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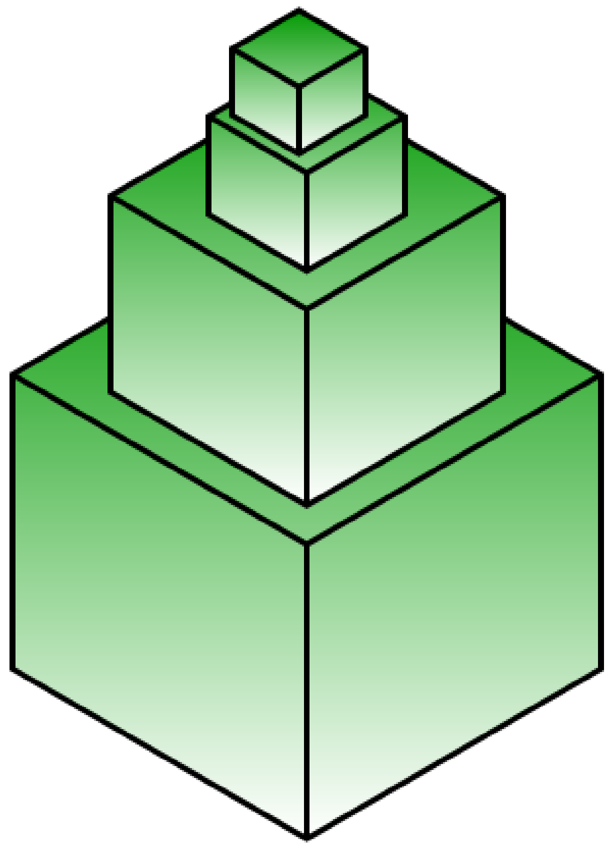
Box Stacking Problem | DP-22

You are given a set of n types of rectangular 3-D boxes, where the i^th box has height h(i), width w(i) and depth d(i) (all real numbers). You want to create a stack of boxes which is as tall as possible, but you can only stack a box on top of another box if the dimensions of the 2-D base of the lower box are each strictly larger than those of the 2-D base of the higher box. Of course, you can rotate a box so that any side functions as its base. It is also allowable to use multiple instances of the same type of box.

Source: http://people.csail.mit.edu/bdean/6.046/dp/. The link also has video for explanation of solution.



Recommended: Please solve it on "*PRACTICE*" first, before moving on to the solution.

The Box Stacking problem is a variation of LIS problem. We need to build a maximum height stack.

Following are the key points to note in the problem statement:
1) A box can be placed on top of another box only if both width and depth of the upper placed box are smaller than width and depth of the lower box respectively.
2) We can rotate boxes such that width is smaller than depth. For example, if there is a box with dimensions {1x2x3} where 1 is height, 2×3 is base, then there can be three possibilities, {1x2x3}, {2x1x3} and {3x1x2}
3) We can use multiple instances of boxes. What it means is, we can have two different rotations of a box as part of our maximum height stack.

Following is the **solution** based on DP solution of LIS problem.

1) Generate all 3 rotations of all boxes. The size of rotation array becomes 3 times the size of original array. For simplicity, we consider depth as always smaller than or equal to width.

2) Sort the above generated 3n boxes in decreasing order of base area.

3) After sorting the boxes, the problem is same as LIS with following optimal substructure property.
MSH(i) = Maximum possible Stack Height with box i at top of stack
MSH(i) = { Max (MSH(j)) + height(i) } where j < i and width(j) > width(i) and depth(j) > depth(i).
If there is no such j then MSH(i) = height(i)

4) To get overall maximum height, we return max(MSH(i)) where 0 < i < n

Following is the implementation of the above solution.

C++

```
/* Dynamic Programming implementation of Box Stacking problem */
#include<stdio.h>
#include<stdlib.h>

/* Representation of a box */
struct Box
{
  // h --> height, w --> width, d --> depth
  int h, w, d;  // for simplicity of solution, always keep w <= d
};

// A utility function to get minimum of two intgers
int min (int x, int y)
{ return (x < y)? x : y; }

// A utility function to get maximum of two intgers
int max (int x, int y)
{ return (x > y)? x : y; }

/* Following function is needed for library function qsort(). We
   use qsort() to sort boxes in decreasing order of base area.
   Refer following link for help of qsort() and compare()
   http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */
int compare (const void *a, const void * b)
{
    return ( (*(Box *)b).d * (*(Box *)b).w ) -
           ( (*(Box *)a).d * (*(Box *)a).w );
}

/* Returns the height of the tallest stack that can be
   formed with give type of boxes */
int maxStackHeight( Box arr[], int n )
{
   /* Create an array of all rotations of given boxes
      For example, for a box {1, 2, 3}, we consider three
      instances{{1, 2, 3}, {2, 1, 3}, {3, 1, 2}} */
```

```
    Box rot[3*n];
    int index = 0;
    for (int i = 0; i < n; i++)
    {
       // Copy the original box
       rot[index].h = arr[i].h;
       rot[index].d = max(arr[i].d, arr[i].w);
       rot[index].w = min(arr[i].d, arr[i].w);
       index++;

       // First rotation of box
       rot[index].h = arr[i].w;
       rot[index].d = max(arr[i].h, arr[i].d);
       rot[index].w = min(arr[i].h, arr[i].d);
       index++;

       // Second rotation of box
       rot[index].h = arr[i].d;
       rot[index].d = max(arr[i].h, arr[i].w);
       rot[index].w = min(arr[i].h, arr[i].w);
       index++;
    }

    // Now the number of boxes is 3n
    n = 3*n;

    /* Sort the array 'rot[]' in non-increasing order
       of base area */
    qsort (rot, n, sizeof(rot[0]), compare);

    // Uncomment following two lines to print all rotations
    // for (int i = 0; i < n; i++ )
    //    printf("%d x %d x %d\n", rot[i].h, rot[i].w, rot[i].d);

    /* Initialize msh values for all indexes
       msh[i] --> Maximum possible Stack Height with box i on top */
    int msh[n];
    for (int i = 0; i < n; i++ )
       msh[i] = rot[i].h;

    /* Compute optimized msh values in bottom up manner */
    for (int i = 1; i < n; i++ )
       for (int j = 0; j < i; j++ )
          if ( rot[i].w < rot[j].w &&
               rot[i].d < rot[j].d &&
               msh[i] < msh[j] + rot[i].h
             )
          {
               msh[i] = msh[j] + rot[i].h;
          }


    /* Pick maximum of all msh values */
    int max = -1;
    for ( int i = 0; i < n; i++ )
       if ( max < msh[i] )
          max = msh[i];

    return max;
}

/* Driver program to test above function */
int main()
{
  Box arr[] = { {4, 6, 7}, {1, 2, 3}, {4, 5, 6}, {10, 12, 32} };
  int n = sizeof(arr)/sizeof(arr[0]);

  printf("The maximum possible height of stack is %d\n",
         maxStackHeight (arr, n) );

  return 0;
}
```

Run on IDE

Java

```
/* Dynamic Programming implementation
of Box Stacking problem in Java*/
import java.util.*;

public class GFG {

    /* Representation of a box */
    static class Box implements Comparable<Box>{

        // h --> height, w --> width,
        // d --> depth
        int h, w, d, area;

        // for simplicity of solution,
        // always keep w <= d

        /*Constructor to initialise object*/
        public Box(int h, int w, int d) {
            this.h = h;
            this.w = w;
            this.d = d;
        }

        /*To sort the box array on the basis
        of area in decreasing order of area */
        @Override
        public int compareTo(Box o) {
            return o.area-this.area;
        }
    }
```

```
    /* Returns the height of the tallest
    stack that can be formed with give
    type of boxes */
    static int maxStackHeight( Box arr[], int n){

        Box[] rot = new Box[n*3];

        /* New Array of boxes is created -
        considering all 3 possible rotations,
        with width always greater than equal
        to width */
        for(int i = 0;i < n;i++){
            Box box = arr[i];

            /* Orignal Box*/
            rot[3*i] = new Box(box.h, Math.max(box.w,box.d),
                                    Math.min(box.w,box.d));

            /* First rotation of box*/
            rot[3*i + 1] = new Box(box.w, Math.max(box.h,box.d),
                                       Math.min(box.h,box.d));

            /* Second rotation of box*/
            rot[3*i + 2] = new Box(box.d, Math.max(box.w,box.h),
                                       Math.min(box.w,box.h));
        }

        /* Calculating base area of
        each of the boxes.*/
        for(int i = 0; i < rot.length; i++)
            rot[i].area = rot[i].w * rot[i].d;

        /* Sorting the Boxes on the bases
        of Area in non Increasing order.*/
        Arrays.sort(rot);

        int count = 3 * n;

        /* Initialize msh values for all
        indexes
        msh[i] --> Maximum possible Stack Height
                   with box i on top */
        int[]msh = new int[count];
        for (int i = 0; i < count; i++ )
            msh[i] = rot[i].h;

        /* Computing optimized msh[]
        values in bottom up manner */
        for(int i = 0; i < count; i++){
            msh[i] = 0;
            Box box = rot[i];
            int val = 0;

            for(int j = 0; j < i; j++){
                Box prevBox = rot[j];
                if(box.w < prevBox.w && box.d < prevBox.d){
                    val = Math.max(val, msh[j]);
                }
            }
            msh[i] = val + box.h;
        }

        int max = -1;

        /* Pick maximum of all msh values */
        for(int i = 0; i < count; i++){
            max = Math.max(max, msh[i]);
        }

        return max;
    }

    /* Driver program to test above function */
    public static void main(String[] args) {

        Box[] arr = new Box[4];
        arr[0] = new Box(4, 6, 7);
        arr[1] = new Box(1, 2, 3);
        arr[2] = new Box(4, 5, 6);
        arr[3] = new Box(10, 12, 32);

        System.out.println("The maximum possible "+
                           "height of stack is " +
                           maxStackHeight(arr,4));
    }
}

// This code is contributed by Divyam
```

Run on IDE

Output:

```
The maximum possible height of stack is 60
```

In the above program, given input boxes are {4, 6, 7}, {1, 2, 3}, {4, 5, 6}, {10, 12, 32}. Following are all rotations of the boxes in decreasing order of base area.

```
   10 x 12 x 32
   12 x 10 x 32
   32 x 10 x 12
   4 x 6 x 7
```

```
4 x 5 x 6
6 x 4 x 7
5 x 4 x 6
7 x 4 x 6
6 x 4 x 5
1 x 2 x 3
2 x 1 x 3
3 x 1 x 2
```

The height 60 is obtained by boxes { {**3**, 1, 2}, {**1**, 2, 3}, {**6**, 4, 5}, {**4**, 5, 6}, {**4**, 6, 7}, {**32**, 10, 12}, {**10**, 12, 32}}

Time Complexity: O(n^2)
Auxiliary Space: O(n)

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