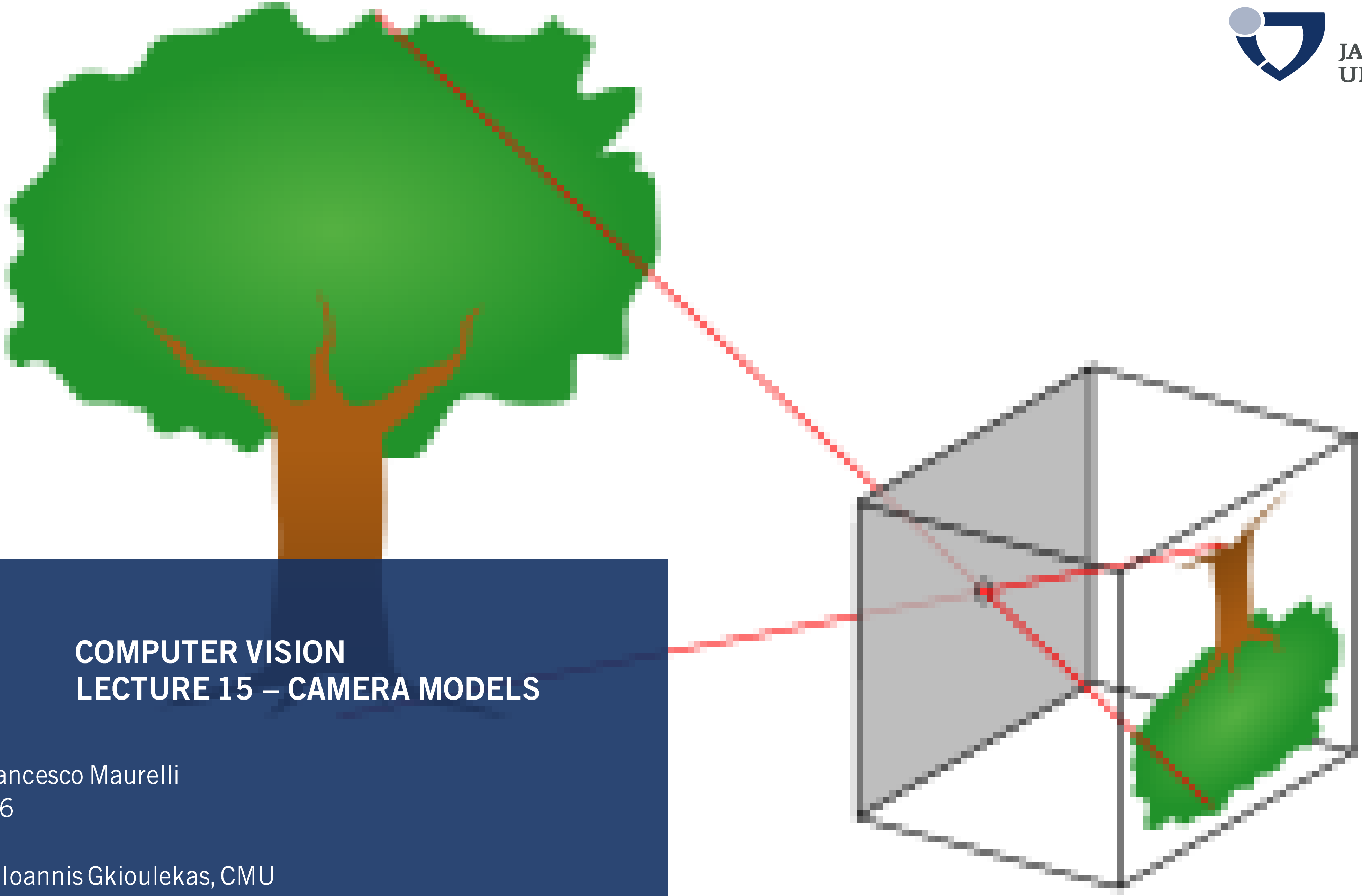


JACOBS UNIVERSITY

COMPUTER VISION
LECTURE 15 – CAMERA MODELS

Prof. Dr. Francesco Maurelli
2018-10-26

Courtesy of Ioannis Gkioulekas, CMU
Fei-Fei Li, Stanford

Overview of today's lecture

- Some motivational imaging experiments.
- Pinhole camera.
- Accidental pinholes.
- Camera matrix.
- Perspective.
- Other camera models.
- Pose estimation.

… and an object we like to photograph

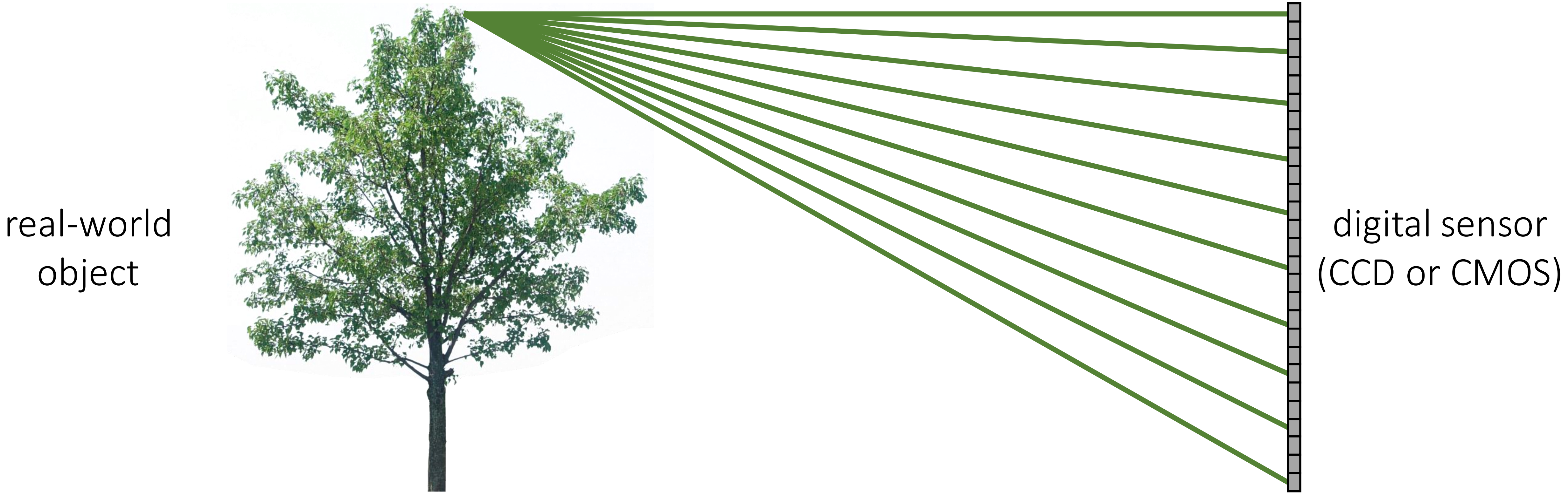
real-world
object

digital sensor
(CCD or CMOS)

What would an image taken like this look like?

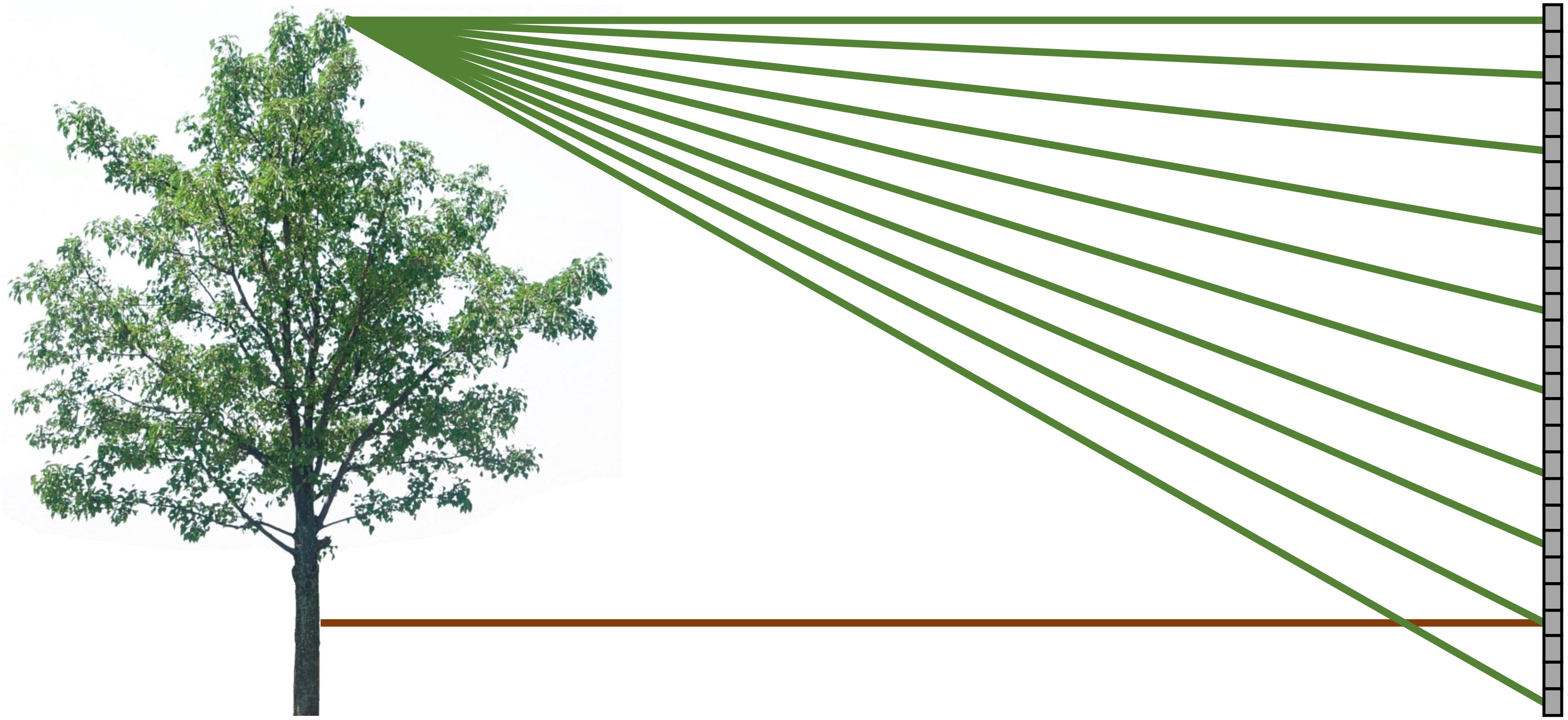
Bare-sensor imaging

Bare-sensor imaging



Bare-sensor imaging

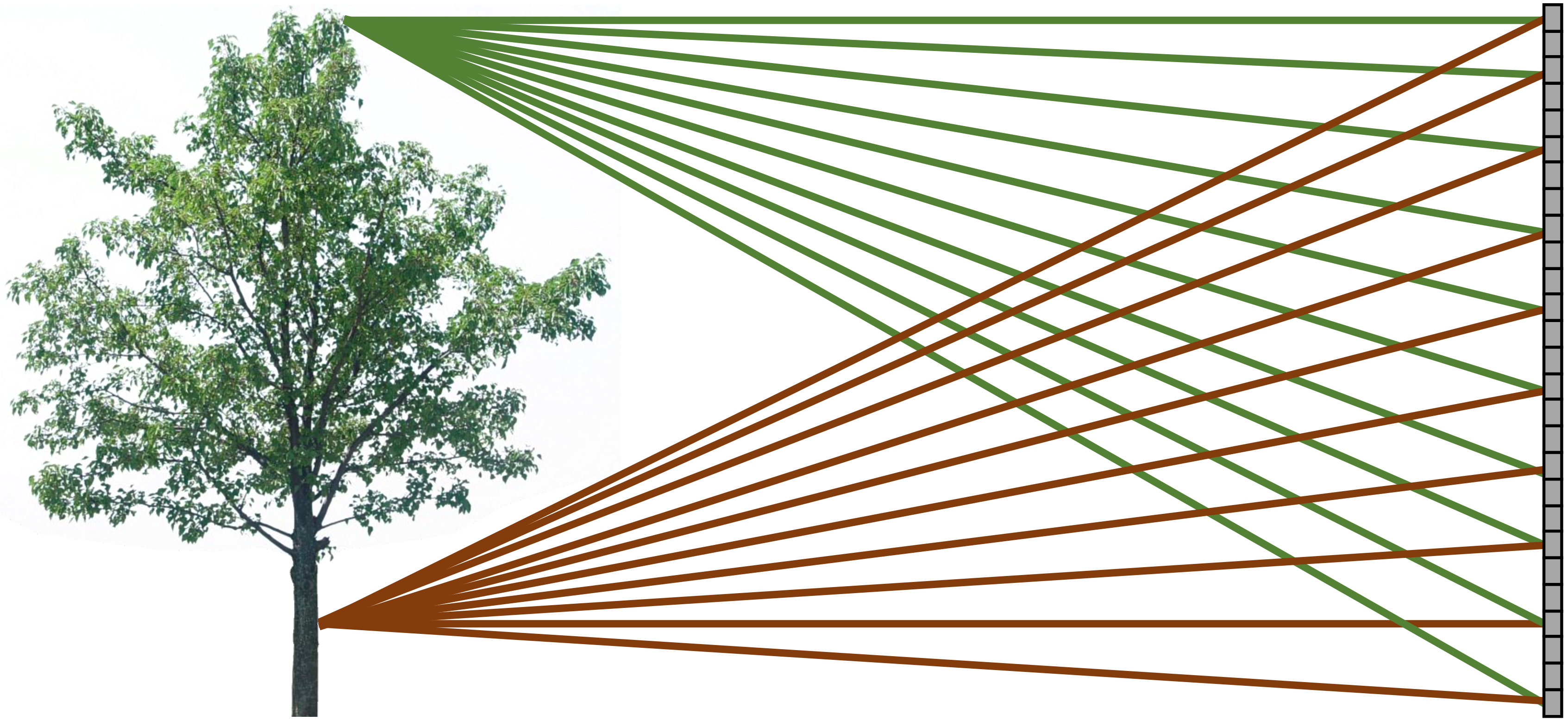
real-world object



digital sensor (CCD or CMOS)

Bare-sensor imaging

real-world object



digital sensor (CCD or CMOS)

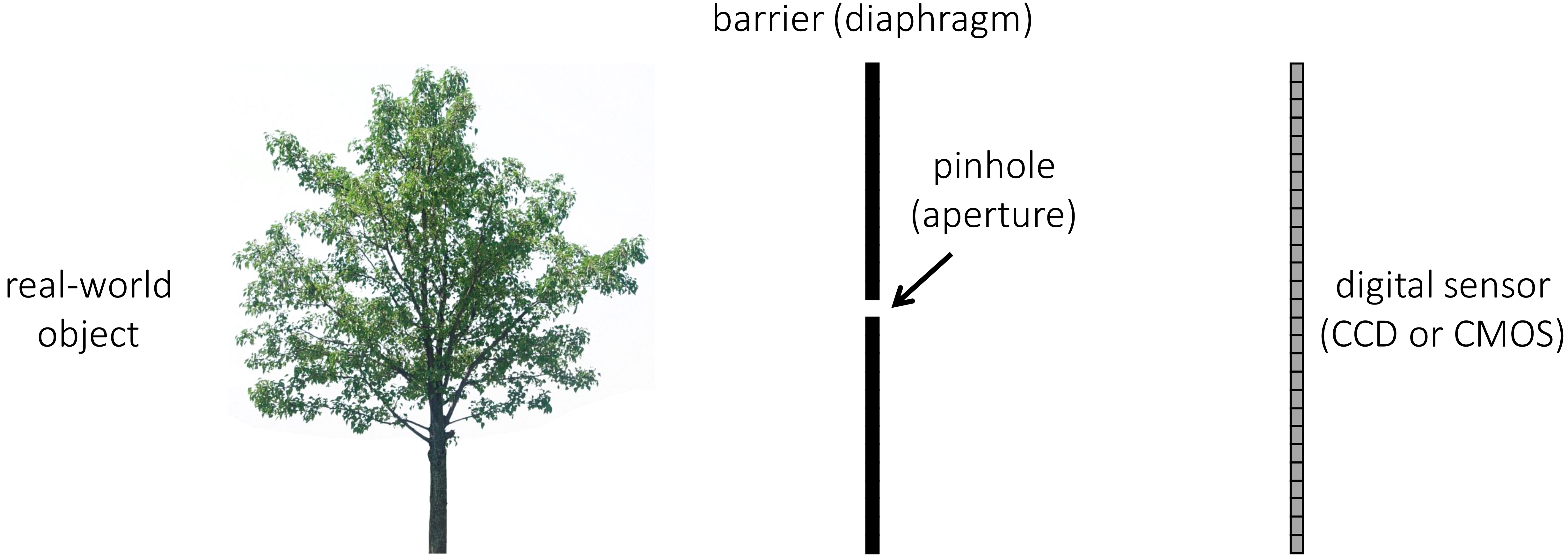
All scene points contribute to all sensor pixels

What does the image on the sensor look like?

Bare-sensor imaging

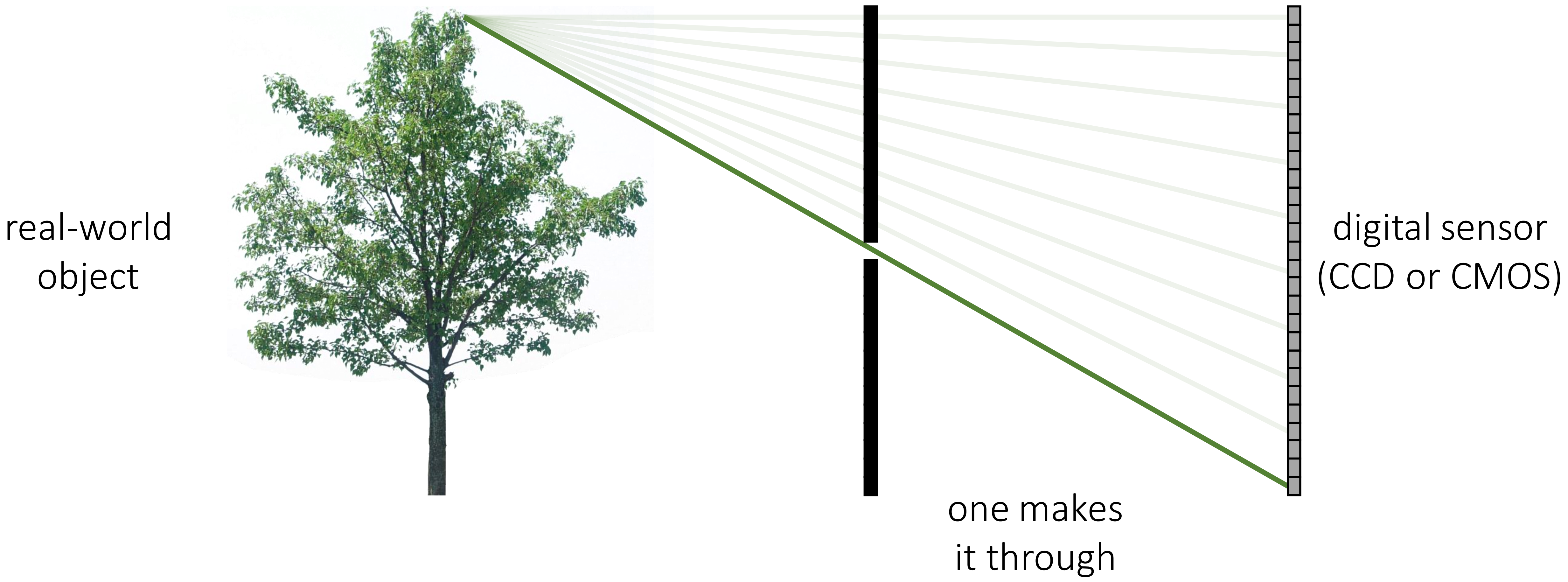
All scene points contribute to all sensor pixels

Let’s add something to this scene

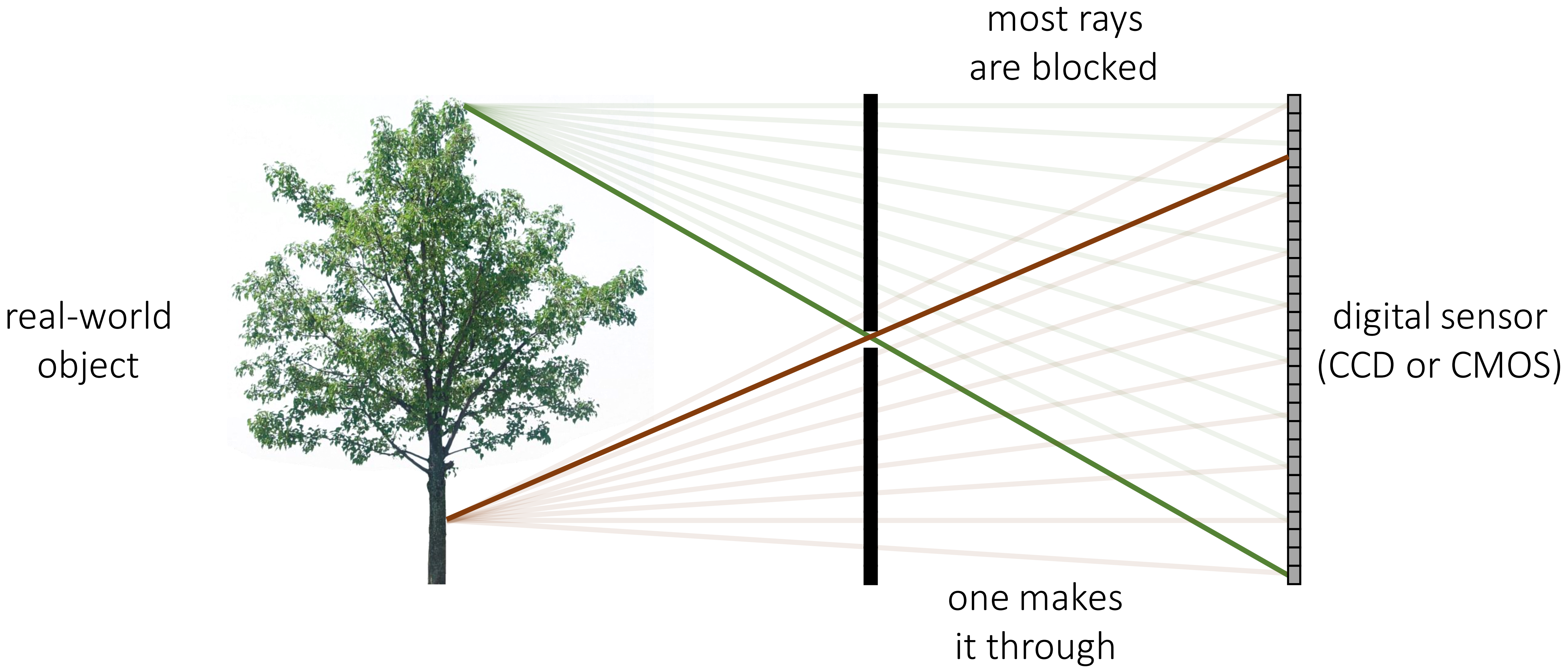


What would an image taken like this look like?

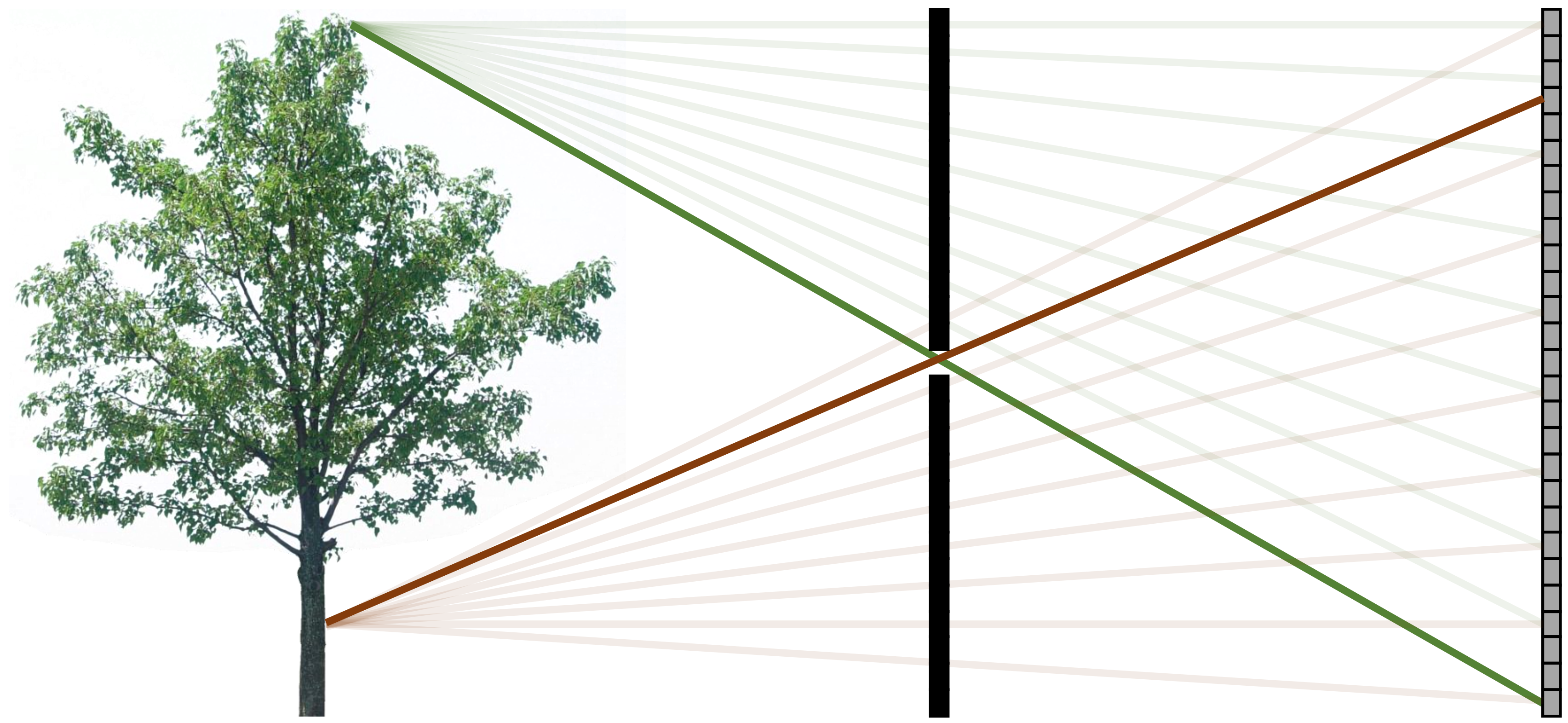
Pinhole imaging



Pinhole imaging



Pinhole imaging

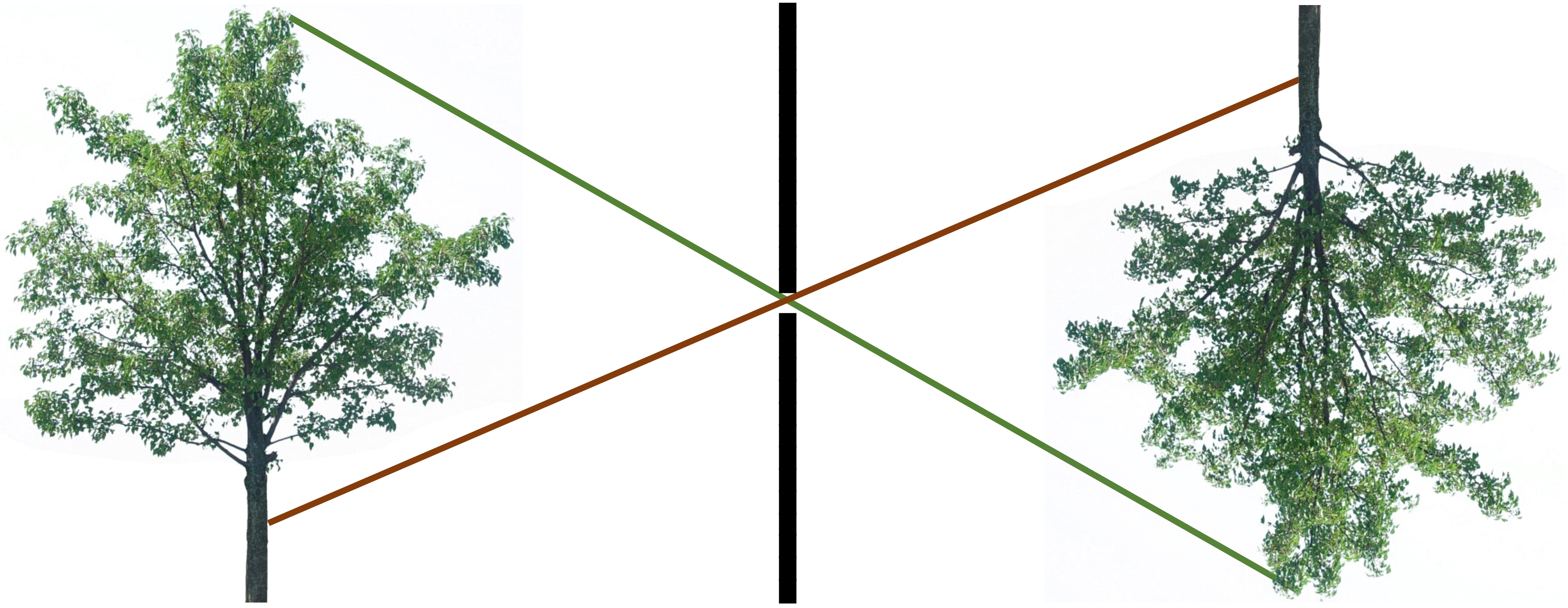


Each scene point contributes to only one sensor pixel

What does the image on the sensor look like?

Pinhole imaging

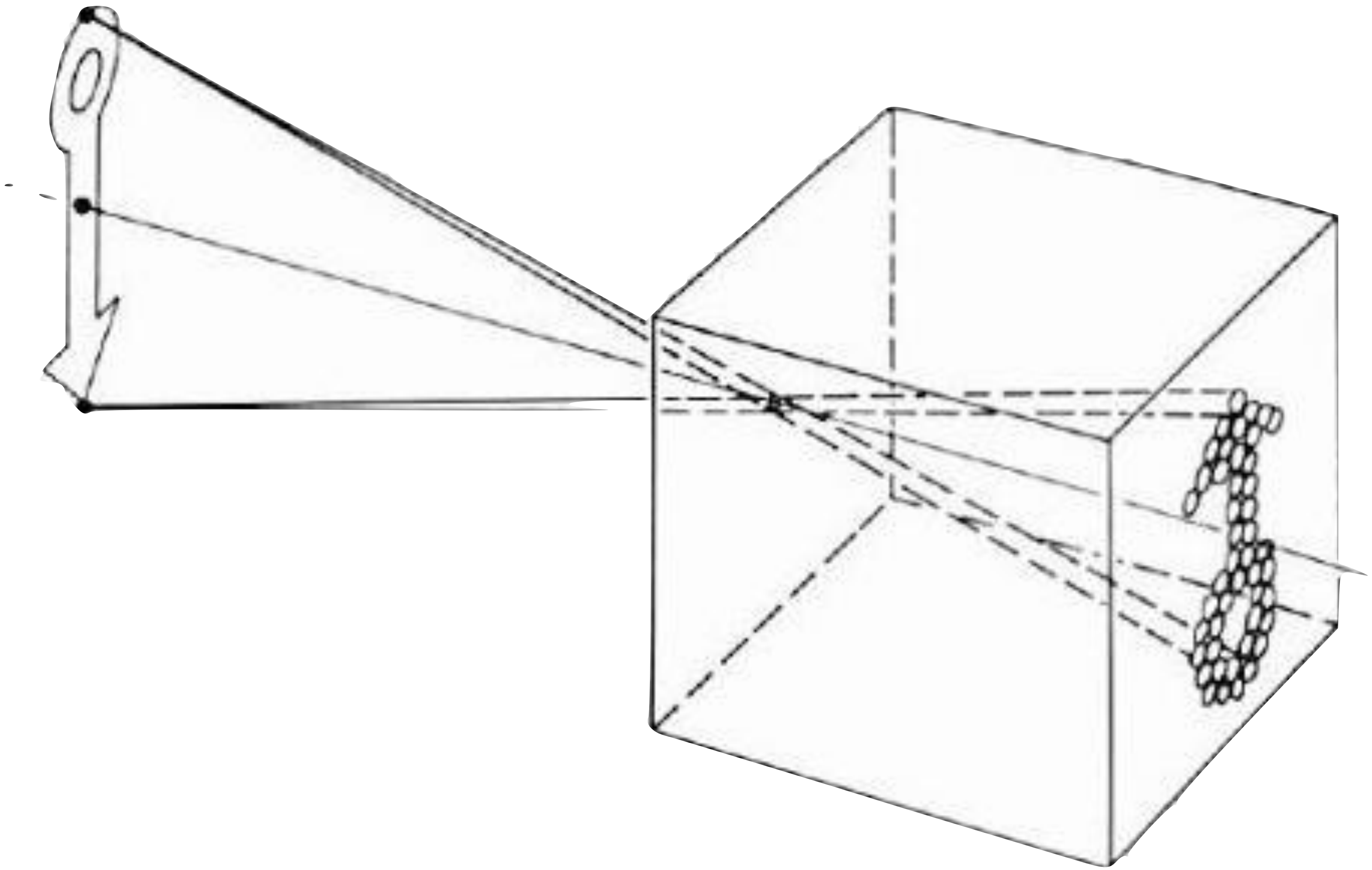
real-world object



copy of real-world object
(inverted and scaled)

Pinhole camera

Pinhole camera a.k.a. camera obscura

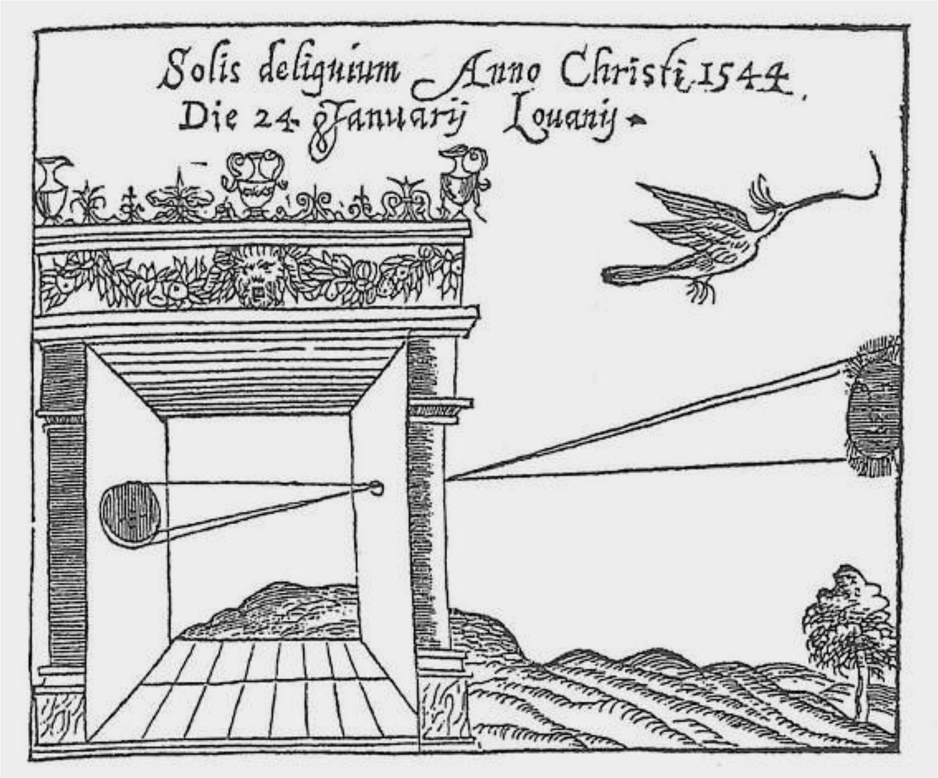


Pinhole camera a.k.a. camera obscura

First mention ...

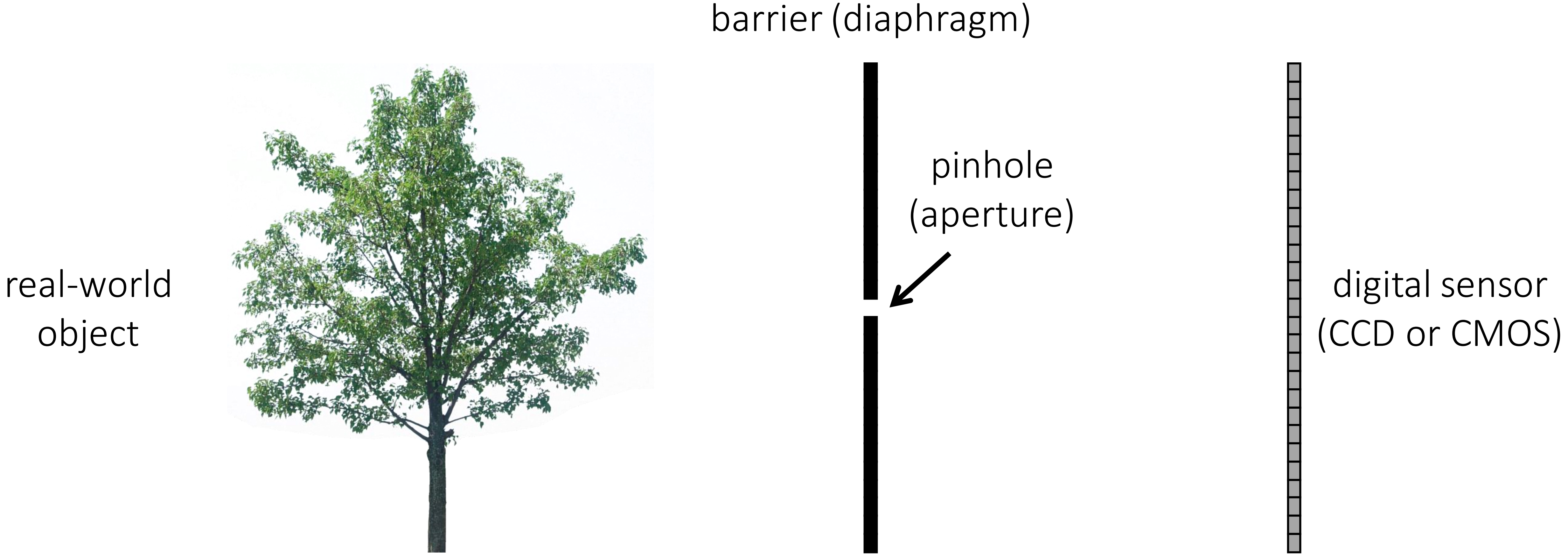
Chinese philosopher Mozi
(470 to 390 BC)

First camera ...

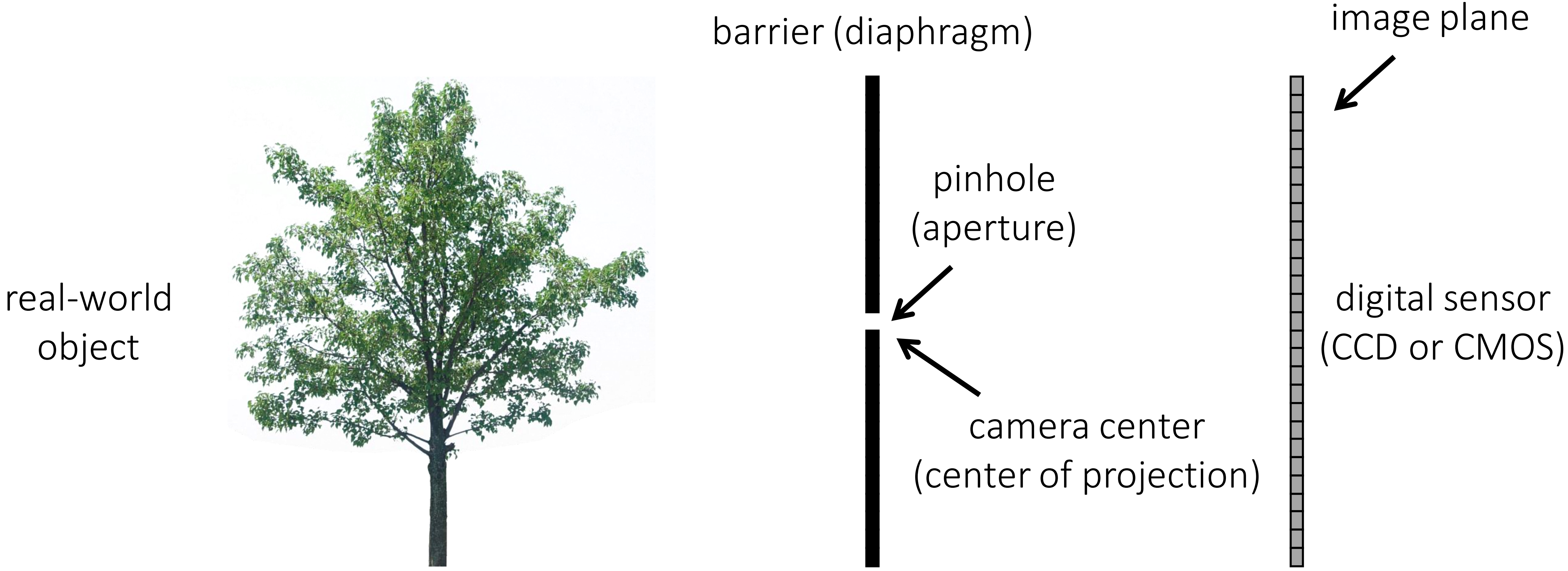


Greek philosopher Aristotle
(384 to 322 BC)

Pinhole camera terms

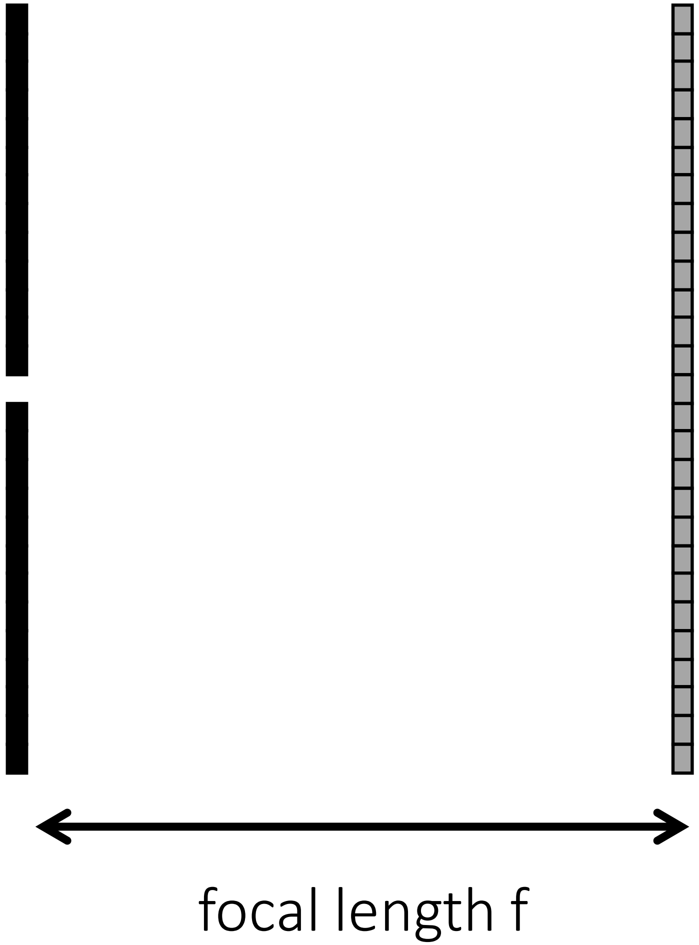


Pinhole camera terms



Focal length

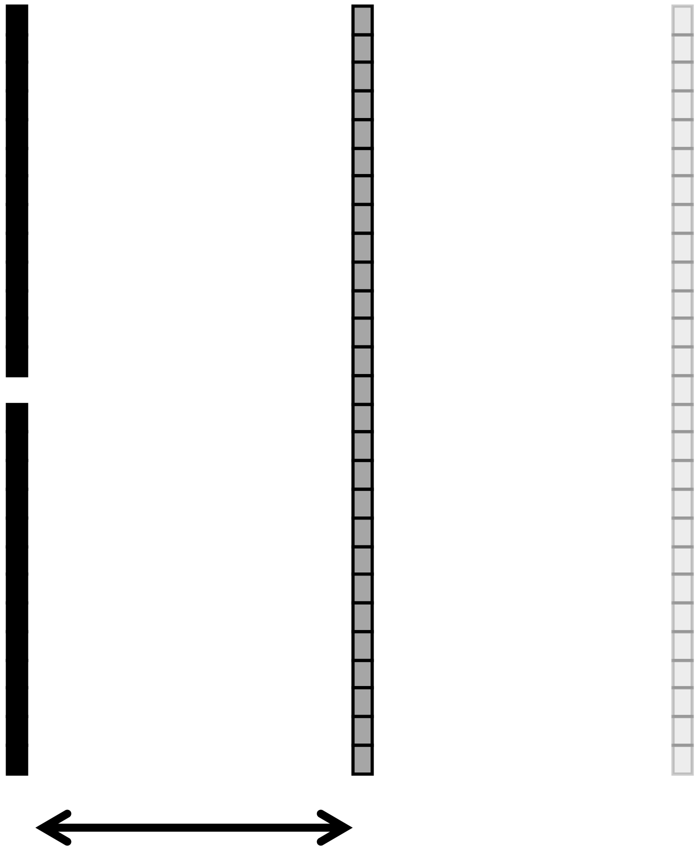
real-world
object



Focal length

What happens as we change the focal length?

real-world
object

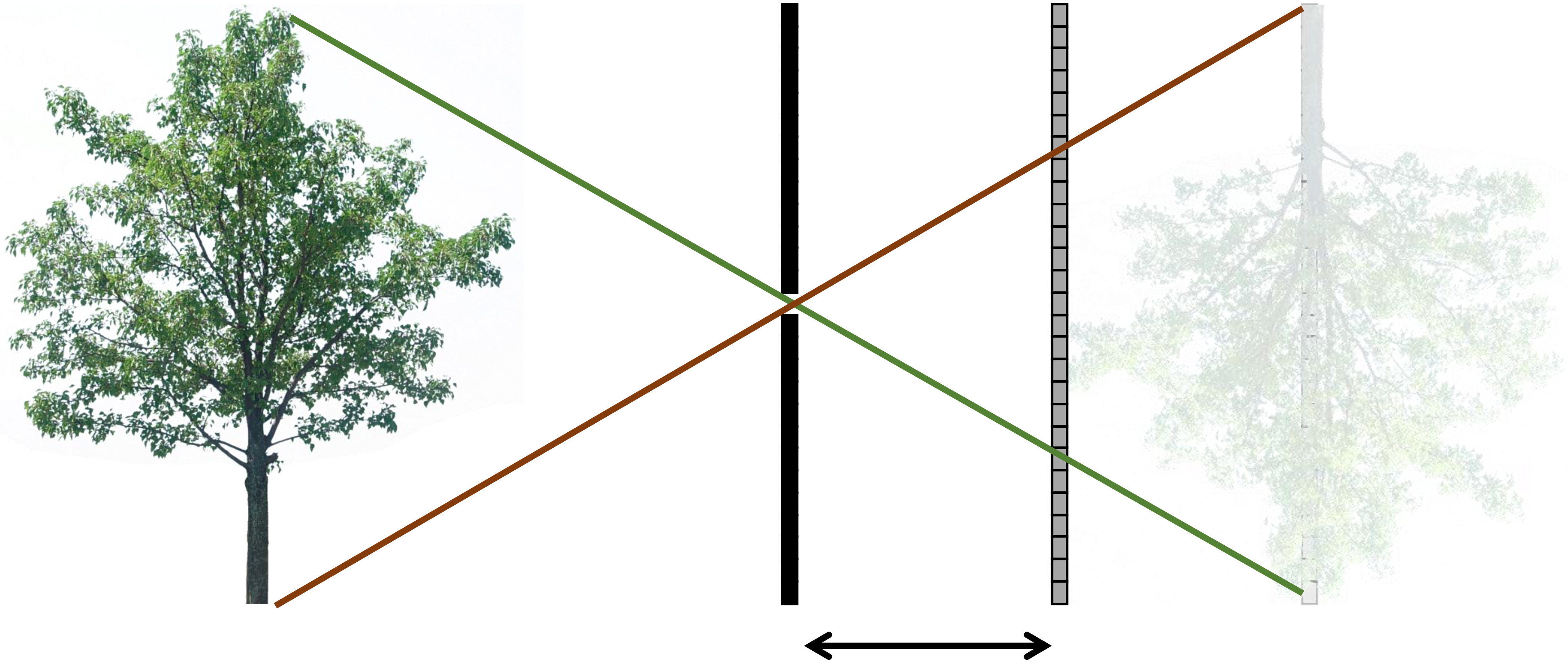


focal length 0.5 f

Focal length

What happens as we change the focal length?

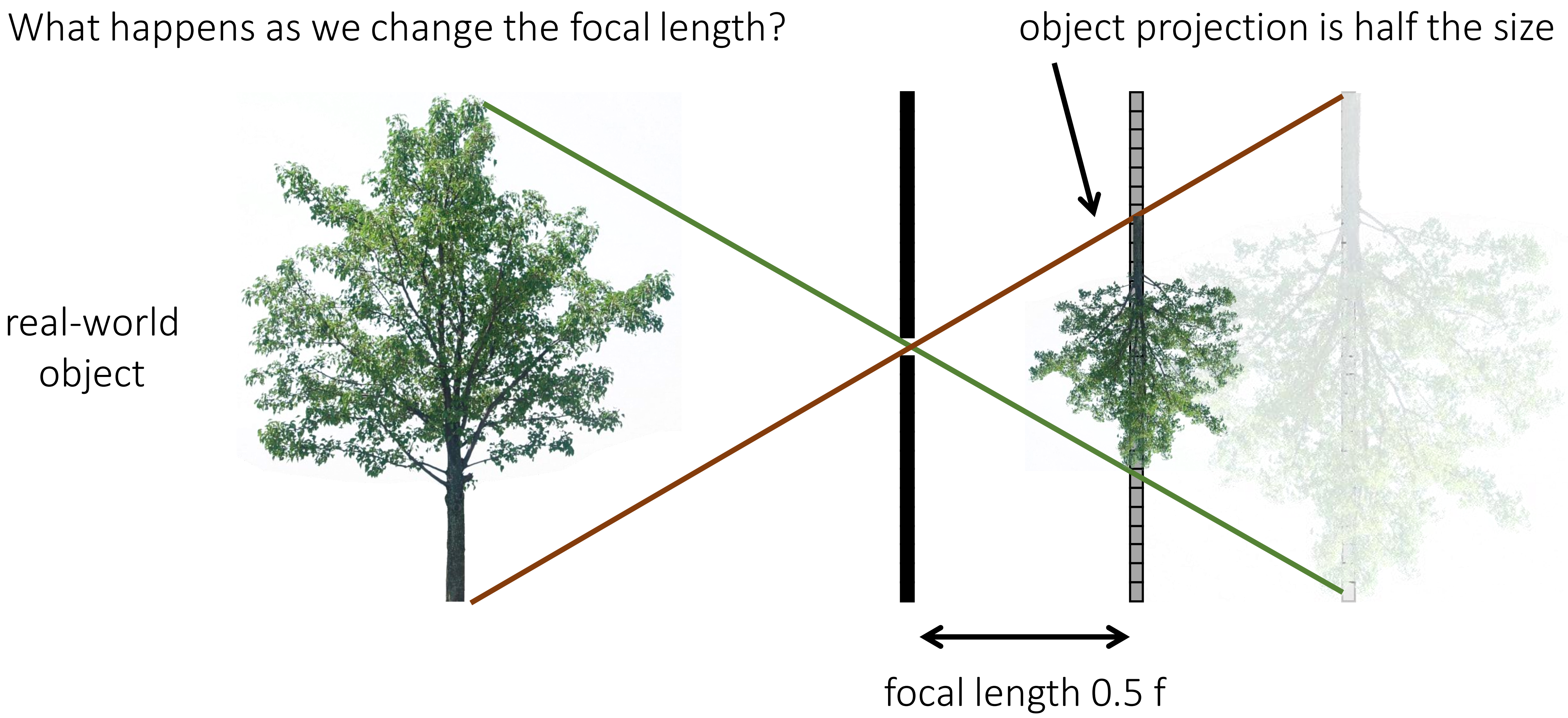
real-world object



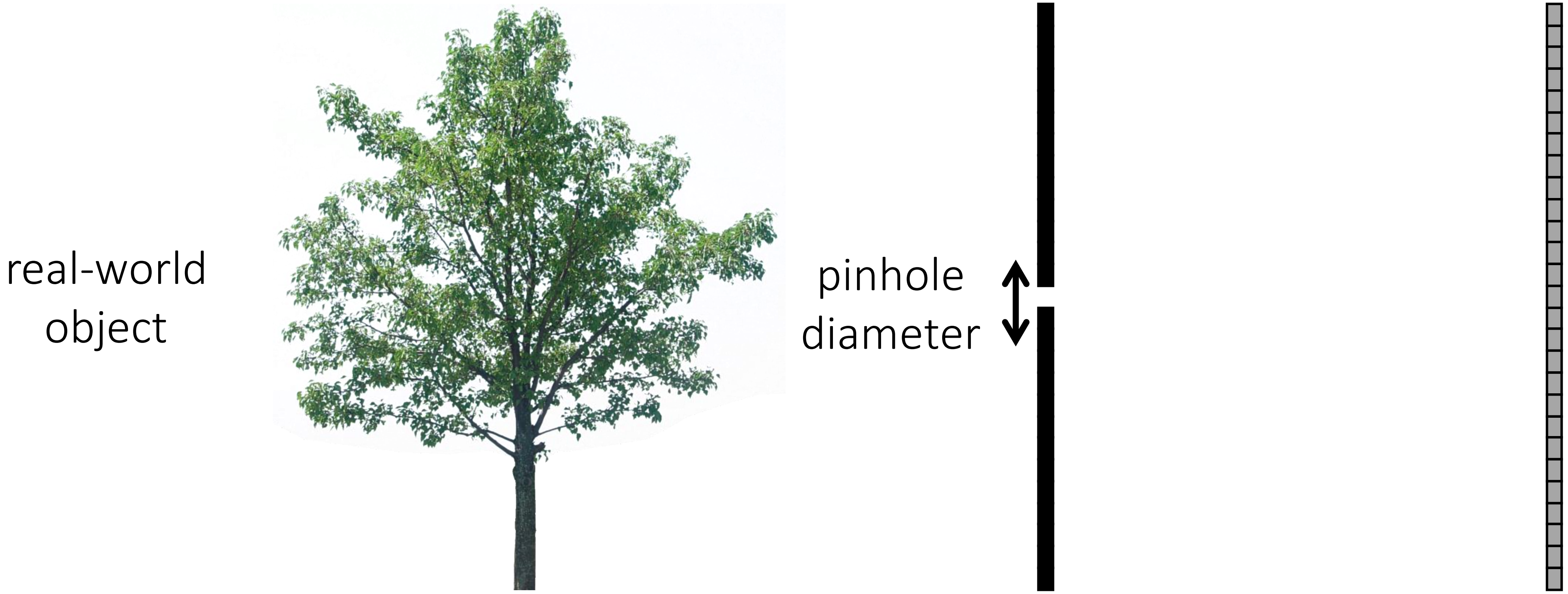
focal length 0.5 f

Focal length

What happens as we change the focal length?



Pinhole size

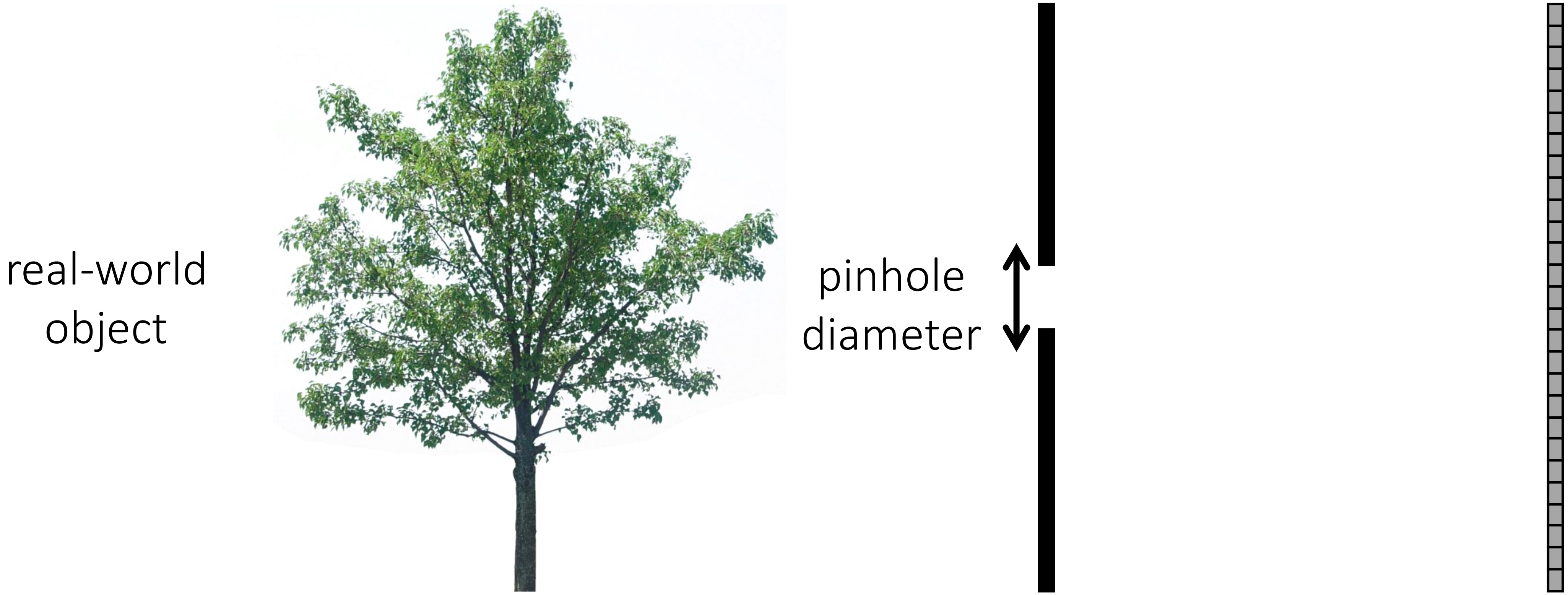


Ideal pinhole has infinitesimally small size

- In practice that is impossible.

Pinhole size

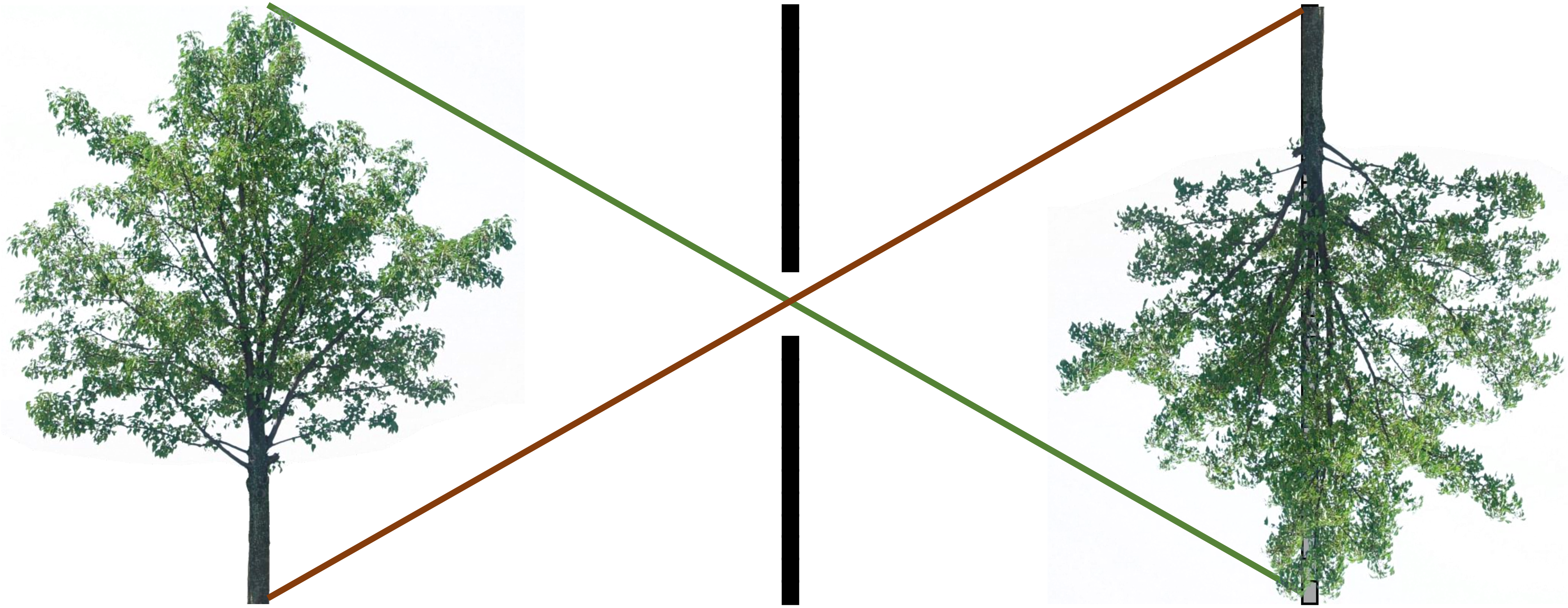
What happens as we change the pinhole diameter?



Pinhole size

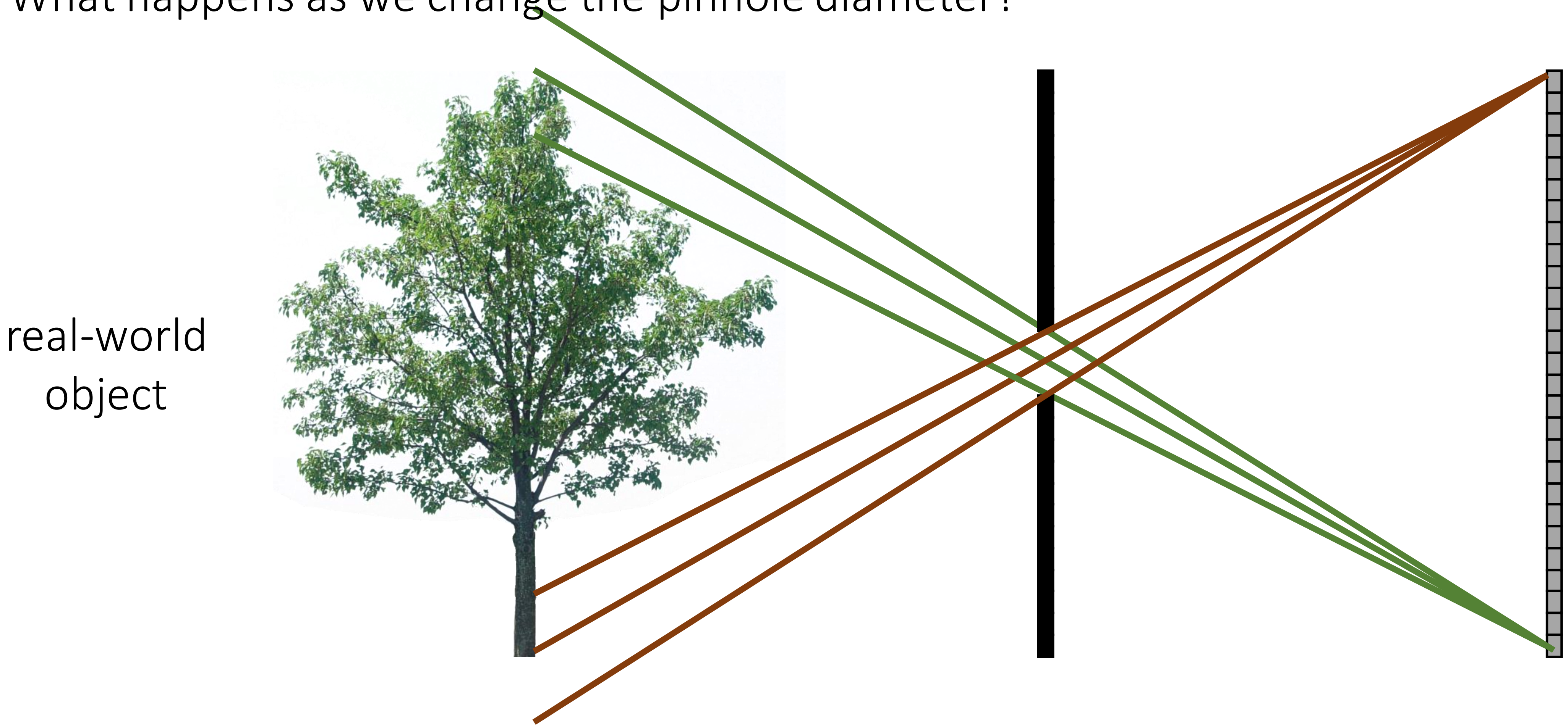
What happens as we change the pinhole diameter?

real-world
object



Pinhole size

What happens as we change the pinhole diameter?

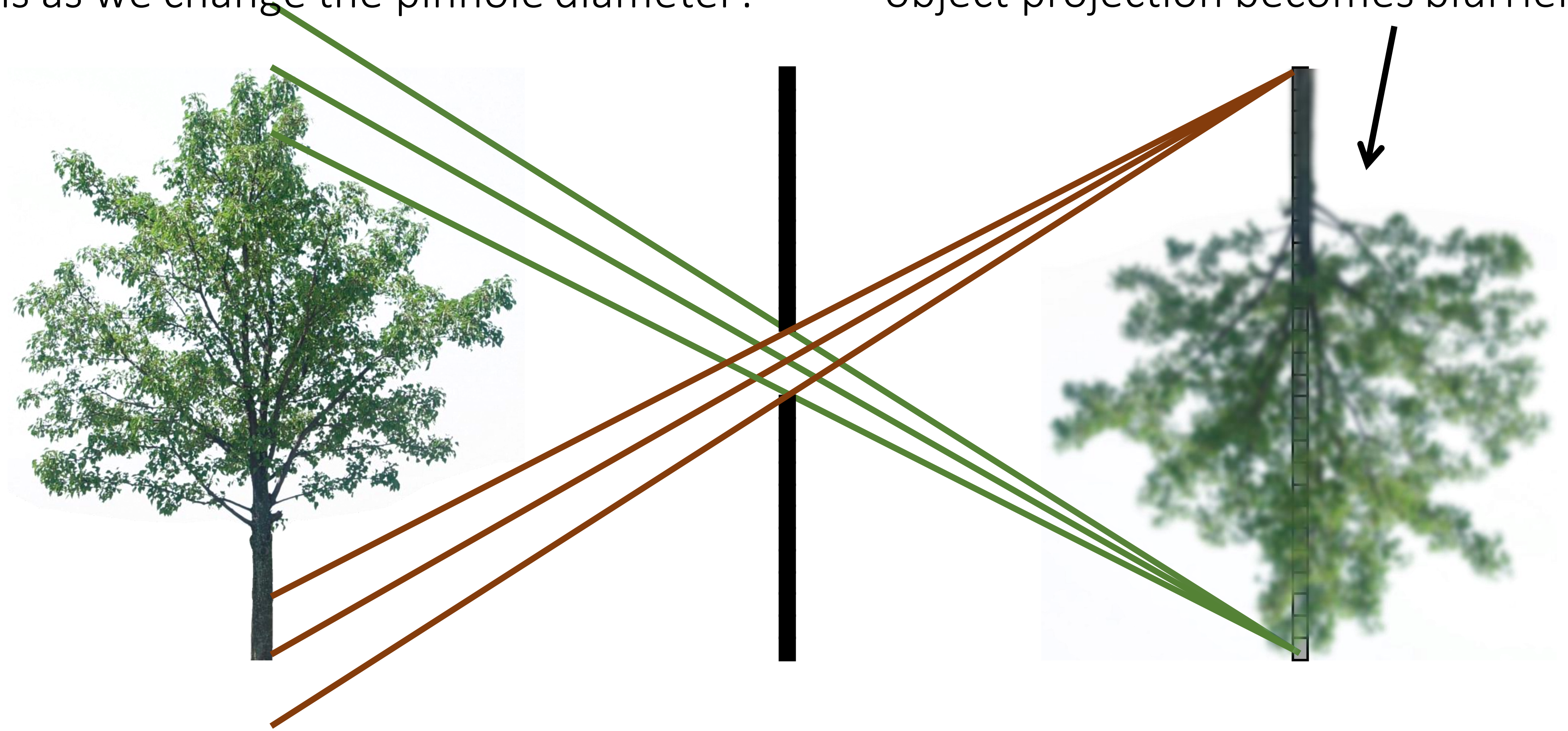


Pinhole size

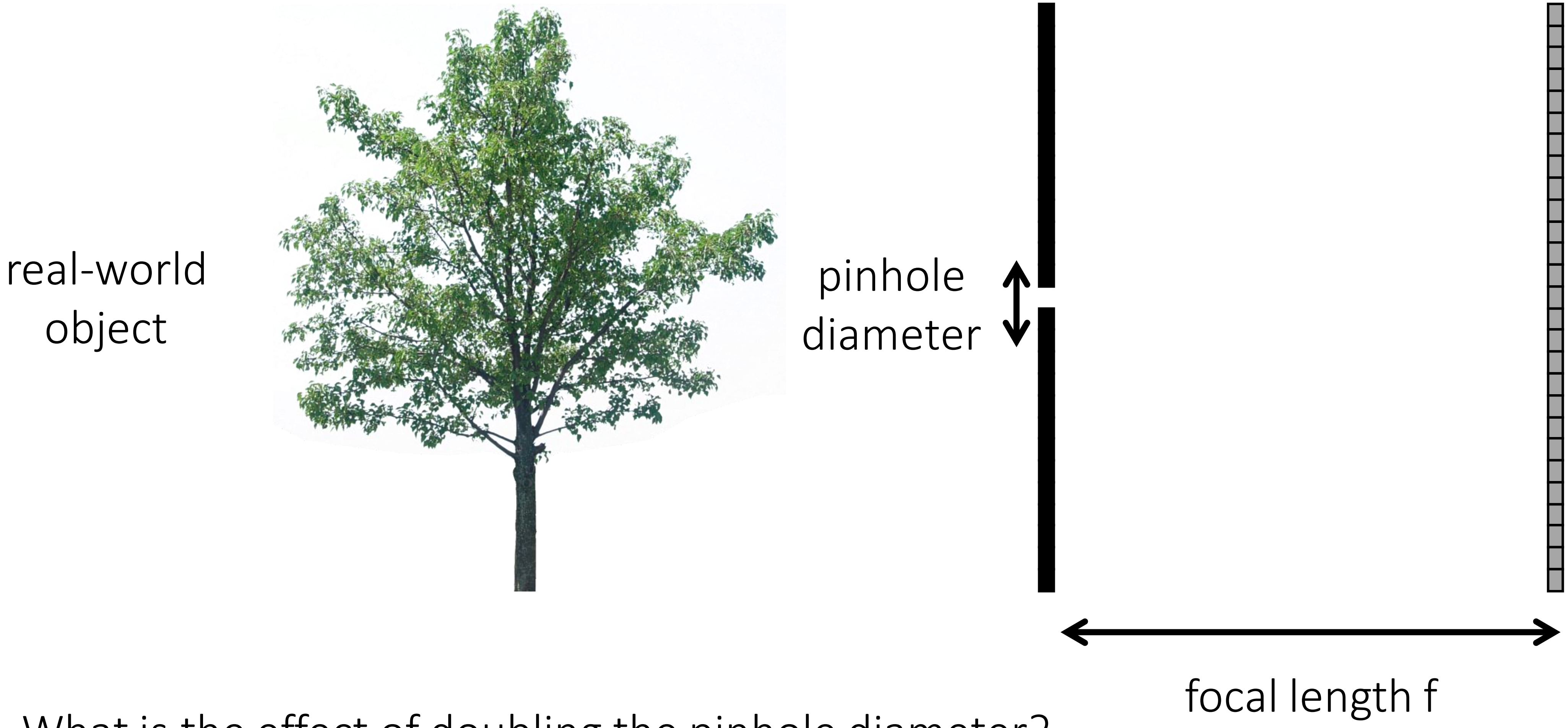
What happens as we change the pinhole diameter?

object projection becomes blurrier

real-world object

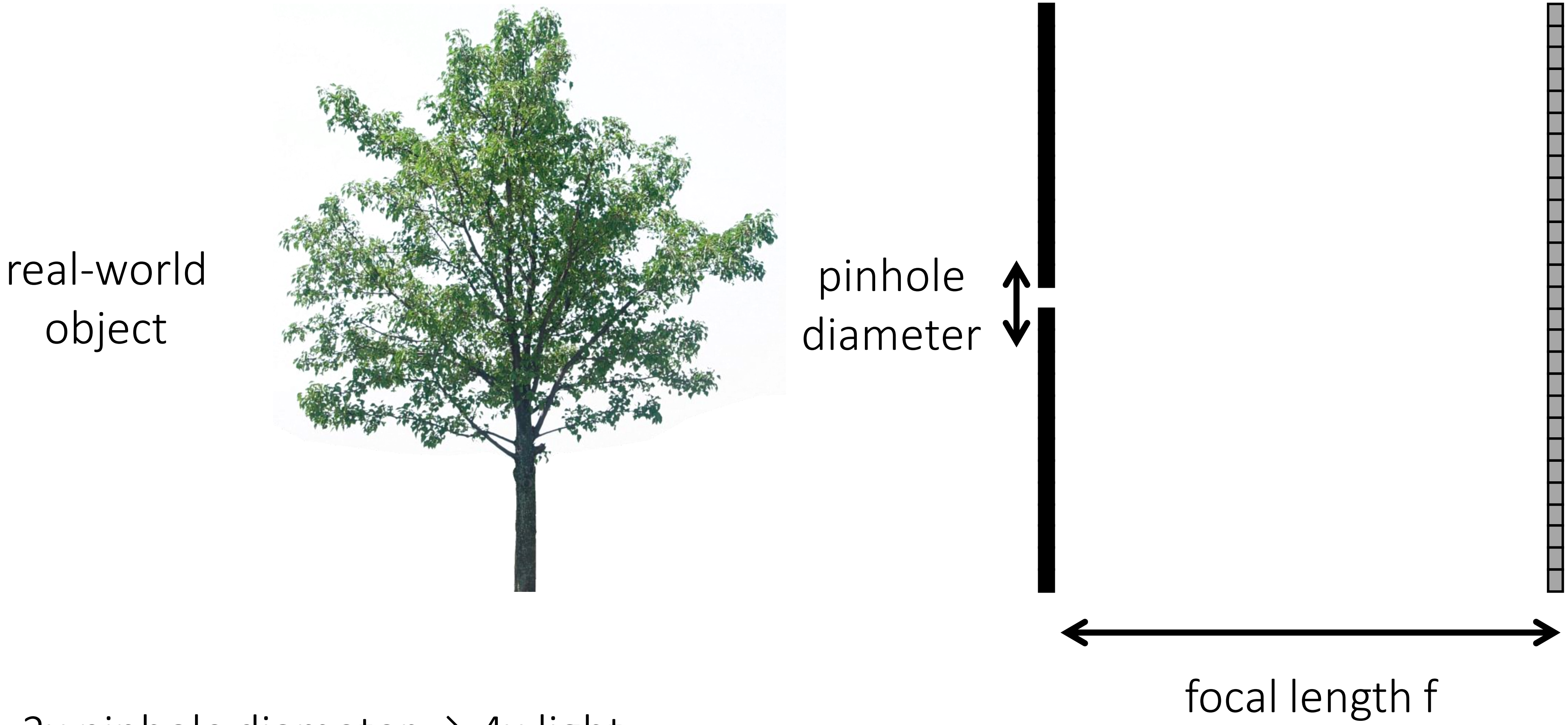


What about light efficiency?



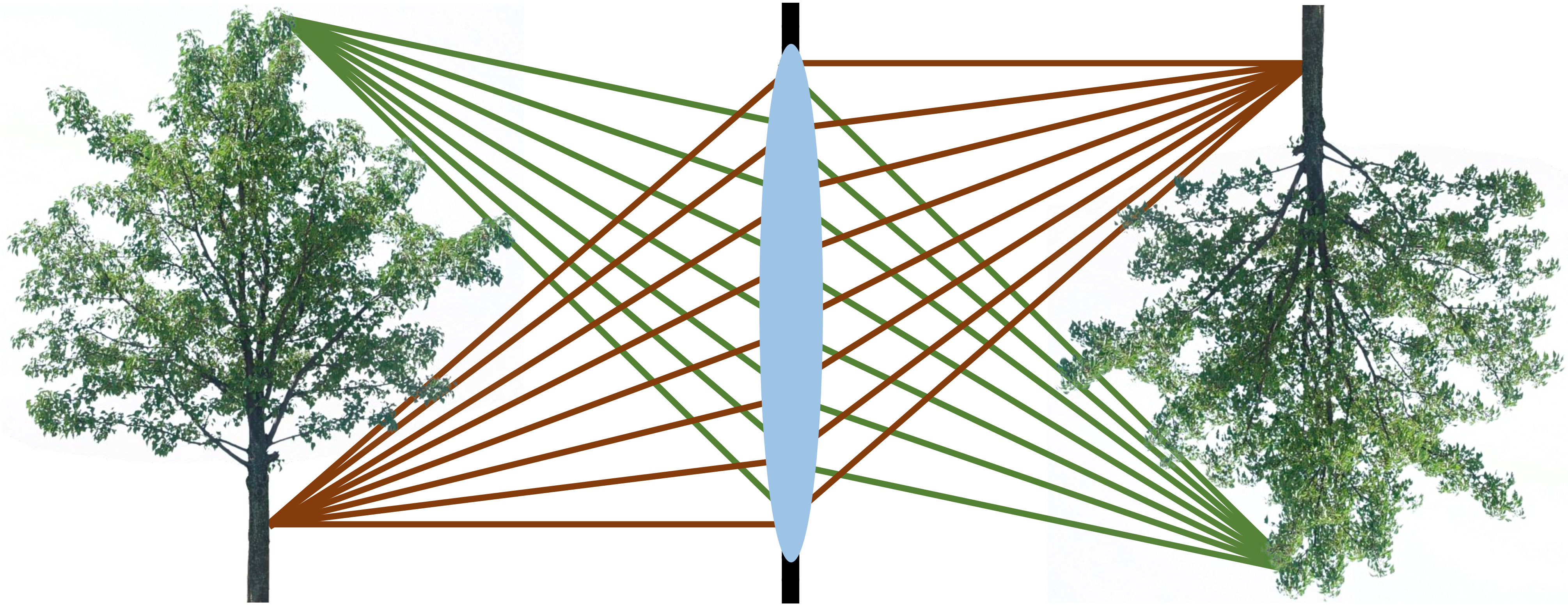
- What is the effect of doubling the pinhole diameter?

What about light efficiency?



- 2x pinhole diameter → 4x light

In practice



Lenses map “bundles” of rays from points on the scene to the sensor.

How does this mapping work exactly?

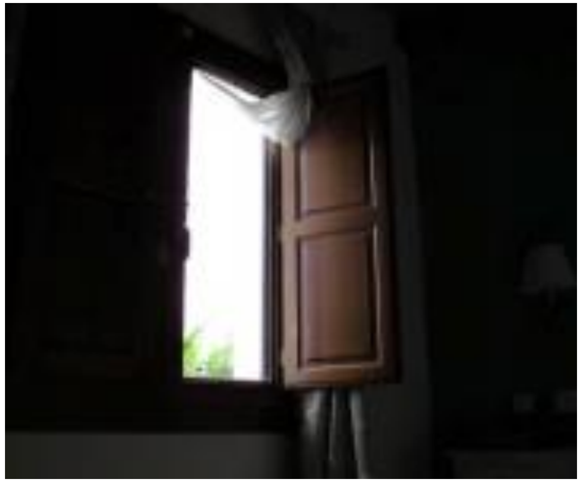
Accidental pinholes

What does this image say about the world outside?

Accidental pinhole camera

Antonio Torralba, William T. Freeman
Computer Science and Artificial Intelligence Laboratory (CSAIL)
MIT
torralba@mit.edu, billf@mit.edu

Accidental pinhole camera



window is an aperture

projected pattern on the wall

upside down

window with smaller gap

view outside window

Camera matrix

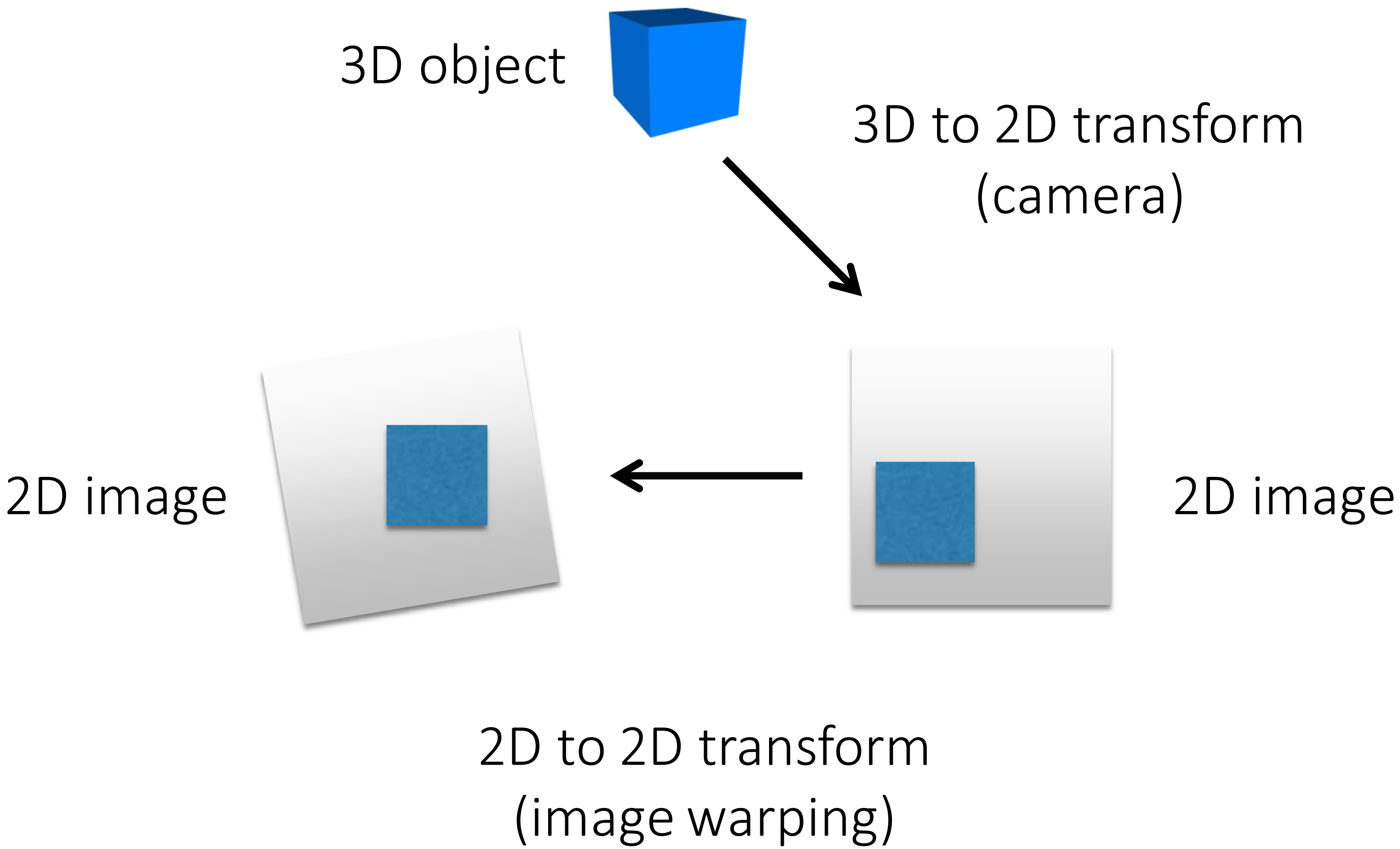
The camera as a coordinate transformation

A camera is a mapping from:

the 3D world

to:

a 2D image



The camera as a coordinate transformation

A camera is a mapping from:

the 3D world

to:

a 2D image

homogeneous coordinates

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

2D image point

camera matrix

3D world point

What are the dimensions of each variable?

The camera as a coordinate transformation

$$\boldsymbol{x} = \mathbf{PX}$$

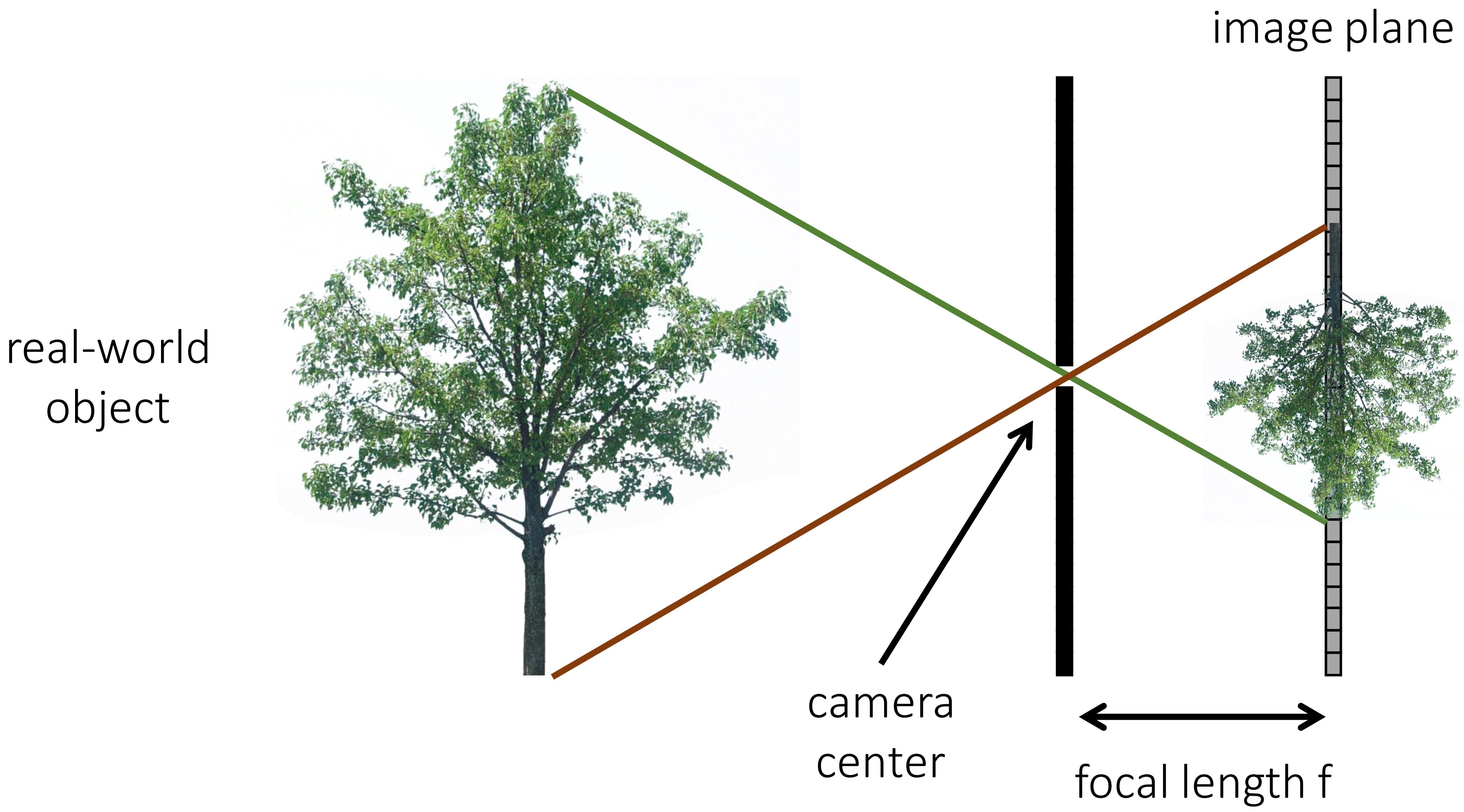
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

homogeneous
image coordinates
3 x 1

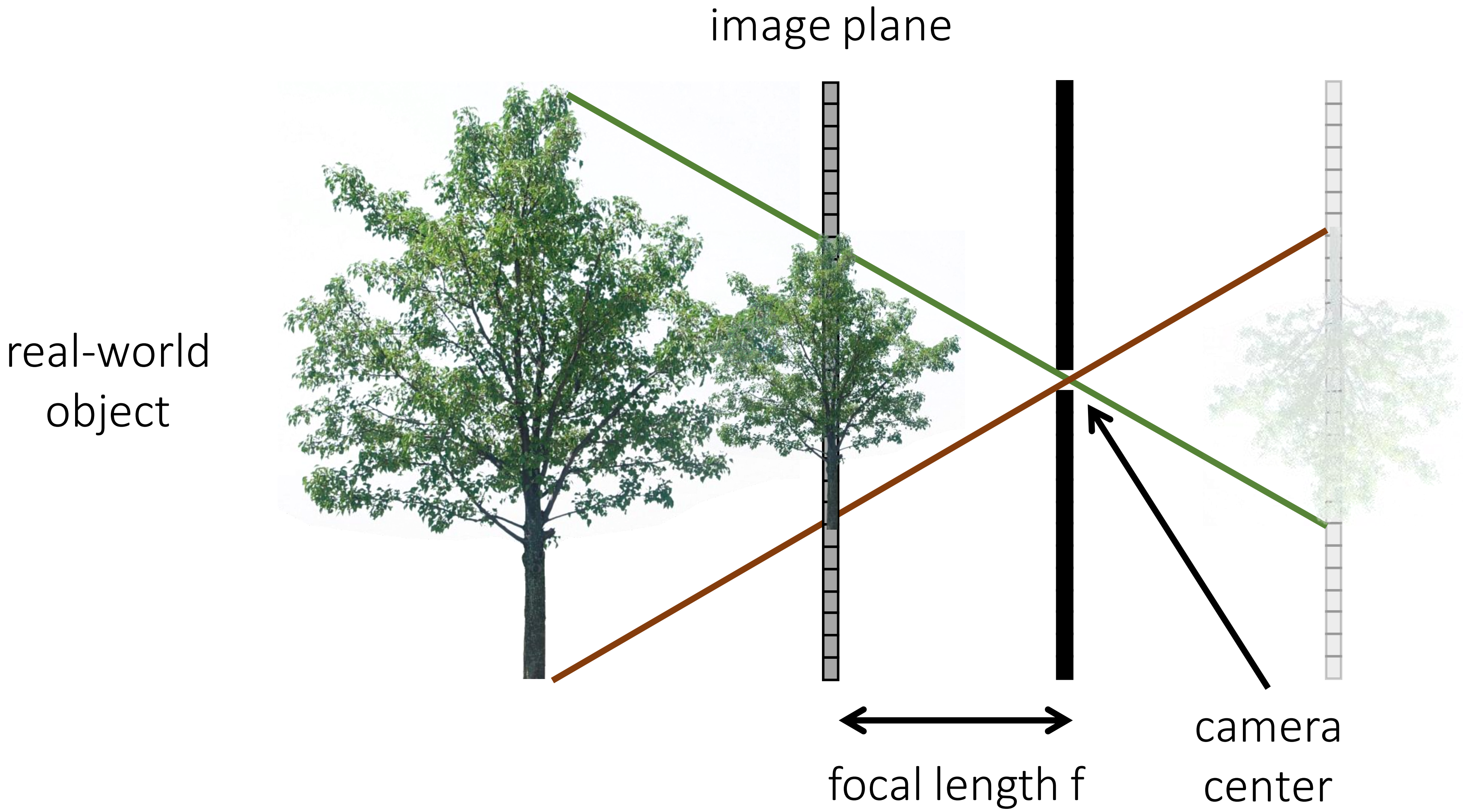
camera
matrix
3 x 4

homogeneous
world coordinates
4 x 1

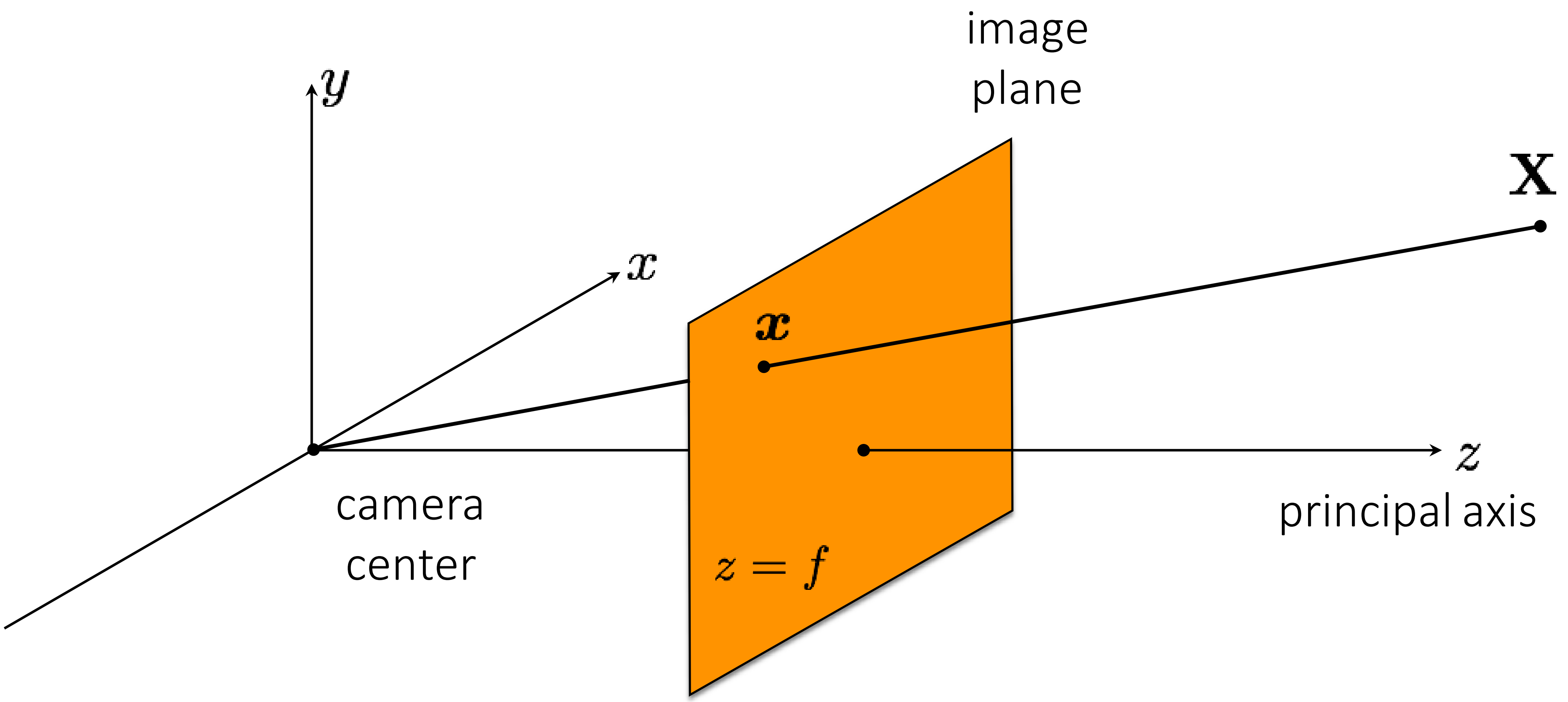
The pinhole camera



The (rearranged) pinhole camera

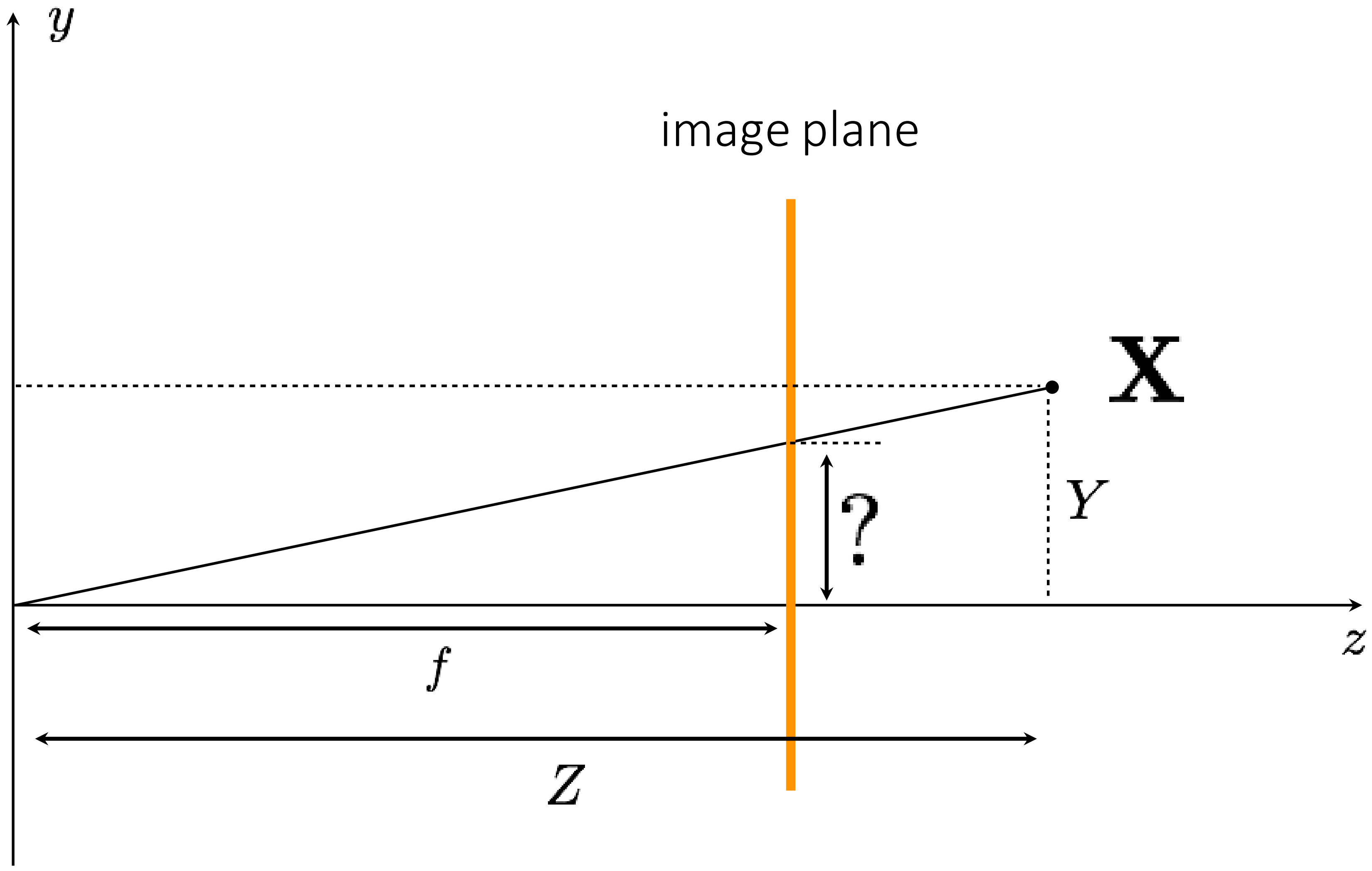


The (rearranged) pinhole camera



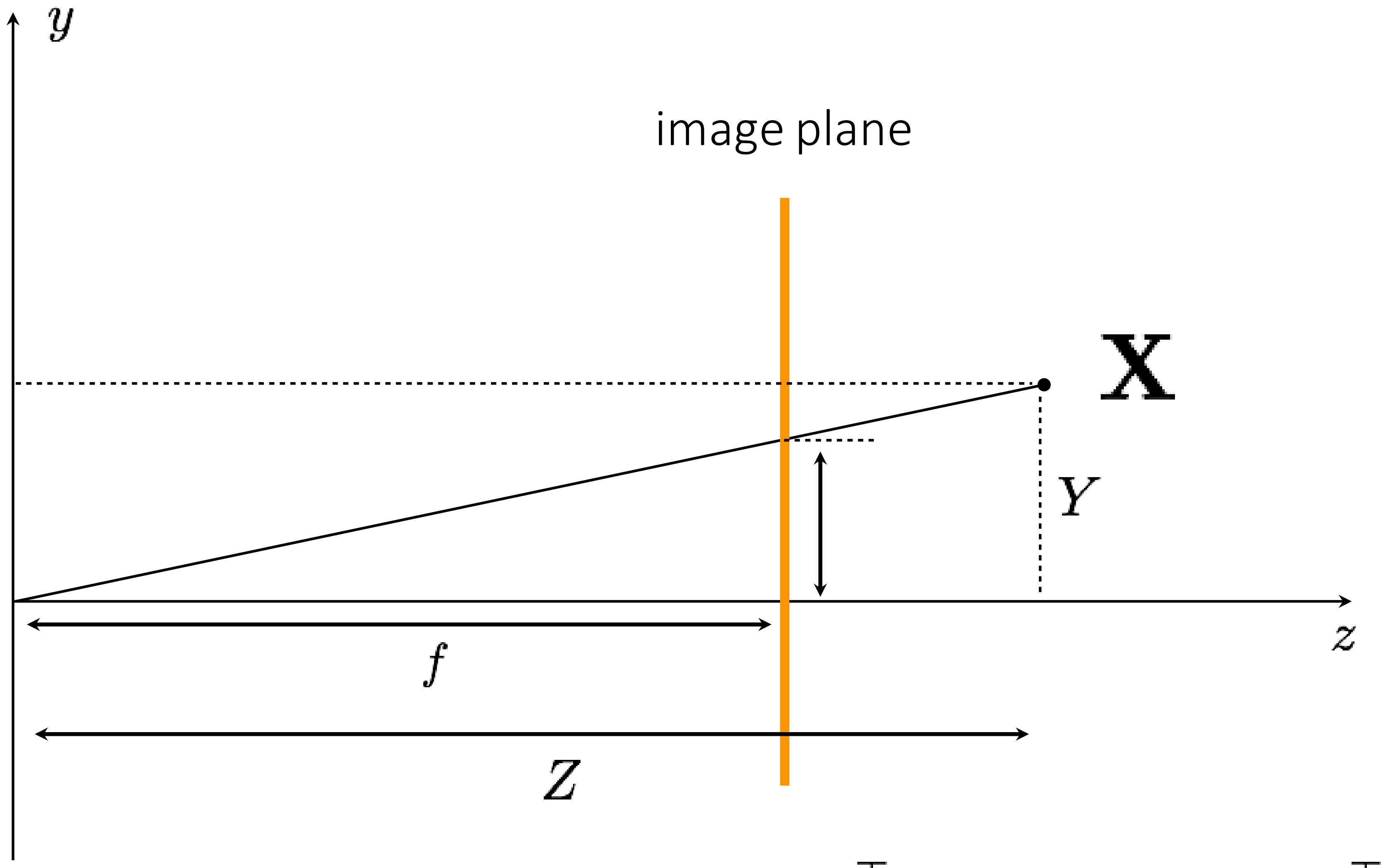
What is the equation for image coordinate **x** in terms of **X**?

The 2D view of the (rearranged) pinhole camera



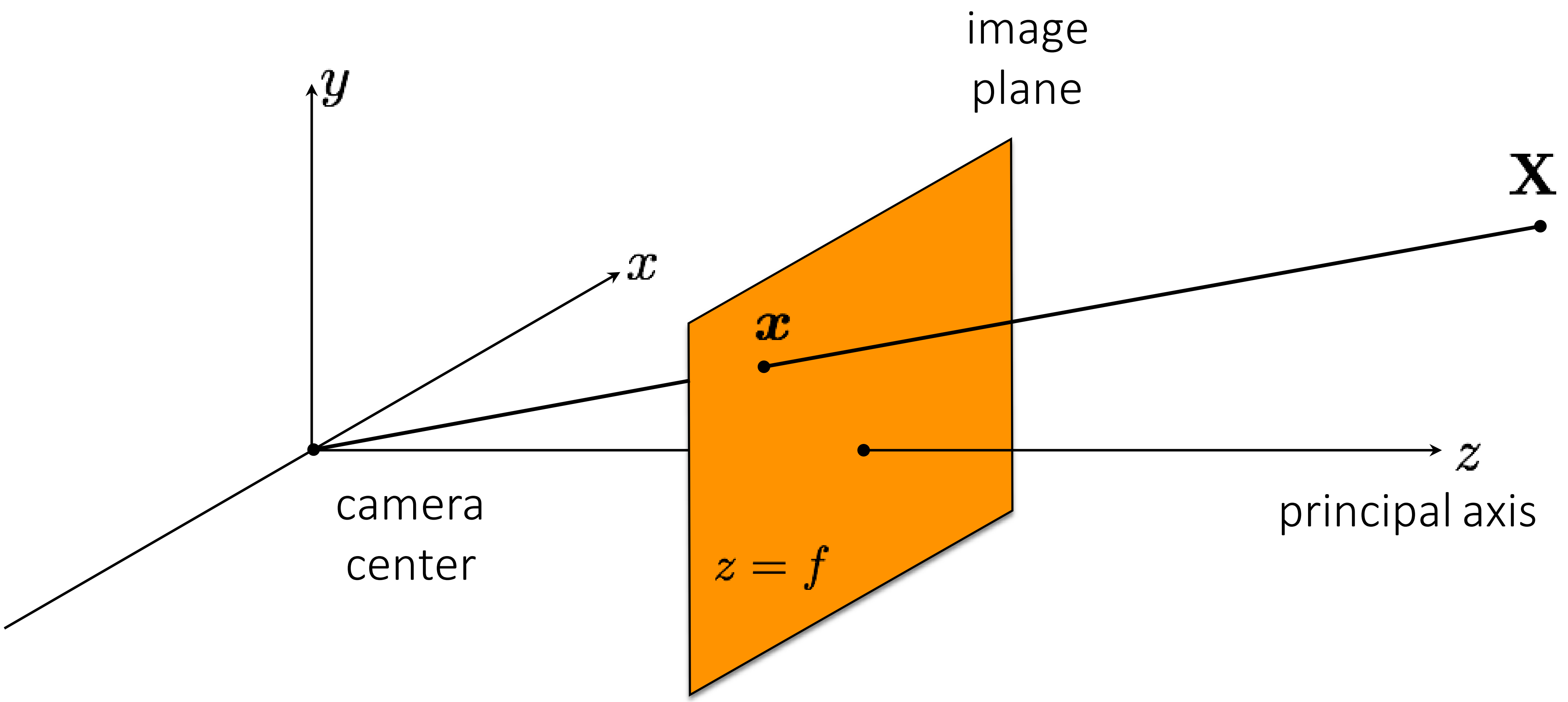
What is the equation for image coordinate **x** in terms of **X**?

The 2D view of the (rearranged) pinhole camera



$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

The (rearranged) pinhole camera



What is the camera matrix **P** for a pinhole camera?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

The pinhole camera matrix

Relationship from similar triangles:

$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

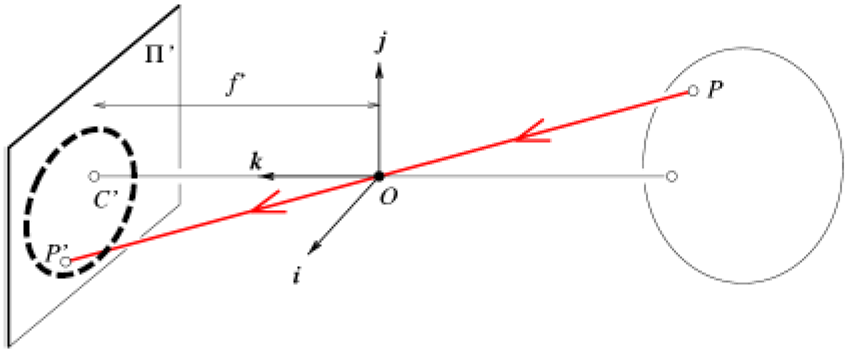
General camera model:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What does the pinhole camera projection look like?

$$\mathbf{P} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$$

Relating a real-world point to a point on the camera



In homogeneous coordinates:

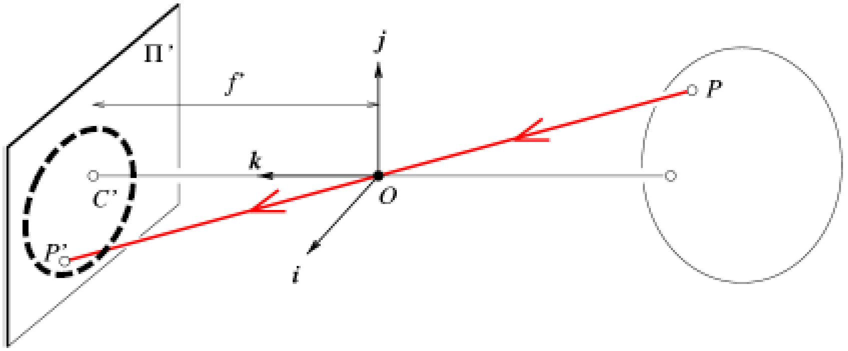
$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

ideal world

Intrinsic Assumptions
- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

Relating a real-world point to a point on the camera



In homogeneous coordinates:

$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = K\begin{bmatrix} I & 0 \end{bmatrix} P$$

K

Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

Remove assumption: known optical center

Intrinsic Assumptions
- ~~Optical center at (0,0)~~
- Optical center at (u_0, v_0)
- Square pixels
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions

- Optical center at (u_0, v_0)
- ~~Square pixels~~
- Rectangular pixels
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

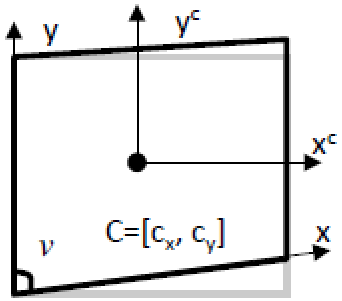
$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: non-skewed pixels

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- ~~No skew~~
- Small skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Remove assumption: non-skewed pixels

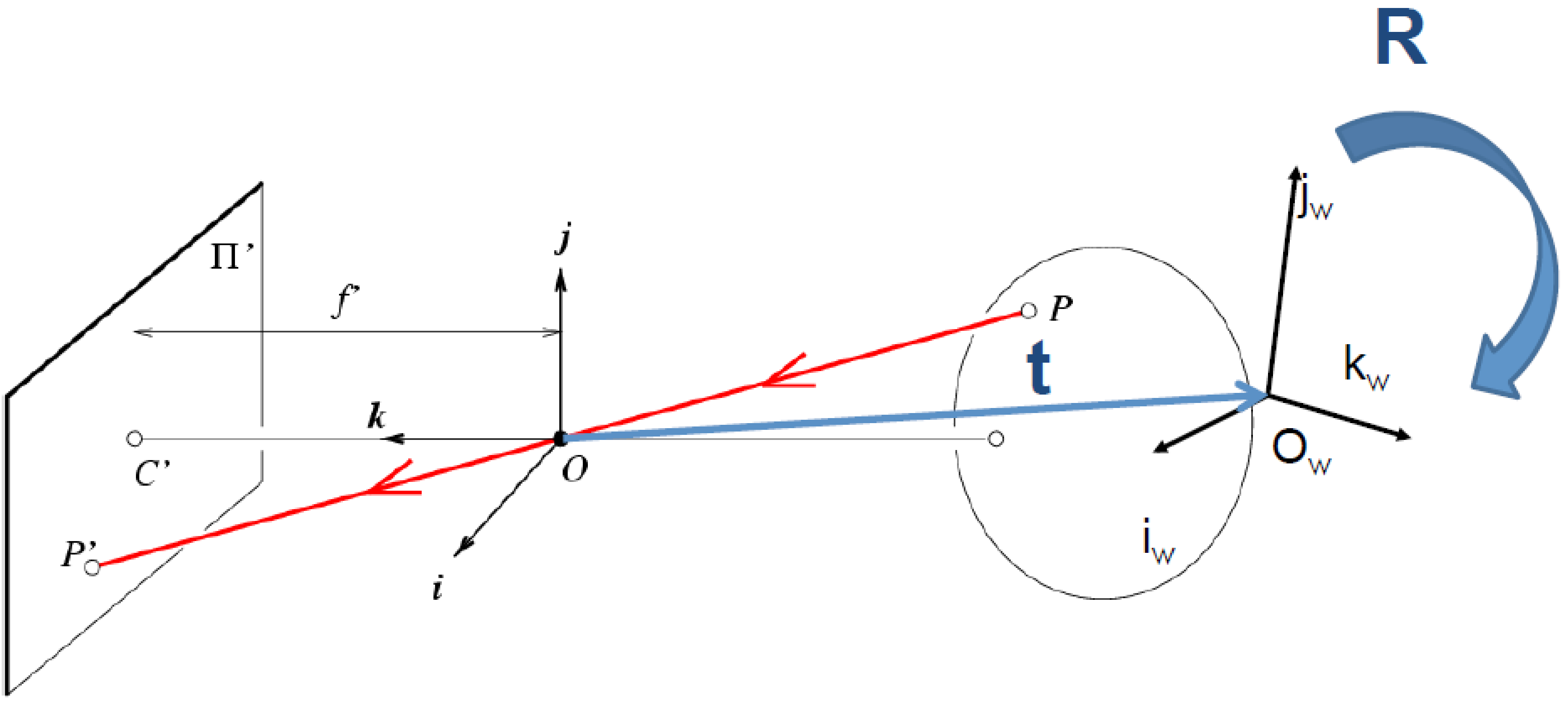
Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

Intrinsic parameters

$$P' = K \begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Real world camera: Translate + Rotate



Remove assumption: allow translation

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- No rotation
- Camera at ~~(0,0,0)~~ -> (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} I & \bar{t} \end{bmatrix} P \quad \Rightarrow \quad w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

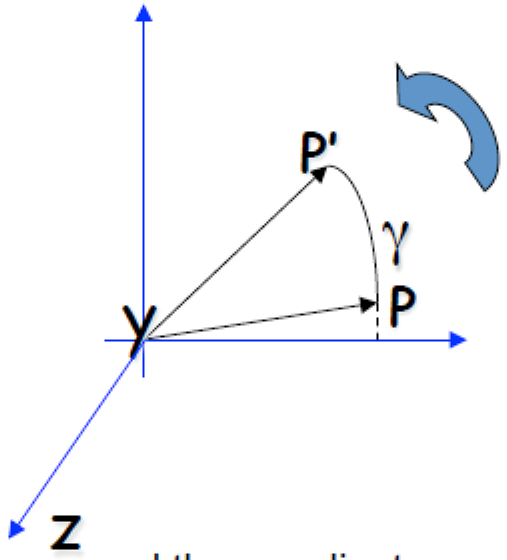
Remove assumption: allow rotation

Intrinsic Assumptions

- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions

- ~~No~~ rotation
- Camera at (t_x, t_y, t_z)



Rotation around the coordinate axes, counter-clockwise

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Remove assumption: allow rotation

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- ~~No~~ rotation
- Camera at (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

A generic projection matrix

Intrinsic Assumptions

- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions

- Allow rotation
- Camera at (t_x, t_y, t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

A generic projection matrix

Intrinsic Assumptions

- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions

- Allow rotation
- Camera at (t_x, t_y, t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Degrees of freedom??

A generic projection matrix

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x, t_y, t_z)

$$P' = K\begin{bmatrix} R & \overline{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{5} \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}}_{6} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Degrees of freedom??

Quiz

The camera matrix relates what two quantities?

Quiz

The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

Quiz

The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

3D points to 2D image points

Quiz

The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

3D points to 2D image points

The camera matrix can be decomposed into?

Quiz

The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

3D points to 2D image points

The camera matrix can be decomposed into?

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

Quiz

The camera matrix relates what two quantities?

$$x = \mathbf{P}\mathbf{X}$$

3D points to 2D image points

The camera matrix can be decomposed into?

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

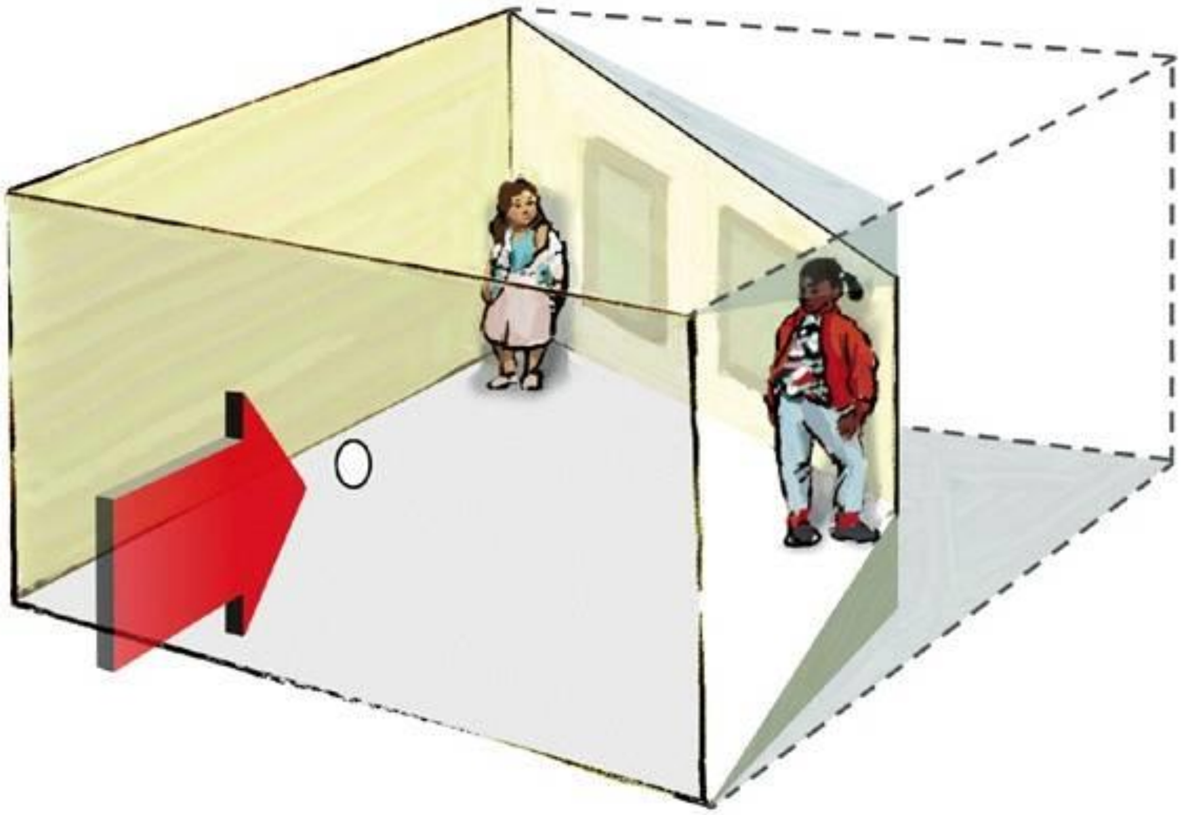
intrinsic and extrinsic parameters

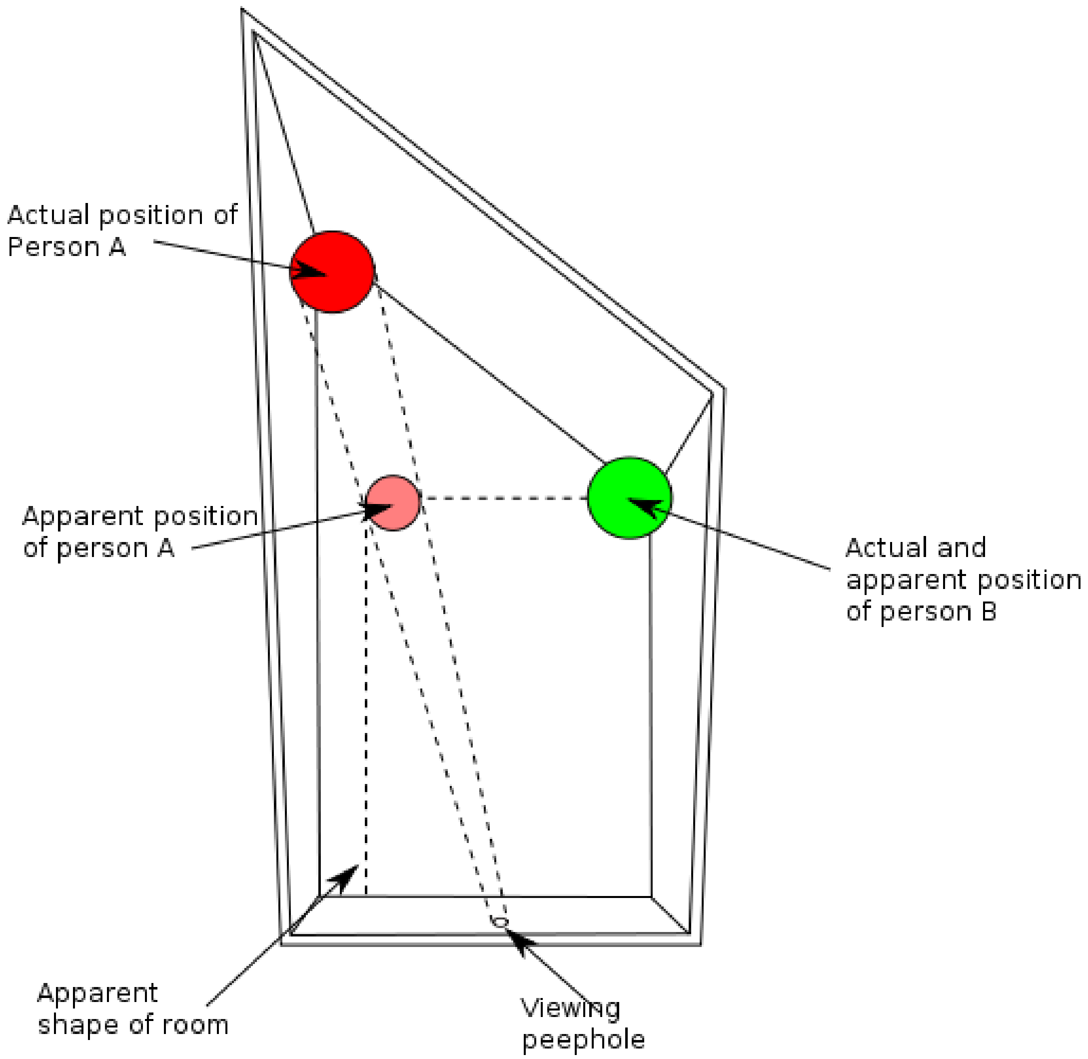
Perspective

Forced perspective

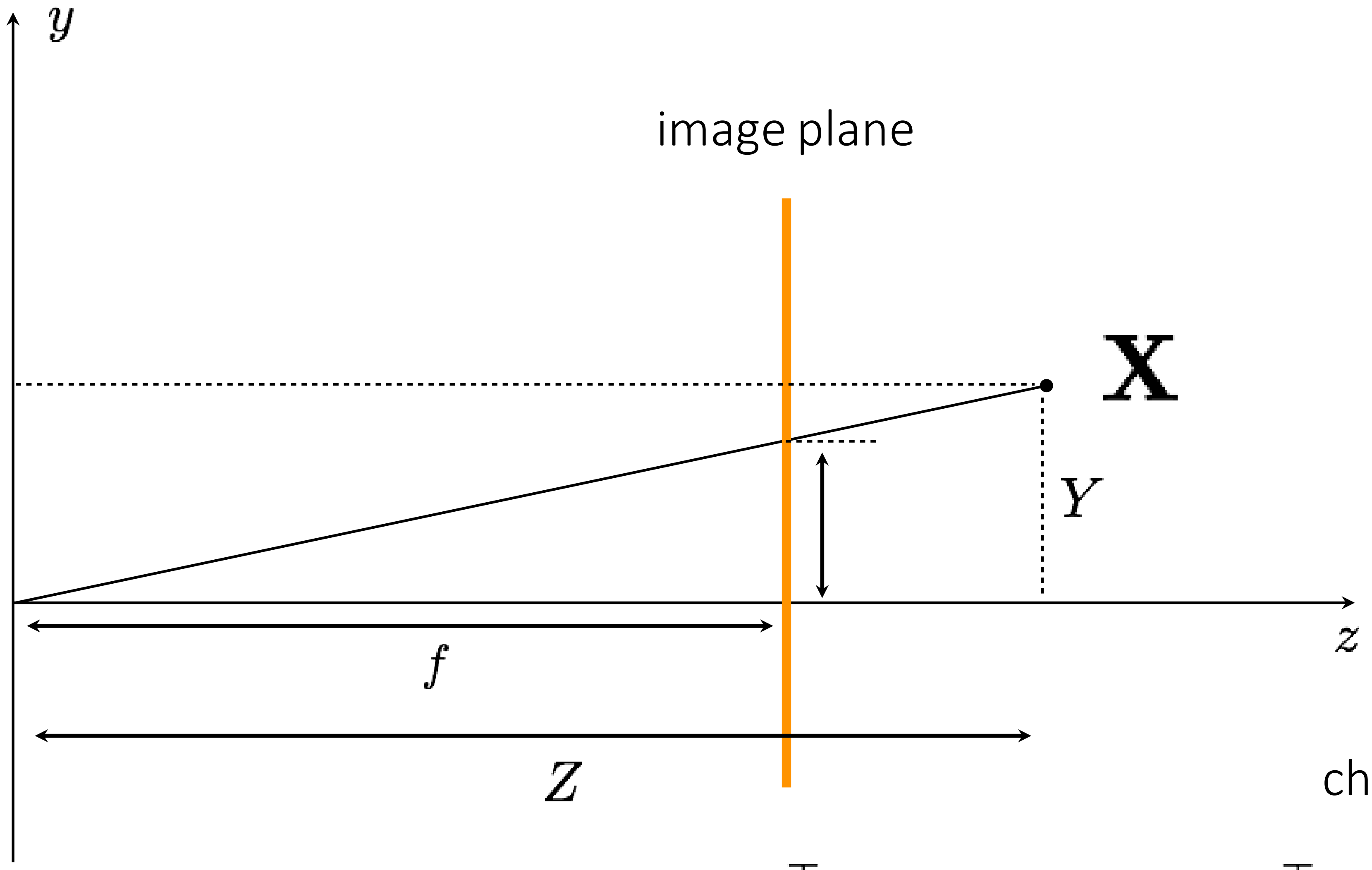
The Ames room illusion

The Ames room illusion





The 2D view of the (rearranged) pinhole camera

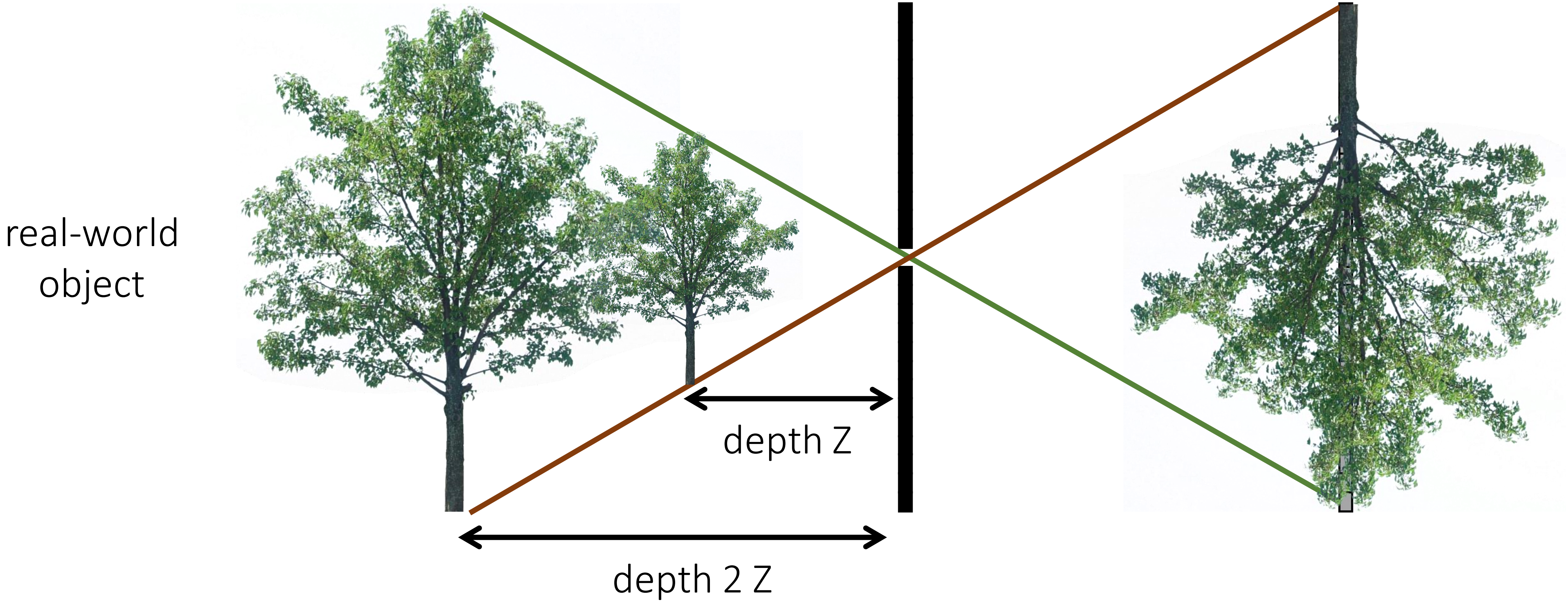


magnification changes with depth

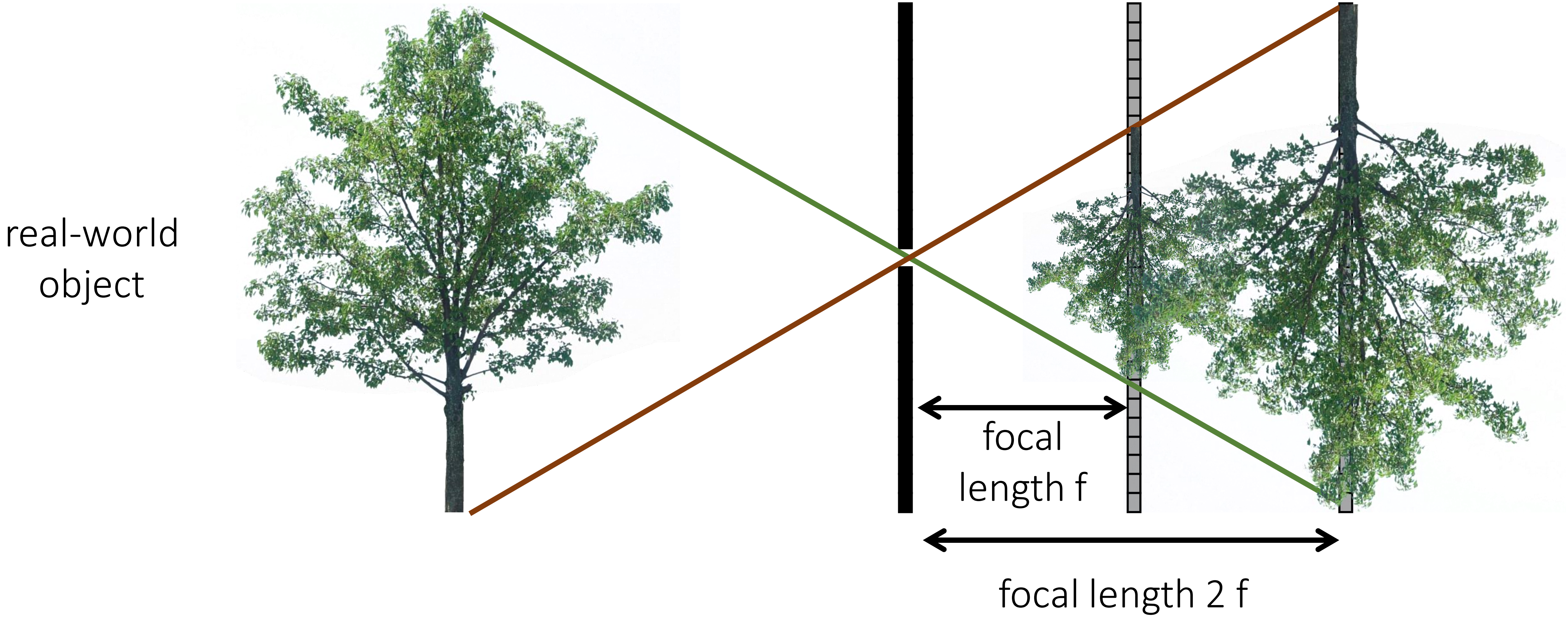
$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

Magnification depends on depth

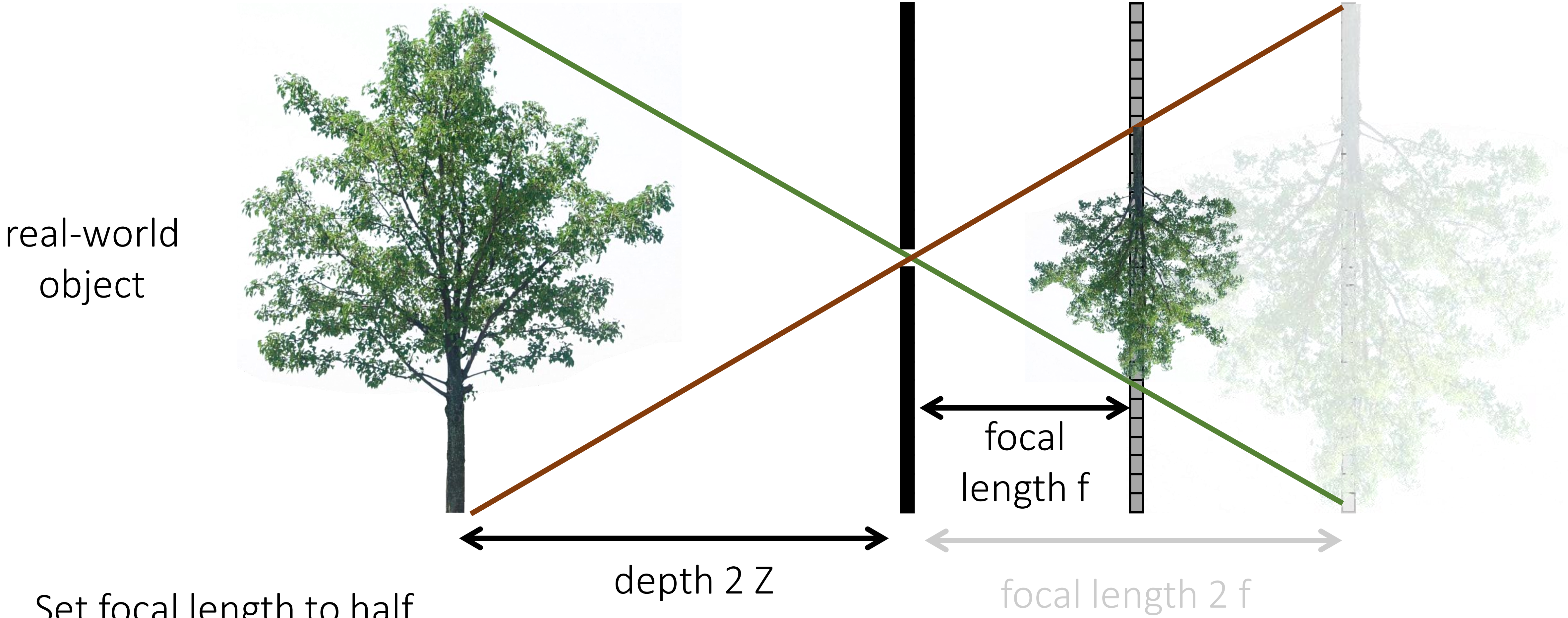
What happens as we change the focal length?



Magnification depends on focal length



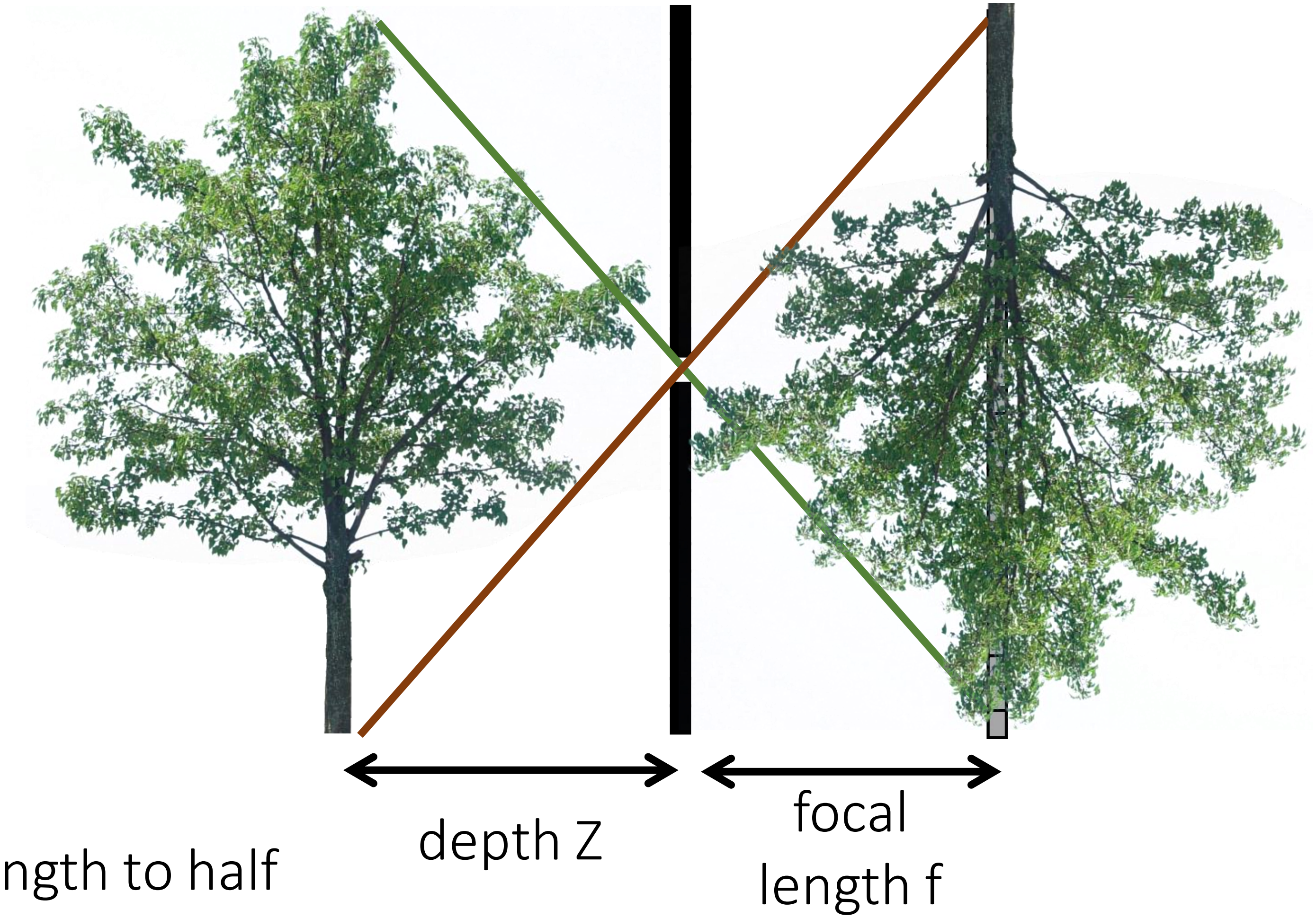
What if...



1. Set focal length to half

What if...

real-world object



Is this the same image as the one I had at focal length 2f and distance 2Z?

1. Set focal length to half
2. Set depth to half

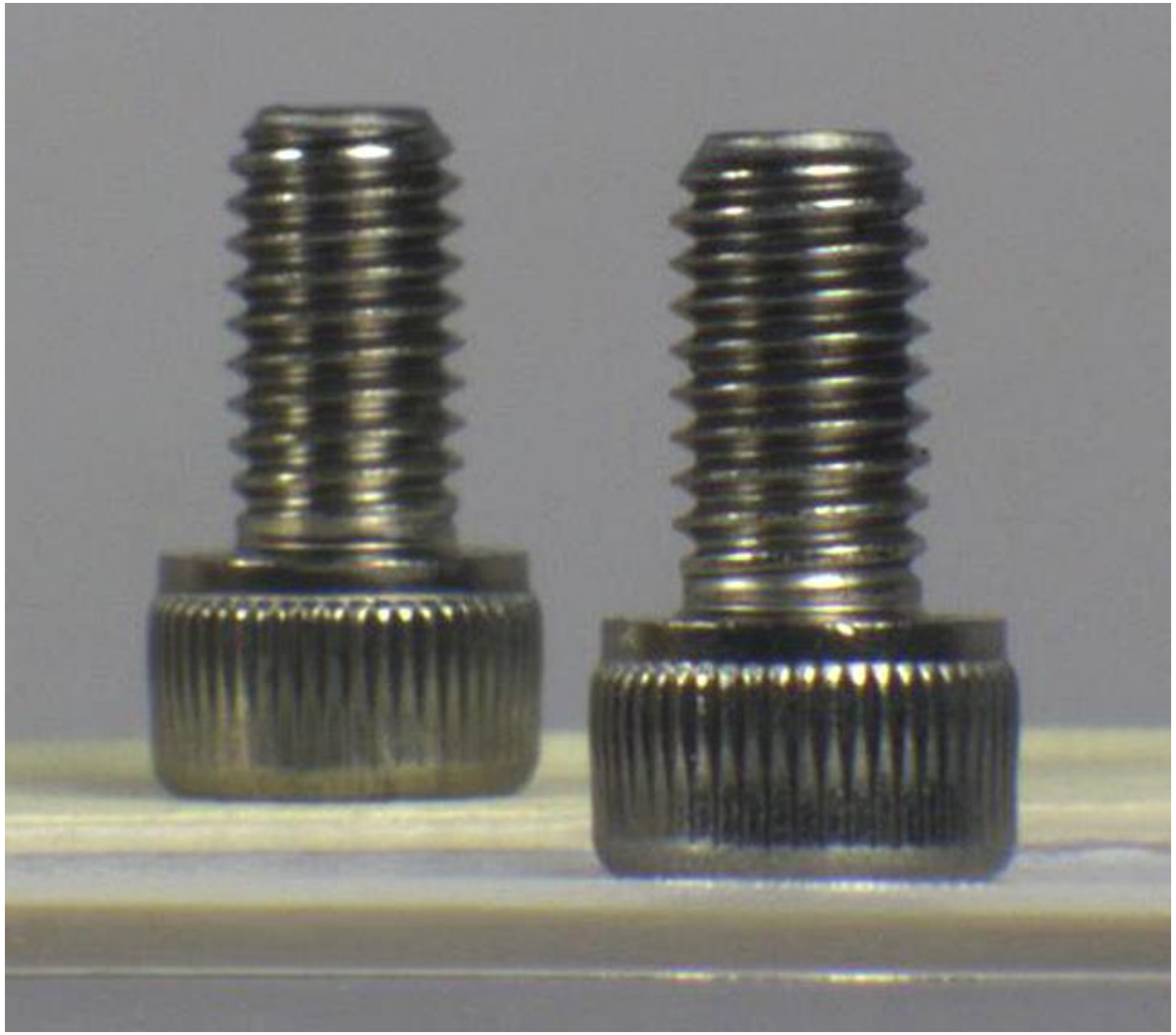
Perspective distortion

long focal length

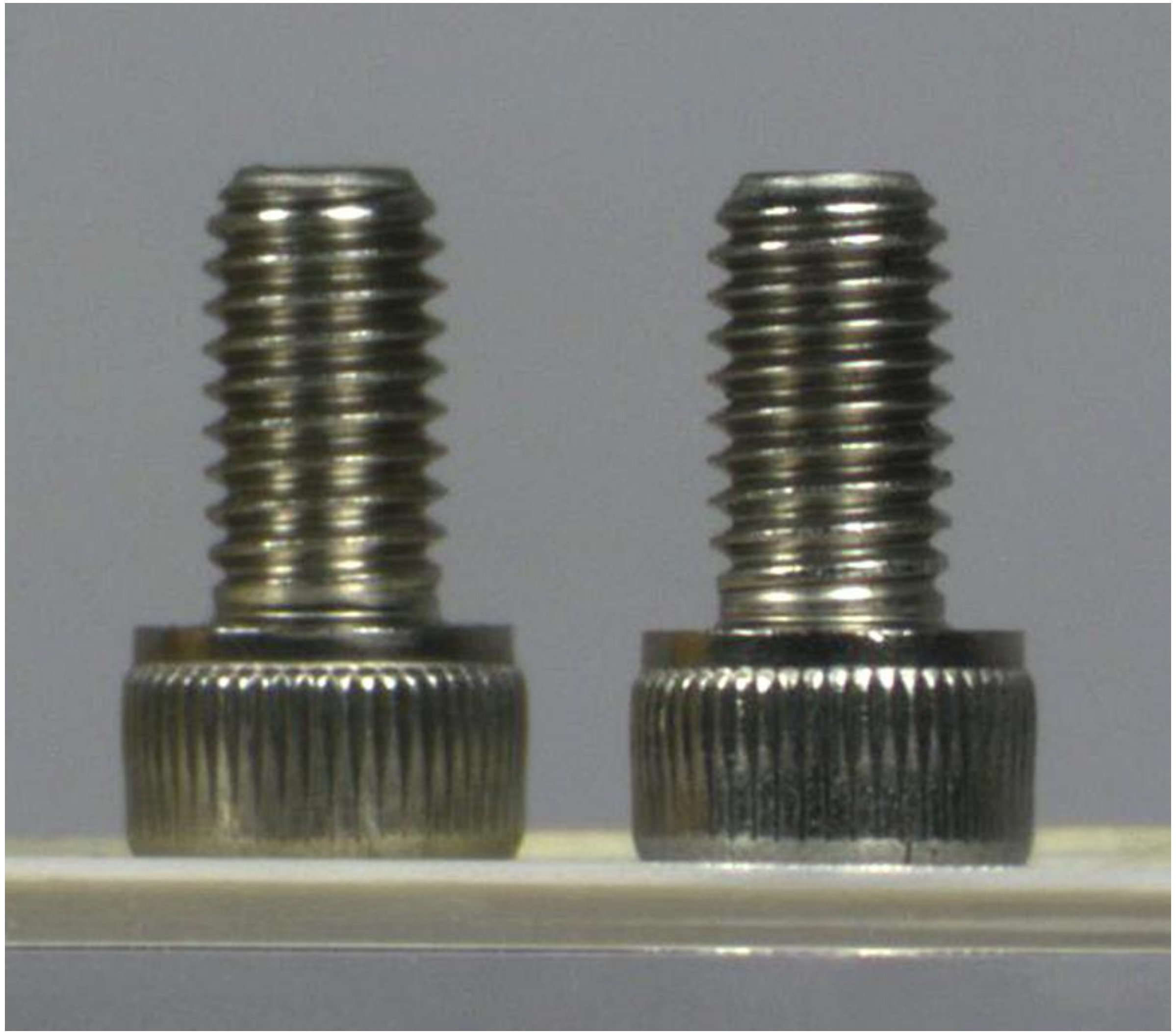
mid focal length

short focal length

Different cameras

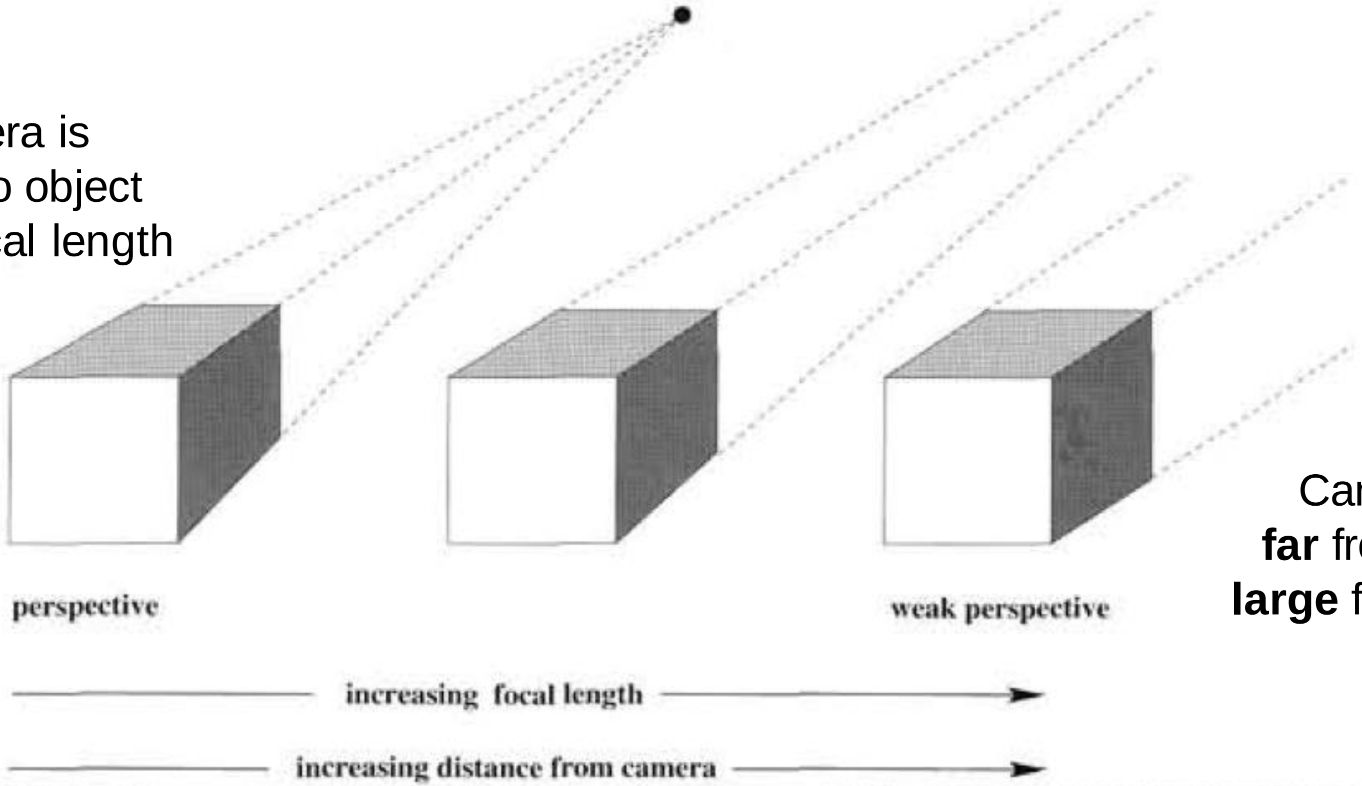


Perspective camera

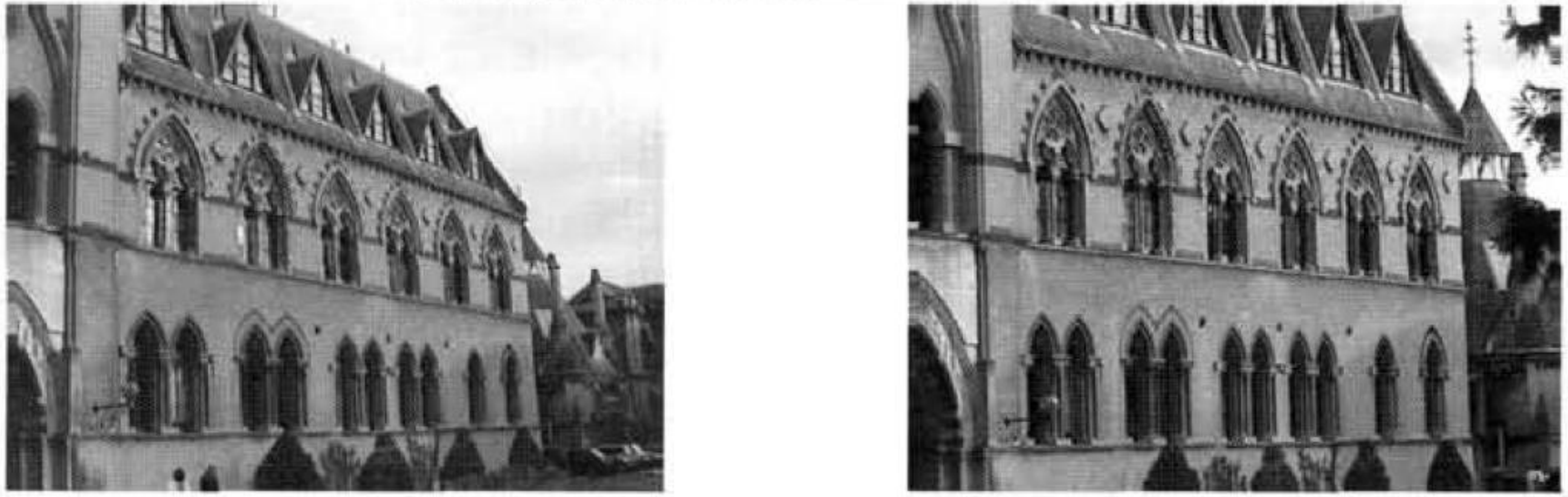


Orthographic camera

Camera is
close to object
small focal length



Camera is
far from object
large focal length



When can you assume a weak perspective camera model?

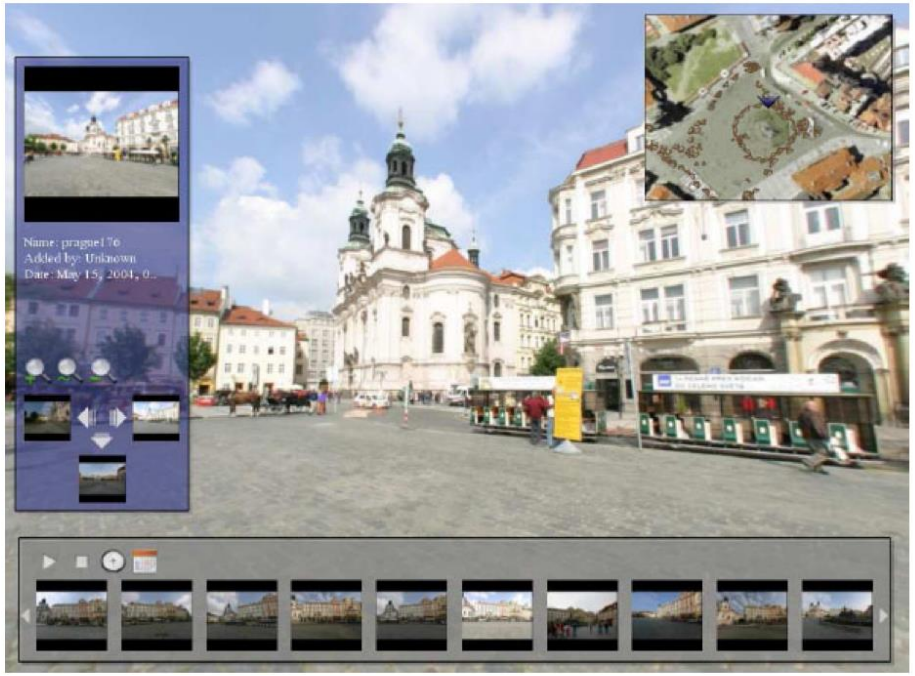
When can you assume a weak perspective camera model?

all the mountains are roughly ‘far away’

Pose estimation

	Structure (scene geometry)	Motion (camera geometry)	**Measurements**
Pose Estimation	known	estimate	3D to 2D correspondences
Triangulation	estimate	known	2D to 2D coorespondences
Reconstruction	estimate	estimate	2D to 2D coorespondences

Pose Estimation



Given a single image,
estimate the exact position of the photographer

3D Pose Estimation

(Resectioning, Geometric Calibration, Perspective n-Point)

Given a set of matched points

$$\{\mathbf{X}_i, \boldsymbol{x}_i\}$$

point in 3D space | point in the image

and camera model

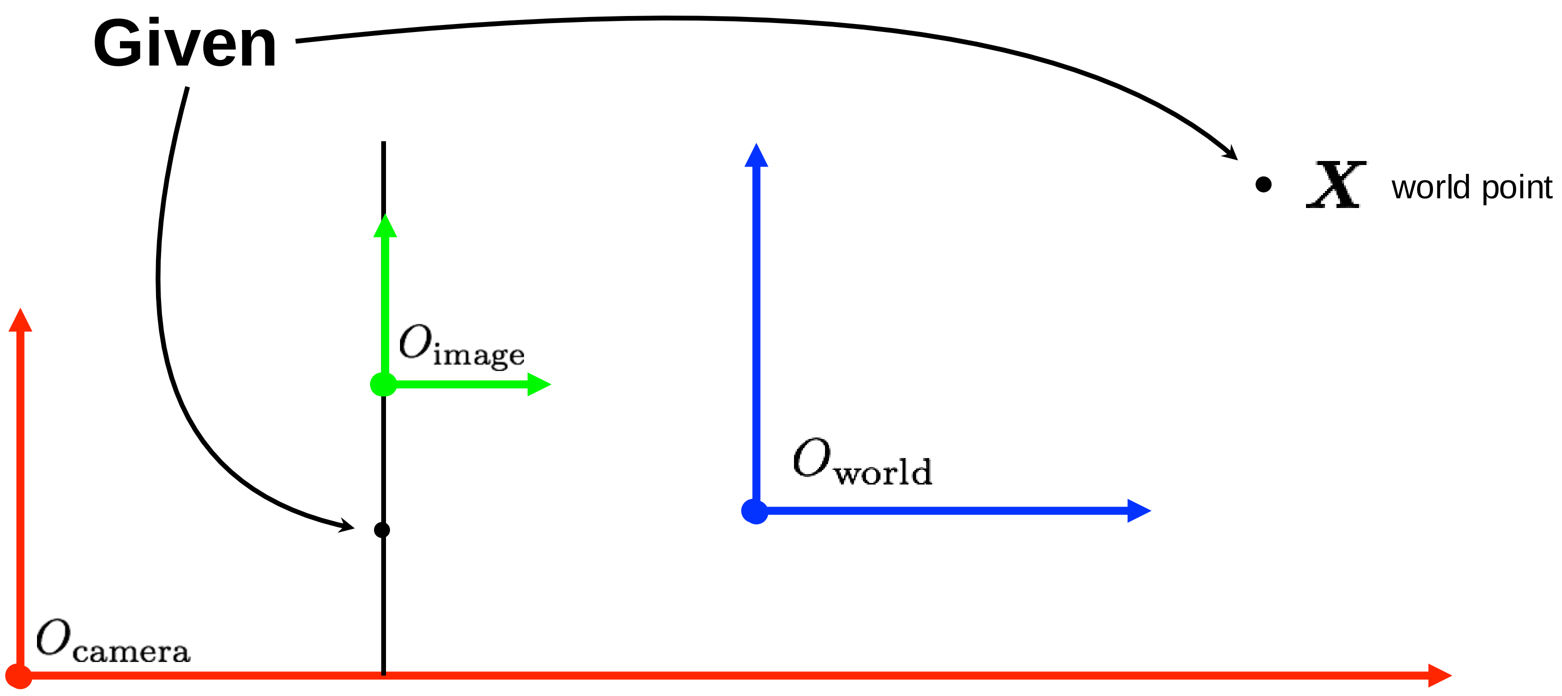
$$\boldsymbol{x} = \boldsymbol{f}(\mathbf{X}; \boldsymbol{p}) = \mathbf{P}\mathbf{X}$$

projection model | parameters | Camera matrix

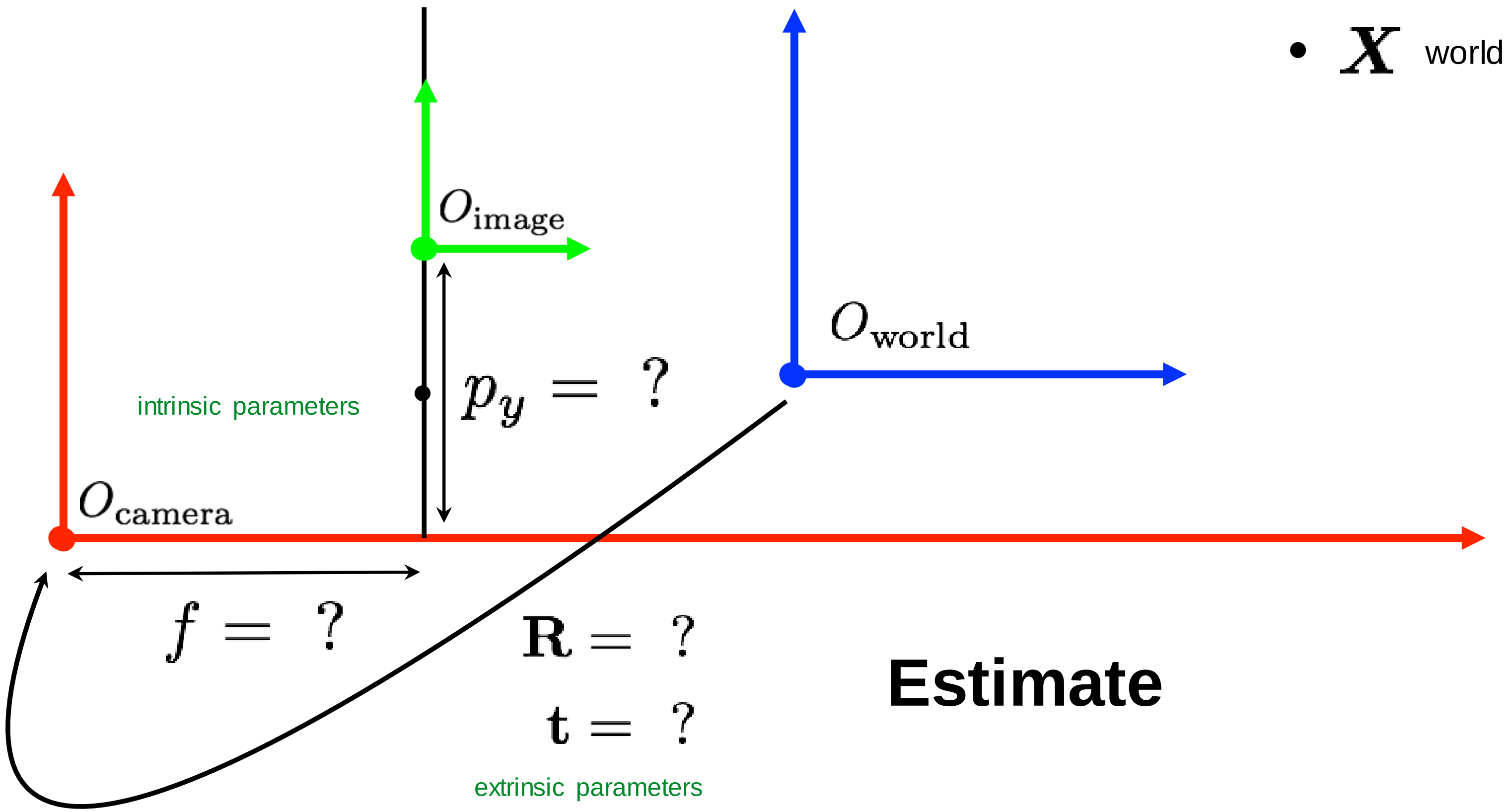
Find the (pose) estimate of

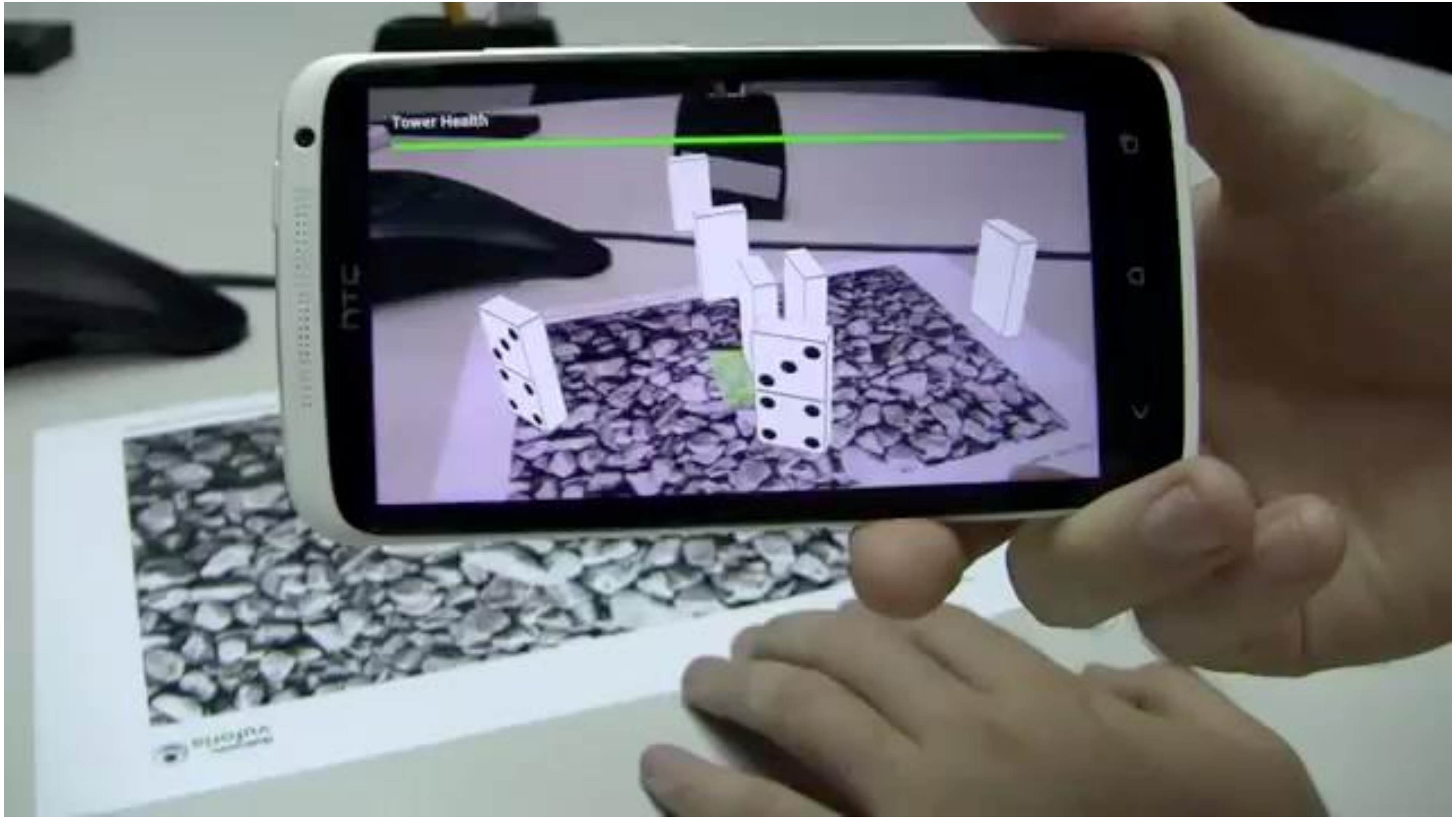
$$\mathbf{P}$$

What is Pose Estimation?

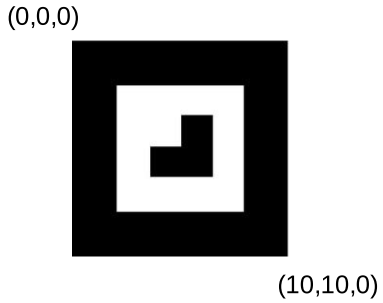


What is Pose Estimation?

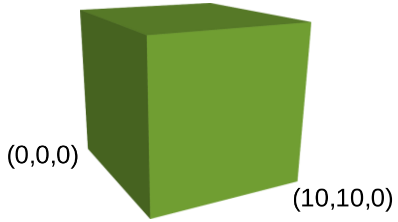



Tower Health

3D locations of planar marker
features are known in advance



3D content prepared in advance



Simple AR program

1. Compute point correspondences (2D and AR tag)

2. Estimate the pose of the camera **P**

3. Project 3D content to image plane using **P**


junaio


PEPSI MAX
PRESENTS

References

Basic reading:

- Szeliski textbook, Section 2.1.5, 6.2.

Additional reading:

- Hartley and Zisserman, “Multiple View Geometry in Computer Vision,” Cambridge University Press 2004.
  chapter 6 of this book has a very thorough treatment of camera models.
- Torralba and Freeman, “Accidental Pinhole and Pinspeck Cameras,” CVPR 2012.
  the eponymous paper discussed in the slides.