Definitions

In[294]:=

$$G0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix};$$

$$G1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix};$$

$$G2 = \begin{pmatrix} 0 & 0 & 0 & -I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ -I & 0 & 0 & 0 \end{pmatrix};$$

$$G3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};$$

$$G5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};$$

In[299]:=

```
$Assumptions = {e ∈ Reals , p0 ∈ Reals , p1 ∈ Reals , p2 ∈ Reals ,
  p3 ∈ Reals, v0 ∈ Reals, z ∈ Reals, L ∈ Reals, e > 0, m > 0, v0 > 0, L > 0, pz > 0,
  pz ∈ Reals, py ∈ Reals, e > m, e - v0 > m, e > v0 + m, hx > 0, hx ∈ Reals}
```

Out[299]=

```
{e ∈ ℝ, p0 ∈ ℝ, p1 ∈ ℝ, p2 ∈ ℝ, p3 ∈ ℝ, v0 ∈ ℝ, z ∈ ℝ, L ∈ ℝ, e > 0, m > 0,
 v0 > 0, L > 0, pz > 0, pz ∈ ℝ, py ∈ ℝ, e > m, e - v0 > m, e > m + v0, hx > 0, hx ∈ ℝ}
```

Dirac’s Eq

In[∘]:=

$$etaD = \frac{G0 + I\,G5}{Sqrt[2]};$$

```
eta = etaD;
```

In[ ]:=

```
Pz = Eigenvectors[ G0 e +  m I G5];
```

In[ ]:=

```
PzA = %
```

Out[ ]=

$$\left\{\left\{0,\ \frac{i\left(-e+\sqrt{e^2-m^2}\right)}{m},\ 0,\ 1\right\},\ \left\{\frac{i\left(-e+\sqrt{e^2-m^2}\right)}{m},\ 0,\ 1,\ 0\right\},\ \left\{0,\ -\frac{i\left(e+\sqrt{e^2-m^2}\right)}{m},\ 0,\ 1\right\},\ \left\{-\frac{i\left(e+\sqrt{e^2-m^2}\right)}{m},\ 0,\ 1,\ 0\right\}\right\}$$

In[ ]:=

```
PzC = PzA /. {Sqrt[e^2 - m^2] → pz}
```

Out[ ]=

$$\left\{\left\{0,\ \frac{i(-e+pz)}{m},\ 0,\ 1\right\},\ \left\{\frac{i(-e+pz)}{m},\ 0,\ 1,\ 0\right\},\ \left\{0,\ -\frac{i(e+pz)}{m},\ 0,\ 1\right\},\ \left\{-\frac{i(e+pz)}{m},\ 0,\ 1,\ 0\right\}\right\}$$

In[ ]:=

```
P = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}};

PzD = PzC.P
```

Out[ ]=

$$\left\{\left\{1,\ 0,\ \frac{i(-e+pz)}{m},\ 0\right\},\ \left\{0,\ 1,\ 0,\ \frac{i(-e+pz)}{m}\right\},\ \left\{1,\ 0,\ -\frac{i(e+pz)}{m},\ 0\right\},\ \left\{0,\ 1,\ 0,\ -\frac{i(e+pz)}{m}\right\}\right\}$$

In[ ]:=

```
col[v_] := List /@ v;

u1 = col[PzD[[3]]];
u2 = col[PzD[[4]]];

u3 = col[PzD[[1]]];
u4 = col[PzD[[2]]];
```

In[∘]:=
```
u1 // MatrixForm
```
Out[∘]//MatrixForm=
$$\begin{pmatrix} 1 \\ 0 \\ -\frac{i\,(e+pz)}{m} \\ 0 \end{pmatrix}$$

In[∘]:=
```
ConjugateTranspose[u1].u2 // FullSimplify
```
Out[∘]=
```
{{0}}
```

In[∘]:=
```
Fu1[ee_, ppz_] := u1 /. {e → ee, pz → ppz}
Fu2[ee_, ppz_] := u2 /. {e → ee, pz → ppz}
Fu3[ee_, ppz_] := u3 /. {e → ee, pz → ppz}
Fu4[ee_, ppz_] := u4 /. {e → ee, pz → ppz}
```
In[∘]:=
```
Fu1[e, p1]
```
Out[∘]=
$$\left\{\{1\}, \{0\}, \left\{-\frac{i\,(e+p1)}{m}\right\}, \{0\}\right\}$$

In[∘]:=
```
psiIN = a Fu1[e, p1];
psiR = b Fu1[e, -p1] + bp Fu2[e, -p1];
psiT = c Fu1[e - v0, p2] + cp Fu2[e - v0, p2];
```
In[∘]:=
```
Xm = G0
```
Out[∘]=
```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}
```

In[∘]:=
```
jin = ConjugateTranspose[psiIN].Xm.psiIN // FullSimplify
```
Out[∘]=
$$\left\{\left\{-\frac{a\,(e-m+p1)\,(e+m+p1)\,\text{Conjugate}[a]}{m^2}\right\}\right\}$$

In[ ]:=

```
jR = ConjugateTranspose[psiR].Xm.psiR // FullSimplify
```

Out[ ]=

$$\left\{\left\{\frac{(e+m-p1)\ (-e+m+p1)\ \left(\mathrm{Abs}[b]^2+\mathrm{Abs}[bp]^2\right)}{m^2}\right\}\right\}$$

In[ ]:=

```
jT = ConjugateTranspose[psiT].Xm.psiT // FullSimplify
```

Out[ ]=

$$\left\{\left\{\frac{(e+m+p2-v0)\ (-e+m-p2+v0)\ \left(\mathrm{Abs}[c]^2+\mathrm{Abs}[cp]^2\right)}{m^2}\right\}\right\}$$

In[ ]:=

```
jR / jin /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[ ]=

$$\left\{\left\{\frac{m^2\ \left(\mathrm{Abs}[b]^2+\mathrm{Abs}[bp]^2\right)}{\left(m^2-2\,e\,\left(e+\sqrt{(e-m)\ (e+m)}\right)\right)\ \mathrm{Abs}[a]^2}\right\}\right\}$$

In[ ]:=

```
jT / jin /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[ ]=

$$\left\{\left\{\frac{\left(e-m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)\ \left(e+m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)\ \left(\mathrm{Abs}[c]^2+\mathrm{Abs}[cp]^2\right)}{2\,a\,\left(-m^2+e\,\left(e+\sqrt{(e-m)\ (e+m)}\right)\right)\ \mathrm{Conjugate}[a]}\right\}\right\}$$

In[ ]:=

$$\mathrm{nt}=\frac{\left(e-m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)\ \left(e+m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)}{2\ \left(-m^2+e\,\left(e+\sqrt{e^2-m^2}\right)\right)};\ (*G3*)$$

$$\mathrm{nr}=\frac{m^2}{\left(m^2-2\,e\,\left(e+\sqrt{e^2-m^2}\right)\right)};$$

In[ ]:=

```
sol1 =
 Solve[a Fu1[e, p1] + b Fu1[e, -p1] + bp Fu2[e, -p1] == c Fu1[e - v0, p2] + cp Fu2[e - v0, p2],
   {a, b, c, bp, cp}] // FullSimplify
```

··· Solve: Equations may not give solutions for all "solve" variables.

Out[ ]=

$$\left\{\left\{a\to\frac{c\ (p1+p2-v0)}{2\ p1},\ b\to\frac{c\ (p1-p2+v0)}{2\ p1},\ bp\to 0,\ cp\to 0\right\}\right\}$$

In[ ]:=

```
sol2 = sol1 /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[ ]=

$$\left\{\left\{a \to \frac{c\left(\sqrt{(e-m)(e+m)}+\sqrt{-m^2+(e-v0)^2}-v0\right)}{2\sqrt{(e-m)(e+m)}},\ b \to \frac{c\left(\sqrt{(e-m)(e+m)}-\sqrt{-m^2+(e-v0)^2}+v0\right)}{2\sqrt{(e-m)(e+m)}},\ bp \to 0,\ cp \to 0\right\}\right\}$$

In[ ]:=

```
Tr1 = (c /. sol2) / (a /. sol2) // FullSimplify
```

Out[ ]=

$$\left\{\frac{2\sqrt{(e-m)(e+m)}}{\sqrt{(e-m)(e+m)}+\sqrt{-m^2+(e-v0)^2}-v0}\right\}$$

In[ ]:=

```
Tr2 = (cp /. sol2) / (a /. sol2) // FullSimplify
```

Out[ ]=

$$\{0\}$$

In[ ]:=

```
Rf1 = (b /. sol2) / (a /. sol2) // FullSimplify
```

Out[ ]=

$$\left\{\frac{\sqrt{(e-m)(e+m)}-\sqrt{-m^2+(e-v0)^2}+v0}{\sqrt{(e-m)(e+m)}+\sqrt{-m^2+(e-v0)^2}-v0}\right\}$$

In[ ]:=

```
Rf2 = (bp /. sol2) / (a /. sol2) // FullSimplify
```

Out[ ]=

$$\{0\}$$

In[ ]:=

```
Ttotal = nt (modsq[Tr1[[1]]] + modsq[Tr2[[1]]]) // FullSimplify
```

Out[ ]=

$$\frac{2(e-m)(e+m)\left(e-m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)\left(e+m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)}{\left(-m^2+e\left(e+\sqrt{(e-m)(e+m)}\right)\right)\left(\sqrt{(e-m)(e+m)}-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)^2}$$

In[ ]:=

```
Limit[Ttotal, v0 → ∞] // FullSimplify
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[ ]=

$$2 - \frac{2\, e \left(e + \sqrt{(e-m)\,(e+m)}\right)}{m^2}$$

In[ ]:=

```
Limit[Ttotal, v0 → 0] // FullSimplify
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[ ]=

$$1$$

In[ ]:=

```
Rtot = nr (modsq[Rf1[[1]]] + modsq[Rf2[[1]]]) // FullSimplify
```

Out[ ]=

$$\frac{m^2 \left(\sqrt{(e-m)\,(e+m)} + v0 - \sqrt{e^2 - m^2 - 2\, e\, v0 + v0^2}\right)^2}{\left(m^2 - 2\, e \left(e + \sqrt{(e-m)\,(e+m)}\right)\right) \left(\sqrt{(e-m)\,(e+m)} - v0 + \sqrt{e^2 - m^2 - 2\, e\, v0 + v0^2}\right)^2}$$

In[ ]:=

```
Limit[Rtot, v0 → ∞] // FullSimplify
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[ ]=

$$-\frac{\left(e + \sqrt{(e-m)\,(e+m)}\right)^2}{m^2}$$

In[ ]:=

```
Limit[Rtot, v0 → 0] // FullSimplify
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[ ]=

$$0$$

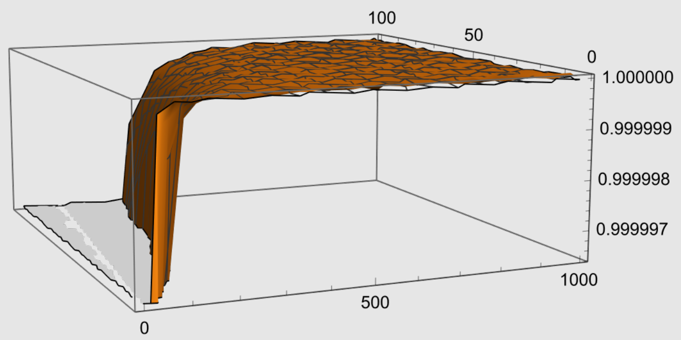
In[ ]:=

```
TotP [ee_, v00_, mm_] :=
 nt (modsq[ Tr1[[1]] ] + modsq[ Tr2[[1]] ] ) + nr ( modsq[ Rf1[[1]] ] + modsq[ Rf2[[1]] ]) /.
  {e → ee, v0 → v00, m → mm}
```

In[ ]:=

```
Plot3D[TotP [e, v0, 1], {e, 2, 1000}, {v0, 1, 100}]
```

Out[ ]=




```
me = 9.1 * 10^-31;
cv = 3 * 10^8;
ev = me * cv^2 / (1.6 * 10^-19);
mev = me * cv^2 / (1.6 * 10^-19);
jev = (1.6 * 10^-19);
vstep = 10^6;
```

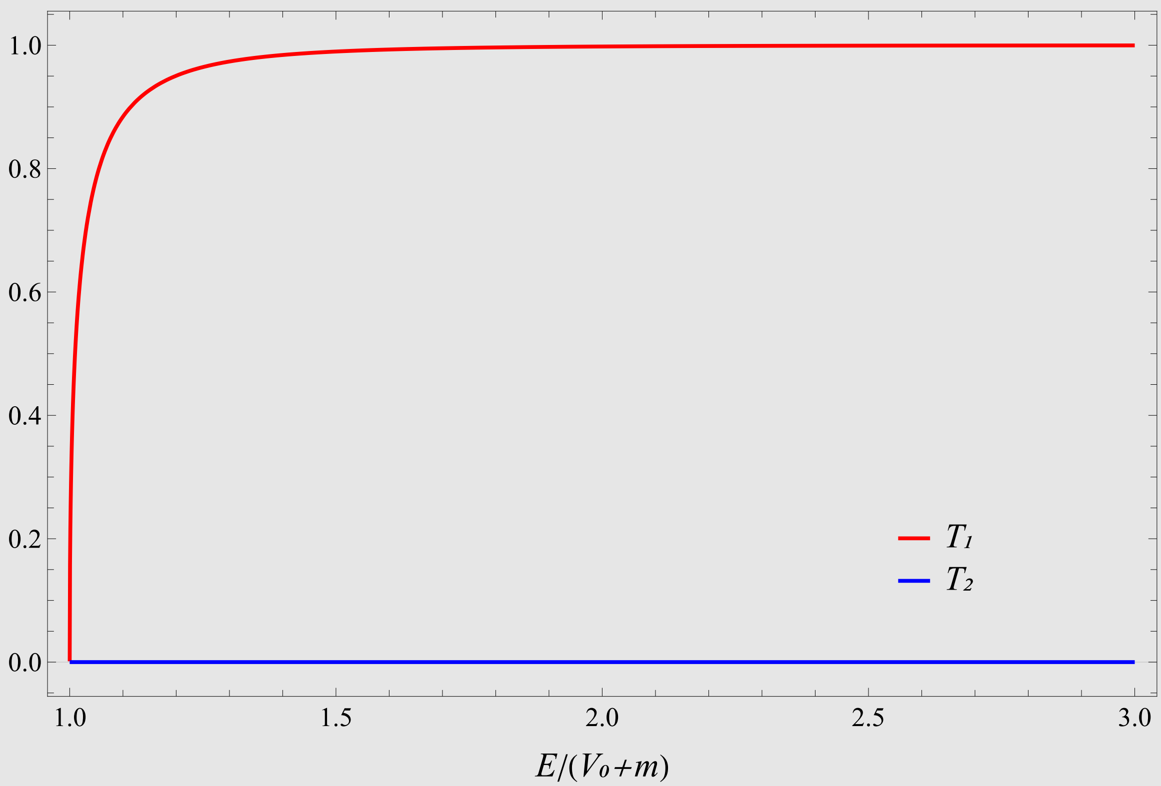
In[ ]:=

```
Tup[ee_, v00_, mm_] := Abs[nt modsq[ Tr1⟦1⟧ ]] /. {e → ee, v0 → v00, m → mm}
Tdn[ee_, v00_, mm_] := Abs[nt modsq[Tr2⟦1⟧ ]] /. {e → ee, v0 → v00, m → mm}
Rup[ee_, v00_, mm_] := Abs[ nr modsq[ Rf1⟦1⟧ ]] /. {e → ee, v0 → v00, m → mm}
Rdn[ee_, v00_, mm_] := Abs[nr modsq[ Rf2⟦1⟧ ]] /. {e → ee, v0 → v00, m → mm}
```

In[∘]:=

```
Plot[{Tup[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
  Tdn[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]},
 {x, 1, 3}, PlotStyle → {{Red, Thick}, {Blue, Thick}}, Frame → True,
 FrameLabel → {Style["E/ (V₀+m)", 18, Italic], None},
 PlotLegends → Placed[{Style["T₁", 18, Italic], Style["T₂", 18, Italic]}, {0.8, 0.2}],
 ImageSize → {600, 600}, FrameTicksStyle → Directive[Black, 14],
 PlotRange → All, PlotTheme → "Scientific"]
```

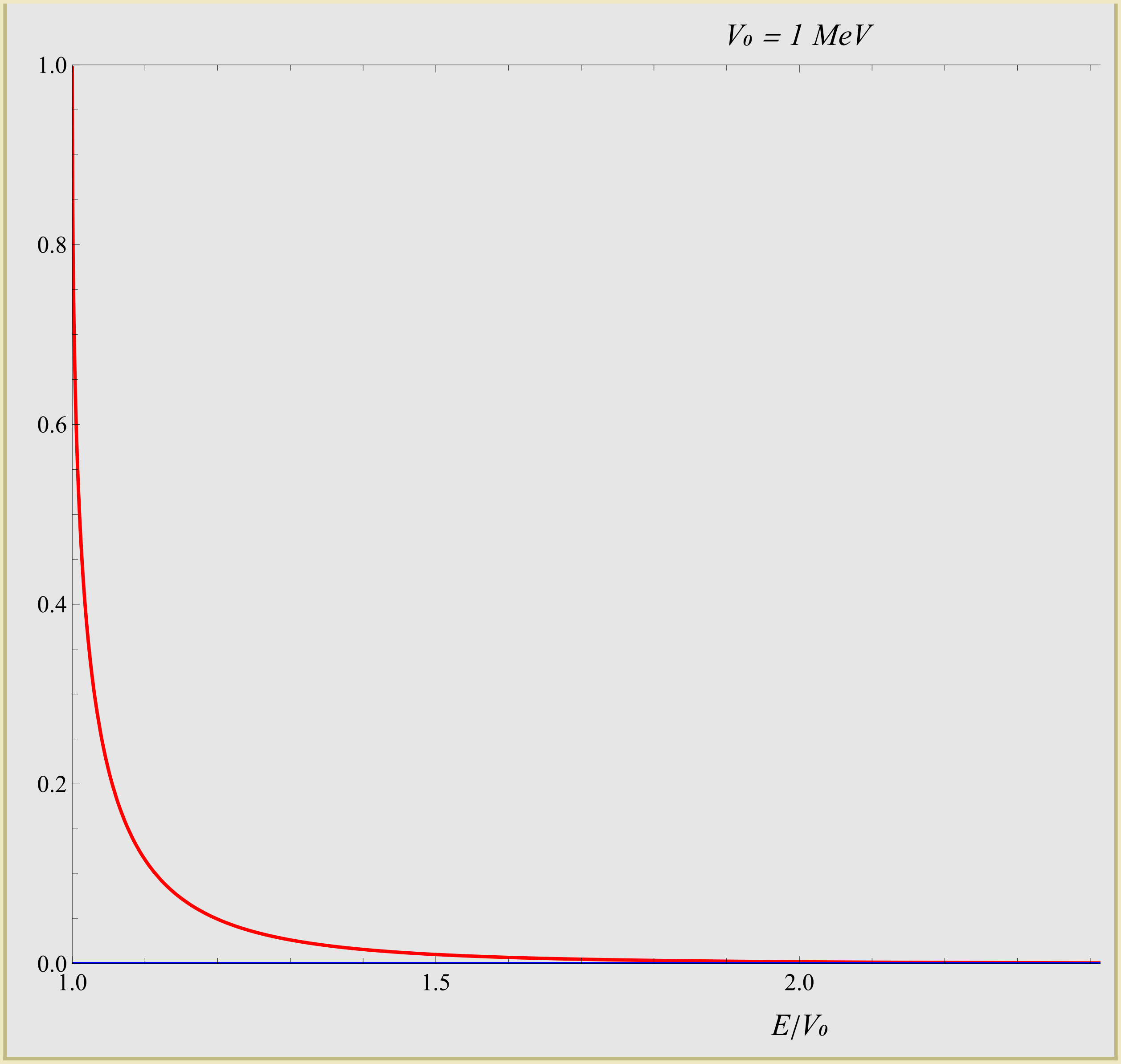
Out[∘]=



In[*]:=

```
Plot[{Rup[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
   Rdn[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]}, {x, 1, 3},
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange → {{1, 3}, {0, 1}},
  Frame → True, FrameLabel → {Style["E/V₀", 18, Italic], None},
  PlotLegends → Placed[{Style["R₁", 18, Italic], Style["R₂", 18, Italic]}, {0.75, 0.3}],
  ImageSize → {900, 600}, FrameTicksStyle → Directive[Black, 14],
  FrameStyle → Directive[Black, 14], (*☑ replaces FrameLabelStyle*)
  PlotTheme → "Scientific", PlotLabel → Style["V₀  = 1 MeV", 18, Italic]]
```

Out[*]=



In[ ]:=
```
Rup[(vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]
```
Out[ ]=
```
-1.
```

Ajaib’s Representation

In[ ]:=
$$\text{etaA} = \frac{-\text{I}}{\text{Sqrt}[2]}(\text{G0.G1.G5} + \text{G2});$$
```
eta = etaA;
```
In[ ]:=
$$\text{x1} = \frac{\text{eta} + \text{ConjugateTranspose}[\text{eta}]}{\text{Sqrt}[2]};$$
$$\text{x2} = \frac{\text{eta} - \text{ConjugateTranspose}[\text{eta}]}{\text{Sqrt}[2]};$$
In[ ]:=
```
Pz = Eigenvectors[ x1 e +  m x2];
```
In[ ]:=
```
PzA = %
```
Out[ ]=
$$\left\{\left\{\frac{\sqrt{e^2-m^2}}{e}, \frac{i\,m}{e}, 0, 1\right\}, \left\{-\frac{i\,m}{e}, -\frac{\sqrt{e^2-m^2}}{e}, 1, 0\right\}, \left\{-\frac{\sqrt{e^2-m^2}}{e}, \frac{i\,m}{e}, 0, 1\right\}, \left\{-\frac{i\,m}{e}, \frac{\sqrt{e^2-m^2}}{e}, 1, 0\right\}\right\}$$
In[ ]:=
$$\text{PzC} = \text{PzA} \;/.\; \left\{\text{Sqrt}\left[e^2 - m^2\right] \to \text{pz}\right\}$$
Out[ ]=
$$\left\{\left\{\frac{pz}{e}, \frac{i\,m}{e}, 0, 1\right\}, \left\{-\frac{i\,m}{e}, -\frac{pz}{e}, 1, 0\right\}, \left\{-\frac{pz}{e}, \frac{i\,m}{e}, 0, 1\right\}, \left\{-\frac{i\,m}{e}, \frac{pz}{e}, 1, 0\right\}\right\}$$

In[ ]:=

```
P = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}};

PzD = PzC.P
```

Out[ ]=

$$\left\{\left\{1, 0, \frac{i\, m}{e}, \frac{pz}{e}\right\}, \left\{0, 1, -\frac{pz}{e}, -\frac{i\, m}{e}\right\}, \left\{1, 0, \frac{i\, m}{e}, -\frac{pz}{e}\right\}, \left\{0, 1, \frac{pz}{e}, -\frac{i\, m}{e}\right\}\right\}$$

In[ ]:=

```
col[v_] := List /@ v;

u1 = col[PzD[[3]]];
u2 = col[PzD[[4]]];

u3 = col[PzD[[1]]];
u4 = col[PzD[[2]]];
```

In[ ]:=

```
u1 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ \frac{i\, m}{e} \\ -\frac{pz}{e} \end{pmatrix}$$

In[ ]:=

```
ConjugateTranspose[u1].u2 // FullSimplify
```

Out[ ]=

{{0}}

In[ ]:=

```
Fu1[ee_, ppz_] := u1 /. {e → ee, pz → ppz}
Fu2[ee_, ppz_] := u2 /. {e → ee, pz → ppz}
Fu3[ee_, ppz_] := u3 /. {e → ee, pz → ppz}
Fu4[ee_, ppz_] := u4 /. {e → ee, pz → ppz}
```

In[ ]:=

```
Fu1[e, p1]
```

Out[ ]=

$$\left\{\{1\}, \{0\}, \left\{\frac{i\, m}{e}\right\}, \left\{-\frac{p1}{e}\right\}\right\}$$

In[ ]:=

```
psiIN = a Fu1[e, p1];
psiR = b Fu1[e, -p1] + bp Fu2[e, -p1];
psiT = c Fu1[e - v0, p2] + cp Fu2[e - v0, p2];
```

In[ ]:=

```
Xm = eta + ConjugateTranspose[eta]
```

Out[ ]=

$$\left\{\left\{0, 0, 0, -\sqrt{2}\right\}, \left\{0, 0, \sqrt{2}, 0\right\}, \left\{0, \sqrt{2}, 0, 0\right\}, \left\{-\sqrt{2}, 0, 0, 0\right\}\right\}$$

In[ ]:=

```
jin = ConjugateTranspose[psiIN].Xm.psiIN // FullSimplify
```

Out[ ]=

$$\left\{\left\{\frac{2\sqrt{2}\ a\ p1\ \text{Conjugate}[a]}{e}\right\}\right\}$$

In[ ]:=

```
jR = ConjugateTranspose[psiR].Xm.psiR // FullSimplify
```

Out[ ]=

$$\left\{\left\{-\frac{2\sqrt{2}\ p1\left(\text{Abs}[b]^2+\text{Abs}[bp]^2\right)}{e}\right\}\right\}$$

In[ ]:=

```
jT = ConjugateTranspose[psiT].Xm.psiT // FullSimplify
```

Out[ ]=

$$\left\{\left\{\frac{2\sqrt{2}\ p2\left(\text{Abs}[c]^2+\text{Abs}[cp]^2\right)}{e-v0}\right\}\right\}$$

In[ ]:=

```
jR / jin /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[ ]=

$$\left\{\left\{-\frac{\text{Abs}[b]^2+\text{Abs}[bp]^2}{\text{Abs}[a]^2}\right\}\right\}$$

In[ ]:=

```
jT / jin /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[ ]=

$$\left\{\left\{\frac{e\sqrt{\frac{e^2-m^2-2\,e\,v0+v0^2}{e^2-m^2}}\left(\text{Abs}[c]^2+\text{Abs}[cp]^2\right)}{(e-v0)\ \text{Abs}[a]^2}\right\}\right\}$$

In[ ]:=

```
nt = e Sqrt[(e^2-m^2-2 e v0+v0^2)/(e^2-m^2)]/(e - v0); (*G3*)
nr = 1;
```

In[ ]:=

```
sol1 =
 Solve[a Fu1[e, p1] + b Fu1[e, -p1] + bp Fu2[e, -p1] == c Fu1[e - v0, p2] + cp Fu2[e - v0, p2],
   {a, b, c, bp, cp}] // FullSimplify
```

••• Solve: Equations may not give solutions for all "solve" variables.

Out[ ]=

$$\left\{\left\{a \to -\frac{i\, bp \left(e^2 (p1+p2)^2 - 2\, e\, p1\, (p1+p2)\, v0 + \left(m^2+p1^2\right) v0^2\right)}{2\, m\, p1\, v0\, (-e+v0)},\right.\right.$$
$$b \to \frac{i\, bp \left(e^2 \left(-p1^2+p2^2\right) + 2\, e\, p1^2\, v0 + (m-p1)\,(m+p1)\, v0^2\right)}{2\, m\, p1\, v0\, (-e+v0)},$$
$$\left.\left. c \to \frac{i\, bp\, (e\,(p1+p2) - p1\, v0)}{m\, v0},\ cp \to bp\right\}\right\}$$

In[ ]:=

```
sol2 = sol1 /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[ ]=

$$\left\{\left\{a \to \frac{i\, bp\, e \left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0\right)}{m \sqrt{(e-m)\,(e+m)}\; v0},\ b \to \frac{i\, bp\, m}{\sqrt{(e-m)\,(e+m)}},\right.\right.$$
$$\left.\left. c \to \frac{i\, bp \left(e \left(\sqrt{(e-m)\,(e+m)} + \sqrt{-m^2+(e-v0)^2}\right) - \sqrt{(e-m)\,(e+m)}\; v0\right)}{m\, v0},\ cp \to bp\right\}\right\}$$

In[ ]:=

```
Tr1 = (c /. sol2) / (a /. sol2) // FullSimplify
```

Out[ ]=

$$\left\{-\frac{-e^2 + m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)}}{e\, v0}\right\}$$

In[ ]:=

```
Tr2 = (cp /. sol2) / (a /. sol2) // FullSimplify
```

Out[ ]=

$$\left\{-\frac{i\, m \sqrt{(e-m)\,(e+m)}\; v0}{e \left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0\right)}\right\}$$

In[]:=

```
Rf1 = (b /. sol2) / (a /. sol2) // FullSimplify
```

Out[]=

$$\left\{\frac{m^2\, v0}{e\left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0\right)}\right\}$$

In[]:=

```
Rf2 = (bp /. sol2) / (a /. sol2) // FullSimplify
```

Out[]=

$$\left\{-\frac{i\, m\sqrt{(e-m)\,(e+m)}\; v0}{e\left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0\right)}\right\}$$

In[]:=

```
Ttotal = nt (modsq[ Tr1[[1]] ] + modsq[ Tr2[[1]] ] ) // FullSimplify
```

Out[]=

$$\frac{2\sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)}}{e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0}$$

In[]:=

```
Limit[Ttotal, v0 → ∞] // FullSimplify
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[]=

$$2 - \frac{2\, e\left(e + \sqrt{e^2 - m^2}\right)}{m^2}$$

In[]:=

```
Limit[Ttotal, v0 → 0] // FullSimplify
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[]=

1

In[]:=

```
Rtot = nr (modsq[ Rf1[[1]] ] + modsq[ Rf2[[1]] ] ) // FullSimplify
```

Out[]=

$$\frac{m^2\, v0^2}{\left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0\right)^2}$$

In[ ]:=

```
Limit[Rtot, v0 → ∞] // FullSimplify
```

··· Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[ ]=

$$\frac{m^2}{\left(e - \sqrt{e^2 - m^2}\right)^2}$$

In[ ]:=

```
Limit[Rtot, v0 → 0] // FullSimplify
```

··· Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[ ]=

```
0
```

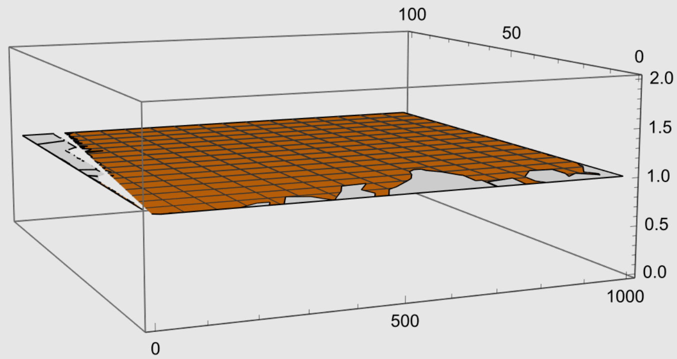
In[ ]:=

```
TotP [ee_, v00_, mm_] :=
 nt (modsq[ Tr1[[1]] ] + modsq[ Tr2[[1]] ] ) + nr ( modsq[ Rf1[[1]] ] + modsq[ Rf2[[1]] ]) /.
  {e → ee, v0 → v00, m → mm}
```

In[ ]:=

```
Plot3D[TotP [e, v0, 1], {e, 2, 1000}, {v0, 1, 100}]
```

Out[ ]=



In[ ]:=

```
me = 9.1 * 10^-31;
cv = 3 * 10^8;
ev = me * cv^2 / (1.6 * 10^-19);
mev = me * cv^2 / (1.6 * 10^-19);
jev = (1.6 * 10^-19);
vstep = 10^6;
```

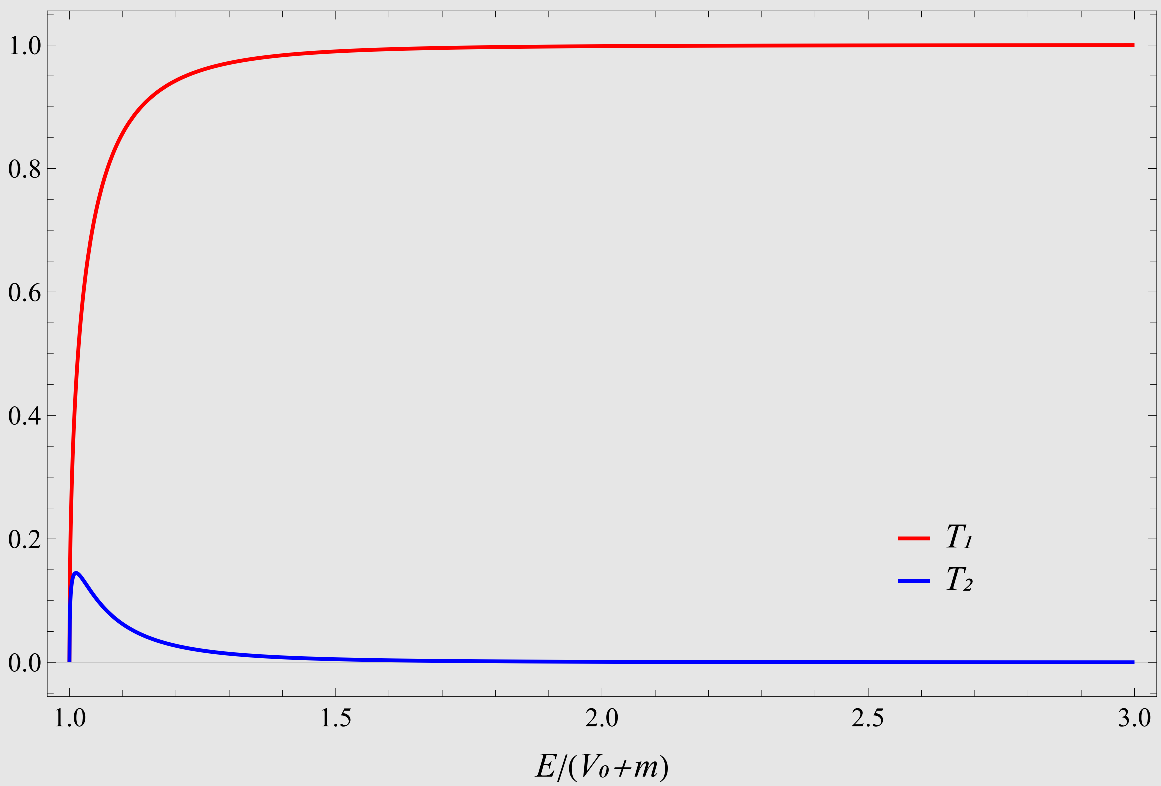
In[ ]:=

```
Tup[ee_, v00_, mm_] := Abs[nt modsq[Tr1[[1]]]] /. {e → ee, v0 → v00, m → mm}
Tdn[ee_, v00_, mm_] := Abs[nt modsq[Tr2[[1]]]] /. {e → ee, v0 → v00, m → mm}
Rup[ee_, v00_, mm_] := Abs[nr modsq[Rf1[[1]]]] /. {e → ee, v0 → v00, m → mm}
Rdn[ee_, v00_, mm_] := Abs[nr modsq[Rf2[[1]]]] /. {e → ee, v0 → v00, m → mm}
```

In[ ]:=

```
Plot[{Tup[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
  Tdn[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]},
 {x, 1, 3}, PlotStyle → {{Red, Thick}, {Blue, Thick}}, Frame → True,
 FrameLabel → {Style["E/(V₀+m)", 18, Italic], None},
 PlotLegends → Placed[{Style["T₁", 18, Italic], Style["T₂", 18, Italic]}, {0.8, 0.2}],
 ImageSize → {600, 600}, FrameTicksStyle → Directive[Black, 14],
 PlotRange → All, PlotTheme → "Scientific"]
```

Out[ ]=



In[ ]:=

```
Export["step_potential.pdf", %]
```

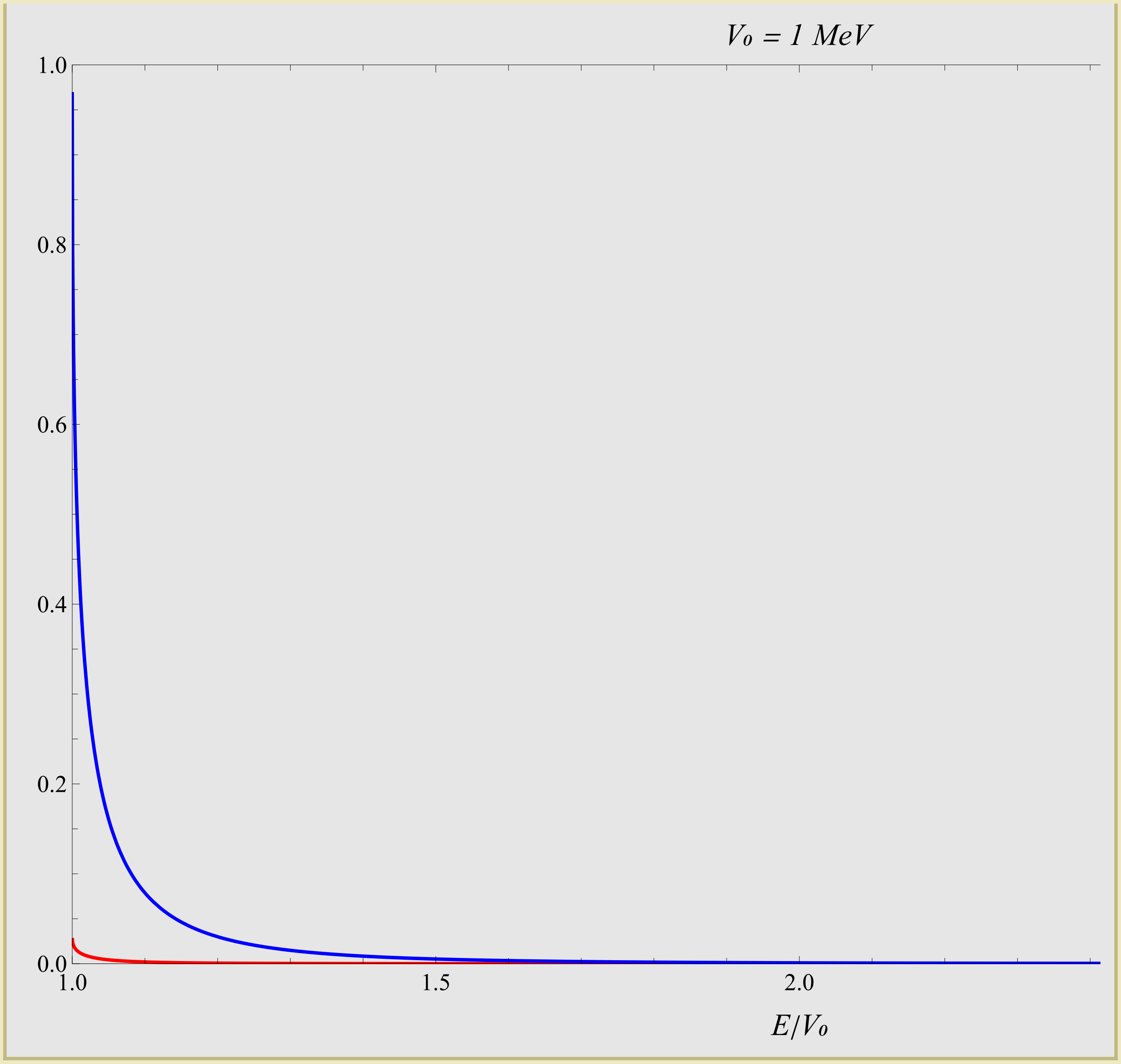
Out[ ]=

```
step_potential.pdf
```

In[*]:=

```
Plot[{Rup[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
   Rdn[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]}, {x, 1, 3},
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange → {{1, 3}, {0, 1}},
  Frame → True, FrameLabel → {Style["E/V₀", 18, Italic], None},
  PlotLegends → Placed[{Style["R₁", 18, Italic], Style["R₂", 18, Italic]}, {0.75, 0.3}],
  ImageSize → {900, 600}, FrameTicksStyle → Directive[Black, 14],
  FrameStyle → Directive[Black, 14], (*☑ replaces FrameLabelStyle*)
  PlotTheme → "Scientific", PlotLabel → Style["V₀ = 1 MeV", 18, Italic]]
```

Out[*]=



Unitary Equivalence

In[∘]:=

```
S1 = 1/Sqrt[2] {{-1, 0, 0, 1}, {0, 1, 1, 0}, {0, 1, -1, 0}, {-1, 0, 0, -1}};
```

In[∘]:=

```
S1 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=

```
S1.x1.ConjugateTranspose[S1] - G0 // FullSimplify
```

Out[∘]=

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

In[∘]:=

```
S1.x2.ConjugateTranspose[S1] - I G5
```

Out[∘]=

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

In[∘]:=

```
S1.ConjugateTranspose[S1] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Non-Unitary: Spin flip & Interference

In[300]:=

```
y1 = {{0, 0, -i, -1}, {0, 0, 1, -i}, {-i, 1, 0, 0}, {-1, -i, 0, 0}}
```

Out[300]=

```
{{0, 0, -i, -1}, {0, 0, 1, -i}, {-i, 1, 0, 0}, {-1, -i, 0, 0}}
```

In[301]:=

```
y2 = {{1, 0, i, 0}, {0, 1, 0, i}, {i, 0, -1, 0}, {0, i, 0, -1}}
```

Out[301]=

$$\{\{1, 0, i, 0\}, \{0, 1, 0, i\}, \{i, 0, -1, 0\}, \{0, i, 0, -1\}\}$$

In[302]:=

$$z1 = \frac{y1 + y2}{\mathrm{Sqrt}[2]};$$
$$z2 = \frac{y1 - y2}{\mathrm{Sqrt}[2]};$$

In[304]:=

```
Eigenvalues[ y1 e + m y2]
```

Out[304]=

$$\left\{-\sqrt{2}\ \sqrt{e}\ \sqrt{m},\ -\sqrt{2}\ \sqrt{e}\ \sqrt{m},\ \sqrt{2}\ \sqrt{e}\ \sqrt{m},\ \sqrt{2}\ \sqrt{e}\ \sqrt{m}\right\}$$

In[305]:=

```
Pz = Eigenvectors[ z1 e + m z2];
```

In[306]:=

$$\mathrm{PzB} = \mathrm{Pz}\ /.\ \left\{1/e^{1/2} \to \frac{\mathrm{Sqrt}[2]\ \mathrm{Sqrt}[m]}{pz}\right\}$$

Out[306]=

$$\left\{\left\{-\frac{(e+m)\left(-e+m+\sqrt{2}\ \sqrt{e^2-m^2}\right)}{-e^2-2\,e\,m+3\,m^2},\ -\frac{2\,i\left(e\,m-m^2-\sqrt{2}\ m\sqrt{e^2-m^2}\right)}{e^2+2\,e\,m-3\,m^2},\ 0,\ 1\right\},\right.$$
$$\left\{-\frac{2\,i\,m\left(-e+m+\sqrt{2}\ \sqrt{e^2-m^2}\right)}{-e^2-2\,e\,m+3\,m^2},\ -\frac{-e^2+m^2+\sqrt{2}\ e\sqrt{e^2-m^2}+\sqrt{2}\ m\sqrt{e^2-m^2}}{e^2+2\,e\,m-3\,m^2},\ 1,\ 0\right\},$$
$$\left\{-\frac{(e+m)\left(-e+m-\sqrt{2}\ \sqrt{e^2-m^2}\right)}{-e^2-2\,e\,m+3\,m^2},\ -\frac{2\,i\left(e\,m-m^2+\sqrt{2}\ m\sqrt{e^2-m^2}\right)}{e^2+2\,e\,m-3\,m^2},\ 0,\ 1\right\},$$
$$\left.\left\{-\frac{2\,i\,m\left(-e+m-\sqrt{2}\ \sqrt{e^2-m^2}\right)}{-e^2-2\,e\,m+3\,m^2},\ -\frac{-e^2+m^2-\sqrt{2}\ e\sqrt{e^2-m^2}-\sqrt{2}\ m\sqrt{e^2-m^2}}{e^2+2\,e\,m-3\,m^2},\ 1,\ 0\right\}\right\}$$

In[307]:=

```
PzC = PzB /. { Sqrt[e^2 - m^2] → pz}
```

Out[307]=

$$\left\{\left\{-\frac{(e+m)\left(-e+m+\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2},\ -\frac{2\,i\left(e\,m-m^2-\sqrt{2}\,m\,pz\right)}{e^2+2\,e\,m-3\,m^2},\ 0,\ 1\right\},\right.$$
$$\left\{-\frac{2\,i\,m\left(-e+m+\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2},\ -\frac{-e^2+m^2+\sqrt{2}\,e\,pz+\sqrt{2}\,m\,pz}{e^2+2\,e\,m-3\,m^2},\ 1,\ 0\right\},$$
$$\left\{-\frac{(e+m)\left(-e+m-\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2},\ -\frac{2\,i\left(e\,m-m^2+\sqrt{2}\,m\,pz\right)}{e^2+2\,e\,m-3\,m^2},\ 0,\ 1\right\},$$
$$\left.\left\{-\frac{2\,i\,m\left(-e+m-\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2},\ -\frac{-e^2+m^2-\sqrt{2}\,e\,pz-\sqrt{2}\,m\,pz}{e^2+2\,e\,m-3\,m^2},\ 1,\ 0\right\}\right\}$$

In[308]:=

```
P = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}};

PzD = PzC.P
```

Out[309]=

$$\left\{\left\{1,\ 0,\ -\frac{2\,i\left(e\,m-m^2-\sqrt{2}\,m\,pz\right)}{e^2+2\,e\,m-3\,m^2},\ -\frac{(e+m)\left(-e+m+\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2}\right\},\right.$$
$$\left\{0,\ 1,\ -\frac{-e^2+m^2+\sqrt{2}\,e\,pz+\sqrt{2}\,m\,pz}{e^2+2\,e\,m-3\,m^2},\ -\frac{2\,i\,m\left(-e+m+\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2}\right\},$$
$$\left\{1,\ 0,\ -\frac{2\,i\left(e\,m-m^2+\sqrt{2}\,m\,pz\right)}{e^2+2\,e\,m-3\,m^2},\ -\frac{(e+m)\left(-e+m-\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2}\right\},$$
$$\left.\left\{0,\ 1,\ -\frac{-e^2+m^2-\sqrt{2}\,e\,pz-\sqrt{2}\,m\,pz}{e^2+2\,e\,m-3\,m^2},\ -\frac{2\,i\,m\left(-e+m-\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2}\right\}\right\}$$

In[310]:=

```
col[v_] := List /@ v;

u1 = col[PzD[[3]]];
u2 = col[PzD[[4]]];

u3 = col[PzD[[1]]];
u4 = col[PzD[[2]]];
```

In[315]:=

```
u1 // MatrixForm
```

Out[315]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ -\frac{2\,i\,\left(e\,m-m^2+\sqrt{2}\,m\,pz\right)}{e^2+2\,e\,m-3\,m^2} \\ -\frac{(e+m)\,\left(-e+m-\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2} \end{pmatrix}$$

In[316]:=

```
u2 // MatrixForm
```

Out[316]//MatrixForm=

$$\begin{pmatrix} 0 \\ 1 \\ -\frac{-e^2+m^2-\sqrt{2}\,e\,pz-\sqrt{2}\,m\,pz}{e^2+2\,e\,m-3\,m^2} \\ -\frac{2\,i\,m\,\left(-e+m-\sqrt{2}\,pz\right)}{-e^2-2\,e\,m+3\,m^2} \end{pmatrix}$$

In[317]:=

```
ConjugateTranspose[u3].u4 // FullSimplify
```

Out[317]=

$$\left\{\left\{\frac{4\,i\,m\,(e+m)\,\left(e^2+m^2+2\,\sqrt{2}\,m\,pz+2\,pz^2-2\,e\,\left(m+\sqrt{2}\,pz\right)\right)}{(e-m)^2\,(e+3\,m)^2}\right\}\right\}$$

In[318]:=

```
ConjugateTranspose[u1].u2 // FullSimplify
```

Out[318]=

$$\left\{\left\{\frac{4\,i\,m\,(e+m)\,\left(e^2+m^2+2\,\sqrt{2}\,e\,pz+2\,pz^2-2\,m\,\left(e+\sqrt{2}\,pz\right)\right)}{(e-m)^2\,(e+3\,m)^2}\right\}\right\}$$

In[319]:=

```
Fu1[ee_, ppz_] := u1 /. {e → ee, pz → ppz}
Fu2[ee_, ppz_] := u2 /. {e → ee, pz → ppz}
Fu3[ee_, ppz_] := u3 /. {e → ee, pz → ppz}
Fu4[ee_, ppz_] := u4 /. {e → ee, pz → ppz}
```

In[323]:=

```
Fu1[e, p1]
```

Out[323]=

$$\left\{\{1\},\,\{0\},\,\left\{-\frac{2\,i\,\left(e\,m-m^2+\sqrt{2}\,m\,p1\right)}{e^2+2\,e\,m-3\,m^2}\right\},\,\left\{-\frac{(e+m)\,\left(-e+m-\sqrt{2}\,p1\right)}{-e^2-2\,e\,m+3\,m^2}\right\}\right\}$$

In[324]:=

```
Fu1[e, -p1]
```

Out[324]=

$$\left\{\{1\},\{0\},\left\{-\frac{2\,i\left(e\,m-m^2-\sqrt{2}\,m\,p1\right)}{e^2+2\,e\,m-3\,m^2}\right\},\left\{-\frac{(e+m)\left(-e+m+\sqrt{2}\,p1\right)}{-e^2-2\,e\,m+3\,m^2}\right\}\right\}$$

In[325]:=

```
Fu2[e, p1]
```

Out[325]=

$$\left\{\{0\},\{1\},\left\{-\frac{-e^2+m^2-\sqrt{2}\,e\,p1-\sqrt{2}\,m\,p1}{e^2+2\,e\,m-3\,m^2}\right\},\left\{-\frac{2\,i\,m\left(-e+m-\sqrt{2}\,p1\right)}{-e^2-2\,e\,m+3\,m^2}\right\}\right\}$$

In[326]:=

```
Fu3[e - v0, p2]
```

Out[326]=

$$\left\{\{1\},\{0\},\left\{-\frac{2\,i\left(-m^2-\sqrt{2}\,m\,p2+m\,(e-v0)\right)}{-3\,m^2+2\,m\,(e-v0)+(e-v0)^2}\right\},\left\{-\frac{(e+m-v0)\left(-e+m+\sqrt{2}\,p2+v0\right)}{3\,m^2-2\,m\,(e-v0)-(e-v0)^2}\right\}\right\}$$

In[327]:=

```
psiIN = a Fu1[e, p1];
psiR = b Fu1[e, -p1] + bp Fu2[e, -p1];
psiT = c Fu1[e - v0, p2] + cp Fu2[e - v0, p2];
```

In[330]:=

```
Xm = G0 + I G2;
```

In[331]:=

```
sol1 =
 Solve[a Fu1[e, p1] + b Fu1[e, -p1] + bp Fu2[e, -p1] == c Fu1[e - v0, p2] + cp Fu2[e - v0, p2],
   {a, b, c, bp, cp}] // FullSimplify
```

••• Solve: Equations may not give solutions for all "solve" variables.

Out[331]=

$$\Big\{\Big\{b \to \Big(a\,\Big(m^2\,\big(-3\,\sqrt{2}\,p1^2 + p2\,\big(3\,\sqrt{2}\,p2 - 4\,v0\big)\big) + \sqrt{2}\,(e\,(p1 - p2) - p1\,v0)\,(e\,(p1 + p2) - p1\,v0) + 2\,m\,\big(\sqrt{2}\,e\,p1^2 - \sqrt{2}\,p1^2\,v0 + e\,p2\,\big(-\sqrt{2}\,p2 + 2\,v0\big)\big)\Big)\Big) \Big/ \Big(((e - m)\,(p1 + p2) - p1\,v0)\,\Big(m\,\big(3\,\sqrt{2}\,p1 + 3\,\sqrt{2}\,p2 - 4\,v0\big) + \sqrt{2}\,(e\,(p1 + p2) - p1\,v0)\Big)\Big),$$

$$c \to \Big(2\,a\,p1\,\Big(\sqrt{2}\,e^2\,p2 + 2\,\sqrt{2}\,e\,m\,p2 - 3\,\sqrt{2}\,m^2\,p2 + \sqrt{2}\,p1\,(e - m - v0)\,(e + 3\,m - v0) - 2\,e\,m\,v0 + 2\,m^2\,v0 - \sqrt{2}\,e\,p2\,v0 - \sqrt{2}\,m\,p2\,v0 + 2\,m\,v0^2\Big)\Big) \Big/ \Big(((e - m)\,(p1 + p2) - p1\,v0)\,\Big(m\,\big(3\,\sqrt{2}\,p1 + 3\,\sqrt{2}\,p2 - 4\,v0\big) + \sqrt{2}\,(e\,(p1 + p2) - p1\,v0)\Big)\Big),$$

$$bp \to \Big(4\,i\,a\,m\,p1\,v0\,\big(-e + m - \sqrt{2}\,p2 + v0\big)\Big) \Big/ \Big((-e\,(p1 + p2) + m\,(p1 + p2) + p1\,v0)\,\Big(m\,\big(3\,\sqrt{2}\,p1 + 3\,\sqrt{2}\,p2 - 4\,v0\big) + \sqrt{2}\,(e\,(p1 + p2) - p1\,v0)\Big)\Big),$$

$$cp \to \Big(4\,i\,a\,m\,p1\,v0\,\big(-e + m - \sqrt{2}\,p2 + v0\big)\Big) \Big/ \Big((-e\,(p1 + p2) + m\,(p1 + p2) + p1\,v0)\,\Big(m\,\big(3\,\sqrt{2}\,p1 + 3\,\sqrt{2}\,p2 - 4\,v0\big) + \sqrt{2}\,(e\,(p1 + p2) - p1\,v0)\Big)\Big)\Big\}\Big\}$$

In[332]:=

```
sol2 = sol1 /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[332]=

$$\Bigg\{\Bigg\{b \to \left(a\,(e-m)\left(-\frac{e+m}{\sqrt{(e-m)\,(e+m)}}+\frac{1}{\sqrt{1-\frac{2\,m}{e+m-v0}}}\right)\left(\sqrt{2}\,e+2\,\sqrt{(e-m-v0)\,(e+m-v0)}-\sqrt{2}\,(m+v0)\right)\right)\Big/$$
$$\left(\sqrt{2}\,e\left(\sqrt{e^2-m^2}+\sqrt{(e-m-v0)\,(e+m-v0)}\right)+3\,\sqrt{2}\,m\left(\sqrt{e^2-m^2}+\sqrt{(e-m-v0)\,(e+m-v0)}\right)-4\,m\,v0-\sqrt{2}\,\sqrt{e^2-m^2}\,v0\right),$$
$$c \to \Big(2\,a\,\sqrt{(e-m)\,(e+m)}\,\Big(\sqrt{2}\,e^2\,\sqrt{-m^2+(e-v0)^2}+2\,\sqrt{2}\,e\,m\,\sqrt{-m^2+(e-v0)^2}-3\,\sqrt{2}\,m^2\,\sqrt{-m^2+(e-v0)^2}+\sqrt{2}\,\sqrt{(e-m)\,(e+m)}\,(e-m-v0)\,(e+3\,m-v0)-$$
$$2\,e\,m\,v0+2\,m^2\,v0-\sqrt{2}\,e\,\sqrt{-m^2+(e-v0)^2}\,v0-\sqrt{2}\,m\,\sqrt{-m^2+(e-v0)^2}\,v0+2\,m\,v0^2\Big)\Big)\Big/$$
$$\Big(\Big((e-m)\left(\sqrt{(e-m)\,(e+m)}+\sqrt{-m^2+(e-v0)^2}\right)-\sqrt{(e-m)\,(e+m)}\,v0\Big)$$
$$\Big(m\left(3\,\sqrt{2}\left(\sqrt{e^2-m^2}+\sqrt{(e-m-v0)\,(e+m-v0)}\right)-4\,v0\right)+\sqrt{2}\left(e\left(\sqrt{(e-m)\,(e+m)}+\sqrt{-m^2+(e-v0)^2}\right)-\sqrt{(e-m)\,(e+m)}\,v0\right)\Big)\Big),$$
$$bp \to \left(2\,i\,a\left(e-m+\sqrt{2}\,\sqrt{(e-m-v0)\,(e+m-v0)}-v0\right)\left(-e-m+\sqrt{\frac{(-e^2+m^2)\,(e+m-v0)}{-e+m+v0}}\right)\right)\Big/$$
$$\left(\sqrt{2}\,e\left(\sqrt{e^2-m^2}+\sqrt{(e-m-v0)\,(e+m-v0)}\right)+3\,\sqrt{2}\,m\left(\sqrt{e^2-m^2}+\sqrt{(e-m-v0)\,(e+m-v0)}\right)-4\,m\,v0-\sqrt{2}\,\sqrt{e^2-m^2}\,v0\right),$$
$$cp \to \left(2\,i\,a\left(e-m+\sqrt{2}\,\sqrt{(e-m-v0)\,(e+m-v0)}-v0\right)\left(-e-m+\sqrt{\frac{(-e^2+m^2)\,(e+m-v0)}{-e+m+v0}}\right)\right)\Big/$$
$$\left(\sqrt{2}\,e\left(\sqrt{e^2-m^2}+\sqrt{(e-m-v0)\,(e+m-v0)}\right)+3\,\sqrt{2}\,m\left(\sqrt{e^2-m^2}+\sqrt{(e-m-v0)\,(e+m-v0)}\right)-4\,m\,v0-\sqrt{2}\,\sqrt{e^2-m^2}\,v0\right)\Bigg\}\Bigg\}$$

Transmission and Reflection Coefficients

In[333]:=

```
jin = ConjugateTranspose[psiIN].Xm.psiIN // FullSimplify
```

Out[333]=

$$\left\{\left\{-\left(\left(2\,a\left(e^4+2\,\sqrt{2}\,e^3\,p1+2\,e\,m\,p1\left(\sqrt{2}\,m+p1\right)+e^2\left(-2\,m^2+4\,\sqrt{2}\,m\,p1+p1^2\right)+m^2\left(m^2-8\,\sqrt{2}\,m\,p1+5\,p1^2\right)\right)\mathrm{Conjugate}[a]\right)\Big/\left((e-m)^2\,(e+3\,m)^2\right)\right)\right\}\right\}$$

In[334]:=

```
jR = ConjugateTranspose[psiR].Xm.psiR // FullSimplify
```

Out[334]=

$$\left\{\left\{\frac{1}{(e-m)^2 (e+3m)^2}\left(bp\,(e-m)\,(e+3m)\,\left((e-m)\,(e+3m)\,\text{Conjugate}[bp]-\left(e-m-\sqrt{2}\,p1\right)\,(2\,i\,m\,\text{Conjugate}[b]+(e+m)\,\text{Conjugate}[bp])\right)+(2\,i\,b\,m-bp\,(e+m))\,\left(e-m-\sqrt{2}\,p1\right)\,\left((e-m)\,(e+3m)\,\text{Conjugate}[bp]+\left(e-m-\sqrt{2}\,p1\right)\,(2\,i\,m\,\text{Conjugate}[b]+(e+m)\,\text{Conjugate}[bp])\right)-(2\,i\,bp\,m+b\,(e+m))\,\left(e-m-\sqrt{2}\,p1\right)\,\left((e-m)\,(e+3m)\,\text{Conjugate}[b]+\left(e-m-\sqrt{2}\,p1\right)\,\text{Conjugate}[2\,i\,bp\,m+b\,(e+m)]\right)+b\,(e-m)\,(e+3m)\,\left((e-m)\,(e+3m)\,\text{Conjugate}[b]+\left(-e+m+\sqrt{2}\,p1\right)\,\text{Conjugate}[2\,i\,bp\,m+b\,(e+m)]\right)\right)\right\}\right\}$$

In[335]:=

```
jT = ConjugateTranspose[psiT].Xm.psiT // FullSimplify
```

Out[335]=

$$\left\{\left\{-2\,c\,\text{Conjugate}[c]-\frac{2\sqrt{2}\,(c+i\,cp)\,p2\,(\text{Conjugate}[c]-i\,\text{Conjugate}[cp])}{e-m-v0}-\frac{(c+i\,cp)\,p2^2\,(\text{Conjugate}[c]-i\,\text{Conjugate}[cp])}{(-e+m+v0)^2}-\frac{1}{(e+3m-v0)^2}(c-i\,cp)\,\left(8\,m^2-4\sqrt{2}\,m\,p2+p2^2\right)\,(\text{Conjugate}[c]+i\,\text{Conjugate}[cp])+\frac{2\,(c-i\,cp)\,\left(4\,m-\sqrt{2}\,p2\right)\,(\text{Conjugate}[c]+i\,\text{Conjugate}[cp])}{e+3m-v0}-2\,cp\,\text{Conjugate}[cp]\right\}\right\}$$

In[336]:=

```
me = 9.1 * 10^-31;
cv = 3 * 10^8;
ev = me * cv^2 / (1.6 * 10^-19);
mev = me * cv^2 / (1.6 * 10^-19);
jev = (1.6 * 10^-19);
vstep = 10^6;

gv = 0.379;
```

In[343]:=

```
mev
```

Out[343]=

```
511 875.
```

In[344]:=

```
jTjin = jT / jin /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[344]=

$$\Bigg\{\Bigg\{-\Bigg(\Bigg((e-m)(e+3m)^2\Bigg(-2\,c\,\mathrm{Conjugate}[c]+\frac{(c+\mathbb{i}\,cp)(e+m-v0)(\mathrm{Conjugate}[c]-\mathbb{i}\,\mathrm{Conjugate}[cp])}{-e+m+v0}-2(c+\mathbb{i}\,cp)\sqrt{2-\frac{4m}{-e+m+v0}}(\mathrm{Conjugate}[c]-\mathbb{i}\,\mathrm{Conjugate}[cp])-\frac{1}{(e+3m-v0)^2}(c-\mathbb{i}\,cp)\left(7m^2-4\sqrt{2}\,m\sqrt{-m^2+(e-v0)^2}+(e-v0)^2\right)(\mathrm{Conjugate}[c]+\mathbb{i}\,\mathrm{Conjugate}[cp])+\frac{1}{e+3m-v0}2(c-\mathbb{i}\,cp)\left(4m-\sqrt{2}\sqrt{-m^2+(e-v0)^2}\right)(\mathrm{Conjugate}[c]+\mathbb{i}\,\mathrm{Conjugate}[cp])-2\,cp\,\mathrm{Conjugate}[cp]\Bigg)\Bigg)\Big/\left(4a\left(e^3+3em\left(m+\sqrt{2}\sqrt{(e-m)(e+m)}\right)+e^2\left(2m+\sqrt{2}\sqrt{(e-m)(e+m)}\right)+2m^2\left(m+2\sqrt{2}\sqrt{(e-m)(e+m)}\right)\right)\mathrm{Conjugate}[a]\right)\Bigg)\Bigg\}\Bigg\}$$

In[345]:=

```
jRjin = jR / jin /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[345]=

$$\Bigg\{\Bigg\{\Bigg(\Bigg(-\mathbb{i}\,bp\,m\left(-4e^2-6em+2m^2+3\sqrt{2}\,e\sqrt{e^2-m^2}+5\sqrt{2}\,m\sqrt{e^2-m^2}\right)+b\left(e^3+2m^2\left(m-2\sqrt{2}\sqrt{(e-m)(e+m)}\right)+3em\left(m-\sqrt{2}\sqrt{(e-m)(e+m)}\right)+e^2\left(2m-\sqrt{2}\sqrt{(e-m)(e+m)}\right)\right)\Bigg)\mathrm{Conjugate}[b]+\Bigg(\mathbb{i}\,b\,m\left(-4e^2-6em+2m^2+3\sqrt{2}\,e\sqrt{e^2-m^2}+5\sqrt{2}\,m\sqrt{e^2-m^2}\right)+bp\left(e^3+2m^2\left(m-2\sqrt{2}\sqrt{(e-m)(e+m)}\right)+3em\left(m-\sqrt{2}\sqrt{(e-m)(e+m)}\right)+e^2\left(2m-\sqrt{2}\sqrt{(e-m)(e+m)}\right)\right)\Bigg)\mathrm{Conjugate}[bp]\Bigg)\Big/\left(\left(e^3+3em\left(m+\sqrt{2}\sqrt{(e-m)(e+m)}\right)+e^2\left(2m+\sqrt{2}\sqrt{(e-m)(e+m)}\right)+2m^2\left(m+2\sqrt{2}\sqrt{(e-m)(e+m)}\right)\right)\mathrm{Abs}[a]^2\right)\Bigg\}\Bigg\}$$

In[346]:=

```
exprT = Expand[jTjin];
```

In[347]:=

```
(*Tcc1 =Total@Cases[expr,x_/;!FreeQ[x,c Conjugate[c]],All]//Simplify*)

Tcc1 = Coefficient[exprT, c * Conjugate[c]] + Coefficient[exprT, Abs[c]^2];
Tcc2 = Tcc1 * c * Conjugate[c] /. sol2 /. bp → 1 ;
Tcc3 [ee_, v00_, mm_, aa_] := Tcc2 /. {e → ee, v0 → v00, m → mm, a → aa};

Tcpcp1 = Coefficient[exprT, cp * Conjugate[cp] + Coefficient[exprT, Abs[cp]^2]];
Tcpcp2 = Tcpcp1 * cp * Conjugate[cp] /. sol2 /. bp → 1;
Tcpcp3 [ee_, v00_, mm_, aa_] := Tcpcp2 /. {e → ee, v0 → v00, m → mm, a → aa};

Tcpc1 = Coefficient[exprT, cp * Conjugate[c]];
Tcpc2 = Tcpc1 * cp * Conjugate[c] /. sol2 /. bp → 1;
Tcpc3 [ee_, v00_, mm_, aa_] := Tcpc2 /. {e → ee, v0 → v00, m → mm, a → aa};

Tccp1 = Coefficient[exprT, c * Conjugate[cp]];
Tccp2 = Tccp1 * c * Conjugate[cp] /. sol2 /. bp → 1 ;
Tccp3 [ee_, v00_, mm_, aa_] := Tccp2 /. {e → ee, v0 → v00, m → mm, a → aa};

Ttot [e_, v0_, m_] :=
  Abs[Tcc3 [e, v0, m, 1] + Tcpcp3 [e, v0, m, 1] + Tcpc3 [e, v0, m, 1] + Tccp3 [e, v0, m, 1]];
```

In[360]:=

```
Ttot [5, 2, 1] // N
```

Out[360]=

```
{{{0.983961}}}
```

In[361]:=

```
exprR = Expand[jRjin];
```

In[362]:=

```
Rbb1 = Coefficient[exprR, b * Conjugate[b]] + Coefficient[exprR, Abs[b]^2];
Rbb2 = Rbb1 * b * Conjugate[b] /. sol2 /. bp → 1 ;
Rbb3 [ee_, v00_, mm_, aa_] := Rbb2 /. {e → ee, v0 → v00, m → mm, a → aa};

Rbpbp1 = Coefficient[exprR, bp * Conjugate[bp]] + Coefficient[exprR, Abs[bp]^2];
Rbpbp2 = Rbpbp1 * bp * Conjugate[bp] /. sol2 /. bp → 1 ;
Rbpbp3[ee_, v00_, mm_, aa_] := Rbpbp2 /. {e → ee, v0 → v00, m → mm, a → aa};

Rbbp1 = Coefficient[exprR, b * Conjugate[bp]];
Rbbp2 = Rbbp1 * b * Conjugate[bp] /. sol2 /. bp → 1 ;
Rbbp3 [ee_, v00_, mm_, aa_] := Rbbp2 /. {e → ee, v0 → v00, m → mm, a → aa};

Rbpb1 = Coefficient[exprR, bp * Conjugate[b]];
Rbpb2 = Rbpb1 * bp * Conjugate[b] /. sol2 /. bp → 1;
Rbpb3 [ee_, v00_, mm_, aa_] := Rbpb2 /. {e → ee, v0 → v00, m → mm, a → aa};

Rtot[e_, v0_, m_] :=
  Abs[Rbb3 [e, v0, m, 1] + Rbpbp3 [e, v0, m, 1] + Rbbp3 [e, v0, m, 1] + Rbpb3 [e, v0, m, 1]];
```

In[375]:=

```
Ptot[e_, v0_, m_] := Rtot[e, v0, m] + Ttot[e, v0, m]
```

In[376]:=

```
Ptot[3.1, 2, 1] // N
```

Out[376]=

```
{{{0.993662}}}
```

In[377]:=

```
Rtot[4, 2, 1] // N
```

Out[377]=

```
{{{0.0281103}}}
```

In[378]:=

```
Ttot[4, 2, 1] // N
```

Out[378]=

```
{{{0.960197}}}
```

In[379]:=

```
Ptot[2.1 (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]
```

Out[379]=

```
{{{0.997187}}}
```

In[380]:=

```
Ttot[1.1 (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]
```

Out[380]=

```
{{{0.818189}}}
```

In[381]:=

```
Ttot[1 000 000, 50 000, 5000] // N
```

Out[381]=

```
{{{0.999999}}}
```

In[382]:=

```
Eigenvalues[I G5]
```

Out[382]=

```
{i, i, -i, -i}
```

In[383]:=

```
Eigenvalues[z2]
```

Out[383]=

```
{i, i, -i, -i}
```

In[384]:=

```
z2.z2
```

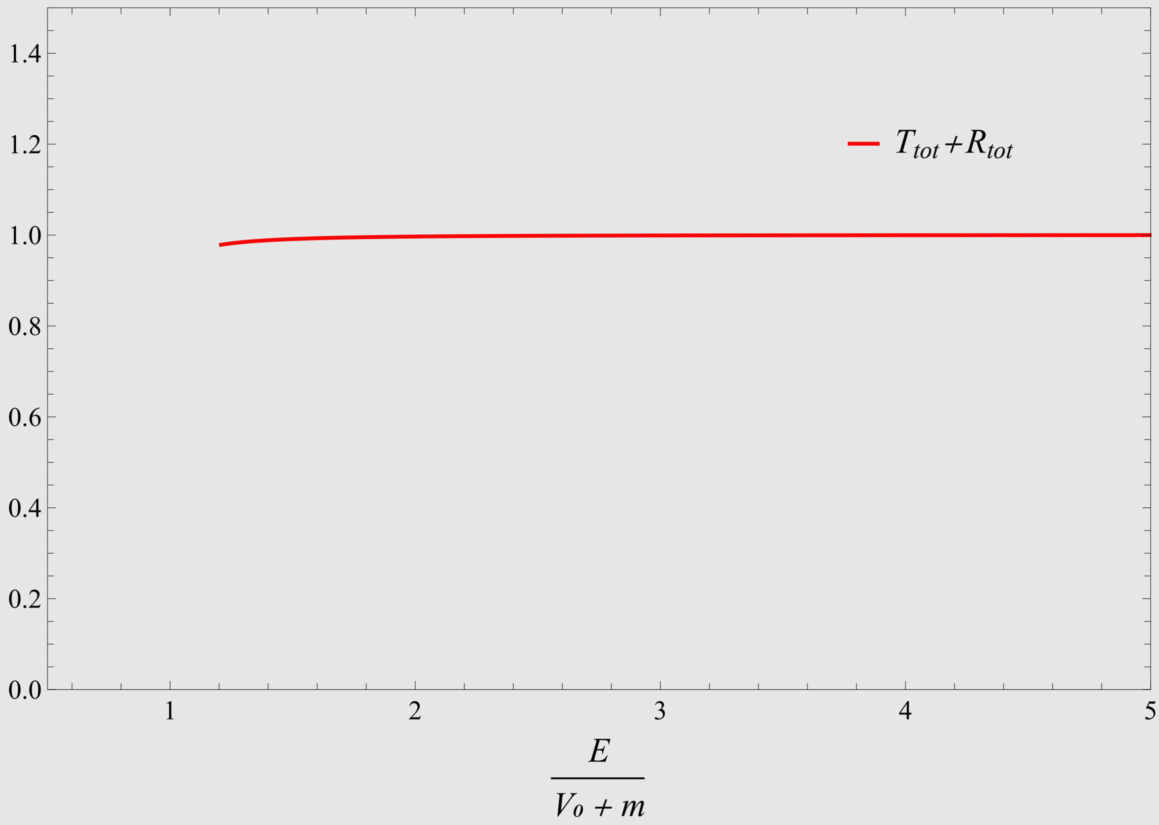
Out[384]=

```
{{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}
```

In[401]:=

```
Plot[{Ptot[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]},
 {x, 1.2, 10}, PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
 Frame → True, FrameLabel → {Style["E/(V0 + m)", 18, Italic], None},
 PlotLegends → Placed[{Style["Ttot+Rtot", 18, Italic]}, {0.8, 0.8}],
 ImageSize → {600, 600}, FrameTicksStyle → Directive[Black, 14],
 PlotRange → {{0.5, 5}, {0, 1.5}}, PlotTheme → "Scientific"]
```

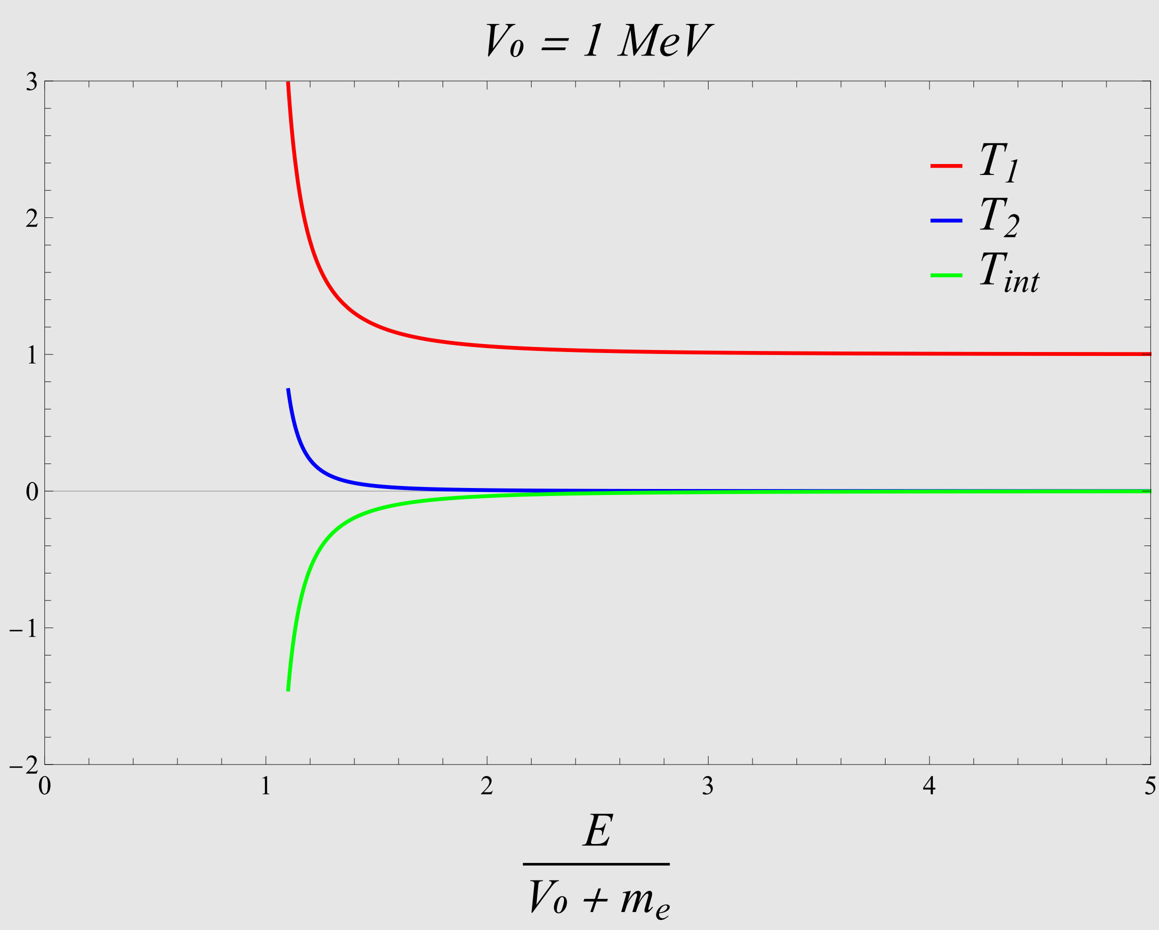
Out[401]=



In[399]:=

```
Plot[{Tcc3[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev, 1],
  Tcpcp3[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev, 1],
  Tccp3[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev, 1]},
 {x, 1.1, 10}, PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
 Frame → True, FrameLabel → {Style["E/(V0 + me)", 25, Italic], None},
 PlotLegends → Placed[{Style["T1", 25, Italic], Style["T2", 25, Italic],
    Style["Tint", 25, Italic]}, {0.85, 0.8}], ImageSize → {600, 600},
 FrameTicksStyle → Directive[Black, 14], PlotRange → {{0, 5}, {-2, 3}},
 PlotTheme → "Scientific", PlotLabel → Style["V0 = 1 MeV", 25, Italic]]
```

Out[399]=



In[387]:=

```
Export["Relativistic-Transmission-nonunitary.jpg", %, "JPEG"]
```

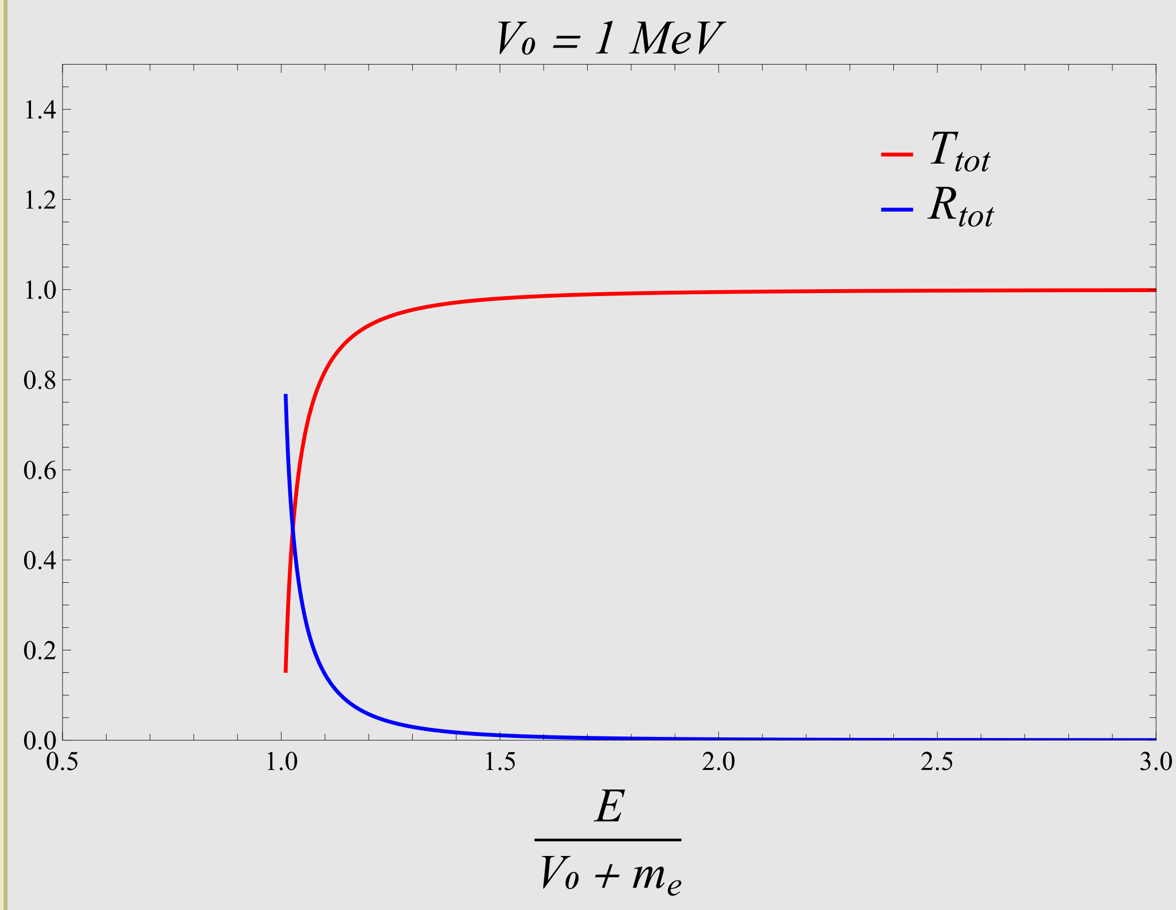
Out[387]=

```
Relativistic-Transmission-nonunitary.jpg
```

In[388]:=

```
Plot[{Ttot[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
   Rtot[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]}, {x, 1.01, 10},
 PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}}, Frame → True,
 FrameLabel → {Style["E/(V0 + me)", 25, Italic], None},
 PlotLegends → Placed[{Style["Ttot", 25, Italic], Style["Rtot", 25, Italic]}, {0.8, 0.83}],
 ImageSize → {600, 600}, FrameTicksStyle → Directive[Black, 14],
 PlotRange → {{0.5, 3}, {0, 1.5}}, PlotTheme → "Scientific",
 PlotLabel → Style["V0 = 1 MeV", 25, Italic]]
```

Out[388]=



In[389]:=

```
Export["Relativistic-total-nonunitary.jpg", %, "JPEG"]
```

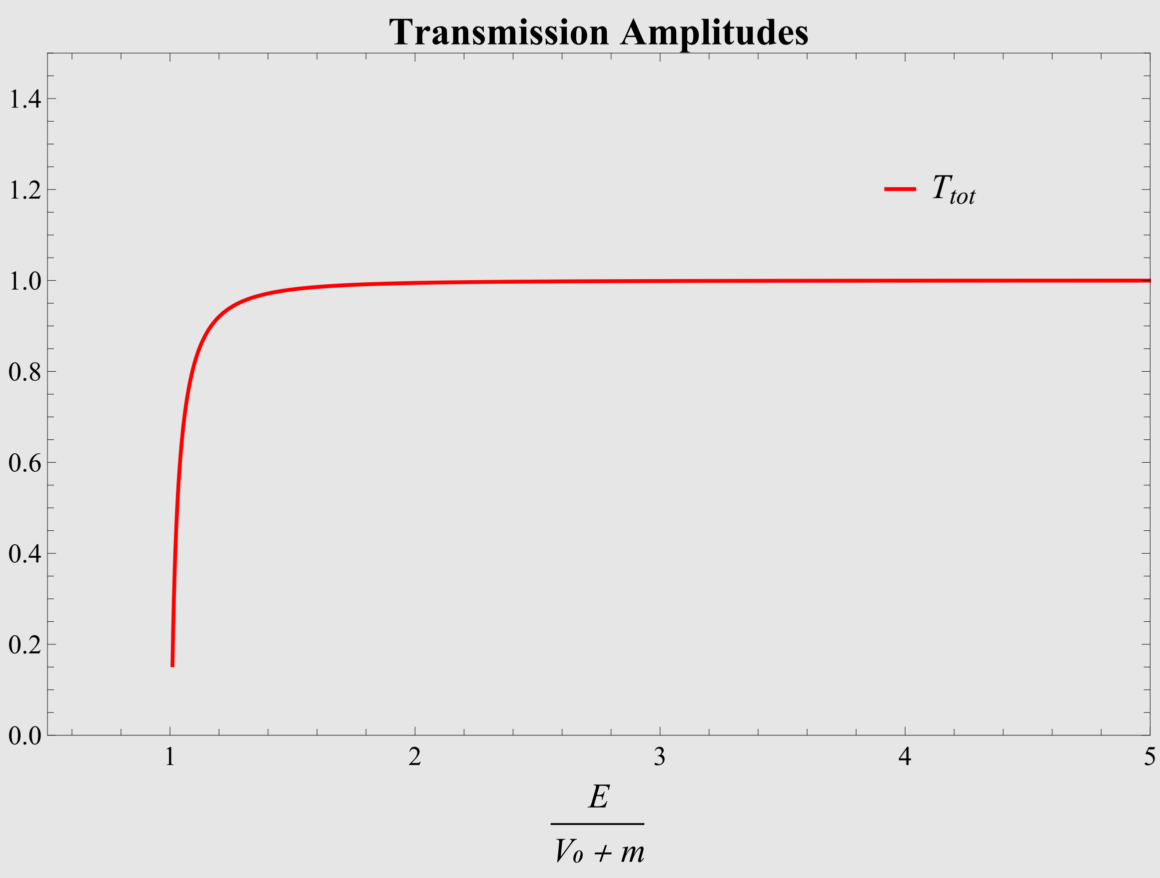
Out[389]=

```
Relativistic-total-nonunitary.jpg
```

In[405]:=

```
Plot[{Ttot[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]},
 {x, 1.01, 10}, PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
 Frame → True, FrameLabel → {Style["E/(V0 + m)", 18, Italic], None},
 PlotLegends → Placed[{Style["Ttot", 18, Italic]}, {0.8, 0.8}], ImageSize → {600, 600},
 FrameTicksStyle → Directive[Black, 14], PlotRange → {{0.5, 5}, {0, 1.5}},
 PlotTheme → "Scientific", PlotLabel → Style["Transmission Amplitudes", Bold, 20]]
```

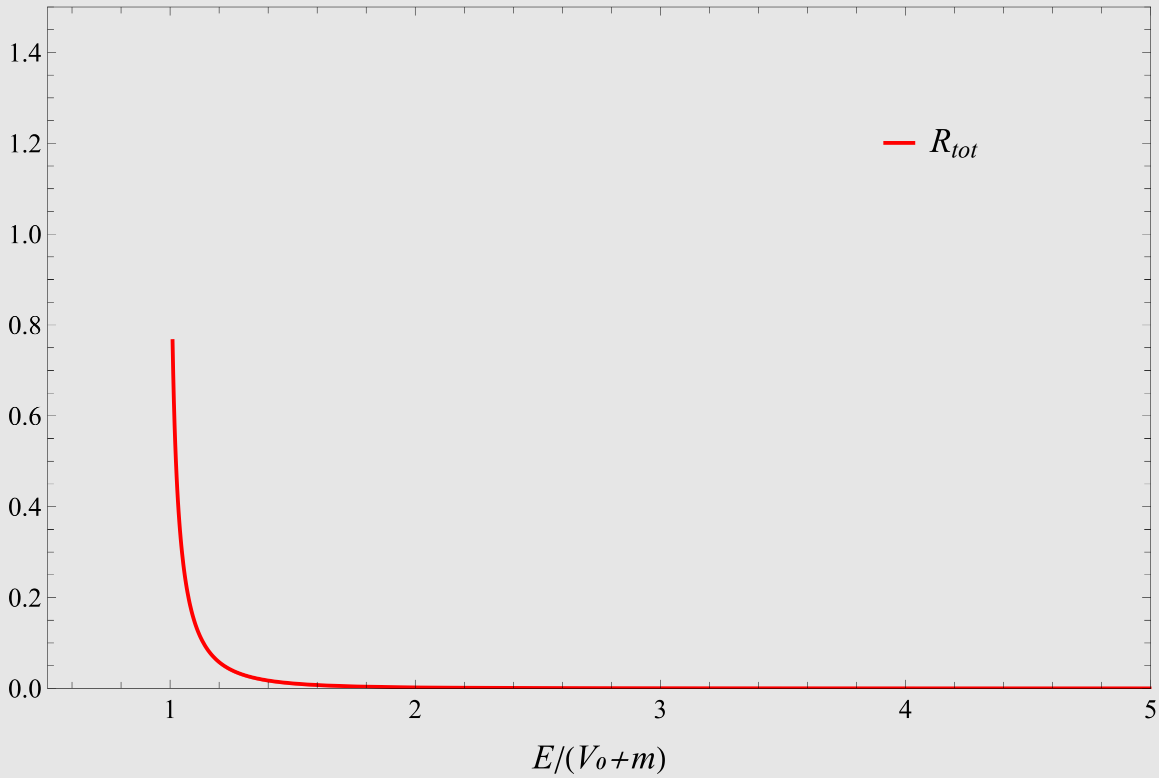
Out[405]=



In[406]:=

```
Plot[{Rtot[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]},
 {x, 1.01, 10}, PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
 Frame → True, FrameLabel → {Style["E/(V0+m)", 18, Italic], None},
 PlotLegends → Placed[{Style["Rtot", 18, Italic]}, {0.8, 0.8}],
 ImageSize → {600, 600}, FrameTicksStyle → Directive[Black, 14],
 PlotRange → {{0.5, 5}, {0, 1.5}}, PlotTheme → "Scientific"]
```

Out[406]=



In[392]:=

```
Rbbp3[1.1 (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev, 1]
```

Out[392]=

```
{{{0.00530692 + 0. i}}}
```

In[393]:=

```
Plot[{Rbb3[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev, 1],
  Rbpbp3[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev, 1],
  Rbbp3[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev, 1]},
 {x, 1.1, 10}, PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
 Frame → True, FrameLabel → {Style["E/(V₀+m)", 18, Italic], None},
 PlotLegends → Placed[{Style["R₁", 18, Italic], Style["R₂", 18, Italic],
    Style["R_int", 18, Italic]}, {0.8, 0.8}], ImageSize → {600, 600},
 FrameTicksStyle → Directive[Black, 14], PlotRange → Full, PlotTheme → "Scientific"]
```

Out[393]=