

Obliczanie granic ciągu

Przykład 1.

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{3n^3 + 7n - 2}{12n^3 - 4n^2 + 2n} &= \lim_{n \rightarrow \infty} \frac{n^3 \left(\frac{3n^3}{n^3} + \frac{7n}{n^3} - \frac{2}{n^3} \right)}{n^3 \left(\frac{12n^3}{n^3} - \frac{4n^2}{n^3} + \frac{2n}{n^3} \right)} = \\ &= \lim_{n \rightarrow \infty} \frac{3 + \overbrace{\frac{7}{n^2}}^{\rightarrow 0} - \overbrace{\frac{2}{n^3}}^{\rightarrow 0}}{12 - \underbrace{\frac{4}{n}}_{\rightarrow 0} + \underbrace{\frac{2}{n^2}}_{\rightarrow 0}} = \frac{3}{12} = \frac{1}{4}.\end{aligned}$$

Przykład 2.

Dany jest ciąg o wyrazie ogólnym:

$$a_n = n - \sqrt{n^2 + 5n},$$

Ile wynosi granica ciągu?

$$\begin{aligned}\lim_{n \rightarrow \infty} \left(n - \sqrt{n^2 + 5n} \right) \cdot \frac{n + \sqrt{n^2 + 5n}}{n + \sqrt{n^2 + 5n}} &= \\ &= \lim_{n \rightarrow \infty} \frac{n^2 - (\sqrt{n^2 + 5n})^2}{n + \sqrt{n^2 + 5n}} = \lim_{n \rightarrow \infty} \frac{n^2 - n^2 - 5n}{n + \sqrt{n^2 + 5n}} = \lim_{n \rightarrow \infty} \frac{-5n}{n + \sqrt{n^2 + 5n}} = \\ &= -\frac{5}{2}\end{aligned}$$

Przykład 3. Oblicz granicę ciągu o wyrazie ogólnym

$$a_n = \sqrt{\frac{3n - 2}{n + 10}},$$

$$\lim_{n \rightarrow \infty} \sqrt{\frac{3n - 2}{n + 10}} = \lim_{n \rightarrow \infty} \sqrt{\frac{n \left(\frac{3n}{n} - \frac{2}{n} \right)}{n \left(\frac{n}{n} + \frac{10}{n} \right)}} = \lim_{n \rightarrow \infty} \sqrt{\frac{3 - \frac{2}{n}}{1 + \frac{10}{n}}} = \sqrt{3}.$$