Computer Graphics
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Homework 1

Exercise 1.1

Solution:

1. Every Cartesian coordinate system is orthogonal. Every orthogonal coordinate system is parallel. **True**
2. Non-uniform scaling transformation is linear. **True**
3. Order of transformations does not matter if one uses homogeneous coordinates. **False**
4. Perspective projection in OpenGL yields vertices coordinates in (x, y) such that x $\in \langle 0,$ screen_width -1$\rangle$, x $\in \langle 0,$ screen_height - 1$\rangle$. **True**
5. OpenGL functions starting with glu prefix are hardware-specific. **False**

Exercise 1.2

Solution:

a) The triangle is scaled with factors 3 and -1 in the x- and y-coordinate, respectively, rotated (clockwise) by 90° around the z-coordinate, and translated by distance bx = 2 units in the direction of the x-coordinate. The transformations are executed in the given order.

- Derive the transformation matrix of each transformation step in homogeneous coordinates.

$$S_z(3) = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_y(-1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\implies \theta = -\pi/2$$

$$R_y(-\pi/2) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$b_x = 2$

$$T_x(b_x) = \begin{bmatrix} 1 & 0 & 0 & b_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_x(2) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

– Compute the combined transformation matrix in homogeneous coordinates.

$$T_x(2)R_y(-\pi/2)S_y(-1)S_x(3) =$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -1 & 0 & 2 \\ -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

– Apply the combined transformation matrix to triangle.

$$= \begin{bmatrix} 0 & -1 & 0 & 2 \\ -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 0 & 0 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 2 & 1 \\ -9 & -6 & -3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

b) The triangle is projected into the given image plane using perspective projections.

– Compute the projection matrix A in homogeneous coordinates for the given example. Leave the depth-related components a = b = 1.

$$A = \begin{bmatrix} h & 0 & 0 & 0 \\ 0 & h & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

h = 1; a = 1; b = 1

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

– Apply the projection matrix A to the triangle and give the results to 2-D Cartesian screen coordinates, with the origin of the screen at (0, 0, 1). Remember that a projection takes two stages!

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ -9 & -6 & -3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 2 & 1 \\ -9 & -6 & -3 \\ 3 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 2 & 2 & 1 \\ -9 & -6 & -3 \\ 3 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 & 0.5 \\ -4.5 & -3 & -1.5 \\ 1.5 & 1.5 & 1.5 \\ 1 & 1 & 1 \end{bmatrix}$$

$$p_1 = (1, -4.5)$$
$$p_2 = (1, -3)$$
$$p_3 = (0.5, -1, 5)$$

According to these coordinates we won't be able to see the triangle on the defined screen.

Exercise 1.3

Solution:

- Compute matrix H_{ry} which will allow you to apply a rotation about the y-axis by angle $\theta = 30°$ and matrix T_x which will allow you to translate the cube along the x-axis by $b_x = 0.5$ units in homogeneous coordinates.
  The vertices of the 3D cube:

$$\langle(-1,1,-1),(1,1,-1),(1,-1,-1),(-1,-1,-1),(-1,1,1),(1,1,1),(1,-1,1),(-1,-1,1)\rangle$$

$$A = \begin{bmatrix} -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$H_{ry} = \begin{bmatrix} cos\theta & 0 & sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta & 0 & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\theta = \pi/6$$

$$H_{ry} = \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  Computing T_x translating along the x axis by $b_x = 0.5$

$$T_x = \begin{bmatrix} 1 & 0 & 0 & b_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Apply the combined transformation matrix to the cube and compute the new vertex coordinates.

$$[T_x][h_ry] = \begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_x][H_{ry}][A] = \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -0.866 & 0.866 & -0.866 & -1.366 & 0.134 & 1.866 & 1.866 & -0.134 & 0.5 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 0 \\ -0.366 & -1.366 & -1.366 & -0.366 & 1.366 & 0.366 & 0.366 & 1.366 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- Assuming $r = -l$ and $t = -b$ pick appropriate camera frustrum parameters r, l, t, b, f and n to complete the matrix and ensure that all points of the cube are in the frustrum ($\equiv$ they will remain in the unit cube after the projection). Remark: frustrum in case of orthographic projection is much simpler than in case of perspective projection!

$$P_{\perp} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & 0 & 0 & 0 \\ 0 & \frac{1}{t} & 0 & 0 \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Assuming the same camera position as in the previous exercise, carry out the projection of the transformed cube's vertices.

- If the cube was to be translated along the z-axis by 1 unit towards the camera, would you have to update your frustrum parameters r, l, t and b? Explain your answer.