T.R. ISTANBUL AYDIN UNIVERSITY
ENGINEERING FACULTY

ROBOTIC ARM CONTROLLING & SIMULATION IN MATLAB SIMULINK

BACHELOR THESIS

OĞUZ GENÇER

DEPARTMENT: ELECTRICAL & ELECTRONICS ENGINEERING
SUPERVISOR: GÖKHAN ERDEMİR

İSTANBUL, 2014

PREFACE & APPRECIATIONS

During this project; providing all the help and sacrifice, shedding light to my works with his knowledge and experience, also allowing me to improve myself by this project, to my supervisor Asst. Assoc. Dr. Gökhan Erdemir,

Encouraged me during the preparation of my thesis and providing moral support to Asst. Assoc. Dr. Saeid Karamzadeh and my dear friends, Erden Temizsoy, Erdem Gençel, İsmail Güneş, Onur Özcan, Özge Meydan and Yavuz Selim Ak,

I owe a debt of thanks.

I dedicated this project to my family who would not spare any support during my entire life.

TABLE OF CONTENTS

TABLE OF FIGURES

ABBREVIATIONS

CAD: Computer Aided Design

DH Parameters: Denavit-Hartenberg Parameters

DH Table: Denavit-Hartenberg Table

DOF: Degree of freedom

RVC Toolbox: Robotics, Vision & Control Toolbox for MATLAB Software

ABSTRACT

In this project a 6-DOF robotic arm has been modelled by using Matlab Simulink software to produce a painter robot. The ABB Robotics' IRB120 model robotic arm has been chosen which has a good stability and mobility specifications for such kind of applications than its alternatives. The solid cad model (can be downloaded from www.abb.com) has been used to create a good visuality of the mathematical model in MATLAB&Simulink. The mathematical model has been created by using the datasheet informations of the robotic arm which is published on the web site of the ABB Robotics.

KEYWORDS

Robot, Robotic Arm, 6DOF, Painter Robot, ABB Robotics, IRB120, Control, Vision, Simulation, Matlab, Smulink, Solidworks, Model, Kinematic, Inverse Kinematic, DH Parameters...

INTRODUCTION & BACKGROUND

The controlling technology is developing day by day. Its generally agreed today that robots have already taken the place of the human in manufacturing. Comparing the human with the robots that located on the production line, it is possible to produce anything faster, cheaper and high quality with the robots than a human. Also in some specific fields, robots are being used in where nobody can work However, there are lots of work which requires creativity and artificial intelligence, is being done by humans. In this sense, my purpose is contributing to develop robotic technology to better.

In this project my main aim is to create a robot that draw a picture with high sensitivity an artist's techniques. However this article includes only the first section of the main project. For the project, I have decided to work with a 6-DOF robotic arm which belongs to ABB Robotics Company and its model name is IRB120. In this section, I have analyzed the mechanical structure of the IRB120 robotic arm mathematically. Then I have calculated the kinematic equations for the IRB120 robotic arm. After that, I have created a mathematical model of the robotic arm from its mechanical model by using MATLAB&Simulink software. In the last part of this section I have created a simulation on MATLAB&Simulink that shows how the IRB120 robotic arm works with its bounded parameter in the workspace. There are also some web links of related projects in the additional contents [1] and [2]. These projects does not work exactly the same way with mine but it will give an idea for this technology. By empirically, examining the 6-DOF robotic arm, we hope to produce a more complete and simple understanding on robotic control & simulation.

MECHANICS & CONTROL OF MECHANIC MANIPULATORS

1. ROBOTS AND VARIOUS TYPES OF ROBOTS

A robot is a computer controlled device that can be programmed easily and includes sensors to adaptivity for various working conditions. There are many types of robots are being used. They differ by application or by their kinematics.

1.1. TYPES OF ROBOTS BY APPLICATION
- 1.1.1. Industrial Robots
- 1.1.2. Domestic or Household Robots
- 1.1.3. Medical Robots
- 1.1.4. Service Robots
- 1.1.5. Military Robots
- 1.1.6. Entertainment Robots
- 1.1.7. Space Robots
- 1.1.8. Hobby & Competition Robots

1.2. TYPES OF ROBOTS BY KINEMATICS
- 1.2.1. Stationary Robots
  - 1.2.1.1. Cartesian Robots
  - 1.2.1.2. Cylindrical Robots
  - 1.2.1.3. Spherical Robots
  - 1.2.1.4. SCARA Robots
  - 1.2.1.5. Articulated Robots (robotic arm)
  - 1.2.1.6. Parallel Robots
- 1.2.2. Wheeled Robots
  - 1.2.2.1. Single Wheel (ball) Robots

- 1.2.2.2. Two Wheel Robots
- 1.2.2.3. Three and more Wheel Robots

1.2.3. Legged Robots

- 1.2.3.1. Bipedal Robots (Humanoid Robots)
- 1.2.3.2. Tripedal Robots
- 1.2.3.3. Quadrupedal Robots
- 1.2.3.4. Hexapod Robots
- 1.2.3.5. Others (octopod, centipede, etc.)

1.2.4. Swimming Robots

1.2.5. Flying Robots

1.2.6. Mobile Spherical Robots

1.2.7. Swarm Robots

1.2.8. Others

2. POSITION, ORIENTATION & FRAME

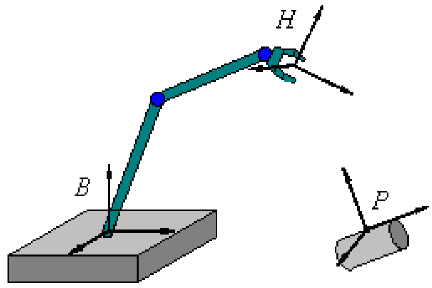

Figure 1: Frame includes point, base and holder

In the study of robotics, it is constantly be concerned with the location of objects in three dimensional space. These objects are the link of the manipulator, the part and tool with which it deals, and other objects in the manipulator's environment. At a crude but important level, these objects are described by just two attributes: position and orientation. In order to describe the position and orientation of a body in space, we will always attach a coordinate system, or frame, rigidly to the object. We then proceed to describe the position and orientation of this frame with respect to some reference coordinate system. [3]

2.1. Description of a Position

Once a coordinate system is established, we can locate any point in the universe with a 3x1 position vector. Becausewe will often define many coordinate systems in addition to the universe coordinate system, vectors must be tagged with information identifying which coordinate system they are defined within. ${}^{A}P$ is represented as a vector and can equivalently be thought of as a position in space, or simply as an ordered set of three numbers. Individual elements of a vector are given the subscripts x, y and z.

$${}^{A}P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

2.2. Description of an Orientation

In order to describe the orientation of a body, we will attach a coordinate system to the body and then give a description of this coordinate system relative to the reference system. A description of {B} relative to {A} now suffices to give the orientation of the body. The coordinate system {B} written in terms of coordinate system {A} as shown below. This matrix called as **rotation matrix** and it describes {B} relative to {A} and its name in notation is ${}^{A}_{B}R$.

$${}^{A}_{B}R = [{}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B] = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

2.3. Description of a Position

The situation of a position and an orientation pair arises so often in robotics that we define an entity called a frame, which is a set of four vectors giving position and orientation information. A frame is a coordinate system where, in addition to the orientation, we give a position vector which locates its origin relative to some other embedding frame. For example, frame {B} is described by ${}^{A}_{B}R$ and ${}^{A}P_{BORG}$, where ${}^{A}P_{BORG}$ is the vector that locates the origin of the frame {B}:

$$\{B\} = \{ {}^{A}_{B}R, \qquad {}^{A}P_{BORG} \}$$

3. MANIPULATOR KINEMATICS

Kinematics is the science of motion that treats the subject without regard to the forces that cause it. Within the science of kinematics, one studies the position, the velocity, the acceleration, and all higher order derivatives of the position variables (with respect to time or any other variable(s)). Hence, the study of the kinematics of manipulators refers to all the geometrical and time-based properties of the motion. A manipulator may be thought of as a set of bodies connected in a chain by joints. These bodies are called links. Joints from a connection between neighboring pair of links. Mechanical-design considerations favor manipulators' generally being constructed from joints that exhibit just one degree of freedom. Most manipulators have revlute joint or have sliding joints called prismatic joints. To design a robot, at least three joints is needed. There are many types of robots which can be used for many different purposes. They vary by their joint types. Usually, two types of joints are used which are prismatic and revolute joints. Robot types are vary by the types of joints:

- Cartesian Robots : Includes three prismatic joints (PPP)
- Cylindrical Robots: Includes a revolute and two prismatic joints (RPP)
- SCARA Robots: Includes two revolute and a prismatic joint. All joints are parallel. (RRP)
- Spherical Robots: Includes two revolute and a prismatic joint. (RRP)
- Articulated Robots: Includes three revolute joints (RRR)

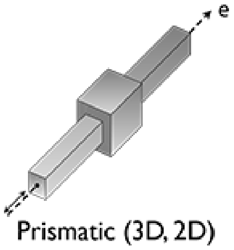

Figure 3

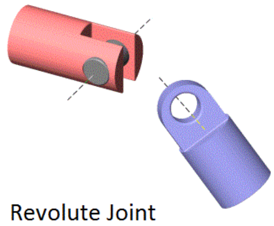

Figure 2

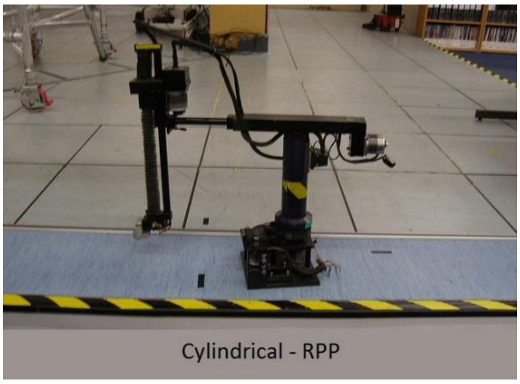

Figure 5

Figure 4

Figure 8

Figure 7

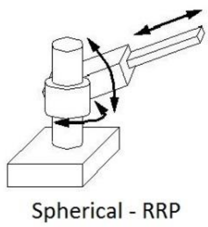

Figure 6

4. DENAVIT-HARTENBERG (DH) PARAMETERS

The Denavit–Hartenberg parameters (also called DH parameters) are the four parameters associated with a particular convention for attaching reference frames to the links of a spatial kinematic chain, or robot manipulator. [11]

d : offset along previous **z** to the common normal

θ : angle about previous **z**, from old **x** to new **x**

r : length of the common normal (this is **a**, but if using this notation, do not confuse with **α**). Assuming a revolute joint, this is the radius about previous **z**.

α : angle about common normal, from old **z** axis to new **z** axis

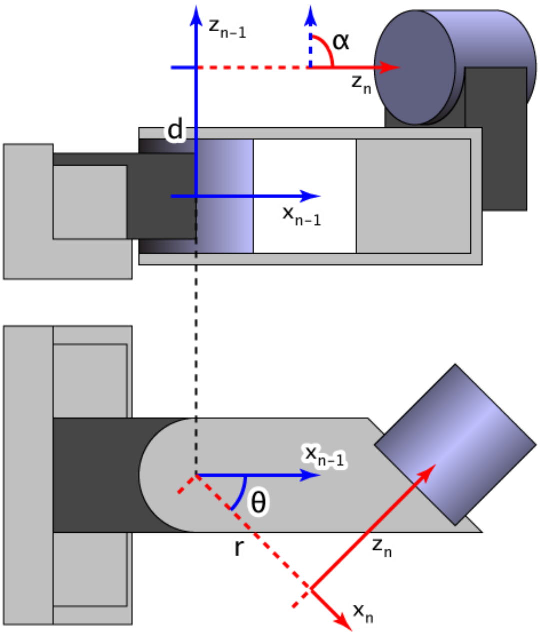

Figure 9: DH Parameters

We can note constraints on the relationships between the axes:

- the X_n-axis is perpendicular to both the Z_{n-1} and Z_n axes
- the X_n-axis intersects both Z_{n-1} and Z_n axes
- the origin of joint **n** is at the intersection of X_n and Z_n
- Y_n completes a right-handed reference frame based on X_n and Z_n

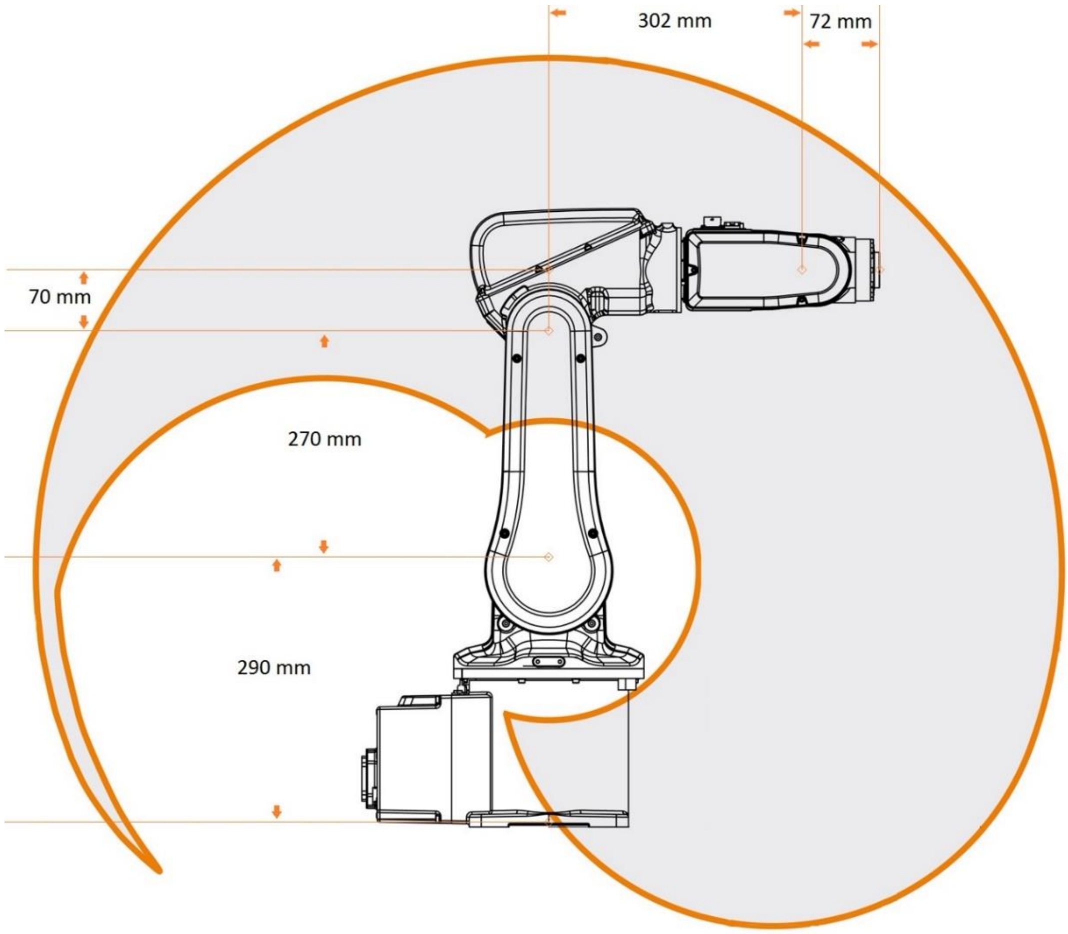

Figure 10: Technical Drawing of the IRB120 Robotic Arm

i	d_i (mm)	θ_i	a_i (mm)	α_i
1	290	θ_1	0	$-\pi/2$
2	0	$\theta_2-\pi/2$	270	0
3	0	θ_3	70	$-\pi/2$
4	302	θ_4	0	$\pi/2$
5	0	θ_5	0	$-\pi/2$
6	72	$\theta_6+\pi$	0	0

Table 1: DH Parameters of IRB120 Robotic Arm

5. FORWARD KINEMATICS OF MANIPULATORS

While we are making relation with links and joints, there are some methods to solve kinematic calculations. One of them is known as Denavit Hartenberg matrix. In this method, two axes is enough to be known. There are some rotational and radial variables in this method. All variables should be calculated before use them in an homogenous transformation matrix. Then the transformation matrix could be calculated.

For X component of i-th joint;

$$[X_i] = Trans_{X_i}\left(r_{i,i+1}\right)Rot_{X_i}(\alpha_{i,i+1})$$

For Z component of i-th joint;

$$[Z_i] = Trans_{Z_i}(d_i)Rot_{Z_i}(\theta_i)$$

If we combine both of them, the transformation formula for i-th joint can be calculated as;

$$^{i-1}T_i = [Z_i][X_i] = Trans_{Z_i}(d_i)Rot_{Z_i}(\theta_i)Trans_{X_i}\left(r_{i,i+1}\right)Rot_{X_i}(\alpha_{i,i+1})$$

There are the transformational and rotational matrix for n-th joint;

$$Trans_{X_n}(r_n) = \begin{bmatrix} 1 & 0 & 0 & r_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Trans_{Z_{n-1}}(d_n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot_{X_n}(\alpha_n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_n & -s\alpha_n & 0 \\ 0 & s\alpha_n & c\alpha_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Rot_{Z_{n-1}}(\theta_n) = \begin{bmatrix} c\theta_n & -s\theta_n & 0 & 0 \\ s\theta_n & c\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The homogenous transformation matrix for n-th joint can be calculated as;

$$ {}^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

There is the kinematic solution matrix below, which has calculated by using DH table information of the IRB120 robot manipulator;

Note that, some abbreviations has been used in equations below;

$$ \mathrm{c}\theta_x : \cos\theta_x $$

$$ \mathrm{s}\theta_x : sin\theta_x $$

$$ \mathrm{c}\theta_{xy} : \cos\theta_x * \cos\theta_y $$

$$ \mathrm{s}\theta_{xy} : \sin\theta_x * \sin\theta_y $$

and also “r” used below instead of to avoid any confusion with “α” ;

The transformation matrix identified as below;

$$ {}^{i}T_j = \begin{cases} A_{i+1} \times A_{i+2} \times \ldots \times A_j, & i < j \\ I, & i = j \\ \left({}^{i}T_j\right)^{-1} = {}^{j}T_i, & i > j \end{cases} $$

$$ {}^{0}T_1 = A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 c\alpha_1 & s\theta_1 s\alpha_1 & r_1 c\theta_1 \\ s\theta_1 & c\theta_1 c\alpha_1 & -c\theta_1 s\alpha_1 & r_1 s\theta_1 \\ 0 & s\alpha_1 & c\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & 290 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_2 = A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 c\alpha_2 & s\theta_2 s\alpha_2 & r_2 c\theta_2 \\ s\theta_2 & c\theta_2 c\alpha_2 & -c\theta_2 s\alpha_2 & r_2 s\theta_2 \\ 0 & s\alpha_2 & c\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 270c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & 270s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}T_3 = A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 c\alpha_3 & s\theta_3 s\alpha_3 & r_3 c\theta_3 \\ s\theta_3 & c\theta_3 c\alpha_3 & -c\theta_3 s\alpha_3 & r_3 s\theta_3 \\ 0 & s\alpha_3 & c\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_3 & 0 & -s\theta_3 & 70c\theta_3 \\ s\theta_3 & 0 & c\theta_3 & 70s\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$
{}^{3}T_4 = A_4 = \begin{bmatrix} c\theta_4 & -s\theta_4 c\alpha_4 & s\theta_4 s\alpha_4 & r_4 c\theta_4 \\ s\theta_4 & c\theta_4 c\alpha_4 & -c\theta_4 s\alpha_4 & r_4 s\theta_4 \\ 0 & s\alpha_4 & c\alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_4 & 0 & s\theta_4 & 0 \\ s\theta_4 & 0 & -c\theta_4 & 0 \\ 0 & 1 & 0 & 302 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}T_5 = A_5 = \begin{bmatrix} c\theta_5 & -s\theta_5 c\alpha_5 & s\theta_5 s\alpha_5 & r_5 c\theta_5 \\ s\theta_5 & c\theta_5 c\alpha_5 & -c\theta_5 s\alpha_5 & r_5 s\theta_5 \\ 0 & s\alpha_5 & c\alpha_5 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_5 & 0 & -s\theta_5 & 0 \\ s\theta_5 & 0 & c\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}T_6 = A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 c\alpha_6 & s\theta_6 s\alpha_6 & r_6 c\theta_6 \\ s\theta_6 & c\theta_6 c\alpha_6 & -c\theta_6 s\alpha_6 & r_6 s\theta_6 \\ 0 & s\alpha_6 & c\alpha_6 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 72 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

If we multiply all matrices;

$$
{}^{0}T_6 = A_1 \times A_2 \times A_3 \times A_4 \times A_5 \times A_6 = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix}
$$

$$
\begin{bmatrix} T_{11} \\ T_{21} \\ T_{31} \\ T_{41} \end{bmatrix} = \begin{bmatrix} c\theta_{123456} - c\theta_{1456} s\theta_{23} + c\theta_{56} s\theta_{14} - c\theta_{126} s\theta_{35} - c\theta_{136} s\theta_{25} - c\theta_{123} s\theta_{46} + c\theta_1 s\theta_{2346} + c\theta_4 s\theta_{16} \\ c\theta_{23456} s\theta_1 - c\theta_{456} s\theta_{123} - c\theta_{156} s\theta_4 - c\theta_{26} s\theta_{135} - c\theta_{36} s\theta_{125} - c\theta_{23} s\theta_{146} + s\theta_{12346} - c\theta_{14} s\theta_6 \\ -c\theta_{3456} s\theta_2 - c\theta_{2456} s\theta_3 + c\theta_6 s\theta_{235} - c\theta_{236} s\theta_5 + c\theta_3 s\theta_{246} + c\theta_2 s\theta_{346} \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} T_{12} \\ T_{22} \\ T_{32} \\ T_{42} \end{bmatrix} = \begin{bmatrix} -c\theta_{12345} s\theta_6 + c\theta_{145} s\theta_{236} - c\theta_5 s\theta_{146} + c\theta_{12} s\theta_{356} + c\theta_{13} s\theta_{256} - c\theta_{1236} s\theta_4 + c\theta_{16} s\theta_{234} + c\theta_{46} s\theta_1 \\ -c\theta_{2345} s\theta_{16} + c\theta_{45} s\theta_{1236} + c\theta_{15} s\theta_{46} + c\theta_2 s\theta_{1356} + c\theta_3 s\theta_{1256} - c\theta_{236} s\theta_{14} + c\theta_6 s\theta_{1234} - c\theta_{146} \\ c\theta_{345} s\theta_{26} + c\theta_{245} s\theta_{36} - s\theta_{2356} + c\theta_{23} s\theta_{56} + c\theta_{36} s\theta_{24} + c\theta_{26} s\theta_{34} \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} T_{13} \\ T_{23} \\ T_{33} \\ T_{43} \end{bmatrix} = \begin{bmatrix} -c\theta_{1234} s\theta_5 + c\theta_{14} s\theta_{235} - s\theta_{145} - c\theta_{125} s\theta_3 - c\theta_{135} s\theta_2 \\ -c\theta_{234} s\theta_{15} + c\theta_4 s\theta_{1235} + c\theta_1 s\theta_{45} - c\theta_{25} s\theta_{13} - c\theta_{35} s\theta_{12} \\ c\theta_{34} s\theta_{25} + c\theta_{24} s\theta_{35} + c\theta_5 s\theta_{23} - c\theta_{235} \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} T_{14} \\ T_{24} \\ T_{34} \\ T_{44} \end{bmatrix}
= \begin{bmatrix} 72(-c\theta_{1234} s\theta_5 + c\theta_{14} s\theta_{235} - s\theta_{145} - c\theta_{125} s\theta_3 - c\theta_{135} s\theta_2) - 302(c\theta_{12} s\theta_3 + c\theta_{13} s\theta_2) + 70(c\theta_{123} - c\theta_1 s\theta_{23}) + 270(c\theta_{12}) \\ 72(-c\theta_{234} s\theta_{15} + c\theta_4 s\theta_{1235} + c\theta_1 s\theta_{45} - c\theta_{25} s\theta_{13} - c\theta_{35} s\theta_{12}) - 302(c\theta_2 s\theta_{13} + c\theta_3 s\theta_{12}) + 70(c\theta_{23} s\theta_1 - s\theta_{123}) + 270(c\theta_2 s\theta_1) \\ 72(c\theta_{34} s\theta_{25} + c\theta_{24} s\theta_{35} + c\theta_5 s\theta_{23} - c\theta_{235}) + 302(s\theta_{23} - c\theta_{23}) - 70(c\theta_3 s\theta_2 + c\theta_2 s\theta_3) - 270(s\theta_2) + 290 \\ 1 \end{bmatrix}
$$

Transformation matrix from base to the end effector is actually a combination of the rotation matrix and the position vector;

$$ {}^{0}T_6 = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_x \\ R_{21} & R_{22} & R_{23} & P_y \\ R_{31} & R_{32} & R_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

We can calculate the transformation matrix for $\theta = \begin{bmatrix} 0 & -\frac{\pi}{2} & 0 & 0 & 0 & \pi \end{bmatrix}$;

$$ {}^{0}T_1 = \begin{bmatrix} c(0) & -s(0)c(-\frac{\pi}{2}) & s(0)s(-\frac{\pi}{2}) & 0c(0) \\ s(0) & c(0)c(-\frac{\pi}{2}) & -c(0)s(-\frac{\pi}{2}) & 0s(0) \\ 0 & s(-\frac{\pi}{2}) & c(-\frac{\pi}{2}) & 290 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 290 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_2 = \begin{bmatrix} c(-\frac{\pi}{2}) & -s(-\frac{\pi}{2})c(0) & s(-\frac{\pi}{2})s(0) & 270c(-\frac{\pi}{2}) \\ s(-\frac{\pi}{2}) & c(-\frac{\pi}{2})c(0) & -c(-\frac{\pi}{2})s(0) & 270s(-\frac{\pi}{2}) \\ 0 & s(0) & c(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -270 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}T_3 = \begin{bmatrix} c(0) & -s(0)c(-\pi/2) & s(0)s(-\pi/2) & 70c(0) \\ s(0) & c(0)c(-\pi/2) & -c(0)s(-\pi/2) & 70s(0) \\ 0 & s(-\pi/2) & c(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 70 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}T_4 = \begin{bmatrix} c(0) & -s(0)c(\pi/2) & s(0)s(\pi/2) & 0c(0) \\ s(0) & c(0)c(\pi/2) & -c(0)s(\pi/2) & 0s(0) \\ 0 & s(\pi/2) & c(\pi/2) & 302 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 302 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}T_5 = \begin{bmatrix} c(0) & -s(0)c(-\pi/2) & s(0)s(-\pi/2) & 0c(0) \\ s(0) & c(0)c(-\pi/2) & -c(0)s(-\pi/2) & 0s(0) \\ 0 & s(-\pi/2) & c(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{5}T_6 = \begin{bmatrix} c(\pi) & -s(\pi)c(0) & s(\pi)s(0) & 0c(\pi) \\ s(\pi) & c(\pi)c(0) & -c(\pi)s(0) & 0s(\pi) \\ 0 & s(0) & c(0) & 72 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 72 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The kinematic solution is;

$$^{0}T_{6} = {}^{0}T_{1} \times {}^{1}T_{2} \times {}^{2}T_{3} \times {}^{3}T_{4} \times {}^{4}T_{5} \times {}^{5}T_{6} = \begin{bmatrix} 0 & 0 & 1 & 374 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 630 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

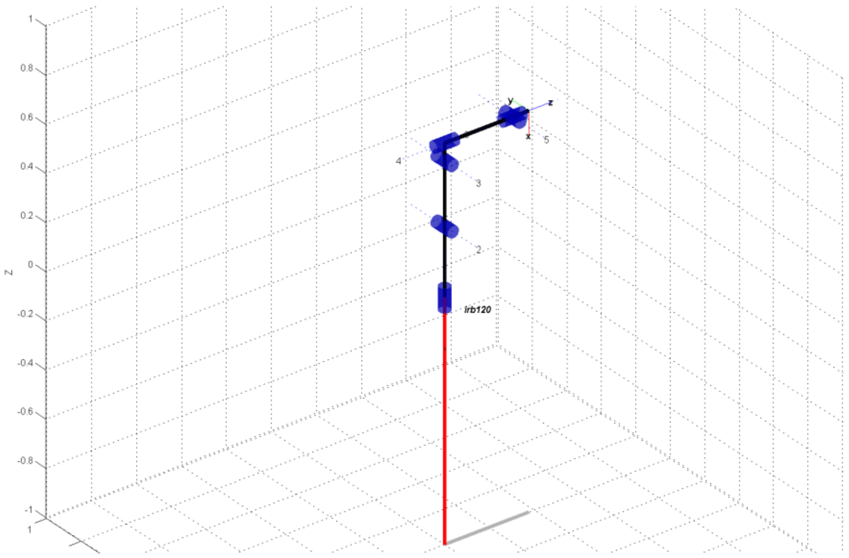

Figure 11: Plotted forward kinematic calculation for fkine([0, –π/2, 0, 0, 0, π]);

6. INVERSE KINEMATICS OF MANIPULATORS

The inverse kinematic solution is a one of the most important process while working with robotics or relative areas. As matematically, it is an inverse matrix of forward kinematic solution. However in reality, there are some boundaries while working with real conditions.

For example; If we have an angle which is;

$$\theta = A\ tan2(-k, p_z)$$

its convertible as mathematically to;

$$\theta = A\ tan2(k, -p_z)$$

In this case the real solution would not be realised because the joint angle does not allow this movement. Basically, the inverse kinematic solution analytical formula should be written as;

$${}^0T_6 = [{}^0T_6]^{-1}$$

hence,

$$T = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} n_x & n_y & n_z & -\left((P_x * n_x) + \left(P_y * n_y\right) + (P_z * n_z)\right) \\ o_x & o_y & o_z & -\left((P_x * o_x) + \left(P_y * o_y\right) + (P_z * o_z)\right) \\ a_x & a_y & a_z & -\left((P_x * a_x) + \left(P_y * a_y\right) + (P_z * a_z)\right) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. JACOBIAN: VELOCITIES AND STATIC FORCES

The Jacobian is a representaton of the geometry of the element of a mechanism in time. It allows the conversion of differential motions or velocities of individual joints to differential motions or velocities of points of interest. It also relates the individual joint motions to overall mechanism motions. Jacobian is time related; since the values of θ_1 and θ_2 vary in time, the magnitude of the elements of the Jacobian vary in time as well.

The Jacobian can be calculated by taking the derivatives of each position equation with respect to all variables.

Suppose that we have a set of equations Y_i in terms of a set of variables x_j:

$$Y_i = f_i(x_1,\ x_2,\ x_3, \dots, x_j)$$

The differential change in Y_i for a differential change in x_i is

$$\begin{cases} \delta Y_1 = \dfrac{\partial f_1}{\partial x_1}\delta x_1 + \dfrac{\partial f_1}{\partial x_2}\delta x_2 + \cdots + \dfrac{\partial f_1}{\partial x_j}\delta x_j, \\ \delta Y_2 = \dfrac{\partial f_2}{\partial x_1}\delta x_1 + \dfrac{\partial f_2}{\partial x_2}\delta x_2 + \cdots + \dfrac{\partial f_2}{\partial x_j}\delta x_j, \\ \qquad \vdots \\ \delta Y_i = \dfrac{\partial f_i}{\partial x_1}\delta x_1 + \dfrac{\partial f_i}{\partial x_2}\delta x_2 + \cdots + \dfrac{\partial f_i}{\partial x_j}\delta x_j \end{cases}$$

We can simply identify the Jacobian equation as below;

$$\begin{bmatrix} dx \\ dy \\ dz \\ \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} = \begin{bmatrix} \\ Robot \\ Jacobian \\ \\ \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ d\theta_4 \\ d\theta_5 \\ d\theta_6 \end{bmatrix} \quad or \ \ [D] = [\,J\,][\,D_\theta\,]$$

8. DYNAMICS

8.1. Lagrangian Mechanics

Lagrangian mechanics is based on differentiation of the energy terms with respect to the system's variables and time, as shown next. For simple cases, it may take longer to use this technique than newtonian mechanics. However, as the complexity of the system increases, the Lagrangian method becomes relatively simpler to use. The lagrangian mechanics is based on the following two generalized equations, one for linear motions, one for rotational motions. First we will define a Lagrangian as;

$$L = K - P$$

Where L is the Lagrangian, K is the kinetic energy of the system, and P is the potential energy of the system. Then;

$$F_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial \dot{x}_i} \text{ ,}$$

$$T_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \dot{\theta}_i} \text{,}$$

Where F is the summation of all external forces for a linear motion, T is the summation of all torques in a rotational motion, and θ and x are system variables. As a result, to get the equations of motion, we need to derive energy equations for the system, and then differentiate the Lagrangian according to F_i and T_i equations.

8.2. Dynamic Equations for Multiple DOF Robots

8.2.1. Kinetic Energy

The kinetic energy of a rigid body simplifies to;

$$K = \frac{1}{2}\, m\, \bar{V}^2 + \frac{1}{2}\, I\, \bar{\omega}^2$$

The velocity of a point along a robot's link can be defined by differentiating the position equation of the point, which is expressed by a frame relative to the robot's base, ${}^0T_n$

$${}^0T_n = {}^0T_1 \times {}^1T_2 \times {}^2T_3 \times \ldots \times {}^{n-1}T_n = A_1 A_2 A_3 \ldots A_n$$

We see that the derive of an A_i matrix for a revolute joint with respect to its joint variable θ_i is;

$$\frac{\partial A_i}{\partial \theta_i} = \frac{\partial}{\partial x}\begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial A_i}{\partial \theta_i} = Q_i A_i$$

Using q_i to represent the joint variables (θ_i, θ_i ... , for revolute joints and θ_i, θ_i ... , for prismatic joints), and extending the same differentiation principle to the ${}^0T_i$ matrix with multiple joint variables ($\theta's\ and\ d's$) , differentiated with respect to only one variable q_j gives;

$$U_{ij} = \frac{\partial\, {}^0T_i}{\partial q_j} = \frac{\partial (A_1 A_2 \ldots A_j \ldots A_i)}{\partial q_j} = A_1 A_2 \ldots Q_j A_j \ldots A_i\,, \quad j \le i$$

Since ${}^0T_i$ is differentiated only with respect to one variable q_i , there is only one Q_i. Higher order derivatives can be formulated similarly from;

$$U_{ijk} = \frac{\partial U_{ij}}{\partial q_k}$$

Using r_i to represent a point on any link i of the robot relative to frame i, we can express the position of the point by premultiplying the vector with the transformation matrix representing its frame;

$$p_i = {}^0T_i r_i$$

The velocity of the point is a function of the velocities of all the joints $\dot{q}_1$, $\dot{q}_2$, ... , $\dot{q}_6$. Therefore, the equation with respect to all the joint variables q_i yields the velocity of the point;

$$V_i = \frac{d({}^0T_i r_i)}{dt} = \sum_{j=1}^{i} \left(\frac{\partial({}^0T_i)}{\partial q_j} \frac{dq_j}{dt} \right) \cdot r_i = \sum_{j=1}^{i} \left(U_{ij} \frac{dq_i}{dt} \right) \cdot r_i$$

The kinetic energy of an element of mass m_i on a link is;

$$dK_i = \frac{1}{2} (\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2) dm$$

Since V_i has three components of $\dot{x}_i,\ \dot{y}_i,\ \dot{z}_i$, it can be written as a matrix, and thus;

$$V_i V_i^T = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} [\dot{x}_i \quad \dot{y}_i \quad \dot{z}_i] = \begin{bmatrix} \dot{x}_i^2 & \dot{x}_i\dot{y}_i & \dot{x}_i\dot{z}_i \\ \dot{y}_i\dot{x}_i & \dot{y}_i^2 & \dot{y}_i\dot{z}_i \\ \dot{z}_i\dot{x}_i & \dot{z}_i\dot{y}_i & \dot{z}_i^2 \end{bmatrix}$$

and,

$$Trace(V_i V_i^T) = Trace \begin{bmatrix} \dot{x}_i^2 & \dot{x}_i\dot{y}_i & \dot{x}_i\dot{z}_i \\ \dot{y}_i\dot{x}_i & \dot{y}_i^2 & \dot{y}_i\dot{z}_i \\ \dot{z}_i\dot{x}_i & \dot{z}_i\dot{y}_i & \dot{z}_i^2 \end{bmatrix} = \dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2$$

Combining these equations, yields the following equation for the kinetic energy of the element;

$$dK_i = \frac{1}{2}\, Trace \left[\left(\sum_{p=1}^{i} \left(U_{ip} \frac{dq_p}{dt} \right) \cdot r_i \right) \left(\sum_{r=1}^{i} \left(U_{ir} \frac{dq_r}{dt} \right) \cdot r_i \right)^T \right] dm_i$$

Where p and r represent the different joint numbers. This allows us to add the contributions made to the final velocity of a point on any link i from other joints' movements. Integrating this equation and rearranging terms yields the total kinetic energy;

$$K_i = \int dK_i = \frac{1}{2}\, Trace \left[\sum_{p=1}^{i} \sum_{r=1}^{i} U_{ip} \left(\int r_i r_i^T \, dm_i \right) U_{ir}^T \dot{q}_p \dot{q}_r \right]$$

The Psuedo Inertia Matrix, representing the $\int r_i r_i^T \, dm_i$ terms, can be written as;

$$J_i = \begin{bmatrix} (-I_{xx} + I_{yy} + I_{zz})/2 & I_{xy} & I_{xz} & m_i\bar{x}_i \\ I_{xy} & (I_{xx} - I_{yy} + I_{zz})/2 & I_{yz} & m_i\bar{y}_i \\ I_{xz} & I_{yz} & (I_{xx} + I_{yy} - I_{zz})/2 & m_i\bar{z}_i \\ m_i\bar{x}_i & m_i\bar{y}_i & m_i\bar{z}_i & m_i \end{bmatrix}$$

Since this matrix is independent of joint angles and velocities, it must be evaluated only once. Substituting equation gives the final form for kinetic energy of the robot manipulator;

$$K = \frac{1}{2}\sum_{i=1}^{n}\sum_{p=1}^{i}\sum_{r=1}^{i} Trace\ (U_{ip}J_iU_{ir}^T)\dot{q}_p\dot{q}_r$$

The kinetic energy of the actuators can also be added to this equation. Assuming that each actuator has an inertia of $I_{i(act)}$, the kinetic energy of the actuator is $1/2\ I_{i(act)}\dot{q}_i^2$, and the total kinetic energy of the robot is;

$$K = \frac{1}{2}\sum_{i=1}^{n}\sum_{p=1}^{i}\sum_{r=1}^{i} Trace\ (U_{ip}J_iU_{ir}^T)\dot{q}_p\dot{q}_r + \frac{1}{2}\sum_{i=1}^{n} I_{i(act)}\dot{q}_i^2$$

8.2.2. Potential Energy

The potential energy of the system is the sum of the potential energies of each link and can be written as;

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} [-m_i g^T \cdot ({}^0T_i \bar{r}_i)],$$

Where $g^T = [g_x \quad g_y \quad g_z \quad 0]$ is the gravity matrix and r is the location of the center of mass of a link relative to the frame representing the link. Obviously, the potential energy must be a scalar quantity, and thus g, which is a (1x4) matrix, multiplied by the position vector (), which is a (4x1) matrix, yields a single scalar quantity. The values in the gravity matrix are dependent on the orientation of the reference frame.

8.2.3. The Lagrangian

The Lagrangian is then;

$$L = K - P = \frac{1}{2} \sum_{i=1}^{n} \sum_{p=1}^{i} \sum_{r=1}^{i} Trace\left(U_{ip} J_i U_{ir}^T\right) \dot{q}_p \dot{q}_r + \frac{1}{2} \sum_{i=1}^{n} I_{i(act)} \dot{q}_i^2 - \sum_{i=1}^{n} [-m_i g^T \cdot ({}^0T_i \bar{r}_i)].$$

9. TRAJECTORY GENERATION

A path is defined as a sequence of robot configurations in a particular order without regard to the timing of these configurations. So, if a root goes from point (and thus, configuration) A to point B to point C, the sequence of the configurations between A and B and C constitutes a path [1]. However, a trajectory is concerned about when each part of the must be attained, thus specifying timing. As a result, regardless of when points B and C are reached, the path is the same, while as a trajectory, depending on the velocities and accelerations, points B and C may be reached at different times, creating different trajectories. In this chapter, we are not only concerned about the path a robot takes, but also its velocities and accelerations. There is the general solution equation for high order trajectories;

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_{n-1} t^{n-1} + c_n t^n ,$$

10. PROGRAMMING & SIMULATION

As a beginning in the project, I created some models of robotic arm to work with it in MATLAB. There are lots of CAD file of ABB robots. I have found the CAD file for SolidWorks software and used it to create a MATLAB Simulink model.

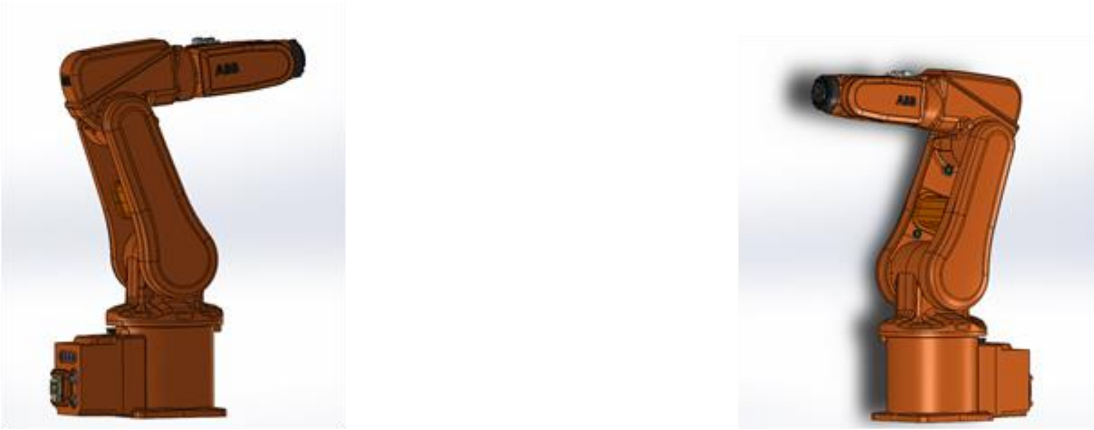

Figure 12: CAD Model of the Robot

This model includes whole body geometry of the robot. For a next step we will import this design into the MATLAB&Simulink to modelling it mathematically.

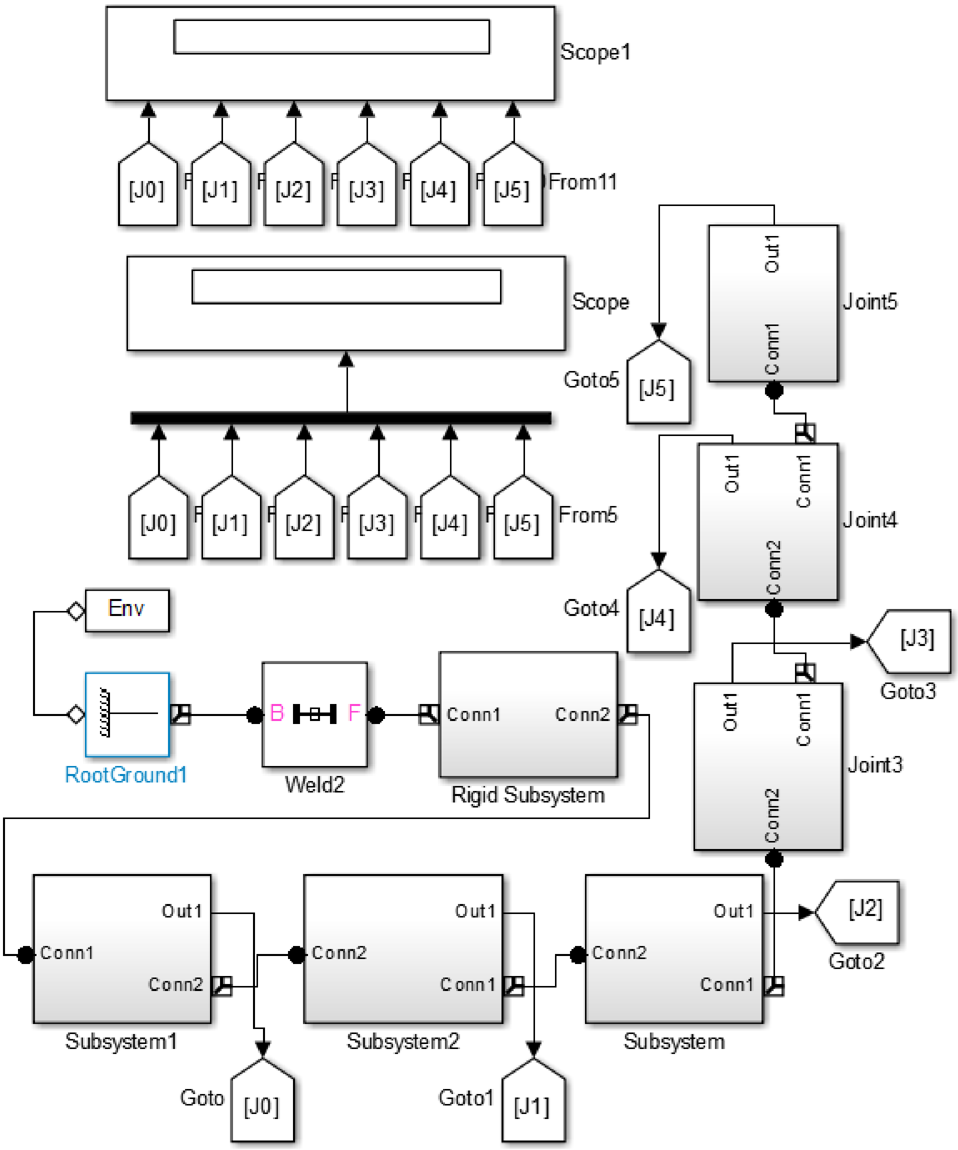

Figure 13: Matlab/Simulink Model of the ABB IRB120 Robotic Arm

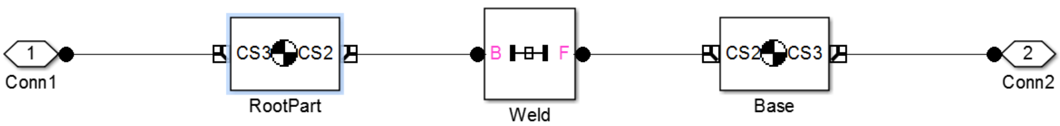

Figure 14: Rigid Subsystem

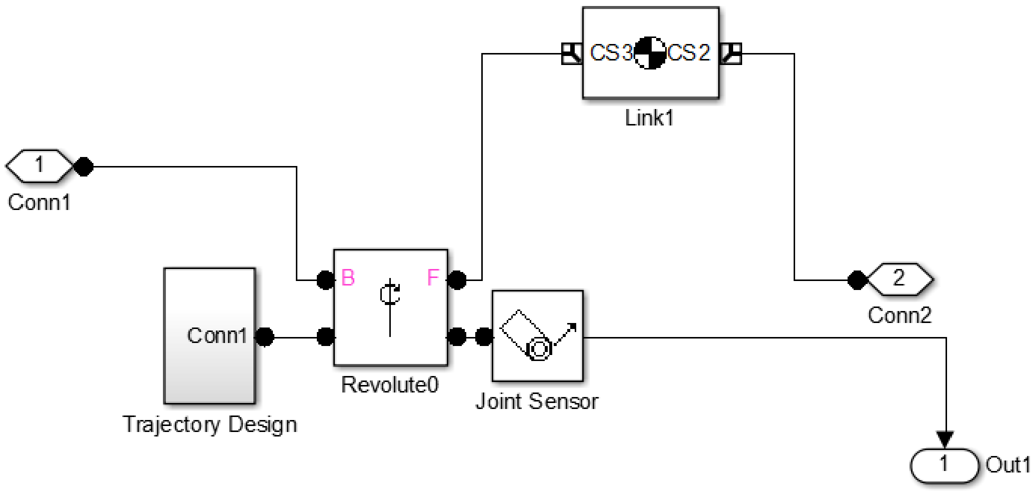

Figure 15: Joint Subsystem

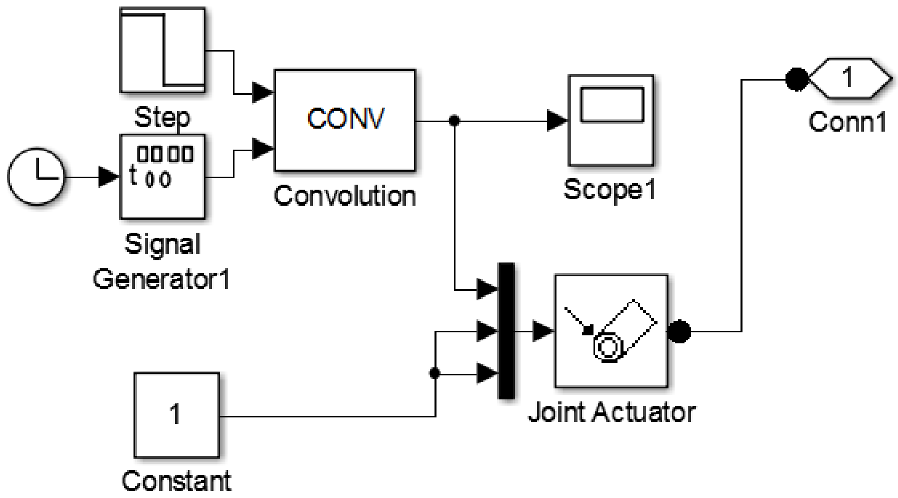

Figure 16: Trajectory Generation Subsystem

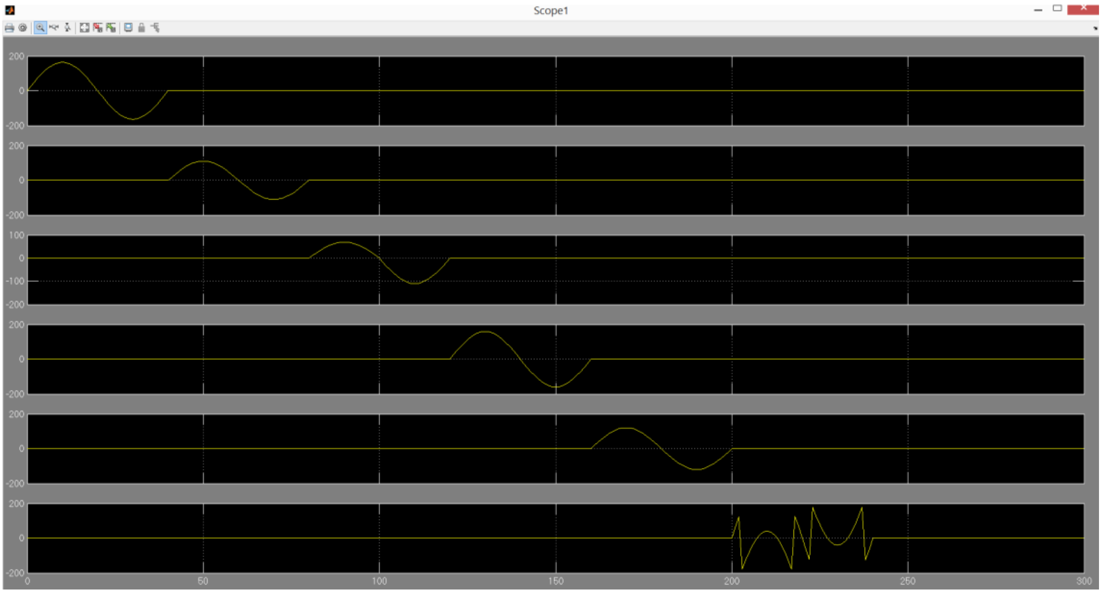

Figure 17: Scope results of all Joints(First Joint at the top, 6th Joint is at the bottom)

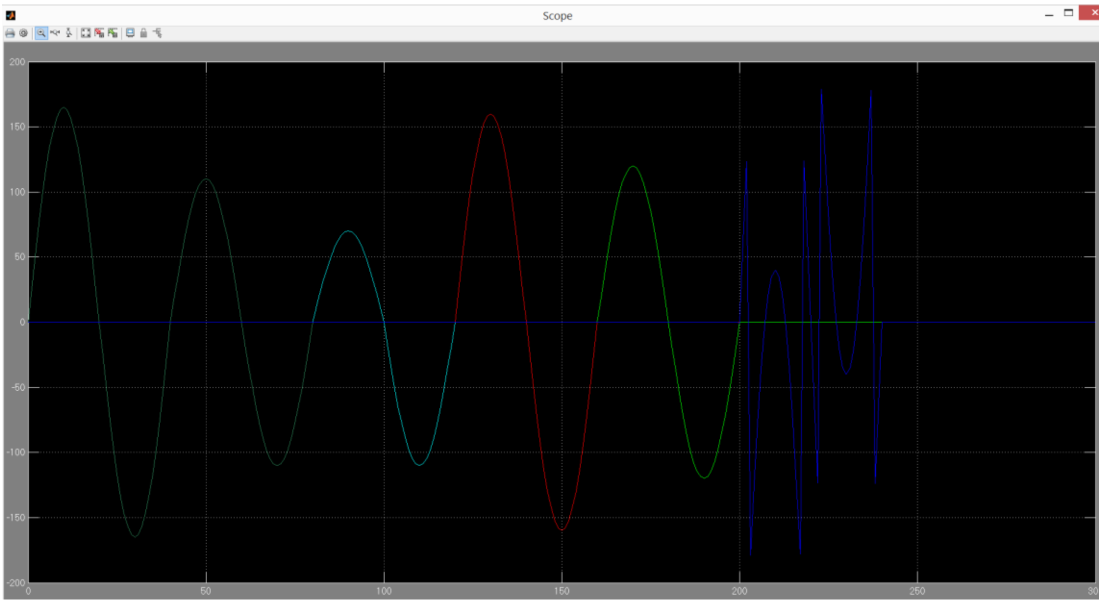

Figure 18: Scope results of all Joints combined together

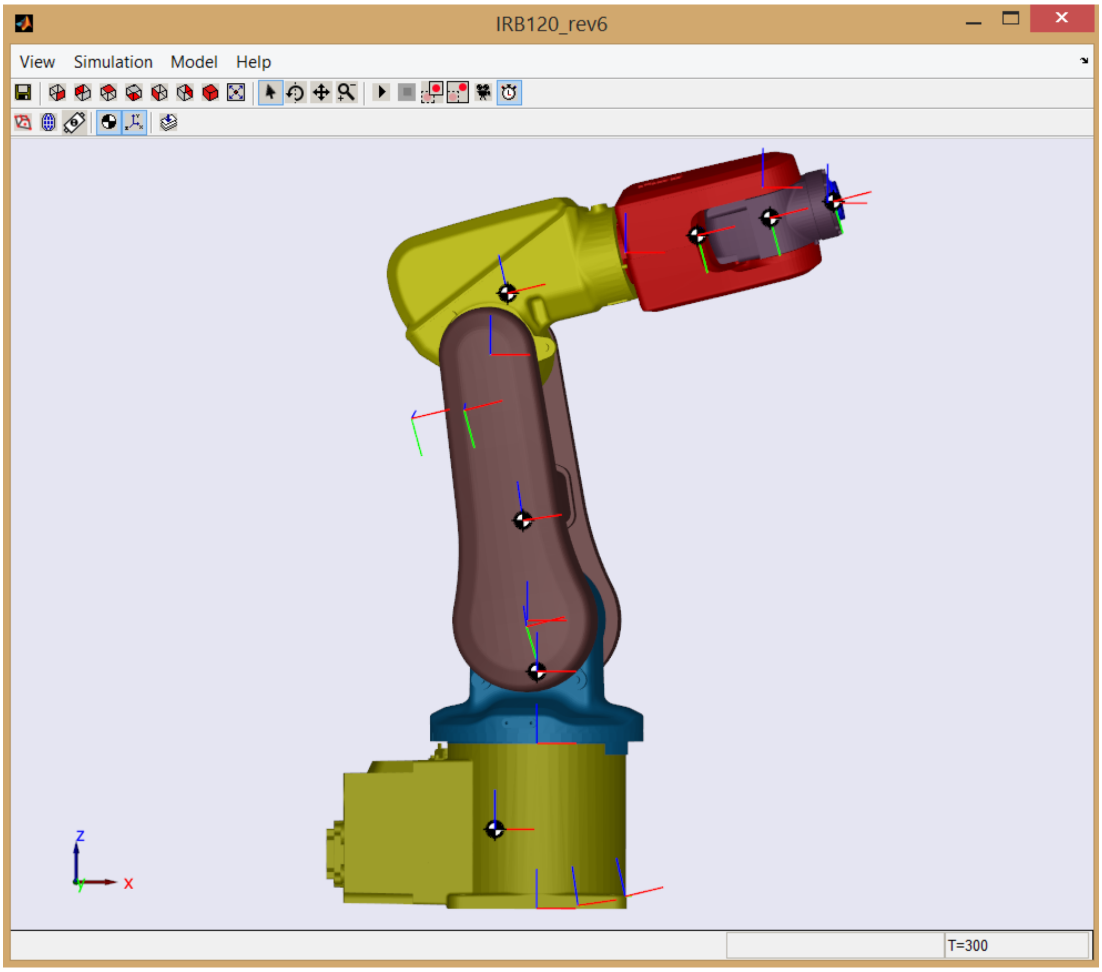

Figure 19: Matlab 3D Model of the Robot

11. MATERIALS & METHODS

The main equipment is ABB IRB 120 (6-DOF) robotic arm will be using in this project. For simulations and a big part of design the MATLAB software will be helpful. In addition this, the Robotics Toolbox for Matlab which has written by Peter Corke, is going to help to make calculations and simulations easily. Of course under the guidance of the RVC Toolbox's manual book.

CITATIONS

[1] - http://www.informatik.uni-konstanz.de/en/edavid/

[2] - http://www.mokafolio.de/#!project=25

[3] - Introduction to Robotics Mechanics and Control - John J. Craig (Pearson)

[4] - http://static.comsol.com/products/multibody/Joint_Picture1_550x309.png

[5] - http://www.mathworks.com/help/releases/R2013b/physmod/sm/mech/ref/disassem_revolute.gif

[6] - http://farm4.staticflickr.com/3494/3257582469_7c41d3dc6c_z.jpg

[7] - http://expo21xx.com/automation21xx/15419_st3_control-equipment/2_p14.jpg

[8] - http://www.societyofrobots.com/images/robot_arm_arm3.jpg

[9] - http://www.toshiba-machine.com/Upload/Product/03c599bcaa.jpg

[10] - http://share.pdfonline.com/4052a0265346463a88eec35c95ca9596/Thesis_santosini_sahu_60603002_images/Thesis_santosini_sahu_6060300224x1.jpg

[11] - http://en.wikipedia.org/wiki/Denavit-Hartenberg_parameters

[12] - http://upload.wikimedia.org/wikipedia/commons/3/3f/Sample_Denavit-Hartenberg_Diagram.png

[13] - http://new.abb.com/products/robotics

[14] - Robot Kinematiği - Dr. Zafer Bingül, Dr. Serdar Küçük (Birsen Yayınevi)

[15] - Robot Dinamiği ve Kontrolü - Dr. Zafer Bingül, Dr. Serdar Küçük (Birsen Yayınevi)