

Complete and Efficient Checking of Transactional Causal Consistency in Databases

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ABSTRACT

Transactional causal consistency (TCC) is a prevalent weak isolation guarantee that has emerged as a successful marriage of the distributed computing and database communities. TCC eschews the performance penalties of strong isolation levels while preventing various undesired data anomalies. However, TCC anomalies have recently been found in many production database engines that claim to provide TCC or even stronger isolation guarantees.

In this paper we present C4, a complete, efficient black-box checker for TCC which provides understandable violating scenarios for detected bugs. C4 builds on a novel, fine-grained characterization of TCC via *transactional anomalous patterns*, for which we establish its soundness and completeness. C4 leverages the joint force of two techniques, namely vectors and tree clocks, to accelerate its TCC checking. Our extensive assessment demonstrates that C4 can successfully reproduce all of the 3097 known TCC anomalies, identify new TCC violations in four production database engines along with their causes, find more anomalies than the state-of-the-art black-box checkers, and efficiently validate TCC under a wide range of workloads.

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Our tool, experimental data, and technical report are available at <https://github.com/dracooooo/C4>.

1 INTRODUCTION

Transactional causal consistency (TCC) [3, 41] has emerged as a successful marriage of the distributed computing and database communities. TCC extends causal consistency [2, 51], the strongest consistency level achievable in an always-available system [5], by providing database transactions. This brings fruitful benefits to both database users and developers. TCC eschews the performance penalties of stronger isolation levels such as snapshot isolation and serializability while avoiding many anomalies such as causality violations and diverging data [40] under weaker isolation levels [9, 12];

the support for transactions simplifies the programming complexity of concurrency by providing an abstraction for executing concurrent computations on shared data in isolation [13]; TCC also provides intuitive semantics that supports session guarantees [57], e.g., *read your writes*, which is favorable to end users.

As an emerging isolation guarantee, TCC has already attracted the attention of both academia and industry. There has been a blossoming of academic advances in the last decade in highly available and performant TCC database systems [3, 10, 23, 24, 40–42, 45, 54, 55]. Facebook advocates the joint force of causal consistency and transactions [43]. Recent production adoptions of TCC and its variants include Neo4j [48], Cosmos DB [46], MongoDB [47], and ElectricSQL [25] (a successful transition from the Cure protocol [3] to production). Unfortunately, TCC anomalies have manifested in many production databases that claim to provide TCC or even stronger isolation levels such as *snapshot isolation* (SI) or *serializability* (SER) [14, 30, 32]. This raises the concerns of whether databases actually deliver the promised isolation levels in practice.

Black-box testing has been adopted as the *de facto* approach to finding isolation bugs and validating isolation fulfilment [14, 30, 35, 56, 60, 61] as database internals are quite often unavailable and hardly understandable. The SIEGE+ principle [30], extending SIEGE [35] with *completeness*, has recently been proposed to guide the design of black-box isolation checkers. Specifically, an ideal checker is expected to be Sound (returning no false positives), Informative (reporting understandable counterexamples), Effective (finding isolation bugs in real-world databases), General (compatible with different kinds of databases and general transaction workloads), Efficient (adding modest checking time even for workloads of high concurrency and large size), and Complete (missing no isolation anomalies).

However, existing isolation checkers, including those for causal consistency, fail to fulfill all of the SIEGE+ criteria. In particular, the state-of-the-art TCC tester dbcop [14] misses TCC bugs, e.g., non-repeatable reads labeled as “critical” by MariaDB [6] (C), and requires a significant amount of time for TCC validation (E); CausalC [60] can only support checking simple transactions of single read/write operation (G). Although checkers for *stronger* isolation guarantees could also be used to detect TCC bugs or validate TCC, they inherently return false positives with respect to TCC (S), e.g., an SER checker may report the lost-update anomalies which do not violate TCC, and are prohibitively expensive due to the high computational complexity of the checking problem *per se* (E), e.g., checking SI is NP-complete in general [14]. Moreover, few checkers can catch all bugs (C) because of either incomplete or coarse-grained characterizations of isolation guarantees [8, 56, 61] or efficiency concerns [29]. Finally, despite the latest trend of utilizing solvers to check isolation guarantees [30, 56, 61], as we have observed through our analysis (Section 5), the checking overhead

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is still significantly high in practice. In addition, solver-based tools return less understandable counterexamples in general (**I**) such as atomic propositions and unsatisfied clauses, rendering the understanding and debugging of the violations hard.

The C4 Checker. We devise a novel black-box TCC checker called C4 to fulfill SIEGE+. C4 builds on two key ideas to particularly address three major challenging criteria: Complete, Efficient, and Informative.

First, driven by the building blocks of TCC such as atomic visibility [9], causal consistency [2, 51], and data convergence [3, 40], and the isolation bugs that manifest in the wild [14, 30, 32], we devise 12 fine-grained *transactional anomalous patterns* (TAPs) to characterize TCC. Our characterization overcomes the C and I challenges. It is *sound* and, more importantly, *complete*, rendering C4 capable of capturing all possible TCC data anomalies in collected execution histories¹ of transactions as in black-box testing of database engines. Thanks to C4’s completeness, we are able to identify significantly more anomalies than the state-of-the-art isolation checkers from the same collection of histories (Section 5). This is highly desirable, especially given that the occurrence of an isolation bug is usually unpredictable due to random test generation commonly adopted by existing black-box checkers.

Our characterization is also fine-grained, with each TAP corresponding to defects of the building blocks underlying a TCC database system. For example, TAP-g (Section 3.2) is typically caused by design or implementation errors on session guarantees (like the incorrect use of vectors) while TAP-h is caused by flaws on atomic commitment of transactions. Such a fine-grained characterization enables C4 to report informative counterexamples with the full violating scenarios, which facilitate developers in locating their root causes. Moreover, these modular TAPs make it flexible to form sound and complete *weaker* isolation checkers by combining an appropriate subset of them, e.g., six TAPs for *read committed* and additionally with two for *read atomicity*.

Second, although TCC is checkable in polynomial time [14], existing tools still suffer from the high checking overhead in practice due to inefficient searching for cycles (which are used to represent TCC violations [14]) in large, dense dependency graphs (constructed from extensive histories) or encoding and solving of graph properties using solvers [30, 56, 60, 61]. C4 leverages two techniques to overcome the E challenge.

Inspired by the decades-long practice of using *vectors* [26] to *design* causally consistent databases, we utilize vectors instead to *black-box check* TCC systems. By capturing and storing the causal dependencies among the transactions in a history, vectors enable C4 to accelerate graph traversals in both reachability checking and cycle detection. Moreover, we adapt the *tree clock* [44] data structure, a recent advance in concurrent program analysis, into our utilization of vectors to further accelerate C4. This adaption significantly reduces the time complexity of both comparing and joining vectors, which arises due to the large number of client sessions and dense dependencies.

Main Contributions. Overall, we provide:

- (1) a new sound and complete characterization of TCC via TAPs which also facilitates the understanding and debugging of violations (Section 3);
- (2) a complete and efficient TAP-based checking algorithm for TCC, leveraging vectors and tree clocks to accelerate the searching for TAPs (Section 4);
- (3) the C4 tool comprising our new checking algorithm and a visualizer for restoring violating scenarios; and
- (4) an extensive evaluation of C4, along with the state-of-the-art black-box isolation checkers, that demonstrates C4’s fulfilment of SIEGE+ (Section 5). In particular, C4 has (i) reproduced all of 3097 known TCC violations, (ii) detected novel TCC bugs in four production databases of different kinds, (iii) identified their causes, (iv) found significantly more anomalies than existing checkers, (v) substantially outperformed the baselines on various benchmarks, and (vi) scaled to large workloads with 1 million transactions, 400 million operations, and 1 billion keys successfully checked less than 200 seconds.

2 TRANSACTIONAL CAUSAL CONSISTENCY

2.1 TCC in a Nutshell

Informally, TCC combines three properties: read atomicity (RA) [9], causal consistency (CC) [2, 51], and convergence [3, 40]. RA ensures that all or none of a transaction’s updates are observed by other transactions. It prohibits the *fractured reads* anomaly, as illustrated below.

Example 1 (Read Atomicity). Joey, Monica, and Rachel are planning a road trip. They share a road trip planner. Rachel would like to invite Ross to the trip after two of them become friends of each other. Without the RA guarantee, Joey and Monica may only see one direction of the friendship (fractured reads), e.g., Ross is a friend of Rachel’s but Rachel is not a friend of Ross’.

CC guarantees that two transactions that are causally related must appear to all client sessions in the same *causal order*. CC is defined by the following three rules:

- **Session Order (SO).** If two transactions t_1 and t_2 are in the same session and t_1 is performed before t_2 , then t_1 is causally before t_2 ;
- **Write-Read Relation (WR).** If transaction t_2 reads values from transaction t_1 , then t_1 is causally before t_2 ;
- **Transitivity.** If transaction t_1 is causally before t_2 and t_2 is causally before t_3 , then t_1 is causally before t_3 .

CC prevents the *causality violation* anomaly, as illustrated below.

Example 2 (Causality). On the road trip planner, Joey sets Los Angeles as the starting city. After seeing it, Monica adds Chicago to the destination. Without causal consistency, Rachel may see the destination but not the starting city.

CC does not constrain transactions that are *not* causally related, which may be observed in different orders by different sessions. As a result, replicas may permanently diverge under concurrent conflicting updates. In TCC, this is forbidden by the *convergence* property which requires replicas eventually converge to the same state even under conflicting updates [3, 40]. In practice, convergence can be achieved by applying the LWW (last-writer-wins) rule [17].

¹A history records the transactional requests to and responses from a database. See Section 3.1 for its formal definition.

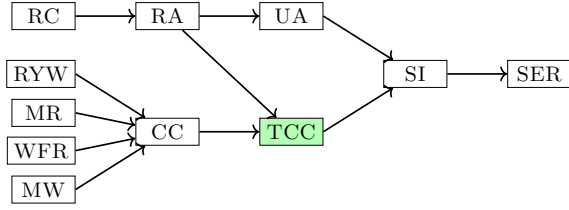


Figure 1: A hierarchy of isolation levels. $A \rightarrow B$: A is strictly weaker than B . RC: read committed; four session guarantees [16, 57]; read your writes (RYW), monotonic reads (MR), writes follow reads (WFR), and monotonic writes (MW).

Although there is a weaker variant of TCC with no convergence guarantee [42], to the best of our knowledge, all of the database systems claiming to support TCC provide convergence in practice.

Example 3 (Convergence). Joey and Ross update the meeting place to their own apartment, respectively. With convergence, these two conflicting updates must be eventually handled in a convergent manner: either Joey’s or Ross’ apartment. Otherwise, Monica and Rachel may head to different apartments.

As shown in Figure 1, TCC is an essential building block of stronger transactional consistency properties such as SI [12] and SER [50]. In particular, both SI and SER comprise TCC and update atomicity (UA) [18] that forbids the lost-update anomaly. Hence, a TCC violation is also a violation for SI and SER. In fact, many isolation bugs found in the production databases that claim to support SI or SER are actually TCC violations [30, 35, 56, 61].

2.2 Checking TCC

Definition 1. The TCC *checking problem* is the decision problem of determining whether a history of transactions satisfies TCC.

Following the common practice in black-box database testing [14, 30, 35, 56, 61], we also make two assumptions. First, every history contains a special transaction (denoted t_\perp) that writes the initial values (denoted \perp_x, \perp_y, \dots) of all keys [14, 19]. This transaction precedes all the other transactions across client sessions. In practice, we can use write-only transactions to populate the database with initial values for each key *before* the actual testing. These transactions can be considered as a single logical write-only transaction.

Second, for each key, every write to the key assigns a unique value [1, 14, 56]. Hence, each read can be uniquely associated with the transaction that issues the corresponding (dictating) write. The TCC checking problem can then be solved in polynomial time [14]; otherwise, it is NP-hard in general [15]. For database testing, we can use, e.g., the client identifier and local counter, to ensure the uniqueness of values written.

3 CHARACTERIZING TCC ANOMALIES

In this section we describe our fine-grained characterization of TCC using *transactional anomalous patterns* (TAPs). We first present the formal basis for our characterization.

3.1 TCC: Formal Definition

We consider a key-value store managing a set of keys $K = \{x, y, z, \dots\}$ associated with values from V . We use $R(x, v)$ to denote a read operation that reads $v \in V$ from $x \in K$ and $W(x, v)$ to denote a write operation that writes $v \in V$ to $x \in K$.

Relations and Orderings. A binary relation R over a set A is a subset of $A \times A$. For $a, b \in A$, we use $(a, b) \in R$ and $a \xrightarrow{R} b$ interchangeably. We use R^+ to denote the transitive closure of R . A relation $R \subseteq A \times A$ is *acyclic* if $R^+ \cap I_A = \emptyset$, where $I_A \triangleq \{(a, a) \mid a \in A\}$ is the identity relation on A . A strict partial order is an irreflexive and transitive relation. A strict total order is a relation that is a strict partial order and total. We write $_$ for irrelevant parts that are implicitly existentially quantified.

Causality and Convergence. Clients interact with the data store by issuing transactions during *sessions*. A *history* records the client-visible results of such interactions.

Definition 2 (History). A *history* is a triple $\mathcal{H} = (T, SO, WR)$, where T is a set of transactions, $SO \subseteq T \times T$ is the session order, and $WR \subseteq T \times T$ is the write-read relation: if transaction t_2 reads the value of key x from transaction t_1 , then $t_1 \xrightarrow{WR} t_2$ (also denoted $t_1 \xrightarrow{WR(x)} t_2$ to make x explicit).

We define the *causal order* as $CO \triangleq (SO \cup WR)^+$ to capture the potential causality between transactions. The convergence property of TCC requires a total ordering of transactions. To this end, we introduce the *version order* $VO \subseteq T \times T$ which is a strict total order such that $CO \subseteq VO$.

Read Atomicity and TCC. Let $r \triangleq R(x, _)$ be a read of transaction t . If r is the first operation on x in t , then it is called an *external* read operation of t ; otherwise, it is an *internal* read operation.

The *internal consistency axiom* INT ensures that, within a transaction, an internal read from a key returns the same value as the last write to or read from this key in the transaction. The *external consistency axiom* EXT ensures that an external read in a transaction t from a key returns the final value written by the last transaction in VO among all the transactions that precede t in terms of CO and write to this key. Note that RA is equivalent to $INT \wedge EXT$ [18].

Definition 3 (TCC). A history $\mathcal{H} = (T, SO, WR)$ satisfies TCC if and only if CO is a strict partial order and there exists a version order VO such that the INT and EXT axioms hold.

Our consideration for basing the TAPs and checking algorithm on this new definition is mainly that it is more suitable for the black-box checking over histories since the WR relation can be extracted straightforwardly from a history. However, in order to establish the correctness of our formal TCC definition, we prove that it is equivalent to the existing TCC definition by Cerone et al. [18]. The proof is given in [37, Appendix A].

Example 4. Figure 2 depicts a visualization of the scenario in Example 2. Since Rachel sees the destination city set by Monica who sees the starting city set by Joey, we have $t_J \xrightarrow{WR} t_{Mr}$ and $t_{Mw} \xrightarrow{WR} t_R$, respectively. By the definition of CO, $t_J \xrightarrow{CO} t_R$. Hence, Rachel must also see the starting city set by Joey.

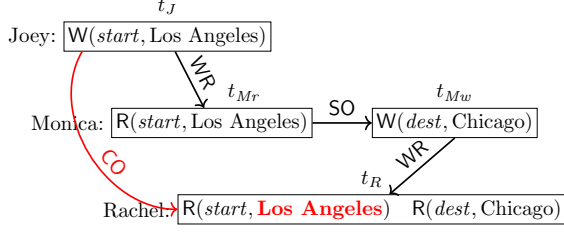


Figure 2: Illustrating TCC's formal definition.

This history satisfies TCC as witnessed by the orders $CO = VO$ such that $t_J \xrightarrow{VO} t_{Mr} \xrightarrow{VO} t_{Mw} \xrightarrow{VO} t_R$.

3.2 Anomalous Patterns for TCC

Arriving at a sound and particularly complete characterization of TCC is non-trivial due to its subtle semantic corner cases. Table 1 describes our characterized 12 TAPs for TCC which are defined in terms of WR, CO, and VO. The accompanying visualization is shown in Figure 3.

We derive these fine-grained TAPs mainly from three studies. First, we extract different kinds of anomalies by studying an extensive collection of formalizations of isolation levels and consistency properties in the literature [1, 12, 15, 17–19, 22, 38, 39]. Our idea of using fine-grained TAPs to capture TCC anomalies is particularly inspired by the “bad patterns” for formalizing non-transactional CC [15], the proscribed phenomena for characterizing isolation levels (excluding TCC) proposed by Berenson et al. [12], and the data anomalies presented in Adya’s thesis [1].

Second, we incrementally devise TAPs in terms of TCC’s four semantic building blocks, namely read committed, no fractured reads, causal consistency, and data convergence (see also Section 2). Specifically, six TAPs (a–f) are defined for read committed and additionally with two TAPs (h and i) for the fractured reads anomaly; two TAPs (g and j) are used to capture the causality relation between transactions; another two TAPs (k and l) characterize the conflicting version orders. This additionally underlies the versatility of our characterization (resp. checker) for defining (resp. check) weaker isolation guarantees. For example, by combining the eight TAPs (a–f, h, and i), we can soundly and completely characterize RA; accordingly, by disabling checking the other four TAPs, our TCC checking algorithm (Section 4) naturally becomes one for RA.

Finally, to validate our devised TAPs, we study 3097 anomalous histories collected from executing production database engines [14, 30, 60, 61] and can successfully associate each TCC anomaly with one TAP. See Table 2, Section 5.3 for the statistics.

Soundness and Completeness. We show that the TAPs defined are necessary and sufficient to characterize TCC. Our proof is given in [37, Appendix B].

Theorem 1. A history satisfies TCC if and only if it does not contain any instances of the TAPs.

In the following, we showcase two kinds of TAPs using a single history. As we will see in Section 5.3, many collected histories in practice contain multiple TAPs. Our tool can identify *all* of them while most of the existing checkers miss certain anomalies.

Example 5 (TAPs). Figure 4 illustrates both TAP-l and TAP-h in one history. First, in Figure 4a, as $t_4 \xrightarrow{WR(y)} t_3$, $t_2 \xrightarrow{SO} t_3$, and both t_2 and t_4 write to x , we have $t_4 \xrightarrow{VO} t_2$. Second, as shown in Figure 4b, $t_2 \xrightarrow{CO} t_5$ (due to $t_2 \xrightarrow{SO} t_3 \xrightarrow{WR(z)} t_5$), $t_4 \xrightarrow{WR(x)} t_5$, and both t_2 and t_4 write to x . Hence, we have $t_2 \xrightarrow{VO} t_4$. Then there is a cycle in VO, i.e., $t_4 \xrightarrow{VO} t_2 \xrightarrow{VO} t_4$. Finally, in Figure 4c, $t_4 \xrightarrow{WR(x)} t_5$ and $t_4 \xrightarrow{CO} t_2$ (due to $t_4 \xrightarrow{WR(x)} t_1 \xrightarrow{SO} t_2$). We also have $t_2 \xrightarrow{WR(y)} t_5$ for $y \neq x$ and both t_2 and t_4 write to x . Hence, we have $t_2 \xrightarrow{VO} t_4$.

In addition to its soundness and completeness, our TAP-based characterization also facilitates the understanding and debugging of the violations found. We devise our TAPs to correspond to different kinds of database defeats. For example, the above TAP-h is typically caused by a flaw in transaction atomicity maintenance while TAP-l usually results from erroneous conflict resolution when replicas are synchronized. In contrast, existing SAT/SMT-solver-based checkers may return incomprehensible unsatisfied clause [56, 60, 61]; tools based on cycle detection may return a coarse-grained cycle [14], rendering locating the root cause hard.

4 THE C4 ALGORITHM

C4 detects *all* TAPs in a given history. Its pseudocode is given in Algorithm 1.

4.1 Overview

C4 identifies TAPs in a directed graph with the node set T and the edge set obtained from SO, WR, CO, and VO. Specifically, in the procedure CHECKTCC (line 1), it first builds CO from SO and WR and checks anomalous patterns TAP-a to TAP-h, TAP-j, and TAP-k. Then it builds VO and checks anomalous patterns TAP-i and TAP-l.

To facilitate efficient reachability checking and cycle detection between transactions in the graph, we associate each transaction t with a vector $Vec[t]$, one element per session. Intuitively, $Vec[t]$ encodes the set of transactions that are able to reach t . Vectors are compared element-wise (similarly for \sqsupseteq): $vec_1 \sqsubseteq vec_2 \iff \forall s \in \mathbb{S}. vec_1(s) \leq vec_2(s)$, where \mathbb{S} is the set of sessions and $vec(s)$ gives the element of vec on session s . The join operator \sqcup on vectors is defined as $vec_1 \sqcup vec_2 \triangleq \langle \lambda s \in \mathbb{S}. \max\{vec_1(s), vec_2(s)\} \rangle$.

C4 maintains the following *invariant* about vectors: transaction t_2 is reachable from t_1 if and only if $Vec[t_1] \sqsubseteq Vec[t_2]$. Therefore, cycle detection amounts to testing whether there are two transactions t_1 and t_2 such that $Vec[t_1] = Vec[t_2]$. To this end, when an edge outgoing from a transaction t is added, we need to join $Vec[t]$ to the vectors of transactions that are reachable from t (line 10). C4 does this in a DFS traversal (line 7 in UPDATEVEC). Due to the vector property, this traversal can backtrack early once it reaches a transaction t' with a larger vector, i.e., $Vec[t'] \sqsupseteq Vec[t]$ (line 8).

Example 6 (Updating Vectors). Consider the history in Figure 5. It consists of two sessions s_1 and s_2 which contain 100 and 2 transactions, respectively. Suppose that the edge $t_{102} \xrightarrow{WR} t_3$ is added first. The second component of vectors of t_3 , t_4 , ..., and t_{100} has been set to 2. Then a VO edge $t_{101} \xrightarrow{VO} t_2$ is added. As a result, we

Table 1: 12 TAPs and their description corresponding to Figure 3.

TAP	Description	TAP	Description
(a)	A transaction reads a value from thin air.	(g)	The relation $SO \cup WR$ is cyclic.
(b)	A transaction reads a value written by an aborted transaction.	(h)	Transaction t_3 reads x from t_1 and $y \neq x$ from t_2 . t_2 also writes to x such that $t_1 \xrightarrow{CO} t_2$. This is a <i>fractured reads</i> anomaly.
(c)	A transaction reads from a future write within the same transaction.	(i)	Transaction t_3 reads x from t_1 and $y \neq x$ from t_2 . t_2 also writes to x such that $t_1 \xrightarrow{VO} t_2$. This is a <i>fractured reads</i> anomaly. It is a general case of (h) which requires $t_1 \xrightarrow{CO} t_2$.
(d)	Transaction t reads value v of x from transaction $t' \neq t$ but t has written $v' \neq v$ to x before this read.	(j)	Transaction t reads the initial value \perp_x of x . However, there exists a transaction $t' \xrightarrow{CO} t$ that writes $v' \neq \perp_x$ to x .
(e)	Transaction t reads value v of x from transaction $t' \neq t$ but v is not the last value written to x by t' (assuming no write to x before this read in t).	(k)	Transaction t_3 reads x from transaction t_1 . There exists a transaction t_2 which also writes to x such that $t_1 \xrightarrow{CO} t_2 \xrightarrow{CO} t_3$. This is undesirable as it would result in a cycle in VO.
(f)	A transaction reads from a key twice with different values, but without writing to it in-between.	(l)	Transaction t_3 reads x from transaction t_1 . There exists a transaction t_2 which also writes to x such that $t_1 \xrightarrow{VO} t_2 \xrightarrow{CO} t_3$. It is a general case of (k) which requires $t_1 \xrightarrow{CO} t_2$.

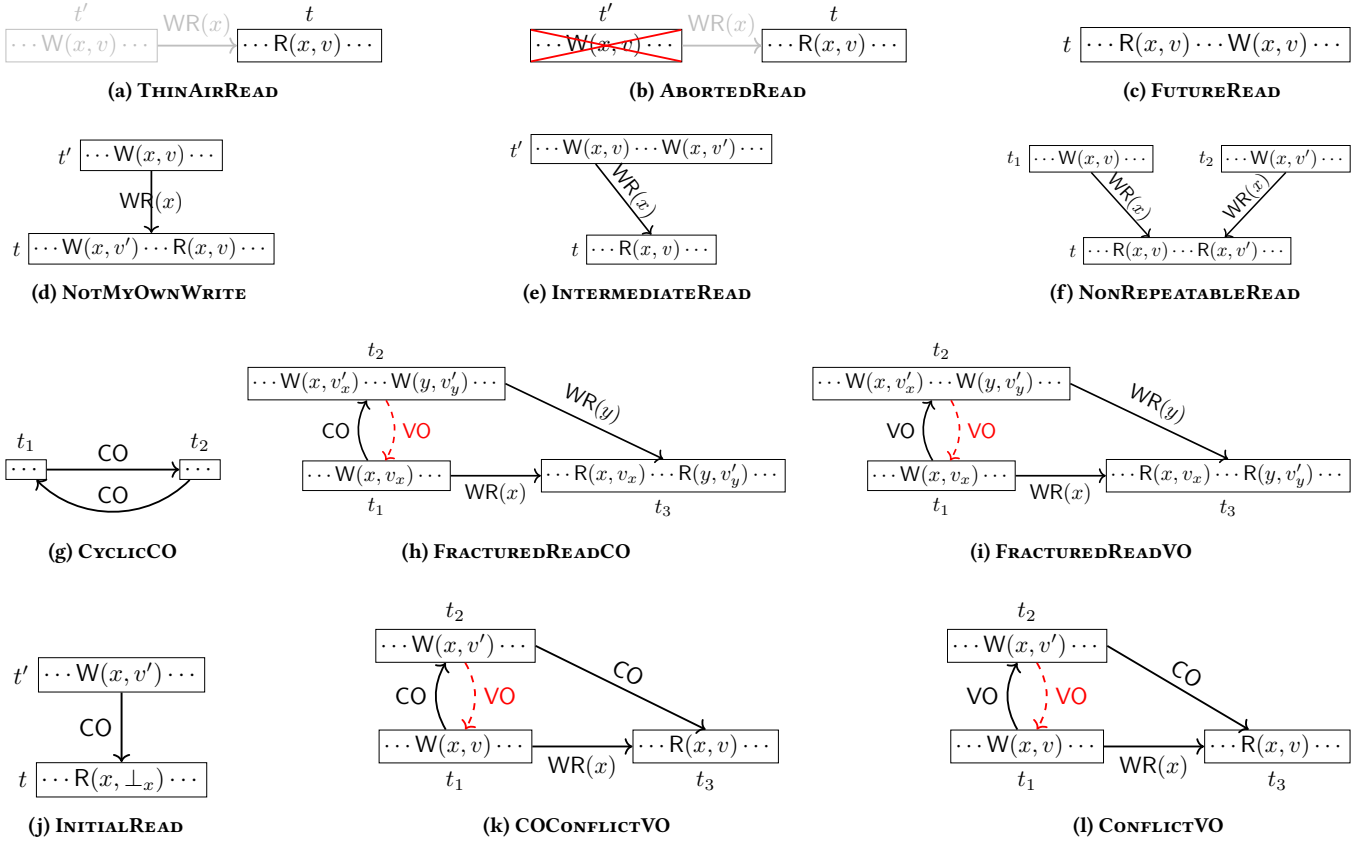


Figure 3: Visualizing 12 TAPs.

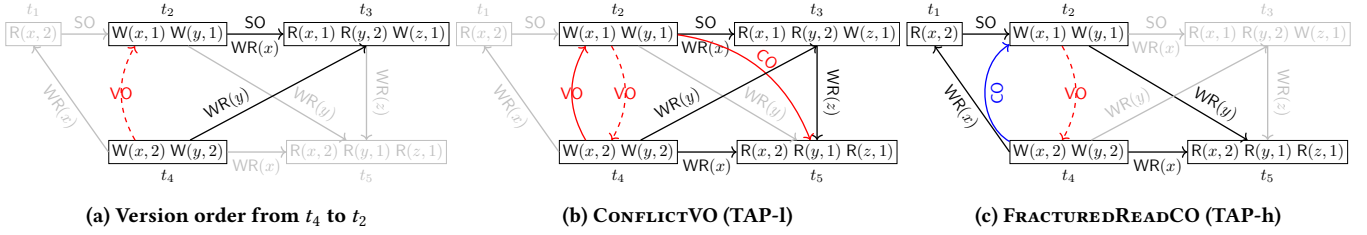


Figure 4: TAP-I and TAP-h coexist in a single history.

Algorithm 1 The C4 algorithm for checking history $\mathcal{H} = (T, \text{SO}, \text{WR})$ against TCC

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 $\text{WT}_x$ : the set of transactions that write to key  $x$ 
 $\text{RT}_x^v$ : the set of transactions that read value  $v$  of key  $x$ 
 $\text{Vec}[t]$ : vector for transaction  $t$ 
 $\text{taps}$ : the set of anomalous patterns found, initially  $\emptyset$ 

1: procedure CHECKTCC()
2:   BUILDVO()
3:   CHECKCOTAP()
4:   BUILDVO()
5:   CHECKVOTAP()

6: procedure UPDATEVEC( $t$ )
7:   for  $t' \in T$  in DFS traversal from  $t$ 
8:     if  $\text{Vec}[t'] \sqsupseteq \text{Vec}[t]$ 
9:       backtrack
10:     $\text{Vec}[t'] \leftarrow \text{Vec}[t'] \sqcup \text{Vec}[t]$ 

11: procedure BUILDVO()
12:   for  $t_1, t_2, t_3 \in T$  such that  $\exists x \in K. t_1 \xrightarrow{\text{WR}(x)} t_3 \wedge t_2 \in \text{WT}_x \wedge \text{Vec}[t_2] \sqsubseteq \text{Vec}[t_3]$ 
13:      $E \leftarrow E \cup \{(t_2, t_1)\}$ 
14:   for  $t \in T$ 
15:     UPDATEVEC( $t$ )

16: procedure CHECKCOTAP()
17:   for  $t \in T$ 
18:     check anomalous patterns TAP-a to TAP-f
19:     if  $\exists x \in K. (t \in \text{RT}_x^v \wedge (\exists t' \neq t. t' \in \text{WT}_x. \text{Vec}[t'] \sqsubseteq \text{Vec}[t]))$ 
20:        $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-j}\}$ 
21:     if  $\exists t_1, t_2 \in T. \text{Vec}[t_1] = \text{Vec}[t_2]$ 
22:        $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-g}\}$ 
23:     if  $\exists x \in K \exists t_1, t_2, t_3 \in T. t_1 \xrightarrow{\text{WR}(x)} t_3 \wedge t_2 \in \text{WT}_x \wedge \text{Vec}[t_1] \sqsubseteq \text{Vec}[t_2] \sqsubseteq \text{Vec}[t_3]$ 
24:       if  $\exists y \neq x \in K. t_2 \xrightarrow{\text{WR}(y)} t_3$ 
25:          $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-h}\}$ 
26:       else
27:          $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-k}\}$ 

28: procedure CHECKVOTAP()
29:   if  $\exists x \in K \exists t_1, t_2, t_3 \in T. t_1 \xrightarrow{\text{WR}(x)} t_3 \wedge t_2 \in \text{WT}_x \wedge \text{Vec}[t_1] = \text{Vec}[t_2]$ 
30:     if  $\exists y \neq x \in K. t_2 \xrightarrow{\text{WR}(y)} t_3$ 
31:        $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-i}\}$ 
32:     else if  $t_2 \xrightarrow{\text{CO}} t_3$ 
33:        $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-l}\}$ 

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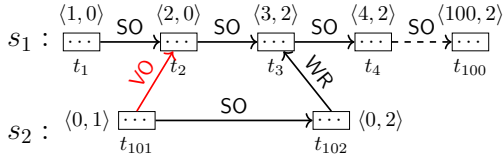


Figure 5: An illustration of updating vectors.

should update the vectors of transactions t_2, t_3, \dots , and t_{100} with $\text{Vec}[t_{101}] = \langle 0, 1 \rangle$. However, since $\text{Vec}[t_3] = \langle 3, 2 \rangle \sqsupseteq \text{Vec}[t_{101}]$, the traversal can terminate early at t_3 .

4.2 Algorithm

We now describe how C4 works in more details. In the following we abuse the notation WR and $\text{WR}(x)$ for transactions to denote the write-read relation between operations.

Procedure BUILDCO. C4 builds CO by examining transactions one by one. Consider a transaction t . It first builds SO and initializes $\text{Vec}[t]$. Then C4 examines each operation of t and builds WR. Finally, C4 calls the procedure **UPDATEVEC** (line 6) where it uses $\text{Vec}[t]$ to update vectors of the transactions reachable from t in case new edges outgoing from t have been added. This ensures the transitivity of CO. Denote the resulting graph by \mathcal{G}_{CO} . See [37, Appendix C] for more details.

Procedure CHECKCOTAP. C4 then checks ten CO-related anomalous patterns in \mathcal{G}_{CO} (line 16). It is straightforward to check TAP-a

to TAP-f given the sets like S_a and S_r constructed during **BUILDCO** (line 18). The details are omitted here. Checking TAP-g, TAP-h, TAP-j, and TAP-k involves reachability checking and cycle detection in terms of CO. Specifically, C4 relies on $\text{Vec}[t'] \sqsubseteq \text{Vec}[t]$ to capture the condition $t' \xrightarrow{\text{CO}} t$ in TAP-j (line 19). Cycle detection in TAP-g is achieved by checking whether $\text{Vec}[t_1] = \text{Vec}[t_2]$ (line 21). Both TAP-h and TAP-k contains a $t_1 \xrightarrow{\text{CO}} t_2 \xrightarrow{\text{CO}} t_3$ chain, which is captured by $\text{Vec}[t_1] \sqsubseteq \text{Vec}[t_2] \sqsubseteq \text{Vec}[t_3]$ (line 23). They differ in whether $t_2 \xrightarrow{\text{CO}} t_3$ is actually a more special case $t_2 \xrightarrow{\text{WR}(y)} t_3$ for some key y (line 24).

Procedure BUILDVO. The remaining two anomalous patterns, namely TAP-i and TAP-l, contain VO edges. In **BUILDVO** (line 11), C4 examines each triple of transactions t_1, t_2 , and t_3 . If $t_1 \xrightarrow{\text{WR}(x)} t_3$, t_2 writes x , and $t_2 \xrightarrow{\text{CO}} t_3$ (in \mathcal{G}_{CO} ; captured by $\text{Vec}[t_2] \sqsubseteq \text{Vec}[t_3]$), then C4 adds an edge $t_2 \xrightarrow{\text{VO}} t_1$. The transitivity of VO is ensured by updating vectors of transactions (line 15). Note that we *cannot* update vectors and add VO edges alternately, because otherwise from $\text{Vec}[t_2] \sqsubseteq \text{Vec}[t_3]$ we can conclude only $t_2 \xrightarrow{\text{VO}} t_3$ instead of $t_2 \xrightarrow{\text{CO}} t_3$ as desired. Denote the resulting graph by \mathcal{G}_{VO} .

Procedure CHECKVOTAP. Both TAP-i and TAP-l contain three transactions t_1, t_2 , and t_3 such that $t_1 \xrightarrow{\text{WR}(x)} t_3$ for some key x , t_2 writes x , and t_1 and t_2 form a VO cycle in \mathcal{G}_{VO} ; the last condition is

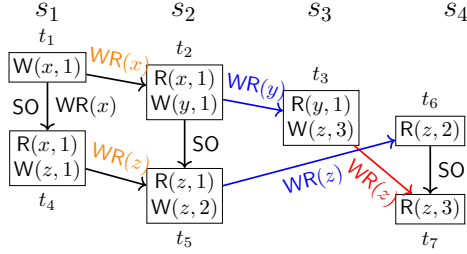


Figure 6: An illustration of tree clocks for updating vectors. Transactions in the same session are vertically aligned.

detected by $Vec[t_1] = Vec[t_2]$ (line 29). They differ in the relation between t_2 and t_3 : TAP-I requires $t_2 \xrightarrow{CO} t_3$ (line 32), while TAP-i requires a more fine-grained condition, i.e., $t_2 \xrightarrow{WR(y)} t_3$ for some key $y \neq x$ (line 30). For TAP-I, we cannot replace $t_2 \xrightarrow{CO} t_3$ with $Vec[t_2] \sqsubseteq Vec[t_3]$, since the latter implies only $t_2 \xrightarrow{VO} t_3$ in \mathcal{G}_{VO} .

4.3 Optimization with Tree Clocks

C4 frequently calls UPDATEVEC to update vectors, where it extensively compares and joins vectors. Each of the two basic operations on vectors requires $\Theta(|S|)$ time, where $|S|$ is the number of sessions. UPDATEVEC becomes a computational bottleneck when $|S|$ is large.

To reduce the time on manipulating vectors, we adapt *tree clocks* [44] into our utilization of vectors. While one tree clock per thread (or session) in concurrent program analysis is sufficient, each transaction in our setting has its own tree clock. This is because transactions that are *not* the latest ones in their sessions may also need to update their vectors.

Comparing tree clocks takes constant time and joining tree clocks takes time proportional to the number of entries being modified. The following example illustrates how C4 uses tree clocks in updating vectors, i.e., the UPDATEVEC procedure.

Example 7. Consider the history in Figure 6 with four sessions. Suppose C4 examines transactions in BUILDCO in the order of t_1, t_2, \dots, t_7 . When dealing with t_7 , C4 adds the edge $t_3 \xrightarrow{WR(z)} t_7$ and then joins $Vec[t_3]$ with $Vec[t_7]$. With simple vectors, the join operation iterates over *all* entries of $Vec[t_3]$ and $Vec[t_7]$. However, via $t_5 \xrightarrow{WR(z)} t_6 \xrightarrow{SO} t_7$, t_7 has learned t_5 on session s_2 which is newer than what t_3 learned about session s_2 , namely t_2 . Transitive, via $t_4 \xrightarrow{WR(z)} t_5$, t_7 has learned t_4 on session s_1 which is newer than what t_3 learned (via $t_1 \xrightarrow{WR(x)} t_2$) about session s_1 , namely t_1 . By recording a transaction’s knowledge of transactions in other sessions in a hierarchical tree structure, tree clocks can avoid examining some entries of vectors. For example, the join of $Vec[t_3]$ with $Vec[t_7]$ stops after examining the entry for session s_2 , without examining the entry for session s_1 .

5 EXPERIMENTS

In this section we conduct a comprehensive evaluation of C4, along with state-of-the-art black-box isolation checkers, and answer the following questions with respect to the SIEGE+ principle:

- (1) **Effective** (Section 5.2): Can C4 detect TCC violations in production database engines?
- (2) **Complete** (Section 5.3): Can C4 identify more anomalies than these checkers?
- (3) **Informative** (Section 5.4): Can C4 report understandable violations?
- (4) **Efficient** (Section 5.5): How efficient is C4 with our design choices? Can C4 outperform the baselines under various workloads? Is C4 scalable for large workloads?

Along with answering these questions, we experimentally validate C4’s **soundness** and demonstrate its **generality** by showing its capability for detecting TCC bugs in production database engines of *different kinds*.

5.1 Setup, Histories, and Benchmarks

Experimental Setup. We employ a PostgreSQL v15.2 instance to generate *valid* histories without SI (thus TCC) violations by setting the isolation level to *repeatable read* (implemented as SI in PostgreSQL [52]). We co-locate the client threads and the PostgreSQL instance (or other target databases) on a local machine with an Intel(R) Xeon(R) E5-2620 v3 CPU and 64GB memory.

We adapt PolySI’s workload generator and history collector into C4: each client issues to the database a stream of transactions, which are transformed from the generated key-value operations to SQL queries for the interactions with relational databases,² and collects the execution history. We prototype C4’s core checking algorithm in around 2k lines of Java code. PolySI’s adaption, together with the workload generator and the history collector/compiler, includes around 2k lines of Rust code. To meet the *unique value* requirement (Section 2), we use counters for values written on each key.

For the actual TCC checking over a history, we set the timeout to 10 minutes for each checker.

Histories. We employ the parametric workload generator to produce *general* transaction workloads. Its parameters are: the number of client sessions (25 by default), the number of transactions per session (200 by default), the number of read/write operations per transaction (20 by default), the read proportion (50% by default), the total number of keys (10k by default), and the key-access distribution including uniform (by default), zipfian, and hotspot (80% operations touching 20% keys). For the performance comparison, the default 5k transactions with 100k operations are already sufficient to distinguish C4 from the competitors (see Section 5.5.1). We further demonstrate C4’s scalability with significantly larger workloads (Section 5.5.4).

Benchmarks. Our experiments also consider six benchmarks commonly used by existing black-box isolation checkers [30, 56, 61]. First, we consider three synthetic benchmarks, each of which contains only serializable histories of at least 10k transactions (also satisfying TCC). TPC-C [58] is a standard benchmark for online transaction processing, consisting of five types of transactions (e.g., new orders and payment). The configuration includes one warehouse, 10 districts, and 30k customers. C-RUBiS [53] is an eBay-like bidding system; users can register and bid for items. The dataset

²For multi-column tables, we represent each table cell as a compound key (i.e., “TableName:PrimaryKey:ColumnName”) and a single value.

Table 2: Distribution of TAPs identified by C4 from 3097 archived anomalous histories and 4 histories for new bugs.

TAP-a	TAP-b	TAP-c	TAP-d	TAP-e	TAP-f	#TAPs
0	3	1	71	1	15	9310
TAP-g	TAP-h	TAP-i	TAP-j	TAP-k	TAP-l	#Hist
2	209	2947	11	3079	2971	3111

contains 20k users and 200k items. C-Twitter [34] is a Twitter clone where users can tweet and follow/unfollow other users and key accesses follow the zipfian distribution.

Additionally, we benchmark the performance of each checker using three representative *general* datasets archived by [30], i.e., GeneralRH (read-heavy with 95% reads), GeneralWH (write-heavy with 70% writes), and GeneralRW (50% reads); each history contains 16k transactions and 320k operations.

5.2 Detecting TCC Violations

Reproducing Known Bugs. C4 successfully reproduces *all* known TCC bugs in a considerable collection of 3097 anomalous histories [14, 30, 60, 61]. These histories were collected from five earlier releases of four production databases of different types, i.e., the relational databases MySQL-Galera and CockroachDB, the graph DBMS Dgraph, and the document store MongoDB. Table 4 depicts the details including their popularity (on GitHub), tested releases, tested isolation levels, etc. It is worth mentioning that TCC bugs already manifest extensively when checking *stronger* isolation levels such as SI and SER.

New Violations Found. We also demonstrate C4’s effectiveness, along with its generality, by examining recent releases of four popular and heavily-tested database engines of *different kinds*, i.e., the emerging key-value database AntidoteDB [4],³ YugabyteDB [59] supporting multiple data models, MariaDB-Galera [21], and CockroachDB [36]. These databases claim to provide TCC or even stronger isolation guarantees; see Table 4 for details.

We have detected and reported novel TCC bugs in all these database engines. In particular, both AntidoteDB and MariaDB have already confirmed and fixed the bugs [6, 7]. MariaDB labeled the bug as “critical” since it affected a wide range of nine releases (v10.3–v10.11), as well as its Galera cluster. AntidoteDB quickly localized the root cause of our reported bug (a Python client issue with session guarantees), thanks to the informative session dependencies in the counterexample. The YugabyteDB and CockroachDB developers are investigating the reported bugs.

5.3 Complete Violation Detection

An ideal checker is expected to identify *all* violations in a given history. This is highly desirable as more bug-coverable detection would (i) potentially catch more defects of a database and (ii) be more efficient and resource-saving as the occurrence of a bug is highly unpredictable due to random workload (or test case) generation. Missing a bug usually results in additional time and resources for checking. Through an empirical analysis, we show that, given the

same number of anomalous histories, C4 can identify significantly more anomalies than existing checkers. With a further investigation, we also show that these checkers are inherently incomplete.

State-of-the-Art Isolation Checkers. Our comparison focuses on the black-box checking of isolation guarantees and considers six state-of-the-art checkers in total. First, our comparison includes two existing (T)CC checkers. CausalC [15, 60] is based on *answer set programming* (ASP) [27], where causal dependencies are encoded into an ASP logic program before running the Clingo solver [49] to compute the solution. Although it is specifically designed for checking CC, CausalC can also be used to check TCC when histories contain only simple transactions of single read/write. dbcop [14] is the most efficient TCC checker to date. Given a history, it constructs the causal dependency graph and then searches for a cycle via DFS without using an off-the-shelf solver.

Second, we consider three SAT/SMT-solving-based checkers, namely PolySI [30], Viper [61], and Cobra [56], which all utilize the MonoSAT solver [11], tailored for efficient checking of graph properties such as acyclicity, and specific optimizations to accelerate the checking procedure. These checkers are designed for checking *stronger* isolation levels (PolySI and Viper for SI while Cobra for SER), which can also be used for detecting TCC violations, as well as TCC validation.

Finally, we also consider Elle [35] which is part of the prevalent testing framework Jepsen [31]. Unlike the above five checkers which only work with read-write registers, Elle can also leverage, e.g., list-specific data models and “append” operations, to efficiently infer transaction dependencies. The latest release of Elle does not support checking TCC yet [33]. In our experiments we therefore use its SI checking component (or Elle-SI), the weakest isolation level supported by Elle which is stronger than TCC. Hence, Elle-SI is expected to detect any TCC bugs.

Finding More Anomalies by C4. Thanks to its completeness, C4 can detect *all* TAPs in a history. Surprisingly, the number of identified TAPs (9310 in total) is far more than the number of histories (3111). As existing black-box checkers (except Elle) terminate upon detecting a violation, the number of anomalies found by them is essentially 3111. That is, C4 in fact finds 6199 more anomalies with the same collection of histories.

We also highlight in Table 4 the anomalies missed by existing checkers in their reported anomalous histories. In particular, C4 captures, in total, seven more types of anomalies than dbcop in testing MySQL-Galera and CockroachDB; the advanced SI checkers PolySI and Viper missed two kinds of anomalies, which can, however, be successfully identified by C4.

Table 2 shows the distribution of TAPs identified by C4 from all these 3111 anomalous histories. Most of them involve *fractured reads* (TAP-h and TAP-i) or *version-order conflicts* (TAP-k and TAP-l). However, simple, yet crucial bugs (TAP-b–TAP-f, 91 in total) are not negligible.⁴ For example, 15 *non-repeatable reads* anomalies (labeled “critical” by MariaDB) are found; many transactions in MySQL-Galera could not read their own writes (TAP-d, 71 in total); MongoDB even reads aborted writes (TAP-b, three in total). See also Table 4,

³The emerging app development platform ElectricSQL [25] bases its core replication layer on AntidoteDB.

⁴We have not found any thin-air reads (TAP-a), which is not surprising. This kind of bug rarely occurs unless there is severe data corruption.

Table 3: Missed bugs by each checker.

Checker	Missed Bugs	Checker	Missed Bugs
C4	none	dbcop	TAP-a, TAP-b, TAP-c
CausalC	all	Viper	TAP-d, TAP-e, TAP-f, TAP-g
PolySI	none	Cobra	TAP-d, TAP-e, TAP-f
		Elle-SI	TAP-c, TAP-g

the column “Violation Cause”, for the distribution of TAPs among different database engines.

Regarding new violations, C4 detects various TAPs in four database engines. In particular, MariaDB-Galera exhibits six different types of anomalies, including simple bugs like reading intermediate writes and non-repeatable reads; three kinds of TAPs manifest in AntidoteDB and YugabyteDB.

(In)completeness. Given that most of the existing checkers terminate by design upon the first detected bug, would it be possible to find more bugs if we run them multiple times for the same history or even make them continue to search for more, different bugs? The answer is unfortunately *no* for almost all of them.

We craft for each TAP a simple anomalous history (or test case) to draw a complete view of the existing checkers’ inherent limitation in finding TCC bugs. See [37, Appendix D] for the 12 test cases. Table 3 shows the results where none of the checkers except C4 and PolySI (also complete by design [30]) can catch all the anomalies. In particular, CausalC fails to pass all the test cases, which is expected as it only supports simple transactions, while our test cases are designed with general transactions; dbcop cannot identify reading aborted reads (TAP-b); Viper and Cobra miss both the *intermediate* and *non-repeatable* reads anomalies (TAP-e and TAP-f); Elle fails to detect CO cycles (TAP-g) and future reads (TAP-c).⁵ Their incompleteness in finding bugs stems mainly from the underlying incomplete or coarse-grained characterizations of isolation guarantees. Hence, missing bugs is inherent to these checkers. Moreover, some checkers such as Elle trade completeness off for less checking overhead in practice [29].

5.4 Understanding Violations Found

In addition to the incompleteness, existing black-box checkers also return *less informative* counterexamples. Both Viper and Cobra report unsatisfied clauses by the MonoSAT solver. CausalC uses atomic propositions to represent a violating cycle. Such solver-returned counterexamples do not produce semantic interpretations of the violations. The dbcop tool returns a coarse-grained cycle upon finding a violation, rendering the debugging difficult, e.g., causality violation (TAP-g) and fractured reads (TAP-h) are indistinguishable.

Thanks to the underlying TAP-based characterization, C4 is capable of returning fine-grained, informative counterexamples and identifying the violation causes in terms of TAPs (see also Table 4). In order to make the counterexamples more understandable, we have integrated into C4 the Graphviz tool [28] to visualize the violating scenarios. Below we showcase four new bugs of different kinds, together with their violating scenarios. We defer representative bugs of other kinds (e.g., TAP-e) to [37, Appendix E].

⁵Elle’s developer quickly confirmed these two bugs [33].

TAP-g in AntidoteDB. As shown in Figure 7a, transaction t_{39} is causally ordered after transaction t_0 via a series of SO and WR dependencies, while t_0 reads t_{39} ’s value written on key 0. This results in a cyclic causality anomaly, i.e., TAP-g.

TAP-f and TAP-h in MariaDB-Galera. Figure 7b presents the NONREPEATABLEREAD anomaly where two consecutive reads on the same key 0 in transaction t_{393} fetch different values written by two separate transactions.

Figure 7c depicts how the *fractured reads* occur. Transaction t_{393} has two reads on keys 0 and 4, respectively. $R(4, 3285)$ fetches the value written by transaction t_{295} ; $R(0, 3328)$ reads the value installed by transaction t_{297} that also writes key 4, i.e., $W(4, 3335)$. Under *read atomicity*, $W(4, 3285)$ must be ordered after $W(4, 3335)$ in the version order. However, $W(4, 3285)$ is issued before $W(4, 3335)$ in the same session.

Interestingly, TAP-f and TAP-h coexist with both transactions t_{297} and t_{393} involved. C4 precisely captures these two anomalies from a *single* history with distinguishable violating scenarios.

TAP-k in YugabyteDB. Figure 7d shows the conflict between the CO and VO orders established from the history. As transaction t_9 reads the value written by transaction t_{15} on key 10 and is also causally ordered (via SO) after transaction t_7 that also writes key 10, t_7 ’s write must be installed before t_{15} ’s in the database. However, t_7 has already been causally ordered after t_{15} . Hence, a CO-VO conflict arises.

5.5 Performance Evaluation

We conduct a comprehensive and in-depth performance analysis of C4. Regarding the competitors, as the non-solver tool dbcop is the only one specifically designed for checking TCC, we also

- extend CausalC to a new tool, CausalC+, for checking general transactions against TCC via the ASP-based solving;
- implement a *strong baseline* checker called TCC-Mono which leverages the advanced MonoSAT solving to search for cycles; and
- involve PolySI as the representative for checking stronger isolation levels.⁶

Our goal is twofold. First, we want to compare C4 with *various* techniques utilized by the state of the art, e.g., with or without a solver *and* which solver to leverage, given the recent trend of applying solving to checking isolation guarantees [30, 56, 60, 61]. Second, although checking stronger isolation levels such as SI is of higher complexity in theory, existing checkers have been shown to be highly efficient in practice with domain-specific optimizations. For example, as we will see, PolySI even outperforms the state-of-the-art TCC checker dbcop. Hence, we want to explore whether stronger isolation checkers could replace C4 for *validating* TCC.⁷

Additionally, we consider Elle, an isolation checker of a different kind that supports list-specific APIs. Given its success in effectively finding isolation bugs in commercial databases and high checking efficiency [32, 35], we want to make C4 compatible with the Jepsen

⁶PolySI has already been shown to outperform Cobra [30].

⁷As a stronger isolation checker inherently returns *false positives with respect to TCC* (e.g., lost updates), validation here means that a history is valid up to the isolation level for which the checker is designed.

Table 4: Summary of tested databases. Multi-model in YugabyteDB refers to relational DBMS, document store, and wide-column store. Anomalies missed by existing checkers are highlighted in yellow.

Database Engine	GitHub Stars	Kind	Checker	Tested Version	Tested Isolation	Violation Cause
New violations:						
AntidoteDB	790	Key-value	C4	v0.2.2	TCC	TAP-g, TAP-h, TAP-k
MariaDB-Galera	4.9k	Relational	C4	v10.4.22	SI	TAP-e, TAP-f, TAP-h, TAP-i, TAP-k, TAP-l
YugabyteDB	8k	Multi-model	C4	v2.11.1	SI	TAP-i, TAP-k, TAP-l
CockroachDB	27.6k	Relational	C4	v21.2.5	SER	TAP-k
Known bugs:						
MySQL-Galera	420	Relational	dbcop CausalC	v25.3.26 v3.2	SI SI	TAP-d , TAP-f , TAP-h , TAP-i , TAP-k, TAP-l TAP-j
Dgraph	19.5k	Graph	PolySI	v21.12	SI	TAP-i , TAP-k, TAP-l
MongoDB	24.3k	Document	Viper	v4.2.6	SI	TAP-b , TAP-c, TAP-g, TAP-l
CockroachDB	27.6k	Relational	dbcop	v2.1	SER	TAP-i , TAP-k, TAP-l

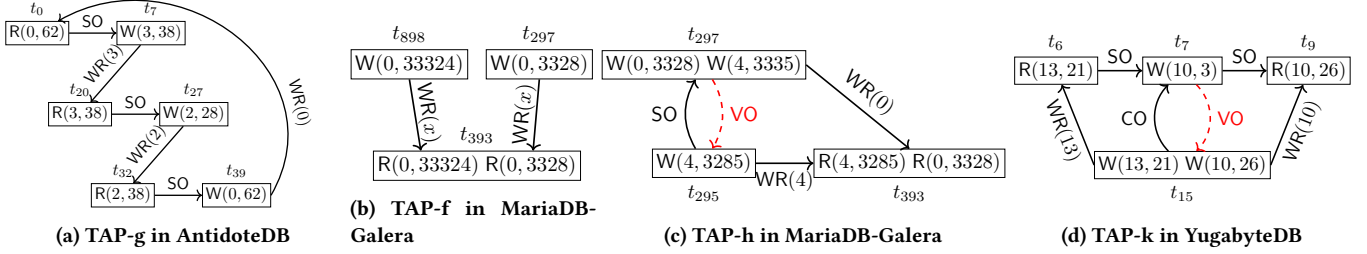


Figure 7: Visualized TCC violations found in production databases by C4. Only involved operations per transaction are shown.

framework, particularly with its produced “append” histories, to complement Elle with TCC checking.

5.5.1 Performance Comparison. As shown in Figure 8, C4 substantially surpasses all the competitors, including the strong baseline TCC-Mono, with respect to checking efficiency under a wide range of workloads. C4’s performance is also fairly stable, with around two seconds checking time, for varying read/write ratio and concurrency such as more sessions, transactions per session, operations per transaction, and skewed key accesses, as well as various benchmarks such as TPC-C. In contrast, the competing tools exhibit exponentially increasing checking time with more concurrency. Note that, in most of the cases, the state-of-the-art TCC checker dbcop and the CausalC extension do not finish within the timeout of 10 minutes. It is not surprising that PolySI outperforms these two checkers due to its domain-specific optimizations [30], especially tailored for certain benchmarks such as C-RUBiS and C-Twitter.

We also measure the memory overhead for all the checkers. Figure 9 shows the comparison results under the same setting as in Figure 8. C4 consumes significantly less memory (for storing generated graphs, vectors, and tree clocks) than the competitors under various workloads and benchmarks. In particular, dbcop, the only competitor that does not rely on solving and therefore stores no constraints, is not competitive with C4. Note that we only plot each tool’s memory usage by timeout. That is why CausalC+ has the “illusion” of less memory overhead.

Overall, our experimental results suggest that (i) vectors and tree clocks are more efficient techniques than solving in checking

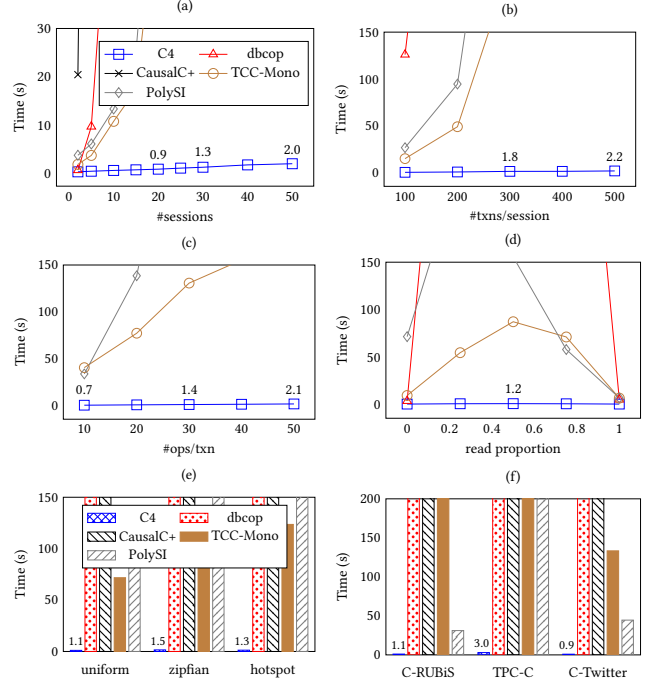


Figure 8: Performance comparison under various workloads. Experiments time out at 10 minutes. For the competitors, data points are not plotted for the timed-out experiments.

TCC, and (ii) compared to C4, the state-of-the-art stronger isolation checker (on read-write registers) is inefficient in validating TCC.

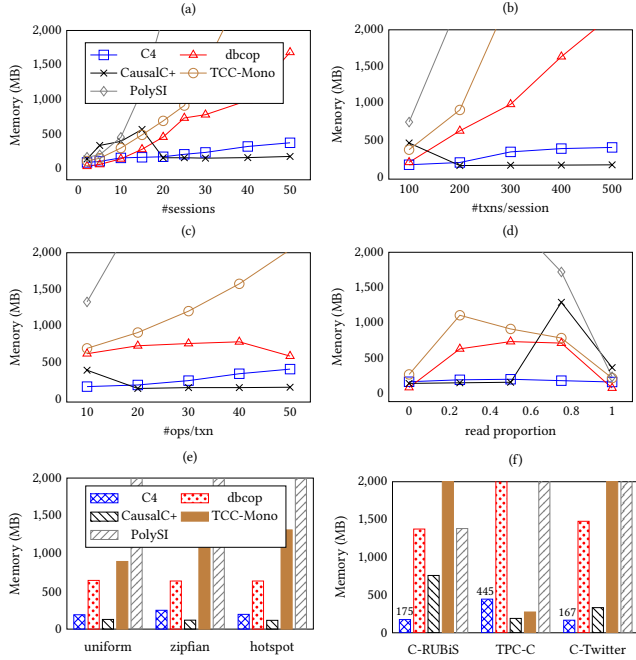


Figure 9: Memory overhead comparison. We only plot each tool’s memory usage by timeout (10 minutes).

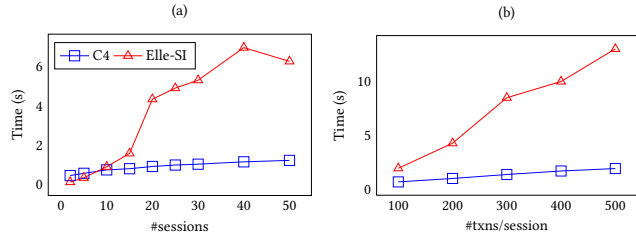


Figure 10: Comparison with Elle-SI (#ops/txn=100).

5.5.2 Compatibility with Jepsen. Elle, as part of the Jepsen framework [31], has shown promising efficiency in checking isolation guarantees. During testing, Jepsen produces workloads of reads and *append* operations, by which Elle can naturally infer the version order of appends (or writes in our case). To make C4 compatible with such “append” histories, we extend it with a compiler that translates a list of values per read into the version order of the corresponding writes. As shown in Figure 10, C4 exhibits high, stable checking efficiency with “append” histories, even when more sessions are involved (a) and the workload becomes large (b). This suggests that C4 can be integrated into Jepsen to complement Elle for checking TCC.

As expected, C4 incurs less checking overhead than Elle-SI. C4 attributes its high efficiency mainly to the less complexity of checking TCC and its acceleration in graph traversals by utilizing vectors and tree clocks as we will show next. Hence, it is fair to say that, for “append” histories, the state-of-the-art stronger isolation checker

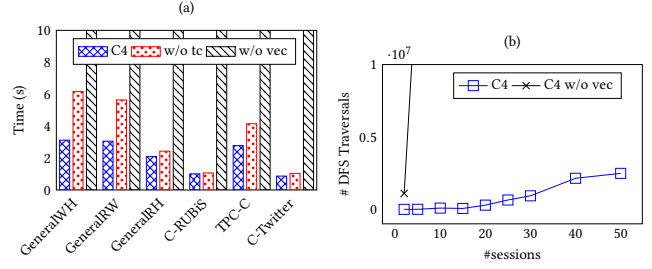


Figure 11: Differential analysis of C4.

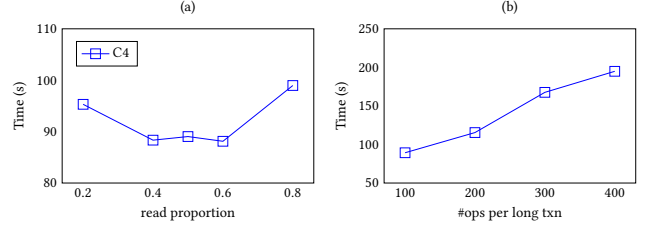


Figure 12: C4’s overhead under large workloads with one billion keys, one million transactions.

cannot replace C4 for validating TCC either (apart from read-write registers as we have seen from the comparison with PolySI).

5.5.3 Differential Analysis. To investigate the contributions of C4’s two major design choices, namely vectors and tree clocks, we experiment with two variants: (i) C4 without tree clocks and (ii) C4 without both tree clocks and vectors. Figure 11a demonstrates the noticeable acceleration produced by each design choice. Note that C-RUBiS or C-Twitter cannot distinguish C4 from (i) since both contain only a small number of sessions and transactions, thus fewer comparing and joining operations over vectors during the graph traversals.

Moreover, thanks to the utilization of vectors, C4 can significantly accelerate reachability checking and cycle detection in the UPDATEVEC procedure by reducing a large number of DFS traversals. As shown in Figure 11b, with more concurrency (in terms of more sessions), the number of DFS traversals performed by (ii) grows dramatically, reaching up to 10^{10} by timeout.

5.5.4 Scalability. To evaluate C4’s scalability, we generate transaction workloads with one billion keys, one million transaction, and up to 400 million operations. We experiment with varying read proportions and operations per long transactions (up to 400). As shown in Figure 12, C4 completes the TCC checking of 400 million operations in less than 200 seconds. We also observe a linear increase of checking time with larger-sized transactions. To conclude, large workloads are fairly manageable for C4.

6 DISCUSSION

Versatility. Our fine-grained characterization of TCC provides *modular* patterns for soundly and completely checking many other isolation guarantees. Regarding weaker isolation levels, we only need to check a subset of all TAPs, e.g., TAP-a-TAP-f for *read committed*, the default isolation level of many databases, and, additionally with TAP-h and TAP-i, for *read atomicity* (RA), a promising isolation guarantee that has recently been layered on Facebook’s

Table 5: Examining existing isolation checkers through the lens of SIEGE+. We omit soundness and effectiveness as all the checkers meet these two criteria. DSG stands for *direct serialization graph* proposed by Adya [1].

Checker	Property Supported	Informative	General	Efficient	Complete
CausalC	TCC	atomic propositions	simple txns only; rw registers	answer set solving	no
dbcop	multiple incl. TCC	coarse-grained cycles	general txns; rw registers	plain graph traversal	no
PolySI	SI	detailed violating scenarios	general txns; rw registers	MonoSAT solving	yes
Viper	SI	unsatisfied clauses	general txns; rw registers	MonoSAT solving	no
Cobra	SER	unsatisfied clauses	general txns; rw registers	MonoSAT solving	no
Elle	multiple excl. TCC	detailed violating scenarios	general txns; rw registers, lists	list-based DSG inference	no
C4	TCC	detailed violating scenarios	general txns; rw registers, lists	vectors & tree clocks	yes

TAO data store [20]. For other isolation levels, one can design and incorporate new TAP(s) correspondingly, e.g., together with new TAP(s) for “lost updates”, the above constructed RA checker would be able to check *update atomicity* [18], and C4 can be used to check SI. Moreover, C4 can be applied directly to check (non-transactional) CC, where each read or write in a history is considered as a transaction of a single operation. This is in contrast to CC checkers such as CausalC which cannot be used to check TCC in general.

C4’s key idea of leveraging vectors and the optimization with tree clocks can also be adapted to *graph-based* approaches for checking other isolation guarantees. This would accelerate the traversals of various dependency graphs, such as *polygraphs* [50] (adopted by Cobra, PolySI, and Viper) and Adya’s *direct serialization graph* [1].

Checking Stronger Isolation Levels. Isolation bugs are subtle and tricky. A large part of the histories produced by the *de facto* randomized testing approach are usually free of anomalies. As testing cannot prove the absence of bugs, developers typically generate a considerable number of histories for checking to gain a higher assurance. This would exhaust stronger isolation checkers due to the high complexity of the checking problems. As we have observed, the state-of-the-art SI checker PolySI requires significant time and memory to validate a moderate-sized history in practice. We argue that, despite being incomplete, a first attempt with C4 is practicable for checking stronger isolation guarantees as it can quickly validate millions of transactions in several minutes against their major building blocks including RC, RA, causality, data convergence, etc. Moreover, as many TCC bugs already manifest when checking SI or SER (see Table 4), C4 can be used to effectively detect all of them.

Alternatively, in order to achieve a complete, efficient validation up to the stronger isolation guarantee, adapting vectors and tree clocks into the checking problem is a promising approach. This is in contrast to the existing ASP-based or SAT/SMT-based solving approaches which have been shown to be significantly less efficient.

7 RELATED WORK

Characterizing TCC. The growing number of TCC databases has attracted the attention of the formal methods community. Cerone et al. [18] propose a framework for declaratively specifying TCC (and other isolation guarantees) with the dual notions of visibility (what transactions can observe) and arbitration (version order in our case). Biswas and Enea [14] present an alternative axiomatic framework for characterizing TCC (and other isolation criteria) which uses the write-read relation as in our case, instead of the

visibility relation as in [18]. This framework is more suitable for black-box testing over database histories as the write-read relation can be extracted straightforwardly from a history. We derive TAP-1 from its TCC formalization.

We base our fine-grained characterization of TCC on the “bad patterns” for formalizing non-transactional causal consistency [15]. Compared to the existing TCC formalizations, our new characterization (i) is sound and complete; (ii) comprises fine-grained patterns that facilitate understanding and debugging of the violations found; and (iii) paves the way for the complete checking of other isolation guarantees via modular patterns.

Black-box Checking Isolation Guarantees. We focus on the black-box checkers for *isolation bugs*. As shown in Table 5, we examine each of them in terms of the SIEGE+ principle [30], which strengthens SIEGE [35] with completeness. Our C4 checker meets all of its criteria, as demonstrated by our assessment. Existing tools are all sound and effective, although stronger isolation checkers inherently report false positives (e.g., the lost-update anomaly) with respect to TCC. CausalC does not support general transactions. Solver-based tools such as CausalC, Viper, and Cobra return to developers incomprehensible counterexamples in the form of atomic propositions or unsatisfied clauses. PolySI improves them by interpreting unsatisfied clauses into detailed violating scenarios. The coarse-grained cycles reported by dbcop are less informative, rendering understanding and debugging violations hard. As shown by our experimental results, C4 significantly outperforms dbcop, as well as the strong baselines that utilize various solving techniques based on answer set programming and MonoSAT. All the checkers except PolySI are incomplete, thereby missing TCC bugs. C4 joins five of the checkers for dealing with read-write registers and additionally supports list-specific data models as for Elle.

8 CONCLUSION

We have presented the black-box isolation checker C4, along with a novel characterization of TCC via TAPs. We have also established its soundness and completeness. Our extensive experimental results demonstrate C4’s fulfillment of SIEGE+’s criteria.

C4 complements the Jepsen [31] testing framework by efficient TCC checking. The modular anomalous patterns and the utilization of vectors and tree clocks can contribute to complete and efficient checking of many other isolation guarantees; building such checkers is the natural future work. The informative counterexamples can facilitate the understanding and debugging of bugs and help the developers rethink their system designs and implementations.

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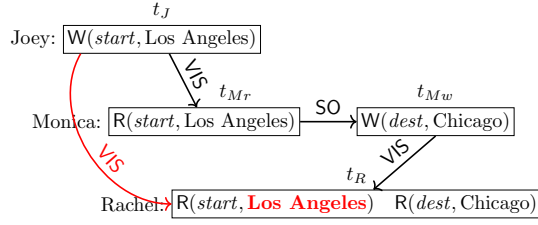


Figure 13: Illustration of the formal definition of TCC.

A TCC: FORMAL DEFINITION IN [18]

The framework in [18] uses the *visibility* relation to capture the potential causality between transactions and the *arbitration* relation to capture the convergence property of TCC.

Definition 4. An *abstract execution* is a tuple $\mathcal{A} = (T, SO, WR, VIS, AR)$, where (T, SO, WR) is a history, *visibility* $VIS \subseteq T \times T$ is a strict partial order such that $(SO \cup WR) \subseteq VIS$, and *version* $AR \subseteq T \times T$ is a strict total order such that $VIS \subseteq AR$.

Note that in this framework, $CO \triangleq (SO \cup WR)^+ \subseteq VIS$. That is, VIS is a general notion of CO .

Definition 5 (TCC [18]). A history $\mathcal{H} = (T, SO, WR)$ satisfies TCC if and only if there exists an abstract execution $\mathcal{A} = (T, SO, WR, VIS, AR)$ such that the INT and EXT axioms hold.

Example 8 (TCC). Consider the scenario in Example 2; see Figure 13. Since Monica sees the starting city set by Joey and Rachel sees the destination city set by Monica, we have $t_J \xrightarrow{VIS} t_{Mr}$ and $t_{Mw} \xrightarrow{VIS} t_R$, respectively. Since $SO \subseteq VIS$ and VIS is transitive, we have $t_J \xrightarrow{VIS} t_R$. Therefore, Monica should also see the starting city set by Joey.

The following theorem establishes the equivalence between Definitions 3 and 5 of TCC.

Theorem 2 (Equivalence of TCC Definitions). Definitions 3 and 5 of TCC are equivalent.

PROOF. Let $\mathcal{H} = (T, SO, WR)$ be a history. We show that \mathcal{H} satisfies Definition 3 of TCC (denoted TCC_3) if and only if \mathcal{H} satisfies Definition 5 of TCC (denoted TCC_5).

(\Rightarrow) Suppose that \mathcal{H} satisfies TCC_3 . Then CO is a strict partial order and there exists a strict total order $VO \supseteq CO$ such that the INT and EXT axioms hold. Let $VIS = CO$ and $AR = VO$ in TCC_5 . Clearly the INT and EXT axioms also hold for TCC_5 . Therefore, \mathcal{H} satisfies TCC_5 .

(\Leftarrow) Suppose that \mathcal{H} satisfies TCC_5 . Then there exists a strict total order VIS and a strict total order AR such that $CO \subseteq VIS \subseteq AR$ and the INT and EXT axioms hold. Since $CO \subseteq VIS$, CO is a strict partial order. Let $VO = AR$ in TCC_3 . Clearly, the INT axiom holds for TCC_3 . In the following, we show that the EXT axiom also holds for TCC_3 .

Let r be an external read operation on key x of transaction t . Suppose r reads from transaction $t' \neq t$. Then t' is the *last* (in AR) transaction among those which precede t in terms of VIS . Since $CO \subseteq VIS$ and $VO = AR$, t' is also the *last* (in VO) transaction among those which precede t in terms of VO . Therefore, the EXT axiom holds for TCC_3 . \square

B PROOF OF THEOREM 1

PROOF. Let $\mathcal{H} = (T, SO, WR)$ be a history. Since TAP-h, TAP-i, TAP-j, and TAP-k are special cases of TAP-l, in the following we show that \mathcal{H} satisfies TCC if and only if it does not contain any of anomalous patterns TAP-a to TAP-g and TAP-l.

(\Rightarrow) Assume that \mathcal{H} satisfies TCC. Therefore, $CO \triangleq (SO \cup WR)^+$ is a strict partial order and there exists a strict total order $VO \subseteq T \times T$ such that the INT and EXT axioms hold. We examine each of the anomalous patterns as follows.

- (TAP-a) THINAIRREAD: Forbidden by the INT and EXT axioms.
- (TAP-b) ABORTEDREAD: Forbidden by the INT and EXT axioms.
- (TAP-c) FUTUREREAD: Forbidden by the INT and EXT axioms.
- (TAP-d) NOTMYOWNWRITE: Forbidden by the EXT axiom.
- (TAP-e) INTERMEDIATEREAD: Forbidden by the EXT axiom.
- (TAP-f) NONREPEATABLEREAD: Forbidden by the INT axiom.
- (TAP-g) CYCLICCO: Since CO is a strict partial order, $SO \cup WR$ acyclic.
- (TAP-l) CONFLICTVO: Since $t_1 \xrightarrow{WR(x)} t_3$ and $WR \subseteq VO$, $t_1 \xrightarrow{VO} t_3$. Since $t_2 \xrightarrow{CO} t_3$ and $CO \subseteq VO$, $t_2 \xrightarrow{VO} t_3$. Since both t_1 and t_2 write to x and $t_1 \xrightarrow{WR(x)} t_3$, by the EXT axiom, we have $t_2 \xrightarrow{VO} t_1$. Since $t_1 \xrightarrow{VO} t_2$, we have $t_1 \xrightarrow{VO} t_2 \xrightarrow{VO} t_1$, contradicting the fact that VO is a strict total order.

(\Leftarrow) Assume that \mathcal{H} does not contain any of anomalous patterns TAP-a to TAP-g and TAP-l. We prove that \mathcal{H} satisfies TCC by showing that $CO \triangleq (SO \cup WR)^+$ is a strict partial order and constructing a strict total order VO such that the INT and EXT axioms hold.

First, since \mathcal{H} does not contain anomalous pattern CYCLICCO (TAP-g), CO is a strict partial order.

Then, we define VO as a relation satisfying

- VO is total;
- $CO \subseteq VO$;
- For any three transactions t_1, t_2 , and t_3 , if
 - both t_1 and t_2 writes to the same key, say x ,
 - $t_2 \xrightarrow{CO} t_3$, and
 - t_3 reads x from t_1 ,
 then $t_2 \xrightarrow{VO} t_1$.

Since \mathcal{H} does not contain the anomalous pattern CONFLICTVO (TAP-l), VO is acyclic. Thus, VO is a strict total order.

In the following, we show that the INT and EXT axioms hold. Let $r \triangleq R(x, v)$ be a read operation of a transaction t . Since \mathcal{H} does not contain anomalous patterns THINAIRREAD (TAP-a) or ABORTEDREAD (TAP-b), r have read x from a committed transaction.

- (INT axiom) Suppose that r is an *internal* read of transaction t . Let o_x be the *last* operation on x which is before r in transaction t . (o_x exists because r is an internal read of transaction t .) We distinguish between two cases according to the type of o_x :
 - Suppose that o_x is a read operation on x . Let $o_x \triangleq R(x, v')$. Since \mathcal{H} does not contain the anomalous pattern NONREPEATABLEREAD (TAP-f), $v' = v$.

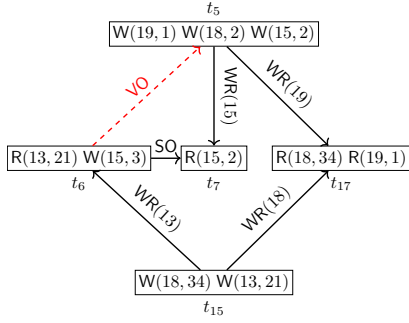


Figure 15: TAP-i found in YugabyteDB

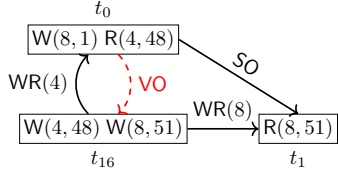


Figure 16: TAP-k found in CockroachDB

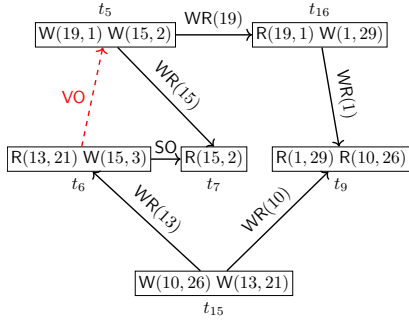


Figure 17: TAP-l found in YugabyteDB

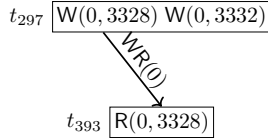


Figure 14: TAP-e found in MariaDB-Galera

- Suppose that o_x is a write operation on x . Let $o_x \triangleq W(x, v')$. Since \mathcal{H} does not contain the anomalous pattern NOTMYOWNWRITE (TAP-d), $v' = v$.

- (EXT axiom) Suppose that r is an *external* read of transaction t . Since \mathcal{H} does not contain the anomalous pattern FUTUREREAD (TAP-c), r reads x from a different transaction $t' \neq t$. Since \mathcal{H} does not contain the anomalous pattern INTERMEDIATEREAD (TAP-e), the write $W(x, v)$ which writes the value v is the last write to x in transaction t' .

Let V_x be the set of transactions that precede t in terms of CO and write to x . Since $t' \xrightarrow{WR(x)} t$, we have $t' \in V_x$. We show that t' is the last (in VO) transaction in V_x . Suppose by contradiction that there is another transaction $t'' \neq t'$ in V_x such that $t' \xrightarrow{VO} t''$. Since $t'' \in V_x$, $t'' \xrightarrow{CO} t$. Then there is an anomalous pattern CONFLICTVO (TAP-l) involving t , t' , and t'' , contradicting the fact that \mathcal{H} does not contain such an anomalous pattern. \square

C THE FULL C4 ALGORITHM

Algorithm 2 gives the full pseudocode of C4.

Procedure BUILD_{CO}. C4 builds CO by examining transactions one by one (line 6). Consider a transaction t . It first builds SO, initializing $Vec[t]$ using its immediate predecessor transaction t' in the same session (lines 10–14). Then C4 examines each operation, denoted by o , of t and builds WR. There are three cases: (1) If o is a read operation but its dictating write has not been encountered (line 17), C4 temporarily adds it to $S_{r'}$; (2) If o is a read operation and its dictating write, denoted t' , has been encountered (line 19), C4 updates $Vec[t]$ using $Vec[t']$ and adds an WR edge (t', t) if t is unreachable from t' in the graph (decided on $Vec[t]$ and $Vec[t']$ at line 22); and (3) If o is a write operation (line 25), C4 adds an WR edge from o to each of its dictated read operations in $S_{r'}$. Finally, C4 uses $Vec[t]$ to update vectors of the transactions reachable from t in case new edges outgoing from t have been added (line 32). This ensures the transitivity of CO. Denote the resulting graph by \mathcal{G}_{CO} .

D TEST CASES FOR TAPS

Table 6 shows 12 test cases (or anomalous histories) that we craft for 12 TAPs, respectively.

E OTHER REPRESENTATIVE BUGS FOUND BY C4

Figures 14–17 depict the violations found by C4 in four production database engines, respectively.

Algorithm 2 The C4 algorithm for checking history $\mathcal{H} = (T, \text{SO}, \text{WR})$ against TCC.

\mathbb{S} : the set of sessions in \mathcal{H}
 WT_x : the set of transactions that write to key x
 RT_x^v : the set of transactions that read value v of key x
 $R(t)$: the set of read operations of transaction t
 $W(t)$: the set of write operations of transaction t
 $O(t)$: the set of operations of transaction t
 $\text{txn}(o)$: the transaction containing operation o
 $\text{key}(o)$: the key accessed by operation o
 $\text{ite}(c, t, e)$: if-then-else with condition c , “then” part t , and “else” part e

$\text{Vec}[t]$: vector for transaction t
 taps : the set of anomalous patterns found, initially \emptyset
 E : the set of SO, WR, and VO edges
 E_{wr} : the set of $\text{WR}(_)$ edges associated with keys
 S_w : the set of write operations encountered, initially \emptyset
 S_r : the set of read operations whose dictating writes are in S_w , initially \emptyset
 $S_{r'}$: the set of read operations whose dictating writes are not in S_w , initially \emptyset
 S_a : the set of operations in aborted transactions encountered, initially \emptyset

```

1: procedure CHECKTCC()
2:   BUILDCO()
3:   CHECKCOTAP()           ▷ TAP-a to TAP-h, TAP-j, and TAP-k
4:   BUILDVO()
5:   CHECKVOTAP()           ▷ TAP-i and TAP-l
6: procedure BUILDCO()
7:   for  $t \in T$ 
8:     if  $t$  is ABORTED
9:        $S_a \leftarrow S_a \cup O(t)$    continue
10:    suppose  $t$  is the  $i$ -th transaction in its session  $s$ 
11:     $\text{Vec}[t] \leftarrow \langle \lambda f \in \mathbb{S}. \text{ite}(f = s, i, 0) \rangle$ 
12:    if  $i > 0$ 
13:      let  $t' \leftarrow$  the  $(i - 1)$ -st transaction in session  $s$ 
14:       $\text{Vec}[t] \leftarrow \text{Vec}[t] \sqcup \text{Vec}[t']$ 
15:       $E \leftarrow E \cup \{(t', t)\}$ 
16:    for  $o \in O(t)$ 
17:      if  $o \in R(t) \wedge \neg(\exists w \in S_w. w \xrightarrow{\text{WR}} o)$ 
18:         $S_{r'} \leftarrow S_{r'} \cup \{o\}$ 
19:      if  $o \in R(t) \wedge \exists w \in S_w. w \xrightarrow{\text{WR}} o$ 
20:         $S_r \leftarrow S_r \cup \{o\}$ 
21:        let  $t' \leftarrow \text{txn}(w)$ 
22:        if  $t' \neq t \wedge \neg(\text{Vec}[t] \supseteq \text{Vec}[t'])$ 
23:           $\text{Vec}[t] \leftarrow \text{Vec}[t] \sqcup \text{Vec}[t']$ 
24:           $E \leftarrow E \cup \{(t', t)\}$     $E_{wr} \leftarrow E_{wr} \cup \{(t', t, \text{key}(o))\}$ 
25:      if  $o \in W(t)$ 
26:         $S_w \leftarrow S_w \cup \{o\}$ 
27:        for  $\forall r \in S_{r'}. o \xrightarrow{\text{WR}} r$ 
28:           $S_{r'} \leftarrow S_{r'} \setminus \{r\}$     $S_r \leftarrow S_r \cup \{r\}$ 
29:          let  $t' \leftarrow \text{txn}(r)$ 
30:          if  $t' \neq t$ 
31:             $E \leftarrow E \cup \{(t, t')\}$     $E_{wr} \leftarrow E_{wr} \cup \{(t, t', \text{key}(o))\}$ 
32:    UPDATEVEC( $t$ )
33: procedure BUILDVO()
34:   for  $t_1, t_2, t_3 \in T$  such that  $\exists x \in K. t_1 \xrightarrow{\text{WR}(x)} t_3 \wedge t_2 \in \text{WT}_x \wedge \text{Vec}[t_2] \sqsubseteq \text{Vec}[t_3]$ 
35:      $E \leftarrow E \cup \{(t_2, t_1)\}$ 
36:   for  $t \in T$ 
37:     UPDATEVEC( $t$ )
38: procedure CHECKCOTAP()
39:   for  $t \in T$ 
40:     check anomalous patterns TAP-a to TAP-f           ▷ details omitted
41:     if  $\exists x \in K. (t \in \text{RT}_x^{\perp x} \wedge (\exists t' \neq t. t' \in \text{WT}_x. \text{Vec}[t'] \sqsubseteq \text{Vec}[t]))$ 
42:        $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-j}\}$            ▷ INITIALREAD
43:     if  $\exists t_1, t_2 \in T. \text{Vec}[t_1] = \text{Vec}[t_2]$ 
44:        $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-g}\}$            ▷ CYCLICCO
45:     if  $\exists x \in K \exists t_1, t_2, t_3 \in T. t_1 \xrightarrow{\text{WR}(x)} t_3 \wedge t_2 \in \text{WT}_x \wedge \text{Vec}[t_1] \sqsubseteq \text{Vec}[t_2] \sqsubseteq \text{Vec}[t_3]$ 
46:       if  $\exists y \neq x \in K. t_2 \xrightarrow{\text{WR}(y)} t_3$ 
47:          $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-h}\}$            ▷ FRACTUREDREADCO
48:       else
49:          $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-k}\}$            ▷ COCONFLICTVO
50: procedure CHECKVOTAP()
51:   if  $\exists x \in K \exists t_1, t_2, t_3 \in T. t_1 \xrightarrow{\text{WR}(x)} t_3 \wedge t_2 \in \text{WT}_x \wedge \text{Vec}[t_1] = \text{Vec}[t_2]$ 
52:     if  $\exists y \neq x \in K. t_2 \xrightarrow{\text{WR}(y)} t_3$ 
53:        $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-i}\}$            ▷ FRACTUREDREADVO
54:     else if  $t_2 \xrightarrow{\text{CO}} t_3$ 
55:        $\text{taps} \leftarrow \text{taps} \cup \{\text{TAP-l}\}$            ▷ CONFLICTVO
56: procedure UPDATEVEC( $t$ )
57:   for  $t' \in T$  in DFS traversal from  $t$ 
58:     if  $\text{Vec}[t'] \supseteq \text{Vec}[t]$ 
59:       backtrack
60:        $\text{Vec}[t'] \leftarrow \text{Vec}[t'] \sqcup \text{Vec}[t]$ 

```

Table 6: Test cases for TAPs. For example, $[\text{w}(1,2,3,4) \text{ r}(2,1,3,4)]$ refers to a transaction of one write operation and one read operation. The write operation writes value 2 to key 1 by transaction 4 of session 3, while the read operation reads value 1 on key 2 by transaction 4 of session 3.

TAP	Test Case
TAP-a	$[\text{r}(1,1,0,0) \text{ r}(2,1,0,0)]$
TAP-b	$[\text{w}(1,1,0,0) \text{ w}(2,1,0,0)] [\text{r}(1,1,0,1)]$
TAP-c	$[\text{r}(1,1,0,0) \text{ w}(1,1,0,0)]$
TAP-d	$[\text{w}(1,1,0,0)] [\text{w}(1,2,1,1) \text{ r}(1,1,1,1)]$
TAP-e	$[\text{w}(1,1,0,0) \text{ w}(1,2,0,0)] [\text{r}(1,1,1,1)]$
TAP-f	$[\text{w}(1,2,0,0)] [\text{w}(1,1,1,1) \text{ r}(1,1,1,1) \text{ r}(1,2,1,1)]$
TAP-g	$[\text{r}(1,1,0,0) \text{ w}(2,1,0,0)] [\text{r}(2,1,1,1)] [\text{w}(1,1,1,2)]$
TAP-h	$[\text{w}(1,2,0,0)] [\text{r}(1,2,1,1)] [\text{w}(1,1,1,2) \text{ w}(2,1,1,2)] [\text{r}(1,2,2,3) \text{ r}(2,1,2,3)]$
TAP-i	$[\text{w}(1,1,0,0) \text{ w}(2,1,0,0)] [\text{w}(1,2,1,1) \text{ w}(2,2,1,1)] [\text{r}(1,1,2,2) \text{ r}(2,2,2,2)]$
TAP-j	$[\text{w}(1,0,0,0)] [\text{w}(1,1,0,1) \text{ w}(2,1,0,1)] [\text{r}(1,0,0,2)]$
TAP-k	$[\text{w}(1,2,0,0)] [\text{r}(1,2,1,1)] [\text{w}(1,1,1,2) \text{ w}(2,1,1,2)] [\text{r}(2,1,2,3)] [\text{r}(1,2,2,4)]$
TAP-l	$[\text{w}(1,2,0,0)] [\text{r}(1,2,1,1)] [\text{w}(1,1,1,2) \text{ w}(2,1,1,2)] [\text{r}(2,1,2,3)] [\text{r}(1,2,2,4)] [\text{r}(1,2,3,5)] [\text{r}(1,1,3,6)]$