Exploring other modal logics

An Introduction to Deontic Logic, Past CTL and PCTL

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MODAL LOGIC

Definition:

A kind of logic used to represents statements about necessity and possibility.

Formally:

It is a formal system which include unary operators such as ◊ and □, which represents the possibility and obligation respectfully.

Example:

$\Diamond P$ *can be read as possibly P*
$\Box P$ *can be read as necessarily P*

Logics:

In this category we have epistemic logic, deontic logic, but also CTL, PAST + CTL or PCTL, etc…

DEONTIC LOGIC

Standardized by *Georg Henrik von Wright - 1951*

Definition:

"Deontic Logic is concerned with the formalization of norms and normative propositions, focusing on the logical relationships between obligations, permissions, and prohibitions." - Chat GPT 4 2024

Objective:

Establishing a logical formalism on the intuition of obligation, possibility and prohibition.

- **Early 20th century:** The formalization of modal logics.
- **1951:** Von Wright introduces the first systematized form of Deontic Logic.
- **1960s-1970s:** Expansion in the study and applications of Deontic Logic.
- **Present:** Diverse applications in computer science, artificial intelligence, and ethical theory.

The whole Logic relies on 3 (2+1)* **concepts :**

φ being a logical proposition

- Obligation (O): What must be the case.
  - *O(φ) means it is obligatory that φ*
- Permission (P): What may be the case.
  - *P(φ) means it is possible that φ*
- Prohibition (F): What must not be the case.*
  - *F(φ) means it is forbidden that φ*

*IF TIME ALLOWS ASK US ABOUT THE 3rd ONE

Possible worlds semantics :

- Like in CTL we will consider scenarios where statements can be true (or not)
- A "world" is a state
- Obligation, Possibilities.. are determined based on world accessibility
    - e.g. in a given world, O(p) is true if p is true in all accessible worlds/states.

- **Crucial in Ethical fields** (law, AI,...)
- **Applications in IT systems for permission management**
- **Challenging when face with Paradoxes or complex real-world scenarios**

CTL REFRESHER

We swear it will help for what's next..

Definition:

CTL is a branching-time logic with possesses quantifiers over paths, and path-specific quantifiers.

Formulae syntax:

$$\begin{aligned}\phi ::= {} & \bot \mid \top \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \Rightarrow \phi) \mid (\phi \Leftrightarrow \phi) \\ \mid \quad & \mathbf{AX}\,\phi \mid \mathbf{EX}\,\phi \mid \mathbf{AF}\,\phi \mid \mathbf{EF}\,\phi \mid \mathbf{AG}\,\phi \mid \mathbf{EG}\,\phi \mid \mathbf{A}\,[\phi\,\mathbf{U}\,\phi] \mid \mathbf{E}\,[\phi\,\mathbf{U}\,\phi]\end{aligned}$$

Where *p* is ranges over a set of atomic formulas.

PCTL

Probabilistic Computation Tree Logic - *Hansson, Hans, and Bengt Jonsson 1994*

Essentially:

- Extension of CTL (yes, like Past CTL)
- Probabilities are added to the logic
  - Now transitions are associated with a probability
- Useful for checking models and verifying systems that don't have a deterministic outcome (no, not like Past CTL, please focus)

Probabilistic CTL A brief history

- **Early 20th century:** The formalization of modal logics.
- **1960s-1970s:** Expansion in the study and applications of temporal Logic.
- **1994:** Hansson & Johnson introduce the PCTL extension of CTL.
- **Present:** Diverse applications in computer science, artificial intelligence, and ethical theory.

Probabilistic CTL — Syntax

- The main addition is that of **probabilistic** operators:
  - `P ≥ 0.9[ F systemFailure ]` means "The probability of eventually experiencing a system failure is at least 90%".

- State / Path formulas are extended with **probabilistic measures**:
  - State formulas in PCTL can quantify the probability of a path formula being satisfied from a given state.
  - Path formulas are evaluated with respect to probabilistic measures.

Probabilistic CTL — Semantics

- Relies on a **probabilistic model** (often a Markov Chain)
- A state in a model **satisfies** a PCTL formula, taking however the probabilities of various outcomes into account .
  - E.g. a state might satisfy a formula if the probability of a certain path formula being true from that state meets a specified threshold.
- In a PCTL formula
  - E.g. evaluating `P≥0.9[F"criticalError"]` involves calculating the probability of eventually reaching a state that satisfies the `"criticalError"` condition and checking if this probability is at least 90%.

Applications / Challenges

- **Quality Assurance** (Probabilistic Model Checking helps building reliable systems)
- **Network Security** (Assessing the risk of security breaches and the effectiveness of security protocols under probabilistic threats.)

- **Model Complexity** (Creating accurate probabilistic models that reflect real-world systems can be highly complex and computationally demanding.)
- **Uncertainty Quantification** (Accurately quantifying the probabilities of various outcomes requires extensive data and sophisticated modeling techniques.)

PAST + CTL

Extending CTL to include the past - *Hansson, Hans, and Bengt Jonsson 1997*

Definition:

An extension of CTL* which add the notion of Past of a state. We add a few operators namely $\mathbf{F}^{-1}$, $\mathbf{X}^{-1}$, **S** and **N**.

The idea:

The past is determined, finit and cumulative.

Moreover you have a partial equivalence between Past+CTL and CTL*.

THANK YOU

Any questions? Remarks ?