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Temperature regulation in a Continuous Stirred Tank Reactor using event triggered sliding mode control

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Abstract: Continuous Stirred Tank Reactor (CSTR) is a typical example of an industrial equipment for chemical processes that exhibit dynamics of a second order nonlinear system. Nonlinear and coupled nature of CSTR pose challenges in design of robust control with larger operating region. Industrial processes require good state estimation and disturbance rejection. Under parameter variations and fast changing dynamics, an event based sliding mode controller is presented in this work to provide robustness to the system with an added benefit of saving energy expenditure. Robustness and efficacy of the controller have been confirmed using numerical simulations.

Keywords: CSTR, event based sliding mode control, matched perturbation, Riemann sampling, Lebesgue sampling, event conditions, triggering rule, inter event time.

1. INTRODUCTION

A Continuous Stirred Tank Reactor (CSTR) exhibits complex nonlinear dynamics and is a benchmark equipment in many process industries (Sinha and Mishra (2016)) that require continuous addition and withdrawal of reactants and products. A CSTR may be assumed to be somewhat opposite of an idealized well-stirred batch and tubular plug-flow reactors. CSTRs are incorporated to achieve optimal productivity of a chemical process by maintaining high conversion rates and thus maximizing economy. The entire set of operating points should show a stable steady state behavior even in presence of disturbances. Linear controller designed for such process may result in undesirable behavior over the whole range of operation particularly outside the linear operating range. The PID controller is the most commonly used controller for CSTR owing to its easy design and tuning properties. The feedback linearization has been also extensively used for CSTR (Yayong Zhai (2012)). The controller mentioned above fails to deliver under varying transient behavior of CSTR, owing to the non-adaptive nature of the controller. A linear controller using Taylor's linearization has been designed assuming bounded uncertainty in (Morari and Zafiriou (1989)). However, by local linearization, global stability cannot be assured. Input-output feedback linearization method proposed in (Colantonio et al. (1995)) failed because it required continuous measurement of states. It is impractical to continuously measure the concentration of feed and reactants.

A method based on state coordination transformation has been used in (Colantonio et al. (1995)) for linear input state behavior. This method guaranteed the asymptotic stability but effect of disturbance was overlooked. Under varying process conditions observer based design method failed to deliver desired response. A high gain controller used to tackle such process showed fast response but resulted in unwanted control effort saturation (Zhao et al. (2014)). While linearizing a system model there remains a part of transformed system which is non linearizable and has zero dynamics which cannot be ignored (Colantonio et al. (1995)). Despite being non-model based control, Adaptive and Fuzzy based techniques also perform suboptimally with widely varying process dynamics.

In this work, we propose a controller based on paradigms of event based sliding mode control. Sliding Mode Control (SMC) (Yan et al. (2017); Edwards and Spurgeon (1998); Sabanovic et al. (2004), Young et al. (1999); Slotine and Li (1991)) is a control scheme which guarantees finite time convergence and provides robust operation over entire regime with complete rejection to matched perturbations. The advantage of using this control is that we can tailor the system dynamical behavior by particular choice of sliding function. SMC used in conjunction with event triggered control retain its robustness as well as event triggering approach aids in saving energy expenditure. When measured variables of a system do not deviate frequently, event based control offers numerous advantages over traditional periodic and time triggered control. To the best of author's knowledge, an event triggered sliding mode control has been applied for the first time in a model of stirred reactor.

The organization of the paper is as follows. Section 1 starts with a brief introduction. Section 2 describes the dynamics of the model. Section 3 presents main results of control synthesis followed by stability analysis. Numerical

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simulations are demonstrated in section 4 and section 5 concludes the paper.

2. PLANT DYNAMIC MODEL

Chemical reactions in the reactor may be endothermic or exothermic and in order to regulate the temperature of the reactor at a set level, energy is required to be added to or removed from the reactor. Usually a CSTR is operated at steady state with contents well mixed. Owing to this quality, modelling does not involve significant variations in concentration, temperature or reaction rate throughout the vessel. Since the temperature and concentration under consideration are identical everywhere within the reaction vessel, they are the same at the exit as they are anywhere else in the tank. As a result, the temperature and concentration at the exit are modelled as being the same as those inside the reactor. In situations where mixing is highly nonideal, the well mixed model is inadequate and one must resort to nonideal CSTR model description.

This section presents a dynamic description of the reactor in which mixing is adequate (Sinha and Mishra (2016); Sinha and Mishra (2017b)). Thus, an ideal CSTR model as found in (Ray (1981)) is adopted. Presence of exponential terms in the modelling equations make the description a nonlinear one. A complex chemical reaction occurs in CSTR. Under the assumption of complete mixing, the reactor gets cooled in a continuous manner. The volume of the chemical product **B** is equal to the volume of the input reactant **A**. The reactor is assumed to be non isothermal and exhibiting an irreversible exothermic first order chemical reaction **A** → **B**.

The dynamic model is then described as

$$\begin{aligned}\frac{dC_A}{dt'} &= \frac{F}{V}(C_{A_f} - C_A) - r,\\ \frac{dT}{dt'} &= \frac{F}{V}(T_f - T) + \frac{(-\Delta H)}{\rho C_p} r - \frac{hA}{V\rho C_p}(T - T_c).\end{aligned} \tag{1}$$

Meaning of the quantities appearing in (1) are given in table (1). In (1),

$$r = k_0 exp(-\frac{E}{RT})C_A, \tag{2}$$

$$hA = \frac{aF_c^{b+1}}{F_c + \frac{aF_c^b}{2\rho_c C_{pc}}}. \tag{3}$$

where a,b are CSTR model parameters and hA is the heat transfer term. A computationally more convenient form of the above modelling equations are presented in state space formulation in (4–5). For original convention and nomenclature, the reader is suggested to refer (Ray (1981)).

$$\dot{x_1} = -x_1 + D_a(1 - x_1)exp(\frac{x_2}{1 + x_2/\gamma}) - d_2 \tag{4}$$

$$\begin{aligned}\dot{x_2} = &-x_2 + BD_a(1 - x_1)exp(\frac{x_2}{1 + x_2/\gamma})\\ &- \beta(x_2 - x_{2_{c_0}}) + \beta u_T + d_1\end{aligned} \tag{5}$$

This formulation is based on dimensionless modelling of CSTR for which the parameters are given in table 2. It should be noted that d_1 and d_2 are taken to be step disturbances and the nominal coolant temperature is $x_{2_{c_0}}$.

Table 1. Symbols & their meaning

Meaning	Symbol	Unit
1^{st} order reaction rate constant	k_0	min^{-1}
inlet concentration of **A**	C_{A_f}	$kmol/m^3$
steady state flow rate of **A**	F	m^3/min
density of the reagent **A**	ρ	g/m^3
specific heat capacity of **A**	C_p	cal/Cg
heat of reaction	ΔH	$cal/kmol$
density of coolant	ρ_c	g/m^3
specific heat capacity of coolant	C_{pc}	cal/Cg
volume of the CSTR	V	m^3
coolant flow rate	F_c	m^3/min
reactor temperature	T	K
feed temperature	T_f	K
coolant temperature	T_c	K
reactor concentration of **A**	C_A	$kmol/m^3$
activation energy	E	J/mol
universal ideal gas constant	R	$J/molK$

Remark 1. *There are two ways to manipulate the observed states (outputs)- coolant temperature and input feed flow. In this study, we have used the coolant temperature as our control input (u_T) to regulate the temperature of the CSTR.*

This is because in industrial environments, temperature becomes more critical to be controlled in order to avoid any secondary reaction in the reactor. It is worthy to note that the control of composition is not discussed in this paper.

Table 2. Dimensionless parameters in CSTR model

Dimensionless Parmeters for CSTR	
ratio of activation energy to average kinetic energy	$\gamma = E/RT_{f_0}$
adiabatic temperature rise	$B = \frac{(-\Delta H)C_{A_{f_0}}\gamma}{\rho C_p T_{f_0}}$
Damkohler number	$D_a = k_0 exp(-\gamma)V/F_0$
heat transfer coefficient	$\beta = hA/\rho C_p F_0$
dimensionless time	$t = t'(F_0/V)$
dimensionless composition	$x_1 = (C_{A_{f_0}} - C_A)/C_{A_{f_0}}$
dimensionless temperature	$x_2 = \gamma(T - T_{f_0})/T_{f_0}$
dimensionless control input	$u_T = \gamma(T - T_{c_0})/T_{f_0}$
dimensionless control input	$u_F = (F - F_0)/F_0$
feed temperature disturbance	$d_1 = \gamma(T_f - T_{f_0})/T_{f_0}$
feed composition disturbance	$d_2 = (C_{A_f} - C_{A_{f_0}})/C_{A_{f_0}}$

3. MAIN RESULTS

3.1 Event based control

There has been a tremendous growth of interest in the area of event based systems due to reduced computational cost and energy expenses. The challenge, however, in this type of control is to maintain performance, stability, optimality, etc. in the presence of uncertainties while ensuring reduced computation/communication. A modern control system consists of a computer and the signal under consideration is sampled periodically to cater the needs of a classic sampled data control system. Under such scheme, the interval between two successive clock pulses is predetermined and fixed. The sampling takes place along the *horizontal* axis, also known as *Riemann sampling*. An alternate and more efficient way is to sample along the *vertical* axis, also known as *Lebesgue sampling* (Astrom and Bernhardsson (2002)). In the later case, the sampling is not periodic rather it depends on the value of previous sample or certain *conditions* that need to be violated to bring forth the next clock pulse. These *conditions* are some noticeable changes (*events* or *event conditions*) on which the next sampling instant depends.

This type of control seems to be a reasonable choice in applications where signal of interest slowly varies. In chemical process industries that contain many production units, primary units are separated by buffer units. Each change in the unit can cause upset and hence it is desirable to keep the change in process variables less frequent. Event based control comes handy in such applications. No action is taken unless there is a huge upset. It is also advantageous to use event based control when the upper bound of a process variable needs to be bounded irrespective of the manner in which the states evolve. For a depth analysis of and earlier works on event based control, readers are requested to refer Astholfi and Marconi (2007), Behera and Bandyopadhyay (2014), Shi et al. (2016), Mazo and Tabuada (2011), Tabuada (2007), Aström (2008), Anta and Tabuada (2010), Tallapragada and Chopra (2013), Lemmon (2010).

3.2 Event triggered sliding modes

Since, next sample instant is dependent on the previous sampling information, the control is held constant between successive events or sampling instants. The control is not updated periodically and is held at the previous value in the interval $[t_k, t_{k+1}[$. This, however, introduces a discretization error between the states of the system.

$$\epsilon(t) = x(t) - x(t_k) \tag{6}$$

such that at $t = t_k$, $\epsilon(t)$ goes to zero. The term t_k is the triggering instant at k^{th} sampling instant. Control gets updated with a new value at t_k instants only. The sampling is not periodic and hence $t_{k+1} - t_k \neq constant$.

The controller in this study has been synthesized without any linearization of the dynamics. An event based sliding mode control has been used to implement the controller. For computational purposes, it is desirable to define the following candidates. The deviation from the desired temperature is given by

$$e_T(t) = x_2(t) - x_{2_d}, \tag{7}$$

where x_{2_d} is the set reference. The control effort must be designed well to achieve accurate set point tracking, reject disturbances and deliver satisfactory results quickly. Stated alternatively, the error variable is required to vanish or at least settle in close vicinity of zero after a transient of acceptable duration.

Sliding mode controller design requires the design of a stable manifold with reduced order dynamics (Young et al. (1999)) onto which the state trajectories need be to confined; and a forcing control effort (Young et al. (1999)) to drive the trajectories on the surface in finite time. In general, the sliding surface takes the form

$$\sigma(t) = \lambda_1 x_1(t) + \lambda_2 x_2(t) \tag{8}$$

where λ_i are the coefficient weight which can be tuned as per performance requirements. For a regulatory control, the surface variable (8) can alternatively be written in error dynamical form as

$$\sigma(t) = \lambda_1 e_{x_1}(t) + \lambda_2 e_{x_2}(t) \tag{9}$$

$e_{x_2}(t)$ is same as $e_T(t)$ described above and $e_{x_1}(t)$ is the error corresponding to the other state variable. Henceforth the sliding variable shall be represented by (8) if no confusion arises. During sliding, $\dot{\sigma}(t) = 0$ and the corrective term used to force the trajectories onto the sliding surface is chosen as $\mu sign(\sigma(t))$, where μ is the adjustable gain.

Theorem 3.1. *For plant dynamics described by* (4–5)*, the stabilizing control law u_T that provides accurate set point tracking is synthesized as*

$$u_T(t) = -\lambda_2^{-1}\beta^{-1}(\lambda^T f(x(t_k)) + \mu sign(\sigma(t_k))) \tag{10}$$

where $f(x) = [f_1(x_1,x_2) - d_2 \quad f_2(x_1,x_2) + d_1]^T$ & $\lambda^T = [\lambda_1 \ \ \lambda_2]$.

Proof. Before proceeding towards a formal proof, it is convenient to express the dynamics $(4-5)$ in functional form to ease computations. Therefore, the dynamics $(4-5)$ can be expressed as

$$\dot{x}_1 = f_1(x_1,x_2) - d_2 \tag{11}$$

$$\dot{x}_1 = f_1(x_1,x_2) + \beta u_T + d_1 \tag{12}$$

where $f_1(\cdot,\cdot) = -x_1 + D_a(1-x_1)exp(\frac{x_2}{1+x_2/\gamma})$
and $f_2(\cdot,\cdot) = -x_2 + BD_a(1-x_1)exp(\frac{x_2}{1+x_2/\gamma}) - \beta(x_2 - x_{2_{c_0}})$.

Assumption 3.1. *We assume the functions in* (11-12) *satisfy a Lipschitz condition with respect to their arguments on some fairly large domain with Lipschitz constant $\bar{L}$.*

From the theory of sliding modes, we have

$$\begin{aligned} &\sigma(t) = \lambda_1 e_{x_1}(t) + \lambda_2 e_{x_2}(t) \\ &\Rightarrow \dot{\sigma}(t) = \lambda_1 \dot{e}_{x_1}(t) + \lambda_2 \dot{e}_{x_2}(t) \\ &\Rightarrow \dot{\sigma}(t) = \lambda_1(\dot{x}_1(t) - \dot{x}_{1_d}) + \lambda_2(\dot{x}_2(t) - \dot{x}_{2_d}). \end{aligned} \tag{13}$$

For set point control, $\dot{x}_{1_d} = \dot{x}_{2_d} = 0$.

$$\therefore \quad \dot{\sigma}(t) = \lambda_1 \dot{x}_1(t) + \lambda_2 \dot{x}_2(t). \tag{14}$$

From (11-12), we can further simplify (14) as

$$\begin{aligned} &\dot{\sigma}(t) = \lambda_1(f_1(x_1,x_2) - d_2) + \lambda_2(f_2(x_1,x_2) + \beta u_T + d_1) \\ &\Rightarrow \dot{\sigma}(t) = \lambda^T f(x(t)) + \lambda_2 \beta u_T(t). \end{aligned} \tag{15}$$

In (15), $f(x) = [f_1(x_1,x_2) - d_2 \quad f_2(x_1,x_2) + d_1]^T$ & $\lambda^T = [\lambda_1 \ \ \lambda_2]$.

Solving for u_T in (15) yields

$$u_T(t) = -\lambda_2^{-1}\beta^{-1}(\lambda^T f(x(t)) + \mu sign(\sigma(t))) \tag{16}$$

However, events are triggered at discrete instants and for time instants between $[t_k, t_{k+1}[$ the states show a tendency to deviate from the sliding surface but remain bounded by a small finite quantity. Hence, the final event triggered control law applied to the system has the following form:

$$u_T(t) = -\lambda_2^{-1}\beta^{-1}(\lambda^T f(x(t_k)) + \mu sign(\sigma(t_k))). \tag{17}$$

This concludes the proof. □

Since the control is applied in a piecewise continuous manner, ideal sliding mode is not possible. It is worthy to investigate the existence of sliding mode in event implementation.

Theorem 3.2. *Consider the dynamics described in* (4-5) *and the control law* (16)*. Sliding mode is said to exist in vicinity of sliding manifold, if the manifold is attractive, i.e., trajectories emanating outside it continuously decrease towards it. Stating alternatively, reachability to the surface is ensured for some reachability constant $\eta > 0$ if gain μ is designed such that $\mu > \sup\{\|\lambda^T\|\bar{L}\|\epsilon(t)\|\}$ is satisfied.*

Proof. Let us consider a Lyapunov candidate of the form $V = \frac{1}{2}\sigma(t)^2$. Taking derivative of this candidate along state trajectories yields

$$\begin{aligned}\dot{V} &= \sigma(t)\dot{\sigma}(t)\\ \Rightarrow \dot{V} &= \sigma(t)(\lambda^T f(x(t)) + \lambda_2 \beta u_T(t)).\end{aligned} \tag{18}$$

Substituing the control (16) into (18), we can simplify the above expression as the following. Thus, $\forall t \in [t_k, t_{k+1}[$, we have

$$\begin{aligned}\dot{V} &= \sigma(t)(\lambda^T f(x(t)) - \lambda^T f(x(t_k)) - \mu sign(\sigma(t_k)))\\ \dot{V} &\le -\sigma(t)\mu sign(\sigma(t_k)) + \|\sigma(t)\|\|\lambda^T\|\|f(x(t)) - f(x(t_k))\|\\ \dot{V} &\le -\sigma(t)\mu sign(\sigma(t_k)) + \|\sigma(t)\|\|\lambda^T\|\bar{L}\|x(t) - x(t_k)\|\\ \dot{V} &\le -\sigma(t)\mu sign(\sigma(t_k)) + \|\sigma(t)\|\|\lambda^T\|\bar{L}\|\epsilon(t)\|.\end{aligned} \tag{19}$$

As long as $\sigma(t) > 0$ or $\sigma(t) < 0$, the condition $\sigma(t) = \sigma(t_k)$ is strictly met $\forall t \in [t_k, t_{k+1}[$. Hence, when trajectories are just outside the sliding surface,

$$\begin{aligned}\dot{V} &\le -\|\sigma(t)\|\mu + \|\sigma(t)\|\|\lambda^T\|\bar{L}\|\epsilon(t)\|\\ \Rightarrow \dot{V} &\le -\|\sigma(t)\|(\mu + \|\lambda^T\|\bar{L}\|\epsilon(t)\|)\\ \Rightarrow \dot{V} &\le -\eta\|\sigma(t)\|\end{aligned} \tag{20}$$

with $\eta > 0$.

This completes the proof of reachability and confirms that the manifold is an essential attractor.

For stability, it is required to be shown that $\dot{V} < 0$. At $t = t_k$, $\|\epsilon(t)\| \to 0$ and the control signal is updated. Thus,

$$\begin{aligned}&\dot{V} \le -\|\sigma(t)\|(\mu + \|\lambda^T\|\bar{L}\|\epsilon(t)\|).\\ &\because \quad \|\epsilon(t)\| \to 0 \quad \Rightarrow \dot{V} < 0.\end{aligned} \tag{21}$$

This completes the proof of stability. □

The triggering instant t_k is completely characterized by a triggering rule. Next sampling instant is by virtue of this criterion. As long as this criterion is respected, next clock pulse is not called upon and the control signal is maintained constant at the previous value. The triggering rule used in this work is given by

$$\delta = \|\zeta e_T + \xi \dot{e}_T^2\| - \psi(m_1 + m_2 e^{-\varsigma t}) \tag{22}$$

with $\zeta > 0$, $\xi > 0$, $\psi \in (0,1)$, $m_1 \ge 0$, $m_2 \ge 0$, $m_1 + m_2 > 0$ and $\varsigma \in (0,1)$.

The term $(m_1 + m_2 e^{-\varsigma t})$ ensures a finite lower bound on inter event execution time and avoids accumulation of samples at same instant, known as *Zeno behaviour* in literature. Moreover, the rule (22) is also dynamic in nature and the accuracy adjustment term $(m_1 + m_2 e^{-\varsigma t})$ is time varying, thereby reducing the controller updates in accordance with the adjustment term and introducing anticipatory action in the system (Sinha and Mishra (2017a,b)).

The following relation completely determines the triggering instants in an iterative manner.

$$t_{k+1} = \inf\ \{t \in [t_k, \infty[\ : \delta > 0\}. \tag{23}$$

The inter event time is given by

$$T_k = t_{k+1} - t_k. \tag{24}$$

For $t_{k+1} - t_k \ge T_K$, lower bound on inter event time is ensured and the triggering instants are admissible.

Theorem 3.3. *Consider the system described by* (4 − 5)*, the control protocol* (16) *and the discretization error* (6)*. The sequence of triggering instants* $\{t_k\}_{k=0}^{\infty}$ *respects the triggering rule given in* (22)*. Consequently,* Zeno *phenomenon is not exhibited and the inter event execution time* T_k *is bounded below by a finite positive quantity such that*

$$T_k \ge \frac{1}{\bar{L}}\ ln\ \left(\frac{\bar{L}\|\epsilon\|_\infty}{\bar{L}(1 + \|\bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T\|)\|x(t_k)\| + \|\bar{B}\|\mu} + 1\right) \tag{25}$$

where $\|\epsilon\|_\infty$ *is the maximum discretization error.*

Proof. Without loss of generality, the system described in $(4-5)$ is recalled here as

$$\dot{x}(t) = f(x) + \bar{B}u_T(t) \tag{26}$$

where $f(x)$ is same as described in theorem 3.1 and $\bar{B} = [0 \quad \beta]^T$. Between k^{th} and $(k+1)^{th}$ sampling instant in the execution of control, the discretization error (6) is non zero. T_k is the time it takes the discretization error to rise from 0 to some finite value. Thus,

$$\frac{d}{dt}\|\epsilon(t)\| \le \|\frac{d}{dt}\bar{\epsilon}(t)\| \le \|\frac{d}{dt}x(t)\| \tag{27}$$

$$\Rightarrow \|\frac{d}{dt}\epsilon(t)\| \le \|f(x(t)) + \bar{B}u_T(t)\|. \tag{28}$$

Substituting the control protocol (16) in the above inequality, we can further simplify (28) as

$$\begin{aligned}\|\frac{d}{dt}\epsilon(t)\| &\le \|f(x(t)) - \bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T f(x(t_k)) - \bar{B}\mu sign(\sigma(t_k))\|\\ &\le \bar{L}\|x(t)\| + \|\bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T\|\bar{L}\|x(t_k)\| + \|\bar{B}\|\mu\\ &\le \bar{L}(\|\epsilon(t)\| + \|x(t_k)\|) + \|\bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T\|\bar{L}\|x(t_k)\| + \|\bar{B}\|\mu\\ &\le \bar{L}\|\epsilon(t)\| + \bar{L}(1 + \|\bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T\|)\|x(t_k)\| + \|\bar{B}\|\mu.\end{aligned} \tag{29}$$

The solution to the differential inequality (29) $\forall t \in [t_k, t_{k+1}[$ can be understood by using Comparison Lemma (Hasan K. Khalil (2002)) with initial condition $\|\epsilon(t)\| = 0$ and is given as

$$\|\epsilon(t)\| \le \frac{\bar{L}(1 + \|\bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T\|)\|x(t_k)\| + \|\bar{B}\|\mu}{\bar{L}}\left(exp\{\bar{L}(t - t_k)\} - 1\right) \tag{30}$$

Comparison Lemma (Hasan K. Khalil (2002); Ramm and Hoang (2011)) is particularly useful when information on bounds on the solution is of greater significance than the solution itself. For triggering time instant t_{k+1},

$$\begin{aligned}\|\epsilon\|_\infty &= \|\epsilon(t_{k+1})\|\\ &\le \frac{\bar{L}(1 + \|\bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T\|)\|x(t_k)\| + \|\bar{B}\|\mu}{\bar{L}}\left(e^{\bar{L}T_k} - 1\right)\end{aligned} \tag{31}$$

$$\therefore\ T_k \ge \frac{1}{\bar{L}}\ ln\ \left(\frac{\bar{L}\|\epsilon\|_\infty}{\bar{L}(1 + \|\bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T\|)\|x(t_k)\| + \|\bar{B}\|\mu} + 1\right) \tag{32}$$

Since the right hand side of (31) is always positive, it is, therefore concluded that inter event execution time is bounded below by a finite positive quantity. This concludes the proof. □

4. RESULTS AND DISCUSSIONS

The efficacy of the proposed control scheme is demonstrated by computer simulation of the given model for two scenarios. Two test cases of regulation of temperatures at $280K$ and $300K$ are shown here. The tempreraturе was allowed to rise from zero to a set reference as quickly as possible. Figure 1 shows the regulation of the temperature of the modelled CSTR at $280K$. It is clear that the reference is achieved quite well using this control. Figure 2 is the plot of inter event execution time T_k which depicts the sampling interval. Clearly the sampling is non uniform and not in a periodic fashion. Sampling has been done on the vertical axis. Controller is updated only at particular instant based on triggering of an event, thereby reducing computational load and saving energy. Figures 3 and 4 depict the repeat case with reference set to $300K$. In figures 2 & 4, vertical axis represents the sampling interval and horizontal axis is time axis. At any given time t, ordinate denotes the length of sampling interval. Following parametric values are used in the experiment.

$\mu = 25$, $\beta = 0.3$, $D_a = 0.078$, $\gamma = 20$, $B = 8$ and $x_{2_c} = 0$. d_1 and d_2 are exogeneous disturbances assumed to be steps of magnitude 0.26 and 0.37 respectively. Moreover these disturbances are fixed by a positive upper bound of magnitude 3, i.e., $|d_1| < |d_2| < 3$. Surface coefficient weights λ_1 and λ_2 are chosen to be 0.1 and 5 respectively. $\zeta = \xi = 0.8$, $m_1 = 10^{-4}$, $m_2 = 0.2025$, $\psi = 0.5$ and $\varsigma = 0.97$ are taken to be parameters of the triggering rule.

5. CONCLUSION

A novel nonlinear controller based on event triggered sliding mode has been devised for a continuous stirred tank reactor. Event triggering technique is one real time control application used for minimum resource utilization while ensuring optimal closed loop behavior. The closed loop performance based on event triggering SMC is stable in Lyapunov sense. The inter event time is separated by a finite discrete time interval. The state under consideration has been regulated at the desired reference with minimal computation of the controller updates. Numerical simulations validated the effectiveness of the proposed event driven sliding mode control.

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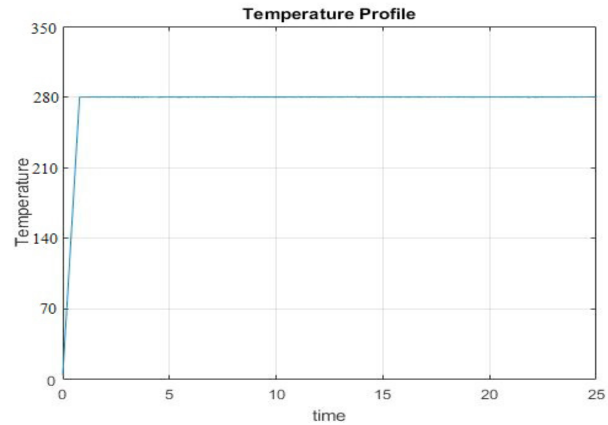

Fig. 1. Regulation at 280K

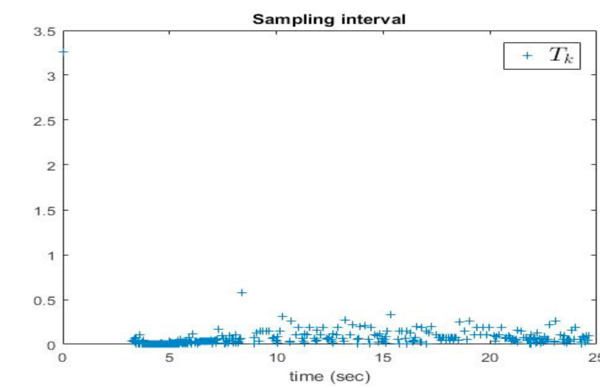

Fig. 2. Sampling interval for regulation at 280K

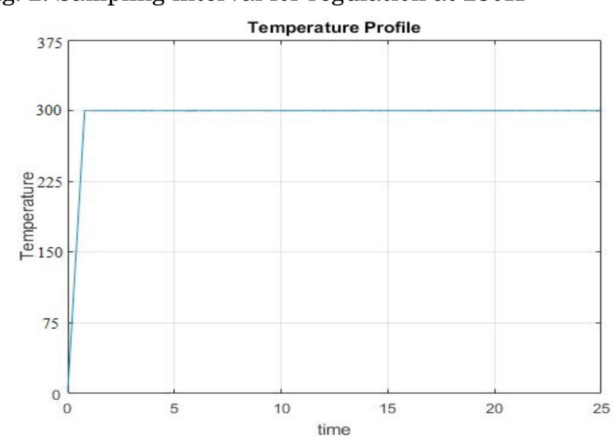

Fig. 3. Regulation at 300K

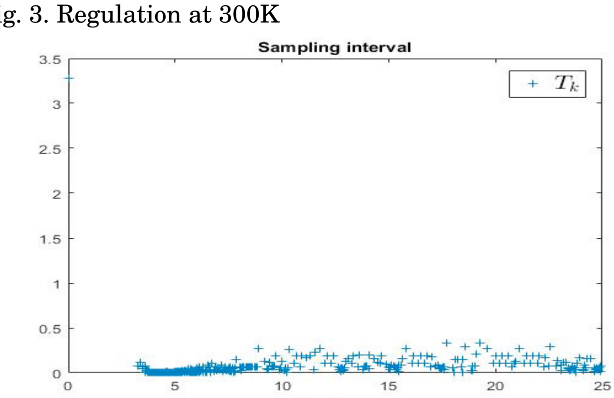

Fig. 4. Sampling interval for regulation at 300K

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