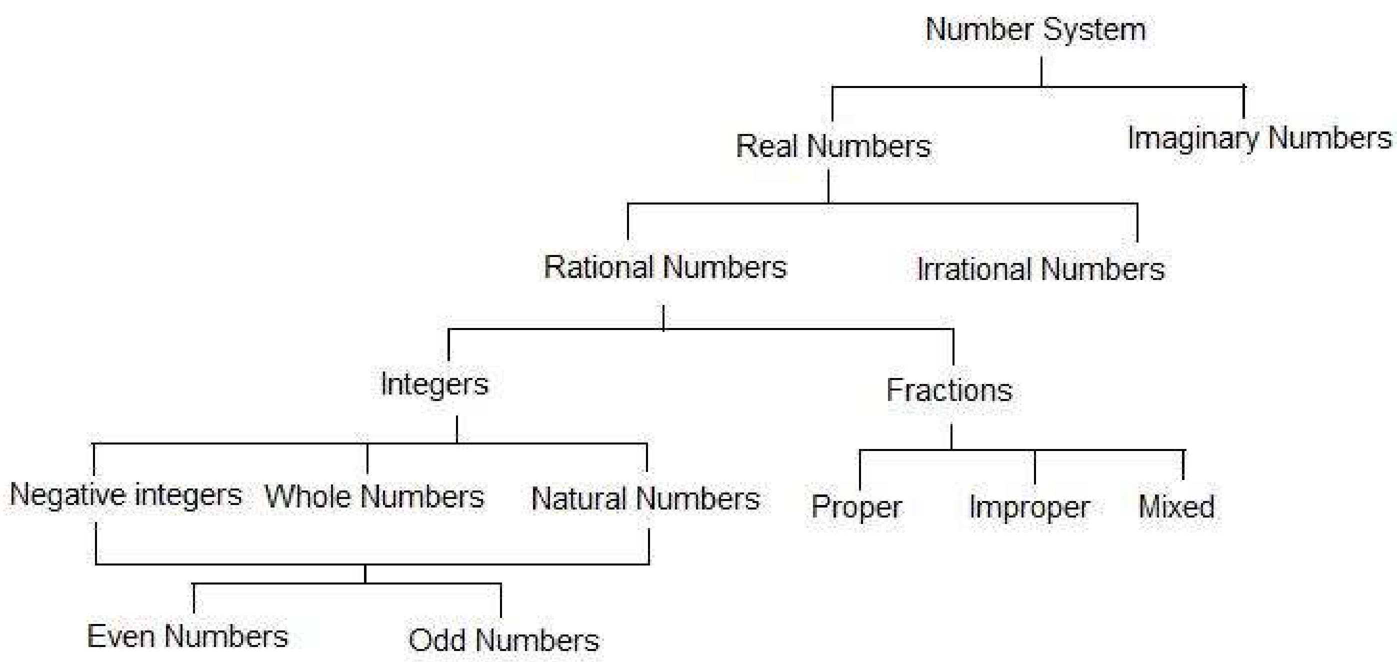
UNIT - I

- Number System
- Test for Divisibility
- Test of prime number
- Division and Remainder
- HCF and LCM of Numbers - Fractions

Number System

- Any system of naming or representing numbers is called as a number system.
- In general, a number system is a set of numbers with one or more operations. Number system includes real numbers, complex numbers, rational numbers, irrational numbers, integers, whole numbers, etc.

Classification of Numbers



Prime Numbers:

Any number greater than 1, which has exactly two factors (1 and itself) is called a prime number.

The only even prime number is 2. All prime numbers greater than 3 can be expressed in the form of $6n \pm 1$, where n is a positive integer.

Examples: 2, 3, 5, 7, 11, 13, 17 ……….

Composite Numbers:

Any number greater than 1 which is not prime is called a composite number.

Examples: 4, 6, 8, 9, 10, ...

- **Note:**

1) 1 is neither prime nor a composite number.
2) There are 25 prime integers between 1 and 100:

   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.
3) The difference between two consecutive prime numbers need not be constant.
4) If p is any composite number, then (p-1)! = (p-1)(p-2)(p-3)......3.2.1 is divisible by p.

To check whether a number is prime or not:

(1) Find the square root of the given number.

(2) Ignore the decimal part of the square root, consider only the integral part.

(3) Find all the prime integers less than or equal to the integral part of the square root.

(4) Check whether the given number is divisible by any of these prime numbers.

(5) If the given number is not divisible by any of these prime numbers then it is a prime number, otherwise it is not a prime number.

To check whether a number is prime or not:

Examples:

(1) Find whether 577 is prime or not.

(i) Find the square root of the given number.

$\sqrt{577} = 24.02$

(ii) Ignore the decimal part of the square root, consider only the integral part.

$\sqrt{577} = 24$

To check whether a number is prime or not:

(iii) Find all the prime integers less than or equal to the integral part of the square root.

Prime integers less than or equal to 24 are 2, 3, 5, 7, 11, 13, 17, 19, and 23.

(iv) Check whether the given number is divisible by any of these prime numbers.

577 is not divisible by any of the above prime numbers 2, 3, 5, 7, 11, 13, 17, 19, and 23.

(v) If the given number is not divisible by any of these prime numbers then it is a prime

number, otherwise it is not a prime number.

So, 577 is a prime integer.

To check whether a number is prime or not:

2) Find whether 687 is prime or not.

$\sqrt{687} = 26.21 = 26$

Prime numbers less than 26 are 2, 3, 5, 7, 11, 13, 17, 19, and 23

Clearly, 687 is divisible by the prime number 3.

Hence 687 is not a prime number.

<u>Note:</u>

If n is a prime number then, $2^n - 1$ is also a prime number.

Example: $2^{23} - 1$ is a prime number since 23 is a prime number.

$2^{10} - 1$ is not a prime number since 10 is not a prime number.

An integer $p > 1$ is prime if, and only if, the term $(p-1)! + 1$ is divisible by p. This is known as Wilson's theorem.

Co-prime numbers:

Two numbers are said to be co-prime or relatively prime if their HCF is 1, or if they have no common factors other than 1.

Example: 8 and 27 are co-prime, but 8 and 36 are not co-prime because both are divisible by 4.

Note that 1 is co-prime to every integer.

Divisibility rules:

Divisible by 2:

A number is divisible by 2, if the last digit is an even number.

Examples:

(a) Consider 1678

The last digit is 8, which is an even number. So, the number is divisible by 2.

(b) Consider 279

The last digit is 9, which is not an even number. So, the number is not divisible by 2.

Divisibility rules(continued):

Divisible by 3:

A number is divisible by 3, if the sum of its individual digits is a multiple of 3.

Examples:

(a) Consider 117

The sum of its individual digits is $1 + 1 + 7 = 9$, which is a multiple of 3. So, the number is divisible by 3.

Divisibility rules(continued):

(b) Consider 218

The sum of its individual digits is 2 + 1 + 8 = 11, which is not a multiple of 3. So, the number is not divisible by 3.

<u>Divisible by 4:</u>

A number is divisible by 4, if the last two digits are divisible by 4.

<u>Examples:</u>

(a) Consider 46728

The last two digits, i.e. 28, are divisible by 4. So, the number is divisible by 4.

(b) Consider 65737

The last two digits, i.e. 37, are not divisible by 4. So, the number is not divisible by 4.

Divisibility rules(continued):

Divisible by 5:

A number is divisible by 5 if the last digit is either 0 or 5.

Examples:

(a) Consider 20505

The last digit is 5. So, the number is divisible by 5.

(b) Consider 263

The last digit is 3. So, the number is not divisible by 5.

Divisibility rules(continued):

Divisible by 7:

To find out if a number is divisible by 7:

(1) Double the last digit of the number.

(2) Subtract it from the remaining leading truncated number.

(3) Repeat the above process until necessary.

(4) If the result is divisible by 7, then the given number is divisible by 7.

Divisibility rules(continued):

Examples:

(a) Consider 684502

(i) Double the last digit of the number.

2 * 2 = 4

- (ii) Subtract it from the remaining leading truncated number.
- 68450 – 4 = 68446
- (iii) Repeat the above process until necessary.
- 6844 – 12 = 6832
- 683 – 4 = 679
- 67 – 18 = 49
- (iv) If the result is divisible by 7, then the given number is divisible by 7.

Divisibility rules(continued):

Clearly, 49 is divisible by 7 and hence the given number is divisible by 7.

(b) Consider 481275

48127 – 10 = 48117

4811 – 14 = 4797

479 – 14 = 465

46 – 10 = 36

Clearly, 36 is not divisible by 7 and hence the given number is not divisible by 7.

Process to find the divisibility rule for prime numbers-

- Process to find the divisibility rule for P, a prime number is as follows:
- Step 1- Find the multiple of P, closest to any multiple of 10. This will be either of the form 10K + 1 or 10K – 1.
- Step 2- If it is 10K – 1, then the divisibility rule will be A + KB, and if it is 10K + 1, then the divisibility rule will be A – KB, where B is the units place digit and A is all the remaining digits.

- Example-
- Find the divisibility rule of 19. The lowest multiple of 17, which is closest to any multiple of 10 is 51 = 10(5) + 1.
- Therefore, the divisibility rule is A – 5B.

Process to find the divisibility rule for prime numbers-

Divisible by 8:

A number is divisible by 8 if the last three digits are divisible by 8.

Examples:

(a) Consider 2568

The last three digits, i.e. 568, are divisible by 8. So, the number is divisible by 8.

(b) Consider 84627

The last three digits, i.e. 627, are not divisible by 8. So, the number is not divisible by 8.

Divisibility rules(continued):

Divisible by 9:

A number is divisible by 9, if the sum of its individual digits is a multiple of 9.

Examples:

(a) Consider 1521.

The sum of its individual digits is 1 + 5 + 2 + 1 = 9, which is a multiple of 9. So, the number is divisible by 9.

(b) Consider 16873

The sum of its individual digits is 1 + 6 + 8 + 7 + 3 = 25, which is not a multiple of 9. So, the number is not divisible by 9.

Divisibility rules(continued):

Divisible by 10:

A number is divisible by 10, if the last digit is 0.

Examples:

(a) Consider 2500

The last digit is 0. So, it is divisible by 10.

(b) Consider 679

The last digit is not 0. So, it is not divisible by 10.

Divisibility rules(continued):

Divisible by 11:

To find out if a number is divisible by 11:

(1) Find the sum of its digits at even places

(2) Find the sum of its digits at odd places

(3) Find the difference between the above two sums in steps (1) and (2)

(4) If the difference is either 0 or a multiple of 11, then the number is divisible by 11.

Divisibility rules(continued):

Examples:

(a) Consider 34155

(1) The sum of its digits in odd places i.e. $3 + 1 + 5 = 9$

(2) The sum of its digits in even places i.e. $4 + 5 = 9$

(3) The difference is $9 - 9 = 0$

(4) Since, the difference is 0, the number is divisible by 11.

Divisibility rules(continued):

(b) Consider 847285

(1) The sum of its digits in odd places i.e. 8 + 7 + 8 = 23

(2) The sum of its digits in even places i.e. 4 + 2 + 5 = 11

(3) The difference is 23 - 11 = 12

(4) Since the difference 12 is not a multiple of 11, the number is not divisible by 11.

Divisibility rules(continued):

<u>Divisible by 13:</u>

To find out if a number is divisible by 13:

(1) Find four times the last digit of the number.

(2) Add it to the remaining leading truncated number.

(3) Repeat the above process until necessary.

(4) If the result is divisible by 13, then the given number is divisible by 13.

Divisibility rules(continued):

- **<u>Examples:</u>**

  (a) Consider 1165502

  (i) Find four times the last digit of the number.

  4 * 2 = 8

  (ii) Add it to the remaining leading truncated number.

  116550 + 8 = 116558

  (iii) Repeat the above process until necessary.

  11655 + 32 = 11687

  1168 + 28 = 1196

  119 + 24 = 143

  14 + 12 = 26

Divisibility rules(continued):

(iv) If the result is divisible by 13, then the given number is divisible by 13.

Clearly, 26 is divisible by 13, hence the given number is divisible by 13.

(b) Consider 113774

11377 + 16 = 11393

1139 + 12 = 1151

Divisibility rules(continued):

115 + 4 = 119

11 + 36 = 47

4 + 28 = 32

Clearly, 32 is not divisible by 13, hence the given number is not divisible by 13.

Divisible by 16:

A number is divisible by 16 if the last four digits are divisible by 16.

Examples:

(a) Consider 13992368

The last four digits i.e. 2368 are divisible by 16. So, the number is divisible by 16.

Divisibility rules(continued):

(b) Consider 143468

The last four digits i.e. 3468 are not divisible by 16. So, the number is not divisible by 16.

<u>Divisible by 17:</u>

To find out if a number is divisible by 17:

(1) Find five times the last digit of the number.

(2) Subtract it from the remaining leading truncated number.

(3) Repeat the above process until necessary.

(4) If the result is divisible by 17, then the given number is divisible by 17.

Divisibility rules(continued):

Examples:

(a) Consider 1523557

(i) Find five times the last digit of the number.

5 * 7 = 35

(ii) Subtract it from the remaining leading truncated number.

152355 – 35 = 152320

(iii) Repeat the above process until necessary.

15232 – 0 = 15232

1523 – 10 = 1513

151 – 15 = 136

Divisibility rules(continued):

(iv) If the result is divisible by 17, then the given number is divisible by 17.

Clearly, 136 is divisible by 17, hence the given number is divisible by 17.

(b) Consider 148761

14876 – 5 = 14871

1487 – 5 = 1482

148 – 10 = 138

Clearly, 138 is not divisible by 17, hence the given number is not divisible by 17.

Divisibility rules(continued):

Divisible by 19:

To find out if a number is divisible by 19:

(1) Double the last digit of the number.

(2) Add it to the remaining leading truncated number.

(3) Repeat the above process until necessary.

(4) If the result is divisible by 19, then the given number is divisible by 19.

Divisibility rules(continued):

Examples:

(a) Consider 1120278

(i) Double the last digit of the number.

2 * 8 = 16

(ii) Add it to the remaining leading truncated number.

112027 + 16 = 112043

(iii) Repeat the above process until necessary.

11204 + 6 = 11210

1121 + 0 = 1121

112 + 2 = 114

11 + 8 = 19

Divisibility rules(continued):

(iv) If the result is divisible by 19, then the given number is divisible by 19.

Clearly, 19 is divisible by 19, hence the given number is divisible by 19.

(b) Consider 1657043

165704 + 6 = 165710

16571 + 0 = 16571

1657 + 2 = 1659

165 + 18 = 183

18 + 6 = 24

Clearly, 24 is not divisible by 19, hence the given number is not divisible by 19.

Divisibility rules(continued):

Divisible by 20:

Any number is divisible by 20, if the tens digit is even and the last digit is 0.

Examples:

(a) Consider 34140

Here, the tens digit is even and the last digit is zero.

So, the given number is divisible by 20.

Divisibility rules(continued):

b) Consider 5678000
Here, the tens digit is even and the last digit is zero.
So, the given number is divisible by 20.

(c) Consider 8472847
Here, the tens digit is even but the last digit is not zero
So, the given number is not divisible by 20.

(d) Consider 8374670
Here, the last digit is zero but the tens digit is not even
So, the given number is not divisible by 20.

Divisibility rules(continued):

- **Divisibility rule for any number of the format $10^n \pm 1$, where n is any natural number-**
- In this method, we will be considering two factors, the first one being the value of n, and the second one being the sign '+' or '-'. The value of 'n' signifies how many digits will be taken one at a time, and the sign signifies in what manner these digits will be taken. Let us see this with the help of some examples.

Divisibility rules(continued):

- When n = 1.

  Case 1: $10^n + 1 = 11$.

  This method tells us that now we will be considering one digit at a time of the number that is to be divided by 11, and since the sign is '+' between 10^n and 1, we will alternately subtract and add starting from the right hand side.

- Example- To check if 523452 is divisible by 11, we will start from the right hand side, by subtracting the first number, and then adding the third, and then subtracting the fourth and so on.

  Case 2: $10^n - 1 = 9$.

  Since the value of n is one, we will take one digit at a time, and due to the '-' sign between 10^n and 1, we will keep on adding the digits from the right hand side.

Divisibility rules(continued):

- When n = 2.

  <u>Case 1:</u> $10^n + 1 = 101$.

  Since the value of 'n' here is two, we will make pairs of digits from the right hand side, and then we will alternately subtract and add each of the pairs.

  Example- To check if 13458975 is divisible by 101, we use the above method.

Divisibility rules(continued):

- Make the pairs first- 13 45 89 75.
- Now we will do this calculation. 75 – 89 + 45 – 13 = 44. This is not a multiple of 101. Hence, the number 13458975 is not divisible by 101.

<u>Case 2:</u> $10^n - 1 = 99$.

- Since the value of ‘n’ is two, we will take two digits at a time and after making the pairs ,due to the ‘-‘ sign between 10^n and 1, we will keep on adding the pairs from the right hand side.
- Example- To check if 452628 is divisible by 99.

Divisibility rules(continued):

- Let us make the pairs first- 45 26 28. Now add the pairs, 45 + 26 + 28 = 99. This is divisible by 99. Hence the number 452628 is divisible by 99.
- And so on, we follow this method for any natural number n.

Divisibility rules(continued):

- **Testing divisibility of other numbers:**
- A number (N) is said to be divisible by another number(N_1), if the number N is divisible by the co-prime factors of the number(N_1).

- **Divisible by 6:**
- A number is divisible by 6 if it is divisible by its co-prime factors i.e. 2 and 3.

Divisibility rules(continued):

- **Examples:**
- (a) Consider 174
- It is divisible by both 2 and 3. So, it is divisible by 6.
- (b) Consider 3194
- It is divisible by 2, but it is not divisible by 3. So, it is not divisible by 6.

- **Divisible by 12:**
- A number is divisible by 12, if it is divisible by its co-prime factors i.e. 3 and 4.

Divisibility rules(continued):

- **Examples:**
- (a) Consider 1716
- It is divisible by both 3 and 4. So, it is divisible by 12.
- (b) Consider 220
- It is not divisible by 3. So, it is not divisible by 12.

- **Divisible by 14:**
- A number is divisible by 14, if it is divisible by its co-prime factors i.e. 7 and 2.

Divisibility rules(continued):

- **<u>Examples:</u>**
- (a) Consider 490
- It is divisible by both 7 and 2. So, it is divisible by 14.
- (b) Consider 1090
- It is not divisible by 7. So, it is not divisible by 14.

- **<u>Divisible by 15:</u>**
- A number is divisible by 15, if it is divisible by its co-prime factors i.e. 5 and 3.

Divisibility rules(continued):

- **<u>Examples:</u>**
- (a)Consider 975
- It is divisible by both 5 and 3. So, it is divisible by 15.
- (b) Consider 2170
  It is not divisible by 3. So, it is not divisible by 15.

- **<u>Divisible by 18:</u>**
- A number is divisible by 18, if it is divisible by its co-prime factors i.e. 2 and 9.

Divisibility rules(continued):

- **Examples:**
- (a)Consider 106128
- It is divisible by both 2 and 9. So, it is divisible by 18.
- (b) Consider 154724
  It is divisible by 2 but the sum of its individual digits = 1 + 5 + 4 + 7 + 2 + 4 = 23, which is not divisible by 9; hence the given number is not divisible by 9.
- So, it is not divisible by 18.

Quotient and Remainder:

- If a given number is divided by another number, then
- Dividend = (Divisor * Quotient) + Remainder (Division algorithm)

- **Example:**

  1) On dividing 186947 by 163, the remainder is 149. Find the quotient.

$$\text{Quotient} = \frac{\text{Dividend} - \text{Remainder}}{\text{Divisor}} = \frac{186947 - 149}{163} = \frac{186798}{163} = 1146$$

Factor:

- Factors of a number N are one or more numbers that divide the number N without a remainder (i.e. the remainder left is zero.)
- Example: Factors of 15 = 15, 5, 3, 1

  Factors of 20 = 20, 10, 5, 2, 1

Multiple:

- If **there are** one or more numbers that divide the number N without a remainder (i.e. the remainder left is zero), then N is called the multiple of those numbers.
- Example:
- 1) Factors of 15 = 15, 5, 3, 1

  Thus 15 is a multiple of 5, 3, 1

- 2) Factors of 20 = 20, 10, 5, 4, 2, 1

  Thus 20 is a multiple of 10, 5, 4, 2, 1

GCD (Greatest Common Divisor)/ HCF (Highest Common Factor:

- The greatest common divisor of any two or more distinct numbers is the greatest natural number that divides each of them exactly.

- **Methods for finding GCD of given numbers:**

  There are two methods for finding the GCD of given numbers.

- **1. Factorisation method:**

  Find all the prime factors of the given numbers.

  Express the given numbers as the product of the prime factors.

  The product of least powers of common prime factors is the GCD of the given numbers.

GCD (Greatest Common Divisor)/ HCF (Highest Common Factor)(continued):

- **Example:**

  1) Find the GCD of 45 and 75

  Prime factors of 45 = 3 * 3 * 5 = $3^2 * 5$

  Prime factors of 75 = 3 * 5 * 5 = $3 * 5^2$

  Thus the GCD of 45 and 75 is 3 * 5 = 15

GCD/HCF Continued

- **2. <u>Division method:</u>**

  Suppose there are two numbers. Divide the larger number by the smaller one.

  Now, divide the divisor by the remainder.

  The process of dividing the preceding divisor by the remainder obtained is repeated till the remainder obtained is zero.

  The last divisor is the GCD of the two given numbers.

  If there are three numbers, first find the GCD of any two numbers. Then find the GCD of the result obtained and the third number. A similar method is followed for obtaining the GCD of more than three numbers.

GCD/HCF Continued

- **<u>Example:</u>**

  1) Find the GCD of 50, 35 and 27.

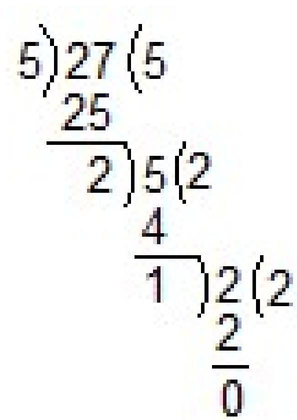
  First find the GCD of 50 and 35.

```
35)50(1
   35
   ---
   15)35(2
      30
      ---
       5)15(3
         15
         ---
         00
```

GCD/HCF Continued

- The GCD of 50 and 35 is 5.

- Now, find the GCD of 5 and 27.



- The GCD of 5 and 27 is 1
- Thus the GCD of the given numbers = 1.

LCM (Least Common Multiple):

- The least common multiple of any two or more distinct numbers is the smallest natural number that is exactly divisible by each one of the given numbers.

- **Methods for finding LCM of given numbers:**

  There are two methods for finding the LCM of given numbers.

- **1. Factorisation method:**

  Find all the prime factors of the given numbers.

  Express the given numbers as the product of the prime factors.

  The product of the highest powers of all factors is the LCM of the given numbers

LCM contd.

- **Example:**

  1) Find the LCM of 45 and 75

  Prime factors of 45 = $3 * 3 * 5 = 3^2 * 5$

  Prime factors of 75 = $3 * 5 * 5 = 3 * 5^2$

  Thus the LCM of 45 and 75 is $3^2 * 5^2 = 9 * 25 = 225$

LCM contd.

- **2. Division method:**
- Arrange the given numbers in a row.
- Divide the given numbers by a number which exactly divides at least two of them and the numbers which are not divisible are carried forward.
- The above process is repeated until no two of the given numbers are divisible by the same number except 1.
- The product of all the divisors and the undivided numbers gives the LCM of the given numbers.

LCM contd.

- **Example:**

  1) Find the LCM of 20, 35, 46 and 50.

2	20	35	48	50
2	10	35	24	25
5	5	35	12	25
	1	7	12	5

Required LCM = 2 * 2 * 5 * 7 * 12 * 5 = 8400

HCF and LCM of fractions:

$$\text{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

$$\text{LCM} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

HCF and LCM of fractions(continued):

- **<u>Example:</u>**

1) Find the HCF and LCM of $\frac{2}{3}, \frac{5}{4}, \frac{1}{3}, \frac{5}{2}$ and $\frac{7}{5}$

Numerators are 2, 5, 1 and 7.
Denominators are 2, 3, 4 and 5.

$$\text{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}} = \frac{\text{HCF of 2, 5, 1 and 7}}{\text{LCM of 2, 3, 4 and 5}} = \frac{1}{60}$$

$$\text{LCM} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}} = \frac{\text{LCM of 2, 5, 1 and 7}}{\text{HCF of 2, 3, 4 and 5}} = \frac{70}{1} = 70$$

HCF and LCM of fractions(continued):

- **Note:**

  HCF * LCM = product of numbers

- **Example:**

  The HCF of two numbers is 12 and their LCM is 72. If one of the numbers is 24, find the other.

  Solution: Let the required number be x. then,

  HCF * LCM = product of numbers

  12 * 72 = 24 * x

  $$x = \frac{12 \times 72}{24} = 36$$

  Thus the other number is 36.