Meta Logics

Understanding Logic Systems

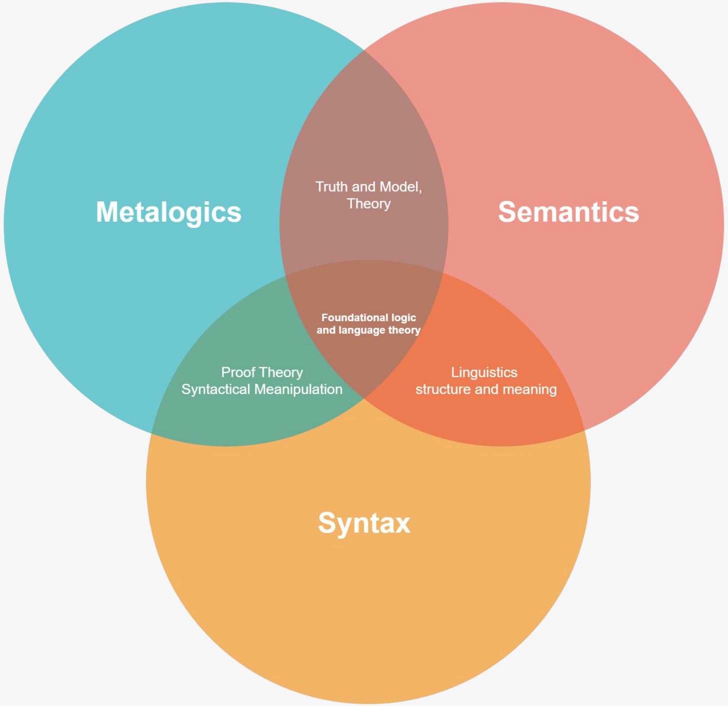
Ethan Arm Dany A. Darghouth – March 2024

Introduction

What are metalogics ? - *"Introduction to Logic" by Patrick Suppes*

Definition:

"The study of the properties, structure, and limitations of logical systems."



- **1879 *Begriffsschrift* - *G. Frege* :** Introduced the concept of a formal system, significantly influencing the development of logic.

- **1931 *Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I* - *K. Gödel* :** The first incompleteness theorem, fundamentally changing the understanding of mathematical logic.

- **1933 *The Concept of Truth in Formalized Languages* - *A. Tarski* :** Groundbreaking advancements in modern logic and semantics

KEY CONCEPTS OF METALOGICS

Sorry but there's going to be some text..

Syntax:

The set of rules that governs the structure of statements in a logical system.

Example:

In propositional logic, a rule might be that if P and Q are propositions, then $P \land Q$ is also a proposition.

Semantics:

The meanings that these statements convey.

Example:

The statement $P \land Q$ is true if and only if both P and Q are true.

Soundness:

A logical system is sound if all statements that can be proven within the system are true in its interpretation.

Example:

In a sound system, if we can prove P→Q and we know P is true, then Q must also be true.

Completeness:

A logical system is complete if every statement that is true in its interpretation can be proven within the system.

Example:

In a complete system, if P∧Q is true, then there exists a proof for P∧Q within the system.

Decidability:

A logical system is decidable if there is a method that can determine, for any statement, whether the statement is true or false within the system.

Example:

The question of whether a given polynomial equation has a solution can be decided by an algorithm.

Consistency:

A logical system is consistent if it does not allow for a statement and its negation to both be provably true at the same time.

Example:

A consistent system cannot prove both P and ¬P for any proposition P.

1st Theorem:

There are true statements in the system that cannot be proven within the system.

Example:

"This statement cannot be proven."

2nd Theorem:

No consistent formal system capable of expressing elementary arithmetic can prove its own consistency.

Example:

If a system could prove its own consistency, it would lead to paradoxical situations, undermining the system's basis.

What’s been done?

The source of night demons

What's been done ? Mathematical Catch-up

- Category theory
- Ordered logic theory
- Model Theory vs. Proof theory
- Metalogic

What's Next?

Maybe liberation

What's Next? Exploration trails

- Link between Meta-logic and Modal logic:
  - Validation of meta-logic properties for modal logic (of the first order)
  - Comparison of several logics on meta-logic properties
- Model theory:
  - Description of modal logic for Model theory
- Completeness for Model checking
  - Exploration of Multi-modal logic

What's Next?

Next Time :

- Investigation on the definition of Meta-logic for first order logic
- Is CTL a first order Logic ? Is an SFDD a person or a set of person ?
- Implication of Meta-logic on Model-theory

THANK YOU

Any questions? Remarks ?