Computability and Complexity
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Homework 2

Exercise 1

Solution:

The idea is to use 2-tape TM: the first tape will be read only and it will be the input tape with the word w written on it, while the second tape will be a binary counter. This ”binary counter” will represent a number in binary form.

How will the increments happen? Well whenever you increment 1, first replace all leading 1s to 0s and write the first 0 you encounter to 1. For example if we have 1110100 on the tape and we want to increment one, it turns into 0001100. Take a look at the next page for a TM which does that (apologize in advance for the quality of the picture, please don't judge us).

The transitions which go from $q0$ to $q1$ and $q0$ to $q3$ are transitions made when there are still 1s in the input tape, while the transition $q0$ to $q2$ is when the decision comes. All numbers which are powers of 2 have this form $0 * 1$. Starting from $q2$ that is being decided: if the number on the write tape is in the form 0*1, then the input word w is of length power of 2.

Exercise 2

Solution:

Solution coppied from Brian Sherif.

(a) Are the functions f(n) = exp(n) and g(n) = exp(2n) polynomially related?
Yes, f(n) and g(n) are polynomially related. After reading, we can find that there exist a p such that $f(n) <= p(g(n))$ while also being $g(n) <= p(f(n))$.
if we consider our p(x)=x^2 we find the ability to make $p(f(n)) == g(n)$ and $f(n) <= p(g(n))$.

Showing indeed that they are both polynomially related

(b)What about f(n) = exp(n) and g(n) = exp(n^2)? When we consider the rate of growth for both the exp(n) and exp(n^2). We can calculate the growth rate by considering the slope (calculating its derivative. The derivative of exp(n) is exp(n) while the exp(n^2) will be equal to 2n(exp(n^2)) showing a much more aggressive growth rate.
To prove it further we can attempt to prove that there exist there exist no $'p' f(n) <= p(g(n))$ while also being $g(n) <= p(f(n))$.
We can attempt this by first trying to equate g(n) and f(n). As lim of n to infinity we can make exp(n^2) = exp(n) when Px = x^2 then P(f(n)) = g(n) as n goes to infinity. But when reversed; that is not the same.
On the other hand; trying to equate g(n) to f(n) by making P(x)=$x^{0.25}$ will lead to an undesired equation if we apply it to f(n) hence showing that f(n) = exp(n) and g(n) = exp(n^2) are not polynomially related.