Computability and Complexity
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Homework 9

Exercise 1

Solution:
Since A is a **finite** nonempty set, that means that the elements are enumerable.

Assume that $A \subseteq \mathcal{A}$, where $\mathcal{A} = \{a_0, a_1 ...\}$. Assume that $\mathcal{A}$ has the property $|\mathcal{A}| = |\mathbb{N}|$. In the this case the elements in A can be enumerated in the following way:

$$
\begin{aligned}
&a_0 \rightarrow < a_0 >= 0 \\
&a_1 \rightarrow < a_1 >= 1 \\
&a_2 \rightarrow < a_2 >= 10 \\
&a_3 \rightarrow < a_3 >= 11 \\
&\quad ... \\
&a_n \rightarrow < a_n >= \text{[binary representation of n]}
\end{aligned}
$$

A set A can then be encoded into the binary representation of $|A|$ followed by # which is followed by the encoded elements in A, separated by #. For example $A = \{1, 3\}$:

$$< A >= 10\#1\#11\#$$

Regarding the function s, it is not necessary to encode it, but since we are using the power of countable sets, we can encode $< s(a) >=< a >$, but this should be figured out by the TM algorithm.

Exercise 2

Solution:
To prove that PARTITION is in NP using a coding scheme consider the following:

$$A = \{1, 2, 3\}$$

Consider using the powerset of A:

$$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

Then, using a 2-tape TM:
First tape storing the powerset of A
Second tape storing all computations

$$\implies |P(A)| = 2^{|A|} \; NP \leq EXP$$