Computability and Complexity
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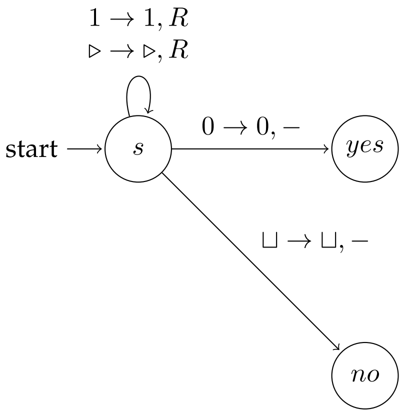
Homework 3

Exercise 1

Solution:



p ∈ K	$\sigma \in \Sigma$	δ(p, σ)
s	0	(yes, 0, -)
s	1	(s, 1, $\rightarrow$)
s	$\sqcup$	(no , $\sqcup$, -)
s	$\triangleright$	(s, $\triangleright$, $\rightarrow$)

This language goes through the tape and checks if there is a 0 on the tape, if there is then it goes to the accepting state *yes*, and if there is no 0 on the tape the it reaches the $\sqcup$ and it goes to the rejecting state *no*.

The RAM program would look like this:

1 read

2 if c(0) = 0 goto 5

3 if c(0) = $\sqcup$ goto 6

4 goto 1

5 store 1 goto 7

6 store 0

7 end

Exercise 2

Solution:

For the sake of beautiful mathematical notation, let us define the function $plus2$ as π_2.

We already know that the successor function σ is a primitive recursive function. Let's use rule (4) from the lecture notes, make a setting and combine it with the successor function.

Let $r = m = 1$, then we have the functions $f : N^r \to N$, $g : N^m \to N$ and $h : N^r \to N$. If we set $f = g = \sigma$ and $h = \pi_2$ and using rule (4) from the lecture notes (substituting) we have:

$$\begin{aligned} h(n) &= f(g_1(n), ..., g_r(n)) \\ \Rightarrow \pi_2(n) &= \sigma(\sigma(n)) \\ \Rightarrow \pi_2(n) &= n + 2 \end{aligned}$$

Exercise 3

Solution:
Let's define our function more "mathematically":

$$\pi_{-1}(n) = \begin{cases} 0 & n = 0 \\ n-1 & \text{otherwise} \end{cases}$$

We already know that the projection function p_i^k ("rule" 3 from lecture notes) and the null function **O** ("rule" 1 from lecture notes) are primitive recursive functions. We can use those 2 functions along with the (5) "rule" from the lecture notes to form a proof that pi_{-1} is also primitive recursive. Observe:

If we set $r = 0$, then we have the functions $f : N^r \to N$, $g : N^{r+2} \to N$ and $h : N^{r+1} \to N$. According to "rule" (5) from the lecture notes, we have:

$$h(0) = f() \tag{1}$$
$$h(n+1) = g(n, h(n)) \tag{2}$$

Before we go any further, we can write (2) from above as $h(n) = g(n-1, h(n-1))$. Now we can make the setting:

$$h = \pi_{-1}$$
$$g(n,m) = p_1^2(n,m)$$
$$f() = \mathbf{O}()$$

Once we substitute, we have:

$$\pi_{-1}(n) = \begin{cases} \mathbf{O}() & n = 0 \\ p_1^2(n-1, \pi_{-1}(n-1)) & \text{otherwise} \end{cases}$$