Homework 6

Exercise 1
Solution:
I will be using the axioms and inference rules from section 7.1 on page 48 in the lecture notes.

call-by-value:

- $$K(KI(KI))y \to KIy \text{ (A3)}$$
  $$KIy \to I \text{ (A3)}$$
  $$K(KI(KI))y \to I \text{ (I3)}$$

- $$K(KI(KIy))y \to K(KII)y \text{ (A3)}$$
  $$K(KII)y \to KIy \text{ (A3)}$$
  $$KIy \to I \text{ (A3)}$$
  $$K(KI(KIy))y \to I \text{ (A3)}$$

- $$SK(KK(KIy))y \to SK(KKI)y \text{ (A3)}$$
  $$SK(KKI)y \to SKKy \text{ (A3)}$$
  $$SKKy \to SK \text{ (A3)}$$

  This one is a dead end.

call-by-name:

- $$K(KI(KI))y \to KI(KI) \text{ (A3)}$$
  $$KI(KI) \to I \text{ (A3)}$$
  $$K(KI(KI))y \to I \text{ (I3)}$$

- $$K(KI(KIy))y \to KI(KIy) \text{ (A3)}$$
  $$KI(KIy) \to I \text{ (A3)}$$
  $$K(KI(KIy))y \to I \text{ (I3)}$$

- $$SK(KK(KIy))y \to Ky((KK(KIy))y) \text{ (A2)}$$
  $$Ky((KK(KIy))y) \to y \text{ (A3)}$$
  $$SK(KK(KIy))y \to y \text{ (I3)}$$

Exercise 2
Solution:

(a) $Cxy = y$, using **call-by-name** evaluation:

$$SKxy \rightarrow Ky(xy) \text{ (A2)}$$
$$Ky(xy) \rightarrow y \text{ (A3)}$$
$$\Rightarrow C = SK$$

(b) $Cxyz = y$ using **call-by-name** evaluation:

$$KKxyz \rightarrow Kyz \text{ (A3)}$$
$$Kyz \rightarrow y \text{ (A3)}$$
$$\Rightarrow C = KK$$

(c) $Cxyz = x$ using **call-by-name** evaluation:

$$S(KK)Kxyz \rightarrow KKx(Kx)yz \text{ (A2)}$$
$$KKx(Kx)yz \rightarrow K(Kx)yz \text{ (A3)}$$
$$K(Kx)yz \rightarrow Kxz \text{ (A3)}$$
$$Kxz \rightarrow x \text{ (A3)}$$
$$\Rightarrow C = S(KK)K$$

Exercise 3
Solution:
The following solution was done with the help of Ana Ambroladze.

$t_2[x, y] = y$
$t_2[x, y] = t_1[x]y = y$
We can see that it is not composite so:
$\implies t_1[x] = I$
$t_1[x] = t_0[\,]x$
And as that is not composite:
$\implies t_0[\,] = KI$

$t_2[x, y] = t_1[x]y = t_0[\,]xy = KIxy = y$
$\implies C = KI$