The measurements for these hypothetical profiles are shown in Table 3.

	$\bar{V}$	$S(\bar{V})$	R	r	δ	LO	MM	α_S	α_V	β	GS	PR	EG	γ
1-proportionality	60.0%	60.9%	1.00	0.8	4.9	11.7%	0.0%	0.0%	0.0%	2.6%	-3.6%	0.0%	10.0%	-1.6%
2-proportionality	60.0%	68.0%	1.92	1.6	1.2	10.3%	0.0%	0.0%	0.0%	4.7%	-2.6%	-9.2%	0.8%	-3.1%
3-proportionality	60.0%	80.0%	2.98	2.4	5.9	10.9%	0.0%	0.0%	0.0%	5.2%	-1.7%	-19.8%	-9.8%	-5.5%
Sweep	64.0%	100%	3.46	0.7	N/A	N/A	0.0%	0.0%	0.0%	0.7%	-0.8%	-34.4%	-20.4%	-37.9%
Competitive	52.0%	83.3%	8.32	8.0	N/A	2.6%	1.3%	0.2%	0.0%	0.3%	0.1%	-14.7%	-12.7%	-0.6%
Competitive even	51.0%	70.0%	6.57	5.0	-0.7	0.8%	-0.5%	-1.5%	-0.3%	-1.3%	-0.4%	-5.6%	-4.6%	-1.6%
Uncompetitive	52.3%	60.0%	4.32	0.1	-15.3	-2.9%	-9.2%	-9.6%	-6.7%	-9.2%	-4.6%	-7.6%	-5.3%	-9.8%
Very uncompetitive	52.3%	60.0%	4.35	0.0	-18.1	-8.0%	-19.2%	-10.0%	-12.6%	-10.0%	-8.0%	-7.7%	-5.4%	-10.0%
Cubic	57.0%	80.0%	3.43	1.9	-30.0	-1.6%	0.0%	0.0%	0.0%	0.9%	-1.4%	-17.0%	-10.0%	-10.4%
Anti-majoritarian	44.3%	60.0%	-0.98	1.6	-29.0	-15.4%	-9.2%	-9.5%	-8.0%	-7.8%	-3.4%	-11.3%	-17.0%	-14.9%
Classic	50.0%	30.0%	N/A	1.3	31.2	8.8%	6.0%	16.1%	4.9%	16.1%	5.4%	16.1%	16.1%	16.1%
Inverted	30.0%	30.0%	1.60	5.0	-24.6	-23.6%	6.0%	13.2%	3.2%	-9.0%	8.3%	12.0%	-8.0%	-68.5%

Table 3 – Measurements for Hypothetical Plans

Warrington evaluated the plans using first-past-the-post accounting, as opposed to the fractional seat probabilities method that I used, so some of these scenarios may not report as crisply here. To simplify the values, I show the percentages for whole seats in the $S(\bar{V})$ column.

4. Analysis of Metrics

This section evaluates the ten metrics shown in Tables 1–3 above as potential measures of $PA|SV$, using the sample plans described in the previous section.

4.1. Measures of Partisan Gerrymandering

The first three metrics measure partisan gerrymandering via packing and cracking: declination (δ), lopsided outcomes (LO), and mean–median (MM) (Warrington 2019). While packing and cracking is an interesting quantity, none of these metrics measures the difference in seat shares. Hence, they are not measures of $PA|SV$ as I have defined it.[33]

These are their detailed definitions.

Given vote shares by district ($v = v_1\ v_2 \ldots v_N$), declination (δ) measures a difference in angles:

$$left = (\frac{180}{\pi})\tan^{-1}(S_B - R_B)/(0.5 - V_B)$$

$$right = (\frac{180}{\pi})\tan^{-1}(R_A - S_B)/(V_A - 0.5)$$

$$\delta = right - left \quad (6)$$

[33] Even though their units of measure invalidate them as measures of $PA|SV$, their measurements sometimes also violate the constraint that super-proportional outcomes can't favor the minority party, e.g., suggesting that the IL 2012 plan favors Republicans.

where:

$$\bar{S} = \sum_1^N p(v_i)$$

$$S_B = \bar{S}/N$$

$$R_A = (1 + S_B)/2$$

$$R_B = S_B/2$$

$$V_A = (\sum_1^n p(1 - v) * (1 - v))/(N - \bar{S})$$

$$V_B = 1 - (\sum_1^n p(v) * v)/\bar{S}$$

$p(v) =$ the fractional seat probability for vote share v

Lopsided outcomes (LO) measures a difference in *vote* shares:

$$LO = (0.5 - V_B) - (V_A - 0.5) \quad (7)$$

Mean–median (MM) also measures a difference in vote shares:

$$MM = \text{mean}(v) - \text{median}(v) \quad (8)$$

These measure partisan gerrymandering via packing & cracking but not $PA|SV$.

4.2. Measures of Partisan Symmetry

The next four metrics measure some aspect of a seats-votes curve: seat bias (α_S), vote bias (α_V), geometric seat bias (β), and global symmetry (GS). Neither vote bias nor global symmetry measures a difference in seat shares; therefore, they are not measures of $PA|SV$.

Both seat bias and geometric seat bias *do* measure differences in seat shares, but they sometimes violate the property that super-proportional outcomes cannot favor minority parties. Among many others, two examples are illustrative: the IL 2012 and TX 2020 plans shown in Fig. 2 and 4, respectively.

- In both cases, seat bias becomes confounded because the seats–votes curves pass the $(0.5, 0.5)$ center point of symmetry[34] on one side of the $45°$ line of proportionality, where $S = V$ before crossing over it and reaching the statewide vote share where one party gets a large majority of the votes and an even larger share of the seats.

- β also sends the wrong signal for these two plans, because the vote shares where the counterfactual minority seats–votes curves are evaluated – Republican (red) and Democratic (blue) – are well outside the zone of uncertainty around the statewide vote share.

[34] All symmetric seats–votes curves pass through this point.