Homework 1, Computer Graphics 2019

Dr. Szymon Krupiński, Jacobs University Bremen

Handed out 14.03.2019, due 22.03.2019 at 11:15)

This is a theoretical assignment, to be solved without using programming (although, of course, you can use a program to verify your results!). Please upload this homework (in pdf - generated in word, latex or simply scanned) using the assignment submission on moodle (`http://moodle.jacobs-university.de`) of our course. In case of technical problem with the platform, notify us and send it to the TAs (m.thanasi@jacobs-university.de, muh.hassan@jacobs-university.de) **before** the deadline.

Problem 1.1 *Computer Graphics (Alternative) Facts* (5 points)

Mark the following statements True (T) or False (F)

1	Every Cartesian coordinate system is orthogonal. Every orthogonal coordinate system is parallel	T	F
2	Non-uniform scaling transformation is linear	T	F
3	Order of transformations does not matter if one uses homogeneous coordinates	T	F
4	Perspective projection in OpenGL yields vertices coordinates in (x, y) such that $x \in \langle 0, \text{screen\_width} - 1\rangle, x \in \langle 0, \text{screen\_height} - 1\rangle$	T	F
5	OpenGL functions starting with `glu` prefix are hardware-specific	T	F

Problem 1.2 *Geometry transformations* (10 points)

We are given a triangle with vertices $p_1 = (3,\ 0,\ 2)$, $p_2 = (2,\ 0,\ 2)$, and $p_3 = (1,\ 1,\ 2)$ expressed in 3-D Cartesian coordinates. Let $v = (0,\ 0,\ 0)$ be the camera origin. Let the screen be the rectangle with the following vertices: $v_1 = (-1,\ -1,\ 1)$, $v_2 = (1,\ -1,\ 1)$, $v_3 = (1,\ 1,\ 1)$, and $v_4 = (-1,\ 1,\ 1)$.

(a) The triangle is scaled with factors 3 and -1 in the x- and y-coordinate, respectively, rotated (clockwise) by 90° around the z-coordinate, and translated by distance b_x =2 units in the direction of the x-coordinate. The transformations are executed in the given order.

- Derive the transformation matrix of each transformation step in homogeneous coordinates.
- Compute the combined transformation matrix in homogeneous coordinates.
- Apply the combined transformation matrix to triangle.

(b) The triangle is projected into the given image plane using perspective projection.

- Compute the projection matrix A in homogeneous coordinates for the given example. Leave the depth-related components $a = b$ =1.
- Apply the projection matrix A to the triangle and give the results to 2-D Cartesian screen coordinates, with the origin of the screen at (0,0,1). Remember that a projection takes two stages!

Problem 1.3 *Parallel projection* (10 points)

Recall the demo cube used on blackboard during the first lectures, with the eight corners at every p =(x,y,z) with $x, y, z \in -1, 1$.

- Compute matrix H_{ry} which will allow you to apply a rotation about the y-axis by angle $\theta = 30°$ and matrix T_x which will allow you to translate the cube along the x-axis by $b_x = 0.5$ units in homogeneous coordinates.
- Apply the combined transformation matrix to the cube and compute the new vertex coordinates.

Ortographic projection (visualised in Fig. 1) is expressed in the follwing way in OpenGL:

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{(f-n)} & -\frac{f+n}{(f-n)} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

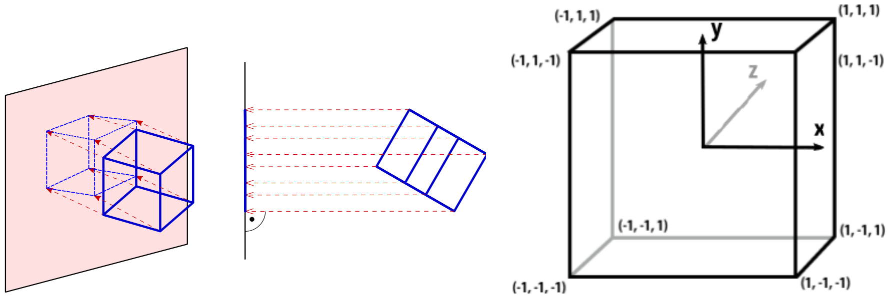

Figure 1: Orthographic projection of a cube visualised and the demo cube.

- Assuming $r = -l$ and $t = -b$ pick appropriate camera frustrum parameters r,l,t,b,f and n to complete the matrix and ensure that all points of the cube are in the frustrum ($\equiv$ they will remain in the unit cube after the projection). Remark: frustrum in case of orthographic projection is much simpler than in case of perspective projection!
- Assuming the same camera position as in the previous exercise, carry out the projection of the transformed cube's vertices.
- If the cube was to be translated along the z-axis by 1 unit towards the camera, would you have to update your frustrum parameters r,l,t and b? Explain your answer.