 2012 plans, except in NC and PA where courts redrew them.

	$\overline{V}$	$\overline{S}$	R	r	δ	LO	MM	α_S	α_V	β	GS	PR	EG	γ
CA	64.2%	83.1%	2.32	1.8	-7	13.4%	-2.8%	-2.6%	-0.8%	6.2%	-3.0%	-18.8%	-4.6%	-8.2%
IL	58.2%	66.6%	2.03	2.9	3	9.6%	0.0%	1.2%	0.5%	2.1%	1.4%	-8.4%	-0.3%	7.3%
MD	62.1%	87.0%	3.05	0.3	-44	2.6%	-1.9%	-9.1%	-1.6%	4.3%	-4.3%	-24.9%	-12.8%	-33.9%
MA	61.4%	96.3%	4.07	1.5	N/A	N/A	3.1%	8.4%	1.3%	-2.3%	2.0%	-35.0%	-23.6%	-28.8%
CO	54.5%	58.7%	1.95	1.5	2	4.6%	-2.0%	0.1%	0.1%	3.7%	1.9%	-4.2%	0.2%	-2.0%
NC	49.4%	41.3%	15.03	1.6	11	1.6%	3.2%	7.6%	3.0%	7.5%	1.9%	8.1%	7.5%	7.7%
OH	46.4%	28.2%	6.03	1.9	30	4.1%	3.0%	11.0%	2.4%	7.8%	4.3%	18.2%	14.6%	14.9%
PA	52.8%	51.1%	0.39	2.9	6	3.9%	0.2%	7.4%	2.4%	7.4%	3.7%	1.7%	4.5%	6.9%
TN	36.7%	22.1%	2.09	0.1	32	-1.7%	6.4%	19.1%	3.6%	-9.9%	8.8%	14.5%	1.2%	27.2%
TX	46.3%	40.7%	2.49	3.1	10	0.8%	0.9%	-2.6%	-0.8%	-2.6%	-2.9%	5.6%	1.8%	-2.1%
SC	43.2%	16.1%	4.98	1.0	48	4.7%	2.2%	10.6%	1.6%	1.1%	4.2%	27.1%	20.3%	27.1%

Table 2 – Metrics for Select 2020 Congressional Plans

Table 2 lists the metrics for these maps. The seat–vote curves for the TX and PA maps below are examples of the R1 and D3 outcomes noted in Fig. 1, respectively.

As the table shows, the metrics again seem to conflict for some of the plans. For example, for the TX plan, seat bias (α_S), vote bias (α_V), and geometric seat bias (β) give one indication

[41] These maps can be found in the Official Maps collection of DRA. The composite is described in "Election Composites" (http://bit.ly/2SeQoDV). The specific elections used in each state's composite are documented in the Supplementary Materials.

(Democratic advantage) while PR and EG suggest the opposite (Republican advantage). This will be discussed in section 4.

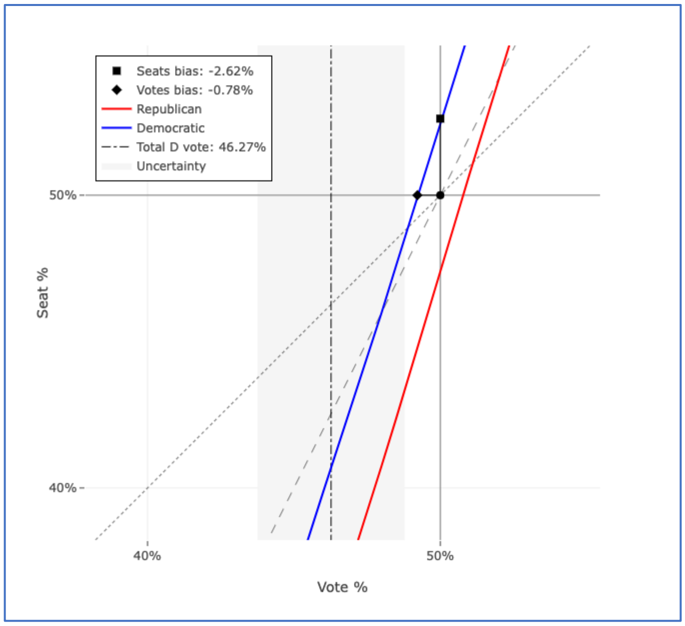

Fig. 4. TX 2020 Congressional

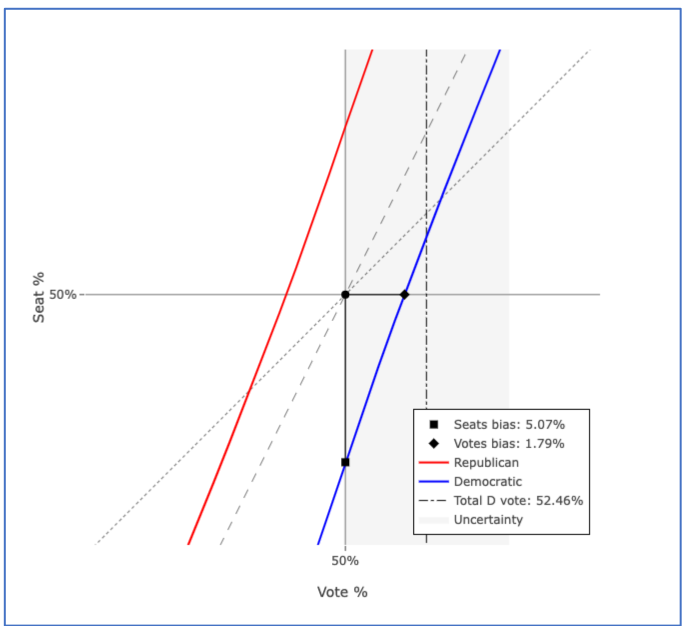

Fig. 5. PA 2020 Congressional

Again, the seats–votes curves for all plans can be found in the Supplementary Materials.

3.3. Warrington’s Hypothetical Plans

Finally, Warrington (2019) created a set of 12 hypothetical plans to study how well various metrics detected partisan gerrymandering. They are described in Appendix B.

The measurements for these hypothetical profiles are shown in Table 3.

	$\bar{V}$	$\bar{S}$	R	r	δ	LO	MM	α_S	α_V	β	GS	PR	EG	γ
1-proportionality	60.0%	60.0%	1.00	0.8	4.9	11.7%	0.0%	0.0%	0.0%	2.6%	-3.6%	0.0%	10.0%	-1.6%
2-proportionality	60.0%	69.2%	1.92	1.6	1.2	10.3%	0.0%	0.0%	0.0%	4.7%	-2.6%	-9.2%	0.8%	-3.1%
3-proportionality	60.0%	79.8%	2.98	2.4	5.9	10.9%	0.0%	0.0%	0.0%	5.2%	-1.7%	-19.8%	-9.8%	-5.5%
Sweep	64.0%	98.4%	3.46	0.7	N/A	N/A	0.0%	0.0%	0.0%	0.7%	-0.8%	-34.4%	-20.4%	-37.9%
Competitive	52.0%	66.7%	8.32	8.0	N/A	2.6%	1.3%	0.2%	0.0%	0.3%	0.1%	-14.7%	-12.7%	-0.6%
Competitive even	51.0%	56.6%	6.57	5.0	-0.7	0.8%	-0.5%	-1.5%	-0.3%	-1.3%	-0.4%	-5.6%	-4.6%	-1.6%
Uncompetitive	52.3%	59.9%	4.32	0.1	-15.3	-2.9%	-9.2%	-9.6%	-6.7%	-9.2%	-4.6%	-7.6%	-5.3%	-9.8%
Very uncompetitive	52.3%	60.0%	4.35	0.0	-18.1	-8.0%	-19.2%	-10.0%	-12.6%	-10.0%	-8.0%	-7.7%	-5.4%	-10.0%
Cubic	57.0%	74.0%	3.43	1.9	-30.0	-1.6%	0.0%	0.0%	0.0%	0.9%	-1.4%	-17.0%	-10.0%	-10.4%
Anti-majoritarian	44.3%	55.6%	-0.98	1.6	-29.0	-15.4%	-9.2%	-9.5%	-8.0%	-7.8%	-3.4%	-11.3%	-17.0%	-14.9%
Classic	50.0%	33.9%	N/A	1.3	31.2	8.8%	6.0%	16.1%	4.9%	16.1%	5.4%	16.1%	16.1%	16.1%
Inverted	30.0%	18.0%	1.60	5.0	-24.6	-23.6%	6.0%	13.2%	3.2%	-9.0%	8.3%	12.0%	-8.0%	-68.5%

Table 3 – Measurements for Hypothetical Plans

Warrington evaluated the plans using first-past-the-post accounting, as opposed to the fractional seat probabilities method that I use, so some of these scenarios may not report as crisply here. To simplify the values, I show the percentages for whole seats in the $\bar{S}$ column.

4. Classification of Metrics

This section inspects the ten study metrics to determine whether they are members of the $PA|SV$ class of metrics. As with classifying animals in zoology, there is no positive or negative value associated with a metric being in this class or not.

4.1. Measures of Packing & Cracking

The first three metrics measure packing & cracking: declination (δ), lopsided outcomes (LO), and mean–median (MM) (Warrington 2019; Wang 2016; McDonald et al. 2018). They were advanced as ways to identify an alleged (US) constitutional harm: partisan gerrymandering. None measures a difference in seat shares though, and they all require district-by-district vote shares to compute (i.e., do not satisfy Property 1). While important historically and in other contexts, they are not measures of $PA|SV$ as defined here. Moreover, since their units differ, these metrics cannot be meaningfully compared with measures of $PA|SV$ and vice versa.

These are their detailed definitions.

Given vote shares by district ($v = v_1\ v_2\ \dots\ v_N$), declination (δ) measures a difference in angles:

$$left = \left(\frac{180}{\pi}\right) \tan^{-1}(S_B - R_B)/(0.5 - V_B)$$

$$right = \left(\frac{180}{\pi}\right) \tan^{-1}(R_A - S_B)/(V_A - 0.5)$$

$$\delta = right - left \tag{9}$$

where:

$$\bar{S} = \textstyle\sum_1^N p(v_i)$$

$$S_B = \bar{S}/N$$

$$R_A = (1 + S_B)/2$$

$$R_B = S_B/2$$

$$V_A = (\textstyle\sum_1^n p(1-v) * (1-v))/(N - \bar{S})$$

$$V_B = 1 - (\textstyle\sum_1^n p(v) * v)/\bar{S}$$

$p(v) =$ the fractional seat probability for vote share v

Lopsided outcomes (LO) measures a difference in *vote* shares:

$$LO = (0.5 - V_B) - (V_A - 0.5) \tag{10}$$

Mean–median (MM) also measures a difference in vote shares:

$$MM = \text{mean}(v) - \text{median}(v) \tag{11}$$

These measure packing & cracking but not $PA|SV$.

4.2. Measures of Partisan Symmetry

The next four metrics measure some aspect of an inferred seats-votes curve: seat bias (α_S), vote bias (α_V), geometric seat bias (β), and global symmetry (GS) (Katz et al. 2020; Nagle and Ramsay 2021). As above, they were suggested as ways to identify the alleged constitutional harm of partisan gerrymandering.

Neither vote bias nor global symmetry measures a difference in seat shares. Therefore, they are not measures of $PA|SV$ as defined here. Again, because they measure different quantities, these metrics cannot be meaningfully compared to measures of $PA|SV$.

Both seat bias and geometric seat bias *do* measure differences in seat shares, but they both require district-by-district vote shares to compute (i.e., do not satisfy Property 1) and, when states are unbalanced politically, they violate locality (Property 4). Among many others, two examples are illustrative: the IL 2012 and TX 2020 plans mentioned above and shown in Fig. 2 and 4, respectively.

Seat bias becomes confounded because the inferred seats–votes curves pass the $(0.5{,}0.5)$ center point of symmetry on one side of the $45°$ line of proportionality (where $S = V$) before crossing over it and reaching the statewide vote share where one party gets a large majority of the votes and an even larger share of the seats. Similarly for β, the vote shares where the counterfactual minority seats–votes curves are evaluated – Republican (red) and Democratic (blue), respectively – are well outside the zone of uncertainty around the statewide vote share. So, neither seat bias nor geometric seat bias are reliable measures of $PA|SV$.

While the notion of partisan symmetry was especially important in the search for a manageable standard of partisan gerrymandering for SCOTUS (Grofman and King 2007), these measures of partisan symmetry are not members of the class $PA|SV$ and their measurements cannot be meaningfully compared to instances of $PA|SV$. Their detailed definitions follow.

Seats bias (α_S) measures a difference in seat shares:[42]

$$\alpha_S = (N/2) - s(0.5) \tag{12}$$

Vote bias (α_V) also measures a difference in vote shares: the vote share required to win 50% of the seats implied by the inferred seats–vote curve.

Geometric seat bias (β) measures a difference in seat shares at statewide vote share $\bar{V}$:

$$\beta = 0.5 * \left(s(1 - \bar{V}) - s(\bar{V})\right) \tag{13}$$

Global symmetry (GS) measures the area of asymmetry between the Democratic (blue) and Republican (red) seats–votes curves – basically the geometric seat bias summed over the entire range of vote shares, normalized by the total seats–votes unit square.

There are also other, newer measures of symmetry not previously studied by Nagle and Ramsay (2021). Of note, Keena et al. (2021, 31) propose another useful measure.[43] As a measure of symmetry, this has much to commend it, especially that it can be easily calculated by hand. While a useful measure of asymmetry, it does not measure a difference in seat shares and it requires district-by-district seat shares (i.e., does not satisfy Property 1). Again, it is not a measure of $PA|SV$.

4.3. Measures of Seats–Votes Partisan Advantage

The last three metrics share a common underlying functional form:

[42] The seats–votes curves for these metrics are inferred from the statewide vote share and district-by-district vote shares using the proportional shift method.

[43] Bias = (Proportion of seats with Democratic vote share 5% more than statewide average) – (Proportion of seats with Republican vote share 5% more than statewide average).

$$\Delta S = m(V - 0.5) - (S - 0.5) \quad (14)$$

where m is the actual or idealized value of responsiveness (r). They all yield the correct units—differences in seat shares—and satisfy the five properties enumerated in section 2.3.

Proportional (PR) measures the difference between the actual (or likely) seat share ($\bar{S}$) and an ideal seat share that matches the statewide vote share ($\bar{V}$):

$$PR = \bar{V} - \bar{S} \quad (15)$$

PR is zero on the $45°$ line, where $S = V$, that is, responsiveness (r) of one.

In contrast, the efficiency gap (EG) embodies a responsiveness (r) of two.[44]

$$EG = 2(\bar{V} - 0.5) - (\bar{S} - 0.5) \quad (16)$$

Finally, gamma (γ) uses responsiveness (r) measured at the statewide vote share:[45]

$$\gamma = r(\bar{V} - 0.5) - (\bar{S} - 0.5) \quad (17)$$

Technically, γ is a valid measure of $PA|SV$. When the measured responsiveness (r) is large, however, almost no plan can be judged as favoring the majority party. Hence, gamma (γ) is of limited practical value.

In addition to the ten prominent metrics studied here, Barton's modified efficiency gap is also a valid measure of $PA|SV$ making it an interesting candidate for a seats–votes benchmark.

Conclusion

While important historically and in other contexts, most prominent measures of the "bias" of redistricting plans cannot be classified as measures of Seats–Votes Partisan Advantage ($PA|SV$). Two are reliable measures of $PA|SV$ across the full range of statewide vote shares: proportionality (PR) and efficiency gap (EG).[46] Barton's modified EG is as well. For Congress, state courts or laws, or redistricting commissions to operationalize what it means for a redistricting plan to favor one party or the other, they simply need to bind $PA|SV$ to a specific ideal seats–votes relationship.

[44] Interpreting a D2 value of EG in the white zone of Fig. 1 can be tricky. It falls below the $EG = 0$ line suggesting the plan favors Republicans even though Democrats won more seats than votes ($\bar{S} > \bar{V}$). An example of this can be seen in the CO 2012 plan, shown in Fig. 3. R2 points in the lower left quadrant present the same challenge. One might deem these as having acceptable levels of bias using EG as the standard.

[45] When the measured responsiveness (r) is very high formula, γ has the analogous issue that EG has. The IL and CO 2012 and IL and TX 2020 plans show examples of this.

[46] Stephanopoulos (2021) concludes the same with respect to the language in Senate bill S2747.

Appendix A. State Prohibitions

Thirteen states prohibit drawing congressional and state legislative districts that unduly favor or disfavor a political party [emphasis added below].

CA	Article XXI, §2(e) – "for the purpose of **favoring** or **discriminating against**" [https://leginfo.legislature.ca.gov/faces/codes_displayText.xhtml?lawCode=CONS&division=&title=&part=&chapter=&article=XXI]
CO	Article V, § 48.1 4(a) – "for the purpose of **protecting** … any political party" [https://advance.lexis.com/documentpage/?pdmfid=1000516&crid=e3a502c8-cda9-41b8-b7fb-0b543874197d&pdistocdocslideraccess=true&config=0143JAAwODgxYWIyNi1mNGJlLTQwYmItYmE4Ni0yOWY2NzQzMjE3MTAKAFBvZENhdGFsb2ecqetP0coiYGhC4QCG46NJ&pddocfullpath=%2Fshared%2Fdocument%2Fstatutes-legislation%2Furn%3AcontentItem%3A61VF-9YD1-DYDC-J4VH-00008-00&pdcomponentid=234165&pdtocnodeidentifier=AABAAGABWAAM&ecomp=p3vckkk&prid=c039a97c-59ee-4c09-aefb-bfea00df4a53]
FL	Article III, §§ 21(a) – "with the intent to **favor** or disfavor a political party" [http://www.leg.state.fl.us/statutes/index.cfm?submenu=3#A3S20]
HI	Statute § 25–2(b)(1) – "so as to unduly **favor** a … political party;" [https://www.capitol.hawaii.gov/hrscurrent/Vol01_Ch0001-0042F/HRS0025/HRS_0025-0002.htm]
IA	Code § 42.4.5 – "for the purpose of **favoring** a political party" [https://www.legis.iowa.gov/docs/code/2016/42.4.pdf]
MI	Article IV, § 6 13(d) – "shall not provide a **disproportionate advantage** to any political party." [http://www.legislature.mi.gov/(S(53rezomyipcv5twgltlmodzr))/mileg.aspx?page=GetObject&objectname=mcl-Article-IV-6]
MT	Code § 5-1-115(3) -- "for the purposes of **favoring** a political party" [https://leg.mt.gov/bills/mca/title_0050/chapter_0010/part_0010/section_0150/0050-0010-0010-0150.html]
NE	LR102(5) – "with the intention of **favoring** a political party" [https://nebraskalegislature.gov/FloorDocs/102/PDF/Intro/LR102.pdf]
NY	Article III, § 4 (c)(5) – "for the purpose of **favoring** or disfavoring … or political parties." [https://www.nysenate.gov/sites/default/files/ckeditor/Oct-21/ny_state_constitution_2021.pdf]
OH	Article XIX, § 1(C)(3)(a) – "that unduly **favors** or disfavors a political party" [https://search-prod.lis.state.oh.us/solarapi/v1/general_assembly_132/resolutions/sjr5/EN/05?format=pdf]
OR	Statute § 188.010(2) – "for the purpose of **favoring** any political party" [https://www.oregonlegislature.gov/bills_laws/ors/ors188.html]
UT	Code § 20A-20-302(5)(f)(iii) – "the purposeful or undue **favoring** or disfavoring of … a political party" [https://le.utah.gov/~2020/bills/static/SB0200.html]
WA	Code § 44.05.090(5) – "purposely to **favor** or **discriminate against** any political party" [https://app.leg.wa.gov/RCW/default.aspx?cite=44.05.090]

Appendix B. Warrington's Hypothetical Plans

Warrington (2019) created a set of 12 hypothetical plans to study how well various metrics detected partisan gerrymandering. Each archetypal plan has an associated partisan profile. These are short descriptions for each.

1. A: 1-proportionality – Designed for seats won to track votes received.
2. B: 2-proportionality – Designed such that $EG = 0$ for all vote shares.
3. C: 3-proportionality – Designed with a responsiveness (r) of three for all vote shares.
4. D: Sweep – Designed so that Democrats win all the seats, even though the statewide vote share is only 64%, e.g., like Massachusetts congressional plans.
5. E: Competitive – Even though the statewide vote share is nearly even (52%) and there are several very competitive races, they all lean slightly towards Democrats.
6. F: Competitive even – Again, the statewide vote share is nearly even (51%) in several competitive districts. Here though none of them fall "in the 'counterfactual window' (i.e., between the majority party's statewide support and 50%)" (Warrington 2019, 12) and they all still lean Democratic.
7. G: Uncompetitive – This profile models an "uncompetitive election as might arise from a bipartisan gerrymander." (Warrington 2019, 12). The average winning margins for both parties are large. The statewide vote share marginally favors Democrats (52.3%).
8. H: Very uncompetitive: This plan is similar to the previous example, except that the average winning margins are even more pronounced.
9. I: Cubic – This profile embodies the classic "cube law" seats–votes relationship.
10. J: Anti-majoritarian – Here, Democrats receive less than half the votes but win more than half the seats.
11. K: Classic – This profile models a classic partisan gerrymander: The statewide vote share is evenly split (50%), but "Republicans win a significant majority through having a number of narrow victories in contrast to their Democratic opponents whose few victories are overwhelming." (Warrington 2019, 12).
12. L: Inverted – This profile is somewhat complementary to the 2-proportionality example, except that the Democratic and Republican vote shares are switched and more extreme.

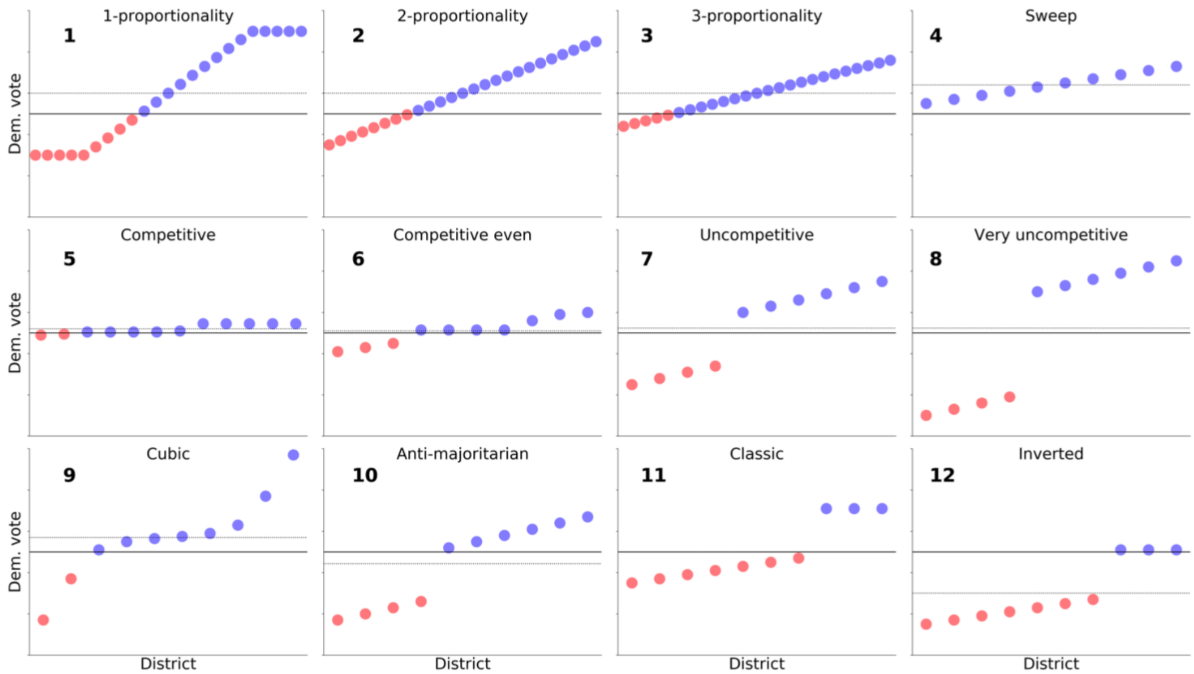

Fig. 6. Rank–Votes Graphs for Warrington's Hypothetical Plans

The essence of each plan is illustrated by the rank-vote graphs shown in Fig. 6. The partisan profiles and seats–votes curves for each can be found in the Supplementary Material.

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