Homework 3 - Jack Brolin, Abhiram Nallamalli

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P. 2 Let $P = \left\{x \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 1, x \geq 0\right\}$ and consider the vector $x = (0, 0, 1)$. Find the set of feasible directions at x.

P. 3 Consider the problem of minimizing $c'x$ over a polyhedron P. Prove the following:

a. A feasible solution x is optimal if and only if $c'd \geq 0$ for every feasible direction d at x.

b. A feasible solution x is the unique optimal solution if and only if $c'd > 0$ for every nonzero feasible direction d at x.

a.

b.

P. 4 Let x be an element of the standard form polyhedron $P = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$. Prove that a vector $d \in \mathbb{R}^n$ is a feasible direction at x if and only if $Ad = 0$ and $d_i \geq 0$ for every i such that $x_i = 0$.

P. 7 Consider a feasible solution x to the standard form problem

$$\begin{aligned} &\text{minimize} c'x \\ &\text{subject to } Ax = b \\ &\qquad\qquad x \geq 0, \end{aligned}$$

and let $Z = \{i : x_i = 0\}$. Show that x is an optimal solution if and only if the linear programming problem

$$\begin{aligned} &\text{minimize} c'd \\ &\text{subject to } Ad = 0 \\ &\qquad\qquad d_i \geq 0, \quad i \in Z, \end{aligned}$$

has an optimal cost of zero.

Proof. Start ■

P. 9 Consider the problem

$$\begin{array}{rl} \text{minimize} & -2x_1 - x_2 \\ \text{subject to} & x_1 - x_2 \leq 2 \\ & x_1 + x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{array}$$