Definitions

In[4863]:=

$$G0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix};$$

$$G1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix};$$

$$G2 = \begin{pmatrix} 0 & 0 & 0 & -I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ -I & 0 & 0 & 0 \end{pmatrix};$$

$$G3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};$$

$$G5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};$$

In[4868]:=

```
$Assumptions = {e ∈ Reals , p0 ∈ Reals , p1 ∈ Reals , p2 ∈ Reals ,
  p3 ∈ Reals, v0 ∈ Reals, z ∈ Reals, L ∈ Reals, e > 0, m > 0, v0 > 0, L > 0, pz > 0,
  pz ∈ Reals, py ∈ Reals, e > m, e - v0 > m, e > v0 + m, hx > 0, hx ∈ Reals}
```

Out[4868]=

```
{e ∈ ℝ, p0 ∈ ℝ, p1 ∈ ℝ, p2 ∈ ℝ, p3 ∈ ℝ, v0 ∈ ℝ, z ∈ ℝ, L ∈ ℝ, e > 0, m > 0,
 v0 > 0, L > 0, pz > 0, pz ∈ ℝ, py ∈ ℝ, e > m, e - v0 > m, e > m + v0, hx > 0, hx ∈ ℝ}
```

Dirac’s Eq

In[4598]:=

$$etaD = \frac{G0 + I\,G5}{Sqrt[2]};$$

```
eta = etaD;
```

In[4600]:=

```
Pz = Eigenvectors[ G0 e +  m I G5];
```

In[4601]:=

```
PzA = %
```

Out[4601]=

$$\left\{\left\{0, \frac{i\left(-e+\sqrt{e^2-m^2}\right)}{m}, 0, 1\right\}, \left\{\frac{i\left(-e+\sqrt{e^2-m^2}\right)}{m}, 0, 1, 0\right\}, \left\{0, -\frac{i\left(e+\sqrt{e^2-m^2}\right)}{m}, 0, 1\right\}, \left\{-\frac{i\left(e+\sqrt{e^2-m^2}\right)}{m}, 0, 1, 0\right\}\right\}$$

In[4602]:=

```
PzC = PzA /. {Sqrt[e^2 - m^2] → pz}
```

Out[4602]=

$$\left\{\left\{0, \frac{i(-e+pz)}{m}, 0, 1\right\}, \left\{\frac{i(-e+pz)}{m}, 0, 1, 0\right\}, \left\{0, -\frac{i(e+pz)}{m}, 0, 1\right\}, \left\{-\frac{i(e+pz)}{m}, 0, 1, 0\right\}\right\}$$

In[4603]:=

```
P = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}};

PzD = PzC.P
```

Out[4604]=

$$\left\{\left\{1, 0, \frac{i(-e+pz)}{m}, 0\right\}, \left\{0, 1, 0, \frac{i(-e+pz)}{m}\right\}, \left\{1, 0, -\frac{i(e+pz)}{m}, 0\right\}, \left\{0, 1, 0, -\frac{i(e+pz)}{m}\right\}\right\}$$

In[4605]:=

```
col[v_] := List /@ v;

u1 = col[PzD[[3]]];
u2 = col[PzD[[4]]];

u3 = col[PzD[[1]]];
u4 = col[PzD[[2]]];
```

In[4610]:=

```
u1 // MatrixForm
```

Out[4610]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ -\frac{i\,(e+pz)}{m} \\ 0 \end{pmatrix}$$

In[4611]:=

```
ConjugateTranspose[u1].u2 // FullSimplify
```

Out[4611]=

```
{{0}}
```

In[4612]:=

```
Fu1[ee_, ppz_] := u1 /. {e → ee, pz → ppz}
Fu2[ee_, ppz_] := u2 /. {e → ee, pz → ppz}
Fu3[ee_, ppz_] := u3 /. {e → ee, pz → ppz}
Fu4[ee_, ppz_] := u4 /. {e → ee, pz → ppz}
```

In[4616]:=

```
Fu1[e, p1]
```

Out[4616]=

$$\left\{\{1\}, \{0\}, \left\{-\frac{i\,(e+p1)}{m}\right\}, \{0\}\right\}$$

In[4617]:=

```
psiIN = a Fu1[e, p1];
psiR = b Fu1[e, -p1] + bp Fu2[e, -p1];
psiT = c Fu1[e - v0, p2] + cp Fu2[e - v0, p2];
```

In[4620]:=

```
Xm = G0
```

Out[4620]=

```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}
```

In[4621]:=

```
jin = ConjugateTranspose[psiIN].Xm.psiIN // FullSimplify
```

Out[4621]=

$$\left\{\left\{-\frac{a\,(e-m+p1)\,(e+m+p1)\,\text{Conjugate}[a]}{m^2}\right\}\right\}$$

In[4622]:=

```
jR = ConjugateTranspose[psiR].Xm.psiR // FullSimplify
```

Out[4622]=

$$\left\{\left\{\frac{(e+m-p1)\,(-e+m+p1)\,\left(\text{Abs}[b]^2+\text{Abs}[bp]^2\right)}{m^2}\right\}\right\}$$

In[4623]:=

```
jT = ConjugateTranspose[psiT].Xm.psiT // FullSimplify
```

Out[4623]=

$$\left\{\left\{\frac{(e+m+p2-v0)\,(-e+m-p2+v0)\,\left(\text{Abs}[c]^2+\text{Abs}[cp]^2\right)}{m^2}\right\}\right\}$$

In[4624]:=

$$\text{jR}/\text{jin}\;/.\;\left\{p1\to\text{Sqrt}\left[e^2-m^2\right],\,p2\to\text{Sqrt}\left[(e-v0)^2-m^2\right]\right\}\;//\;\text{FullSimplify}$$

Out[4624]=

$$\left\{\left\{\frac{m^2\,\left(\text{Abs}[b]^2+\text{Abs}[bp]^2\right)}{\left(m^2-2\,e\,\left(e+\sqrt{(e-m)\,(e+m)}\right)\right)\,\text{Abs}[a]^2}\right\}\right\}$$

In[4625]:=

$$\text{jT}/\text{jin}\;/.\;\left\{p1\to\text{Sqrt}\left[e^2-m^2\right],\,p2\to\text{Sqrt}\left[(e-v0)^2-m^2\right]\right\}\;//\;\text{FullSimplify}$$

Out[4625]=

$$\left\{\left\{\frac{\left(e-m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)\,\left(e+m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)\,\left(\text{Abs}[c]^2+\text{Abs}[cp]^2\right)}{2\,a\,\left(-m^2+e\,\left(e+\sqrt{(e-m)\,(e+m)}\right)\right)\,\text{Conjugate}[a]}\right\}\right\}$$

In[4626]:=

$$nt=\frac{\left(e-m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)\,\left(e+m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)}{2\,\left(-m^2+e\,\left(e+\sqrt{e^2-m^2}\right)\right)};\;\text{(*G3*)}$$

$$nr=\frac{m^2}{\left(m^2-2\,e\,\left(e+\sqrt{e^2-m^2}\right)\right)};$$

In[4628]:=

```
sol1 =
 Solve[a Fu1[e, p1] + b Fu1[e, -p1] + bp Fu2[e, -p1] == c Fu1[e - v0, p2] + cp Fu2[e - v0, p2],
   {a, b, c, bp, cp}] // FullSimplify
```

··· Solve: Equations may not give solutions for all "solve" variables.

Out[4628]=

$$\left\{\left\{a\to\frac{c\,(p1+p2-v0)}{2\,p1},\,b\to\frac{c\,(p1-p2+v0)}{2\,p1},\,bp\to 0,\,cp\to 0\right\}\right\}$$

In[4629]:=

```
sol2 = sol1 /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[4629]=

$$\left\{\left\{a \to \frac{c\left(\sqrt{(e-m)(e+m)}+\sqrt{-m^2+(e-v0)^2}-v0\right)}{2\sqrt{(e-m)(e+m)}},\right.\right.$$
$$\left.\left. b \to \frac{c\left(\sqrt{(e-m)(e+m)}-\sqrt{-m^2+(e-v0)^2}+v0\right)}{2\sqrt{(e-m)(e+m)}},\ bp \to 0,\ cp \to 0\right\}\right\}$$

In[4630]:=

```
Tr1 = (c /. sol2) / (a /. sol2) // FullSimplify
```

Out[4630]=

$$\left\{\frac{2\sqrt{(e-m)(e+m)}}{\sqrt{(e-m)(e+m)}+\sqrt{-m^2+(e-v0)^2}-v0}\right\}$$

In[4631]:=

```
Tr2 = (cp /. sol2) / (a /. sol2) // FullSimplify
```

Out[4631]=

$\{0\}$

In[4632]:=

```
Rf1 = (b /. sol2) / (a /. sol2) // FullSimplify
```

Out[4632]=

$$\left\{\frac{\sqrt{(e-m)(e+m)}-\sqrt{-m^2+(e-v0)^2}+v0}{\sqrt{(e-m)(e+m)}+\sqrt{-m^2+(e-v0)^2}-v0}\right\}$$

In[4633]:=

```
Rf2 = (bp /. sol2) / (a /. sol2) // FullSimplify
```

Out[4633]=

$\{0\}$

In[4635]:=

```
Ttotal = nt (modsq[ Tr1[[1]] ] + modsq[ Tr2[[1]] ] ) // FullSimplify
```

Out[4635]=

$$\frac{2(e-m)(e+m)\left(e-m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)\left(e+m-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)}{\left(-m^2+e\left(e+\sqrt{(e-m)(e+m)}\right)\right)\left(\sqrt{(e-m)(e+m)}-v0+\sqrt{e^2-m^2-2\,e\,v0+v0^2}\right)^2}$$

In[4636]:=

```
Limit[Ttotal, v0 → ∞] // FullSimplify
```

··· Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[4636]=

$$2 - \frac{2\,e\left(e + \sqrt{(e-m)\,(e+m)}\right)}{m^2}$$

In[4637]:=

```
Limit[Ttotal, v0 → 0] // FullSimplify
```

··· Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[4637]=

$$1$$

In[4639]:=

```
Rtot = nr (modsq[Rf1[[1]]] + modsq[Rf2[[1]]]) // FullSimplify
```

Out[4639]=

$$\frac{m^2\left(\sqrt{(e-m)\,(e+m)} + v0 - \sqrt{e^2 - m^2 - 2\,e\,v0 + v0^2}\right)^2}{\left(m^2 - 2\,e\left(e + \sqrt{(e-m)\,(e+m)}\right)\right)\left(\sqrt{(e-m)\,(e+m)} - v0 + \sqrt{e^2 - m^2 - 2\,e\,v0 + v0^2}\right)^2}$$

In[4640]:=

```
Limit[Rtot, v0 → ∞] // FullSimplify
```

··· Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[4640]=

$$-\frac{\left(e + \sqrt{(e-m)\,(e+m)}\right)^2}{m^2}$$

In[4641]:=

```
Limit[Rtot, v0 → 0] // FullSimplify
```

··· Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[4641]=

$$0$$

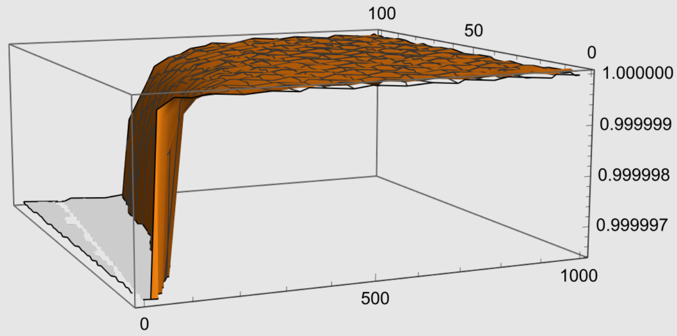
In[4642]:=

```
TotP [ee_, v00_, mm_] :=
 nt (modsq[ Tr1[[1]] ] + modsq[ Tr2[[1]] ] ) + nr ( modsq[ Rf1[[1]] ] + modsq[ Rf2[[1]] ]) /.
  {e → ee, v0 → v00, m → mm}
```

In[4645]:=

```
Plot3D[TotP [e, v0, 1], {e, 2, 1000}, {v0, 1, 100}]
```

Out[4645]=




```
me = 9.1 * 10^-31;
cv = 3 * 10^8;
ev = me * cv^2 / (1.6 * 10^-19);
mev = me * cv^2 / (1.6 * 10^-19);
jev = (1.6 * 10^-19);
vstep = 10^6;
```

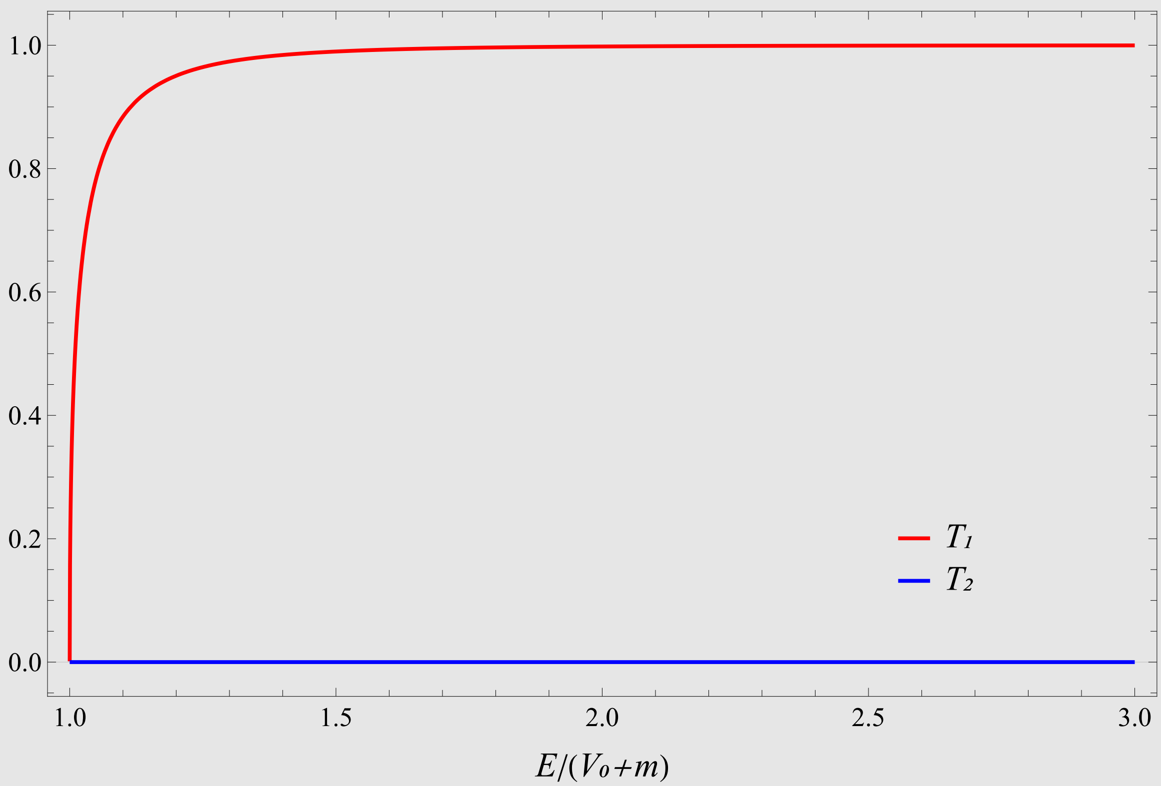
In[4669]:=

```
Tup[ee_, v00_, mm_] := Abs[nt modsq[ Tr1[[1]] ]] /. {e → ee, v0 → v00, m → mm}
Tdn[ee_, v00_, mm_] := Abs[nt modsq[Tr2[[1]] ]] /. {e → ee, v0 → v00, m → mm}
Rup[ee_, v00_, mm_] := Abs[ nr modsq[ Rf1[[1]] ]] /. {e → ee, v0 → v00, m → mm}
Rdn[ee_, v00_, mm_] := Abs[nr modsq[ Rf2[[1]] ]] /. {e → ee, v0 → v00, m → mm}
```

In[4673]:=

```
Plot[{Tup[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
   Tdn[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]},
  {x, 1, 3}, PlotStyle → {{Red, Thick}, {Blue, Thick}}, Frame → True,
  FrameLabel → {Style["E/(V₀+m)", 18, Italic], None},
  PlotLegends → Placed[{Style["T₁", 18, Italic], Style["T₂", 18, Italic]}, {0.8, 0.2}],
  ImageSize → {600, 600}, FrameTicksStyle → Directive[Black, 14],
  PlotRange → All, PlotTheme → "Scientific"]
```

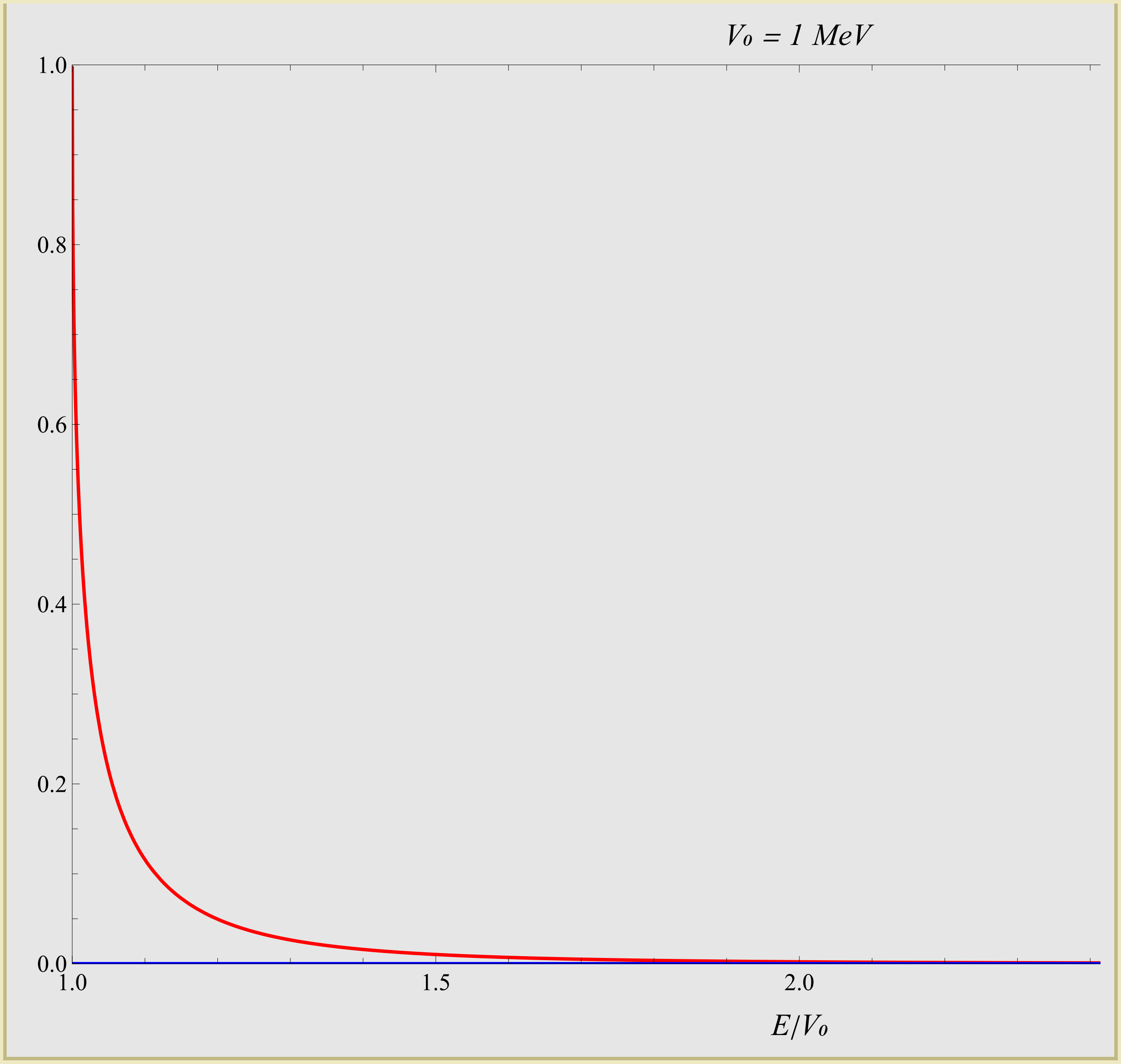
Out[4673]=



In[4674]:=

```
Plot[{Rup[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
  Rdn[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]}, {x, 1, 3},
 PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange → {{1, 3}, {0, 1}},
 Frame → True, FrameLabel → {Style["E/V₀", 18, Italic], None},
 PlotLegends → Placed[{Style["R₁", 18, Italic], Style["R₂", 18, Italic]}, {0.75, 0.3}],
 ImageSize → {900, 600}, FrameTicksStyle → Directive[Black, 14],
 FrameStyle → Directive[Black, 14], (*☑ replaces FrameLabelStyle*)
 PlotTheme → "Scientific", PlotLabel → Style["V₀ = 1 MeV", 18, Italic]]
```

Out[4674]=



In[4667]:=

```
Rup[(vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]
```

Out[4667]=

-1.

Ajaib’s Representation

In[4869]:=

$$\text{etaA} = \frac{-\text{I}}{\text{Sqrt}[2]}\,(\text{G0.G1.G5} + \text{G2});$$

```
eta = etaA;
```

In[4871]:=

$$\text{x1} = \frac{\text{eta} + \text{ConjugateTranspose}[\text{eta}]}{\text{Sqrt}[2]};$$

$$\text{x2} = \frac{\text{eta} - \text{ConjugateTranspose}[\text{eta}]}{\text{Sqrt}[2]};$$

In[4873]:=

```
Pz = Eigenvectors[ x1 e +  m x2];
```

In[4874]:=

```
PzA = %
```

Out[4874]=

$$\left\{\left\{\frac{\sqrt{e^2-m^2}}{e}, \frac{i\,m}{e}, 0, 1\right\}, \left\{-\frac{i\,m}{e}, -\frac{\sqrt{e^2-m^2}}{e}, 1, 0\right\}, \left\{-\frac{\sqrt{e^2-m^2}}{e}, \frac{i\,m}{e}, 0, 1\right\}, \left\{-\frac{i\,m}{e}, \frac{\sqrt{e^2-m^2}}{e}, 1, 0\right\}\right\}$$

In[4875]:=

$$\text{PzC} = \text{PzA} \;/.\; \left\{\text{Sqrt}\left[e^2 - m^2\right] \to \text{pz}\right\}$$

Out[4875]=

$$\left\{\left\{\frac{pz}{e}, \frac{i\,m}{e}, 0, 1\right\}, \left\{-\frac{i\,m}{e}, -\frac{pz}{e}, 1, 0\right\}, \left\{-\frac{pz}{e}, \frac{i\,m}{e}, 0, 1\right\}, \left\{-\frac{i\,m}{e}, \frac{pz}{e}, 1, 0\right\}\right\}$$

In[4876]:=

```
P = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}};

PzD = PzC.P
```

Out[4877]=

$$\left\{\left\{1, 0, \frac{i m}{e}, \frac{pz}{e}\right\}, \left\{0, 1, -\frac{pz}{e}, -\frac{i m}{e}\right\}, \left\{1, 0, \frac{i m}{e}, -\frac{pz}{e}\right\}, \left\{0, 1, \frac{pz}{e}, -\frac{i m}{e}\right\}\right\}$$

In[4878]:=

```
col[v_] := List /@ v;

u1 = col[PzD[[3]]];
u2 = col[PzD[[4]]];

u3 = col[PzD[[1]]];
u4 = col[PzD[[2]]];
```

In[4883]:=

```
u1 // MatrixForm
```

Out[4883]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ \frac{i m}{e} \\ -\frac{pz}{e} \end{pmatrix}$$

In[4884]:=

```
ConjugateTranspose[u1].u2 // FullSimplify
```

Out[4884]=

{{0}}

In[4885]:=

```
Fu1[ee_, ppz_] := u1 /. {e → ee, pz → ppz}
Fu2[ee_, ppz_] := u2 /. {e → ee, pz → ppz}
Fu3[ee_, ppz_] := u3 /. {e → ee, pz → ppz}
Fu4[ee_, ppz_] := u4 /. {e → ee, pz → ppz}
```

In[4889]:=

```
Fu1[e, p1]
```

Out[4889]=

$$\left\{\{1\}, \{0\}, \left\{\frac{i m}{e}\right\}, \left\{-\frac{p1}{e}\right\}\right\}$$

In[4890]:=

```
psiIN = a Fu1[e, p1];
psiR = b Fu1[e, -p1] + bp Fu2[e, -p1];
psiT = c Fu1[e - v0, p2] + cp Fu2[e - v0, p2];
```

In[4893]:=

```
Xm = eta + ConjugateTranspose[eta]
```

Out[4893]=

$$\{\{0, 0, 0, -\sqrt{2}\}, \{0, 0, \sqrt{2}, 0\}, \{0, \sqrt{2}, 0, 0\}, \{-\sqrt{2}, 0, 0, 0\}\}$$

In[4894]:=

```
jin = ConjugateTranspose[psiIN].Xm.psiIN // FullSimplify
```

Out[4894]=

$$\left\{\left\{\frac{2\sqrt{2}\ a\ p1\ \mathrm{Conjugate}[a]}{e}\right\}\right\}$$

In[4895]:=

```
jR = ConjugateTranspose[psiR].Xm.psiR // FullSimplify
```

Out[4895]=

$$\left\{\left\{-\frac{2\sqrt{2}\ p1\ \left(\mathrm{Abs}[b]^2 + \mathrm{Abs}[bp]^2\right)}{e}\right\}\right\}$$

In[4896]:=

```
jT = ConjugateTranspose[psiT].Xm.psiT // FullSimplify
```

Out[4896]=

$$\left\{\left\{\frac{2\sqrt{2}\ p2\ \left(\mathrm{Abs}[c]^2 + \mathrm{Abs}[cp]^2\right)}{e - v0}\right\}\right\}$$

In[4897]:=

```
jR / jin /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[4897]=

$$\left\{\left\{-\frac{\mathrm{Abs}[b]^2 + \mathrm{Abs}[bp]^2}{\mathrm{Abs}[a]^2}\right\}\right\}$$

In[4898]:=

```
jT / jin /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[4898]=

$$\left\{\left\{\frac{e\sqrt{\frac{e^2 - m^2 - 2\,e\,v0 + v0^2}{e^2 - m^2}}\ \left(\mathrm{Abs}[c]^2 + \mathrm{Abs}[cp]^2\right)}{(e - v0)\ \mathrm{Abs}[a]^2}\right\}\right\}$$

In[4899]:=

```
nt = e Sqrt[(e^2-m^2-2 e v0+v0^2)/(e^2-m^2)]/(e - v0); (*G3*)
nr = 1;
```

$$nt = \frac{e\sqrt{\frac{e^2-m^2-2\,e\,v0+v0^2}{e^2-m^2}}}{(e-v0)};\ (*G3*)$$

In[4901]:=

```
sol1 =
 Solve[a Fu1[e, p1] + b Fu1[e, -p1] + bp Fu2[e, -p1] == c Fu1[e - v0, p2] + cp Fu2[e - v0, p2],
   {a, b, c, bp, cp}] // FullSimplify
```

··· Solve: Equations may not give solutions for all "solve" variables.

Out[4901]=

$$\left\{\left\{a \to -\frac{i\,bp\left(e^2\,(p1+p2)^2 - 2\,e\,p1\,(p1+p2)\,v0 + \left(m^2+p1^2\right)\,v0^2\right)}{2\,m\,p1\,v0\,(-e+v0)},\right.\right.$$
$$b \to \frac{i\,bp\left(e^2\left(-p1^2+p2^2\right) + 2\,e\,p1^2\,v0 + (m-p1)\,(m+p1)\,v0^2\right)}{2\,m\,p1\,v0\,(-e+v0)},$$
$$\left.\left.c \to \frac{i\,bp\,(e\,(p1+p2) - p1\,v0)}{m\,v0},\ cp \to bp\right\}\right\}$$

In[4902]:=

```
sol2 = sol1 /. {p1 → Sqrt[e^2 - m^2], p2 → Sqrt[(e - v0)^2 - m^2]} // FullSimplify
```

Out[4902]=

$$\left\{\left\{a \to \frac{i\,bp\,e\left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\,v0\right)}{m\sqrt{(e-m)\,(e+m)}\,v0},\ b \to \frac{i\,bp\,m}{\sqrt{(e-m)\,(e+m)}},\right.\right.$$
$$\left.\left.c \to \frac{i\,bp\left(e\left(\sqrt{(e-m)\,(e+m)} + \sqrt{-m^2+(e-v0)^2}\right) - \sqrt{(e-m)\,(e+m)}\,v0\right)}{m\,v0},\ cp \to bp\right\}\right\}$$

In[4903]:=

```
Tr1 = (c /. sol2) / (a /. sol2) // FullSimplify
```

Out[4903]=

$$\left\{-\frac{-e^2+m^2+\sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)}}{e\,v0}\right\}$$

In[4904]:=

```
Tr2 = (cp /. sol2) / (a /. sol2) // FullSimplify
```

Out[4904]=

$$\left\{-\frac{i\,m\sqrt{(e-m)\,(e+m)}\,v0}{e\left(e^2-m^2+\sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\,v0\right)}\right\}$$

In[4905]:=

```
Rf1 = (b /. sol2) / (a /. sol2) // FullSimplify
```

Out[4905]=

$$\left\{\frac{m^2\, v0}{e\left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0\right)}\right\}$$

In[4906]:=

```
Rf2 = (bp /. sol2) / (a /. sol2) // FullSimplify
```

Out[4906]=

$$\left\{-\frac{\mathrm{i}\, m\, \sqrt{(e-m)\,(e+m)}\; v0}{e\left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0\right)}\right\}$$

In[4907]:=

```
Ttotal = nt (modsq[ Tr1[[1]] ] + modsq[ Tr2[[1]] ] ) // FullSimplify
```

Out[4907]=

$$\frac{2\sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)}}{e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0}$$

In[4908]:=

```
Limit[Ttotal, v0 → ∞] // FullSimplify
```

··· Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[4908]=

$$2 - \frac{2\, e\left(e + \sqrt{e^2 - m^2}\right)}{m^2}$$

In[4909]:=

```
Limit[Ttotal, v0 → 0] // FullSimplify
```

··· Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[4909]=

1

In[4910]:=

```
Rtot = nr (modsq[ Rf1[[1]] ] + modsq[ Rf2[[1]] ] ) // FullSimplify
```

Out[4910]=

$$\frac{m^2\, v0^2}{\left(e^2 - m^2 + \sqrt{(e-m)\,(e+m)\,(e-m-v0)\,(e+m-v0)} - e\, v0\right)^2}$$

In[4911]:=

```
Limit[Rtot, v0 → ∞] // FullSimplify
```

⋯ Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[4911]=

$$\frac{m^2}{\left(e - \sqrt{e^2 - m^2}\right)^2}$$

In[4912]:=

```
Limit[Rtot, v0 → 0] // FullSimplify
```

⋯ Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[4912]=

0

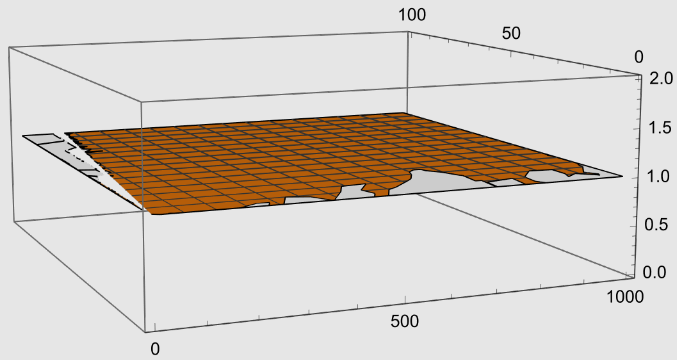
In[4913]:=

```
TotP [ee_, v00_, mm_] :=
 nt (modsq[ Tr1〚1〛 ] + modsq[ Tr2〚1〛 ] ) + nr ( modsq[ Rf1〚1〛 ] + modsq[ Rf2〚1〛 ]) /.
  {e → ee, v0 → v00, m → mm}
```

In[4914]:=

```
Plot3D[TotP [e, v0, 1], {e, 2, 1000}, {v0, 1, 100}]
```

Out[4914]=



In[4915]:=

```
me = 9.1 * 10^-31;
cv = 3 * 10^8;
ev = me * cv^2 / (1.6 * 10^-19);
mev = me * cv^2 / (1.6 * 10^-19);
jev = (1.6 * 10^-19);
vstep = 10^6;
```

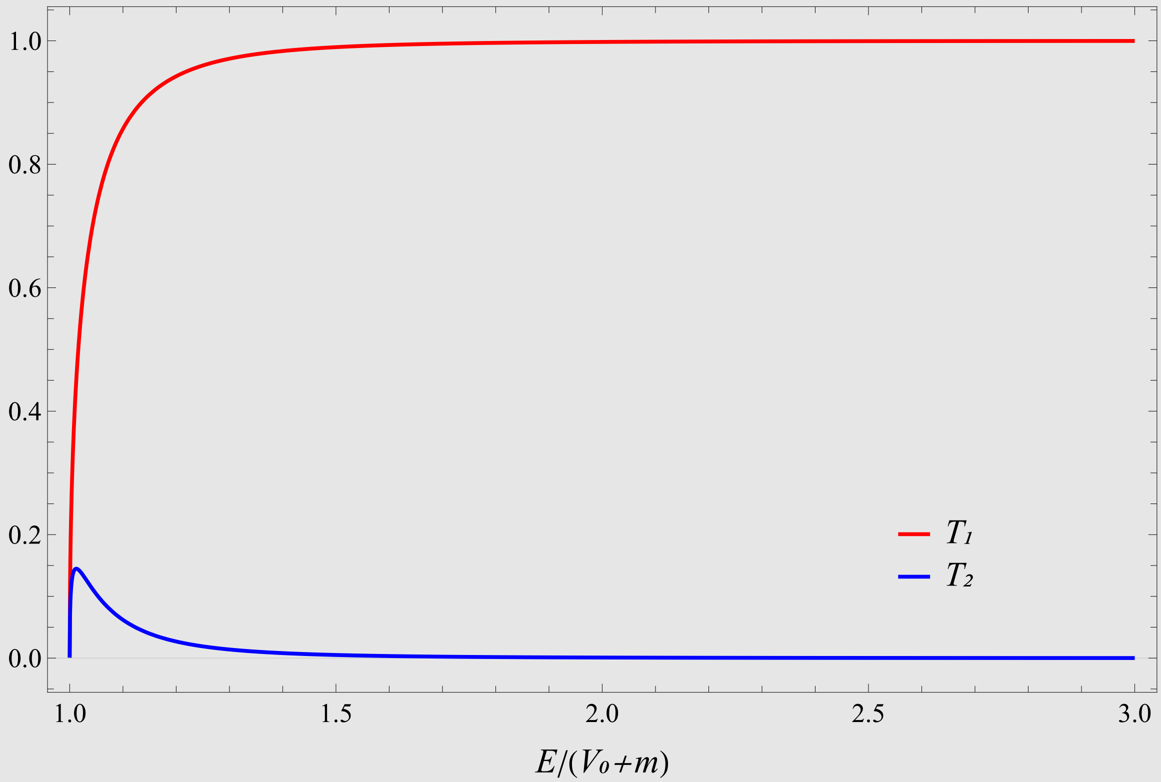
In[4921]:=

```
Tup[ee_, v00_, mm_] := Abs[nt modsq[ Tr1⟦1⟧ ]] /. {e → ee, v0 → v00, m → mm}
Tdn[ee_, v00_, mm_] := Abs[nt modsq[Tr2⟦1⟧ ]] /. {e → ee, v0 → v00, m → mm}
Rup[ee_, v00_, mm_] := Abs[ nr modsq[ Rf1⟦1⟧ ]] /. {e → ee, v0 → v00, m → mm}
Rdn[ee_, v00_, mm_] := Abs[nr modsq[ Rf2⟦1⟧ ]] /. {e → ee, v0 → v00, m → mm}
```

In[4928]:=

```
Plot[{Tup[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
  Tdn[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]},
 {x, 1, 3}, PlotStyle → {{Red, Thick}, {Blue, Thick}}, Frame → True,
 FrameLabel → {Style["E/(V₀+m)", 18, Italic], None},
 PlotLegends → Placed[{Style["T₁", 18, Italic], Style["T₂", 18, Italic]}, {0.8, 0.2}],
 ImageSize → {600, 600}, FrameTicksStyle → Directive[Black, 14],
 PlotRange → All, PlotTheme → "Scientific"]
```

Out[4928]=



In[4929]:=

```
Export["step_potential.pdf", %]
```

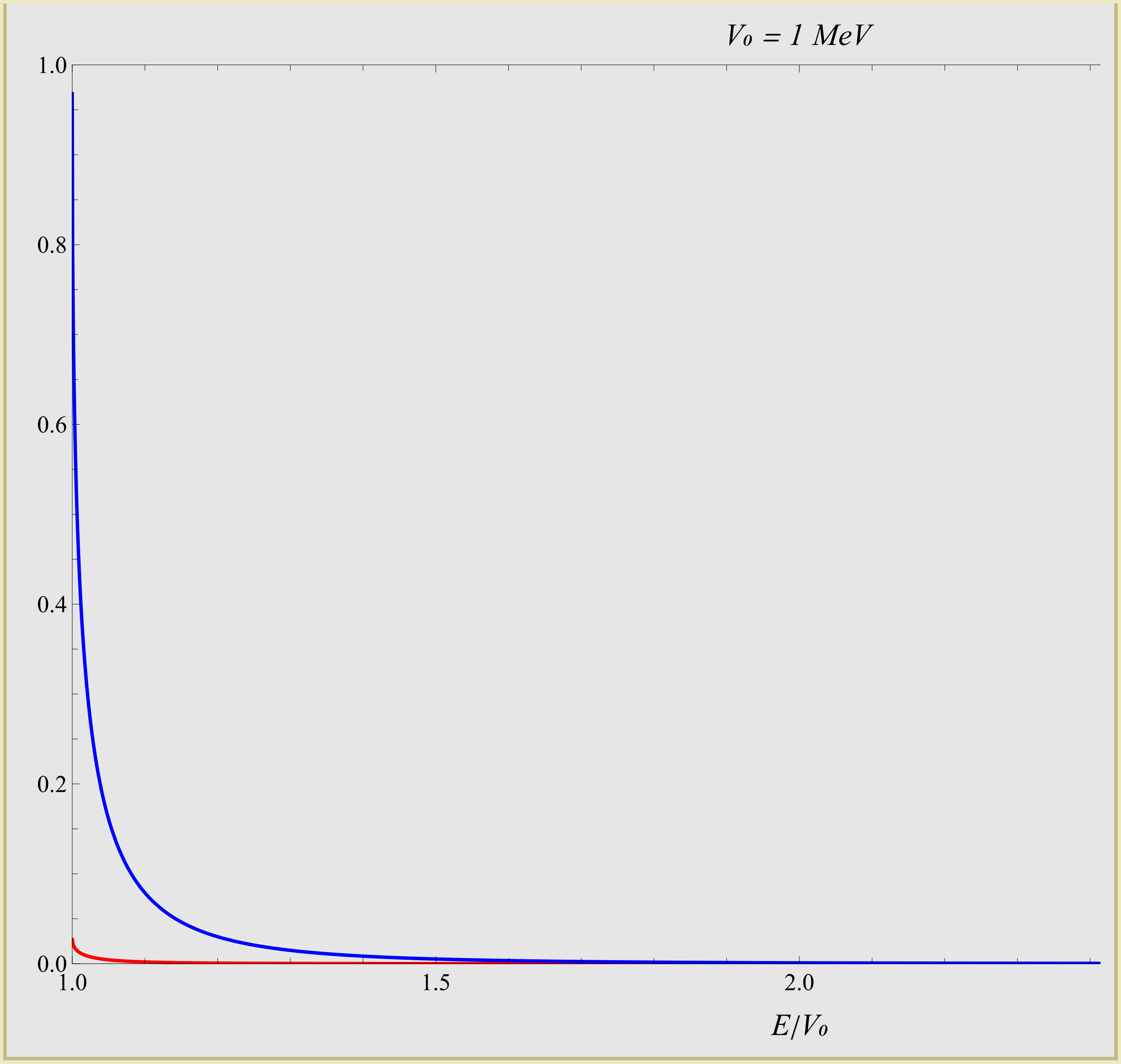
Out[4929]=

```
step_potential.pdf
```

In[4927]:=

```
Plot[{Rup[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev],
   Rdn[x * (vstep + mev), (vstep + 1 / 2 mev), 1 / 2 mev]}, {x, 1, 3},
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange → {{1, 3}, {0, 1}},
  Frame → True, FrameLabel → {Style["E/V₀", 18, Italic], None},
  PlotLegends → Placed[{Style["R₁", 18, Italic], Style["R₂", 18, Italic]}, {0.75, 0.3}],
  ImageSize → {900, 600}, FrameTicksStyle → Directive[Black, 14],
  FrameStyle → Directive[Black, 14], (*☑ replaces FrameLabelStyle*)
  PlotTheme → "Scientific", PlotLabel → Style["V₀ = 1 MeV", 18, Italic]]
```

Out[4927]=



Unitary Equivalence

In[4939]:=

```
S1 = 1/Sqrt[2] {{-1, 0, 0, 1}, {0, 1, 1, 0}, {0, 1, -1, 0}, {-1, 0, 0, -1}};
```

In[4933]:=

```
S1 // MatrixForm
```

Out[4933]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & -1 \end{pmatrix}$$

In[4944]:=

```
S1.x1.ConjugateTranspose[S1] - G0 // FullSimplify
```

Out[4944]=

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

In[4945]:=

```
S1.x2.ConjugateTranspose[S1] - I G5
```

Out[4945]=

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

In[4941]:=

```
S1.ConjugateTranspose[S1] // MatrixForm
```

Out[4941]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$