Implications of Meta-logic

Linking our previous works

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Orders of logics

What are the classes of logics ?

Orders of logics   Zeroth order

0th order

- Propositional logic

- No variables

1st order

- Propositional calculus augmented with quantifiers $\forall$ and $\exists$

- Variables over a specific domain

- Permits quantifying propositions, functions, …

- Quantifying over nested sets

Higher order

Orders of logics

0th order

1st order

Higher order

Properties over orders

Specifically soundness and completeness

Soundness:

A logical system is sound if all statements that can be proven within the system are true in its interpretation.

Example:

In a sound system, if we can prove P→Q and we know P is true, then Q must also be true.

Completeness:

A logical system is complete if every statement that is true in its interpretation can be proven within the system.

Example:

In a complete system, if $P \wedge Q$ is true, then there exists a proof for

$P \wedge Q$ within the system.

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0th order

- **Completeness**: If writable then it's provable

- **Soundness**: Straightforward as they are only propositions
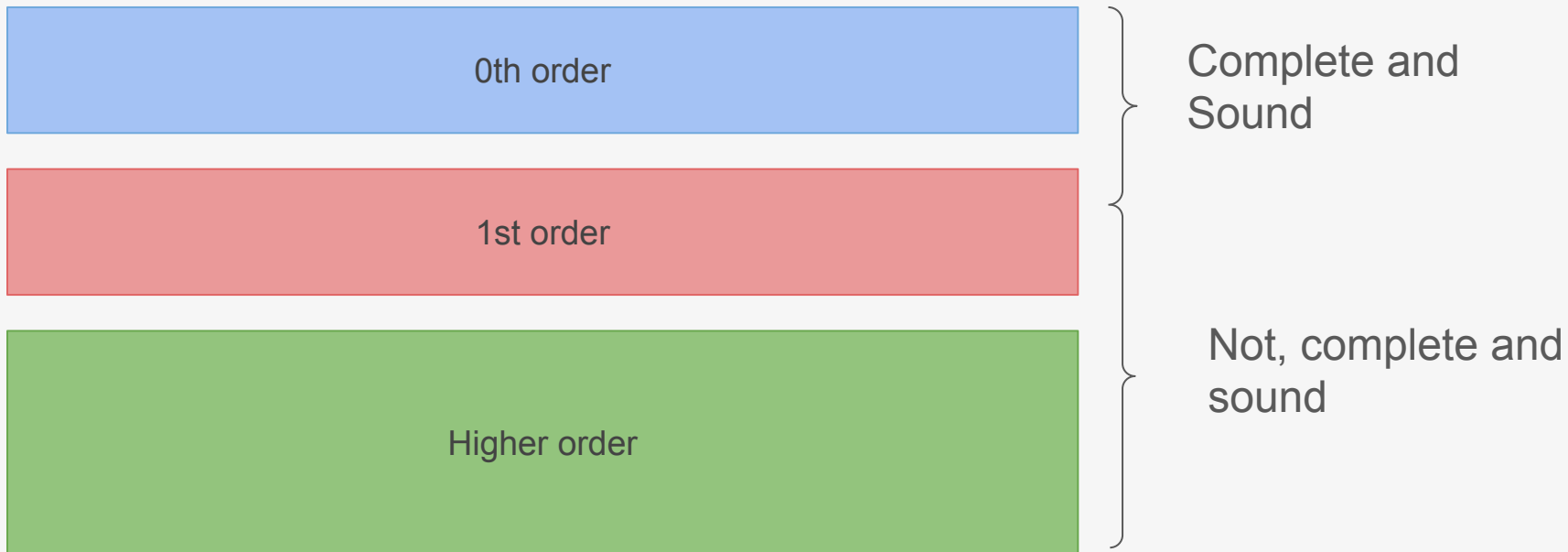
Properties over orders

1st order

- Partial completeness and soundness

- Semidecidable

- Generally working on sound and complete fragments

- No deductive systems that are complete, sound and effective

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Higher order
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Properties over orders

0th order

1st order

Higher order

Complete and Sound

Not, complete and sound

Applications to modal logics

How does that affects our work?

Definition:

It is a formal system which include unary operators such as ◊ and □, which represents the possibility and obligation respectfully.

It's an extension of classical logic

Properties over orders

Modal logics

0th order

Deontic logic

CTL*

1st order

Higher order

Complete and Sound

Not complete and sound

What's Next?

- Study of expressivity (efficiency) of the various logics

  - CTL*

  - Deontic

  - PCTL

- Applying logics to a concrete problem e.g. dining philosophers problem

THANK YOU

Any questions? Remarks ?