Metrics -- UPDATE

This section evaluates the ten metrics shown in Tables 1–3 above as potential measures of , using the sample plans described in the previous section.

## 4.1. Measures of Partisan Gerrymandering

The first three metrics measure partisan gerrymandering via packing and cracking: declination (), lopsided outcomes (), and mean–median () (Warrington 2019). While packing and cracking is an interesting quantity, none of these metrics measures the difference in seat shares. Hence, they are not measures of as I have defined it.[[1]](#footnote-1)

These are their detailed definitions.

Given vote shares by district , declination () measures a difference in angles:

(6)

where:

the fractional seat probability for vote share

Lopsided outcomes () measures a difference in *vote* shares:

(7)

Mean–median () also measures a difference in vote shares:

(8)

These measure partisan gerrymandering via packing & cracking but not .

## 4.2. Measures of Partisan Symmetry

The next four metrics measure some aspect of a seats-votes curve: seat bias (), vote bias (), geometric seat bias (), and global symmetry (). Neither vote bias nor global symmetry measures a difference in seat shares; therefore, they are not measures of .

Seats bias () measures a difference in seat shares:

(9)

Vote bias () also measures a difference in vote shares: the vote share required to win 50% of the seats implied by the inferred seats–vote curve.

Formula

Geometric seat bias () measures a difference in seat shares at statewide vote share :

(10)

Global symmetry () measures the area of asymmetry between the Democratic (blue) and Republican (red) seats–votes curves – basically the geometric seat bias summed over the entire range of vote shares, normalized by the total seats–votes unit square.

Formula

## 4.3. Measures of Seats–Votes Partisan Advantage

The last three metrics share a common underlying functional form:

(11)

where is the actual or idealized value of responsiveness (). They all yield differences in seat shares and, with one slight modification, none violate the constraint that super-proportional outcomes cannot favor the minority party.

Proportional () measures the difference between the actual (or likely) seat share () and an ideal seat share that matches the statewide vote share ():

(12)

is zero on the line, where , that is, responsiveness () of one.

In contrast, the efficiency gap () embodies a constant responsiveness () of two.

(13)

The measurements for both CO plans are highlighted in Tables 1 and 2, as are the values for the 1-proportionality hypothetical plan in Table 3. Here’s why:

The dashed 2-proportionality line in Fig. 1 is, where . Above that line, values are negative (indicating Democratic bias), and below that line, they are positive (indicating Republican bias). However, the formula formalizes the notion that a two-time winner’s bonus is acceptable; therefore, so to say that a point just below the 2-proportionality line immediately favors Republicans is to, in some sense, to contradict the essential framework. Hence, I argue that when the winner’s bonus (R) is between one and two inclusive – in the white regions in Fig. 1 – a map is not biased *with respect to the efficiency gap ideal.* Hence, I have highlighted these values.

An example of this can be seen in the CO 2012 plan, shown in Fig. 3.

The thus modified (or interpreted) is a valid measure of .

Finally, gamma () uses responsiveness () measured at the statewide vote share:

(14)

This formula has the analogous “doesn’t acknowledge acceptable bias” issue as , when the responsiveness () is very high. The IL and CO 2012 and IL and TX 2020 plans are examples which is why these values have been highlighted.

Technically, an appropriately interpreted is a valid measure of . However, when the measured responsiveness () is large, almost no plan can be judged as favoring the majority party. Hence, gamma () is of limited practical value.

1. Even though their units of measure invalidate them as measures of , their measurements sometimes also violate the constraint that super-proportional outcomes can’t favor the minority party, e.g., suggesting that the IL 2012 plan favors Republicans. [↑](#footnote-ref-1)