COMPUTER VISION
LECTURE 8 – CORNER DETECTION

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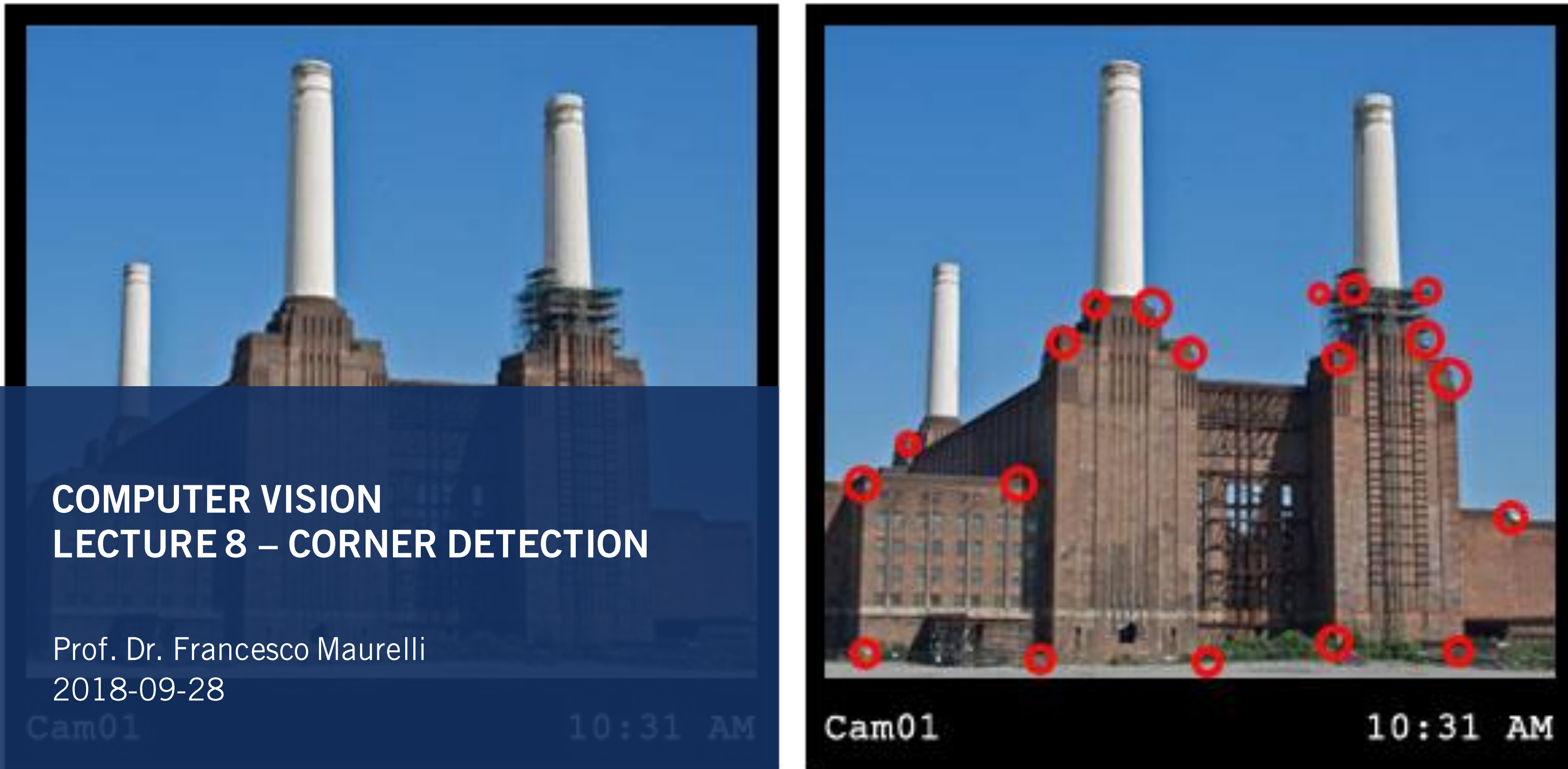
2018-09-28



A: Original image
B: Detected image

WHAT WE WILL LEARN TODAY

- Intro to Features
- Keypoint localization
  - Harris corner detector

Some background reading:
Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

Problem 1:

- Detect the same point *independently* in both images

No chance to match!

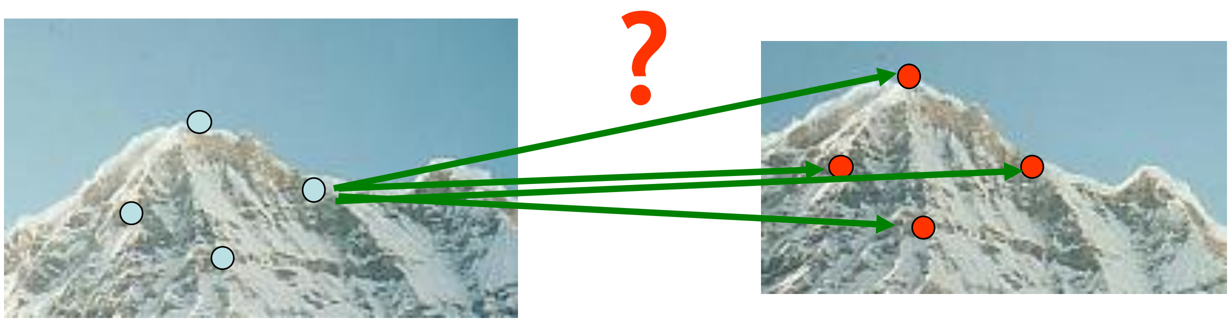
We need a repeatable detector!

Problem 1:

- Detect the same point *independently* in both images

Problem 2:

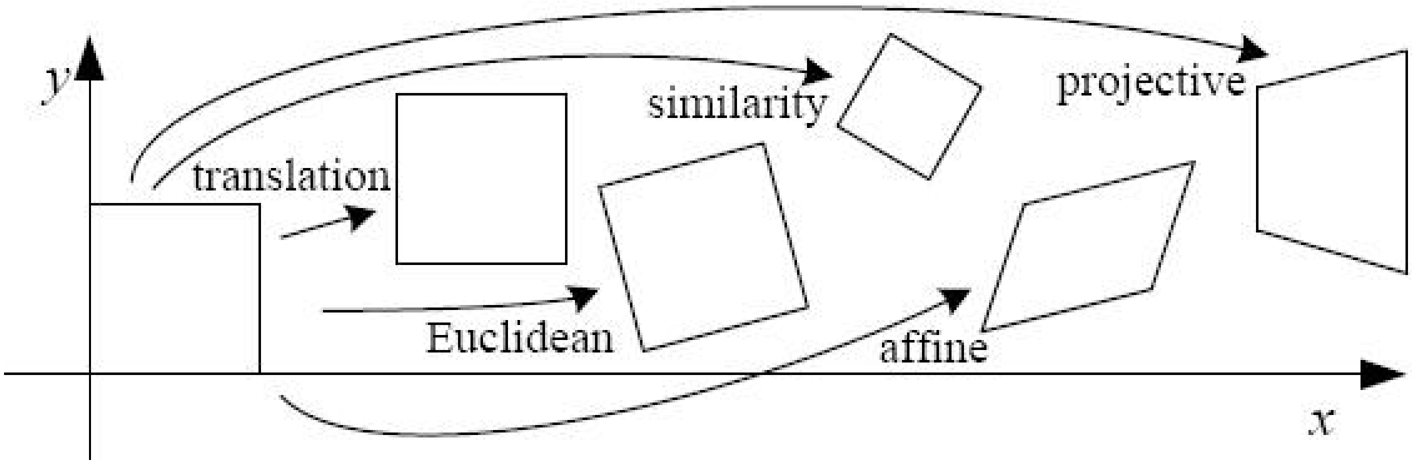
- For each point correctly recognize the corresponding one



We need a reliable and distinctive descriptor!

INVARIANCE: GEOMETRIC TRANSFORMATIONS

LEVELS OF GEOMETRIC INVARIANCE



REQUIREMENTS

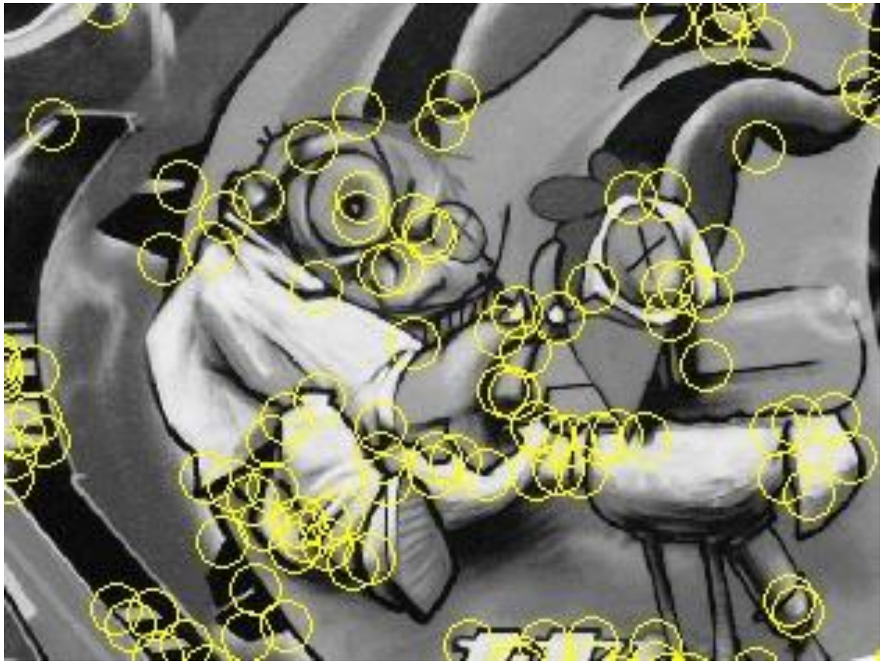
1. Region extraction needs to be repeatable and accurate
   - Invariant to translation, rotation, scale changes
   - Robust or covariant to out-of-plane (≈affine) transformations
   - Robust to lighting variations, noise, blur, quantization
2. Locality: Features are local, therefore robust to occlusion and clutter.
3. Quantity: We need a sufficient number of regions to cover the object.
4. Distinctivenes: The regions should contain “interesting” structure.
5. Efficiency: Close to real-time performance.

MANY DETECTORS AVAILABLE

- Hessian & Harris — [Beaudet '78], [Harris '88]
- Laplacian, DoG — [Lindeberg '98], [Lowe '99]
- Harris-/Hessian-Laplace — [Mikolajczyk & Schmid '01]
- Harris-/Hessian-Affine — [Mikolajczyk & Schmid '04]
- EBR and IBR — [Tuytelaars & Van Gool '04]
- MSER — [Matas '02]
- Salient Regions — [Kadir & Brady '01]
- Others…

- *Those detectors have become a basic building block for many recent applications in Computer Vision.*

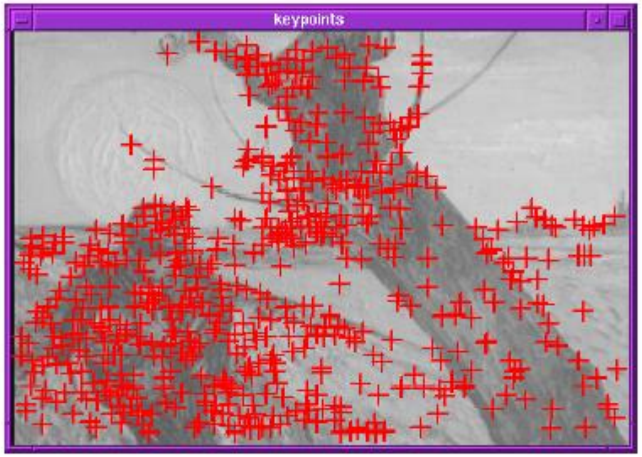
KEYPOINT LOCALIZATION

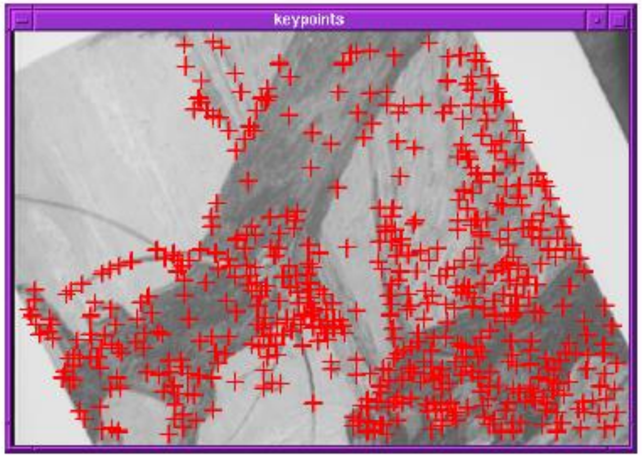


Goals:

- Repeatable detection
- Precise localization
- Interesting content

$\Rightarrow$ *Look for two-dimensional signal changes*





Key property:

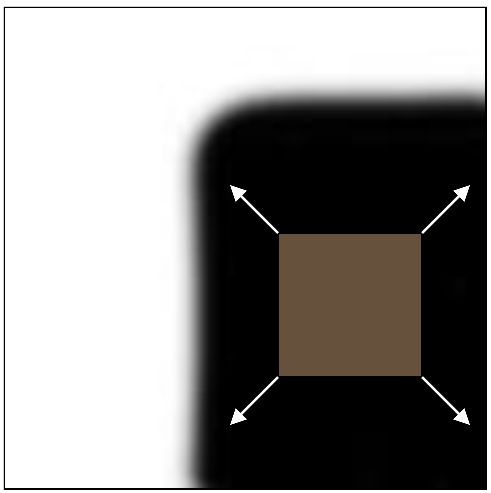
- In the region around a corner, image gradient has two or more dominant directions

Corners are *repeatable* and *distinctive*

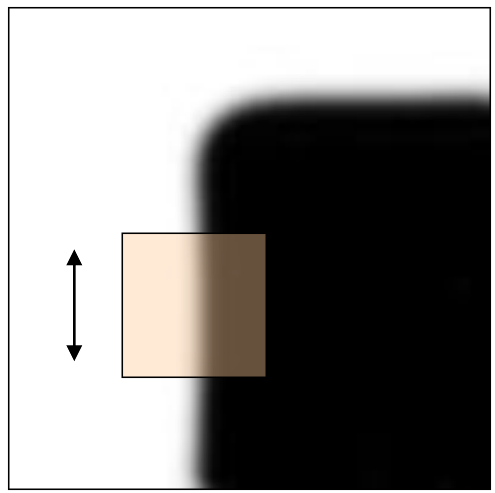
C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*, 1988.

Design criteria

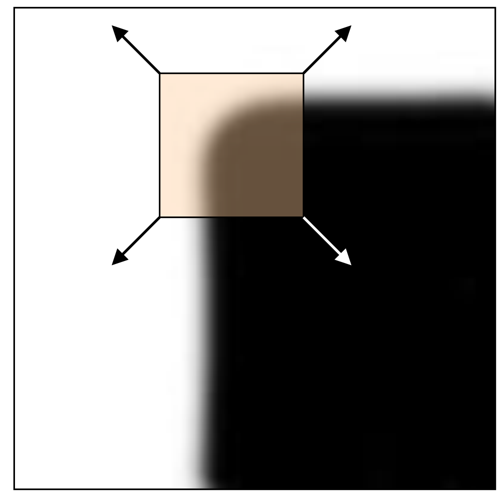
- We should easily recognize the point by looking through a small window (*locality*)
- Shifting the window in *any direction* should give *a large change* in intensity (*good localization*)



"flat" region:
no change in all directions

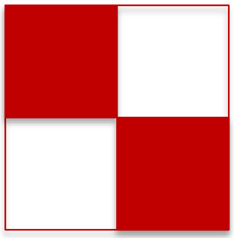


"edge":
no change along the edge direction



"corner":
significant change in all directions

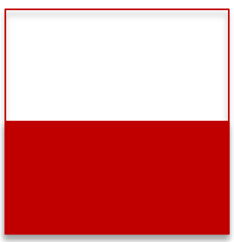
CORNERS VERSUS EDGES



$$\sum I_x^2 \longrightarrow \text{Large}$$

$$\sum I_y^2 \longrightarrow \text{Large}$$

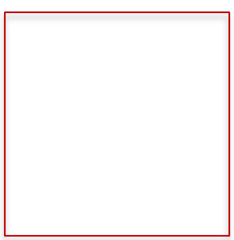
Corner



$$\sum I_x^2 \longrightarrow \text{Small}$$

$$\sum I_y^2 \longrightarrow \text{Large}$$

Edge

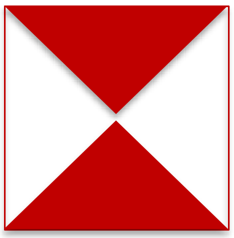


$$\sum I_x^2 \longrightarrow \text{Small}$$

$$\sum I_y^2 \longrightarrow \text{Small}$$

Nothing

CORNERS VERSUS EDGES



$$\sum I_x^2 \longrightarrow ??$$

$$\sum I_y^2 \longrightarrow ??$$

Corner

HARRIS DETECTOR FORMULATION

Change of intensity for the shift [u,v]:

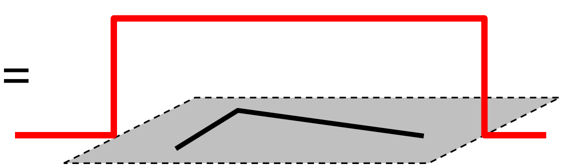
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

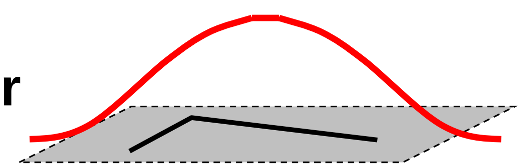
Shifted intensity

Intensity

Window function $w(x,y) =$



or



1 in window, 0 outside

Gaussian

Slide credit: Rick Szeliski

HARRIS DETECTOR FORMULATION

This measure of change can be approximated by:

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region – the area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y]$$

HARRIS DETECTOR FORMULATION

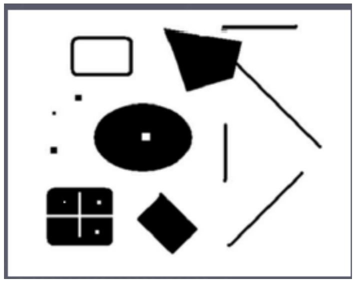
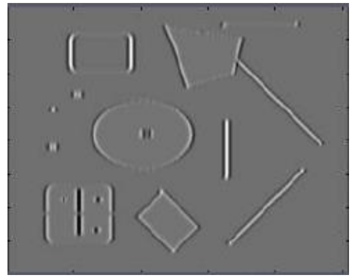
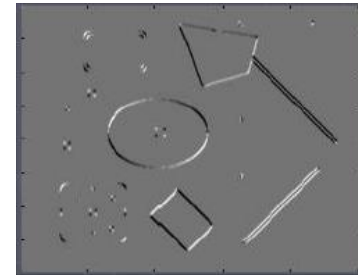


Image I



I_x

I_y



I_xI_y

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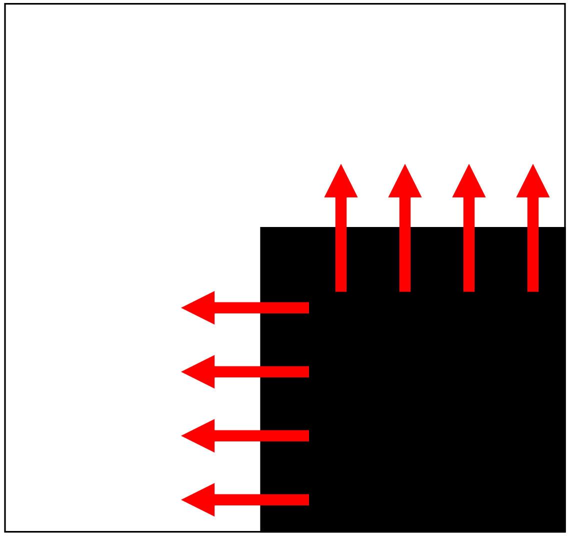
WHAT DOES THIS MATRIX REVEAL?

First, let's consider an axis-aligned corner:

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



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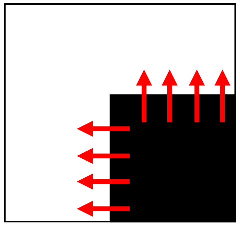


This means:

- Dominant gradient directions align with x or y axis
- If either λ is close to 0, then this is not a corner, so look for locations where both are large.

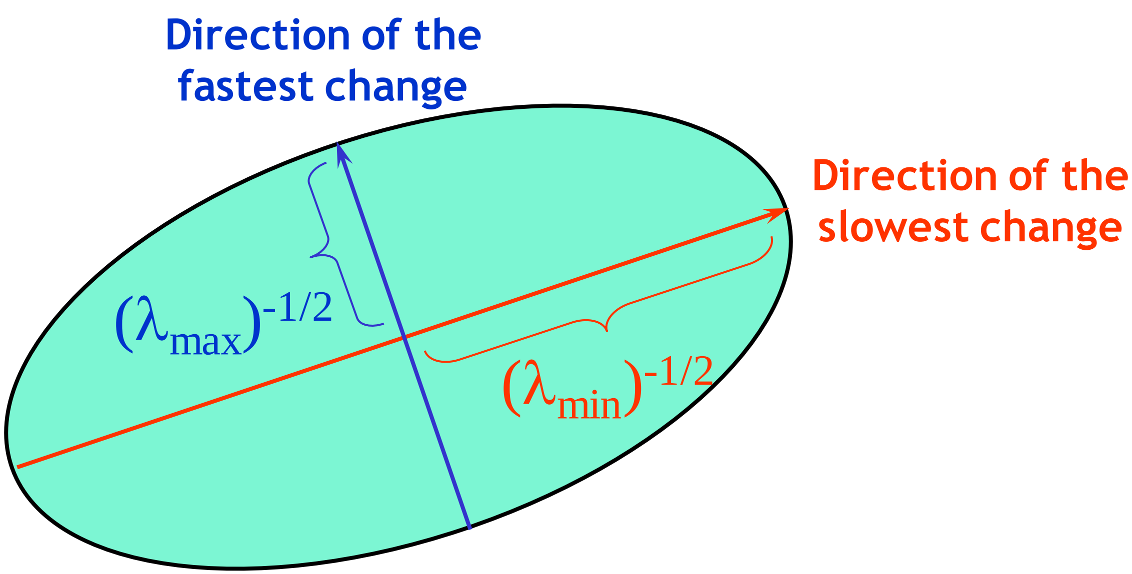
What if we have a corner that is not aligned with the image axes?

GENERAL CASE

- Since M is symmetric, we have $M = R^{-1}\begin{bmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{bmatrix}R$

  (Eigenvalue decomposition)

- We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R



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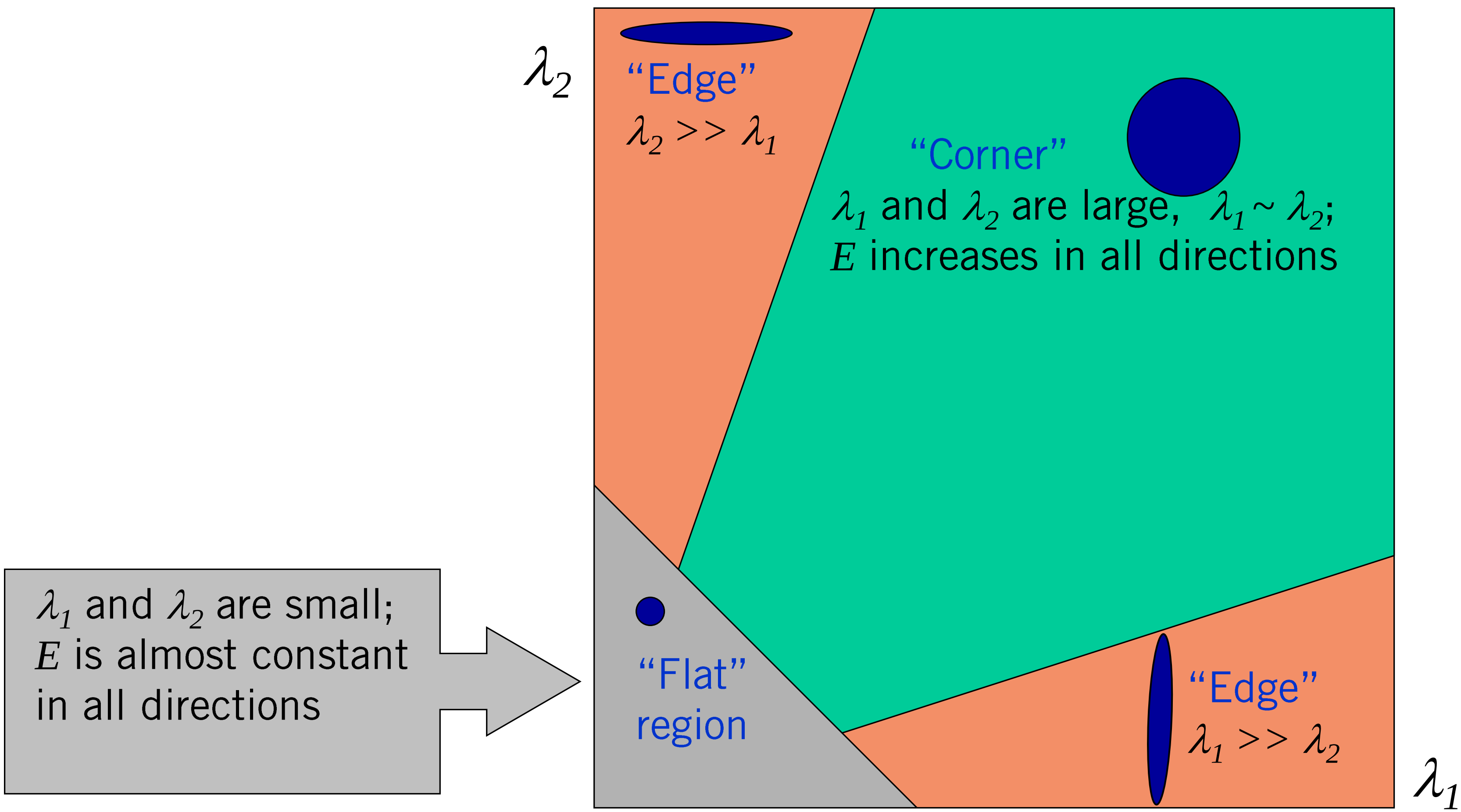
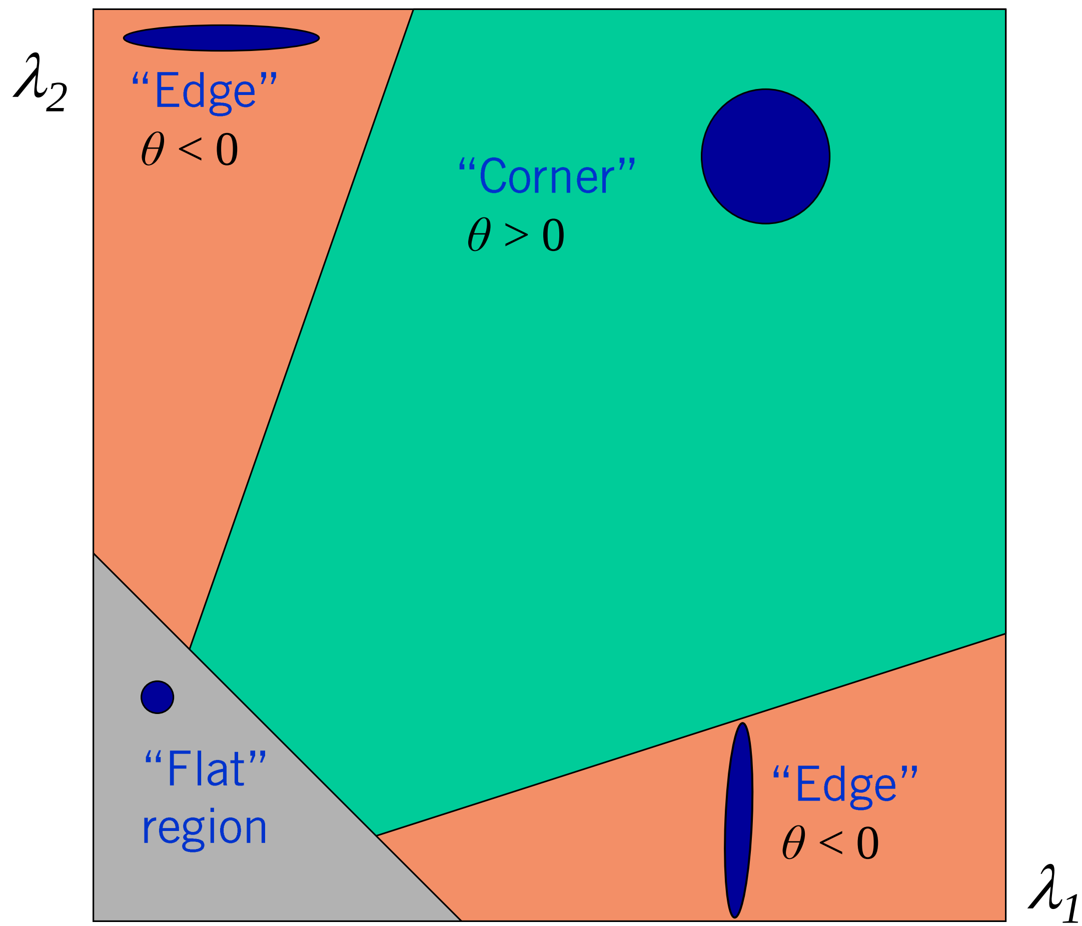
Classification of image points using eigenvalues of M:



CORNER RESPONSE FUNCTION

$$\theta = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$



Fast approximation

- Avoid computing the eigenvalues
- α: constant (0.04 to 0.06)

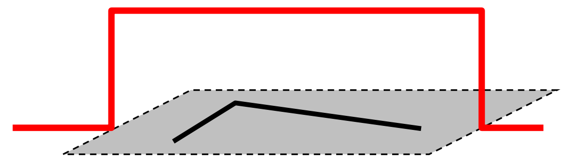
WINDOW FUNCTION *W(X,Y)*

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Option 1: uniform window

- Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



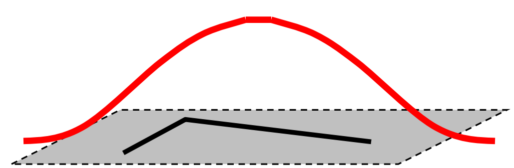
1 in window, 0 outside

- Problem: not rotation invariant

Option 2: Smooth with Gaussian

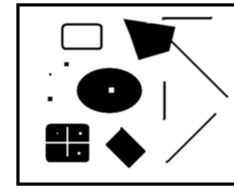
- Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Gaussian

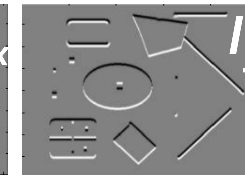
- Result is rotation invariant



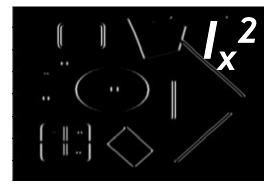
Compute second moment matrix (autocorrelation matrix)

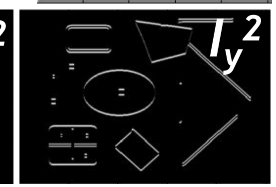
$$M(\sigma_I,\sigma_D)=g(\sigma_I)*\begin{bmatrix} I_x^2(\sigma_D) & I_xI_y(\sigma_D) \\ I_xI_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

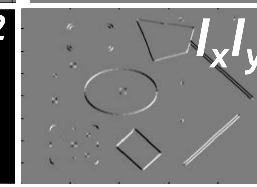
1. Image derivatives



2. Square of derivatives







3. Gaussian filter $g(\sigma_I)$

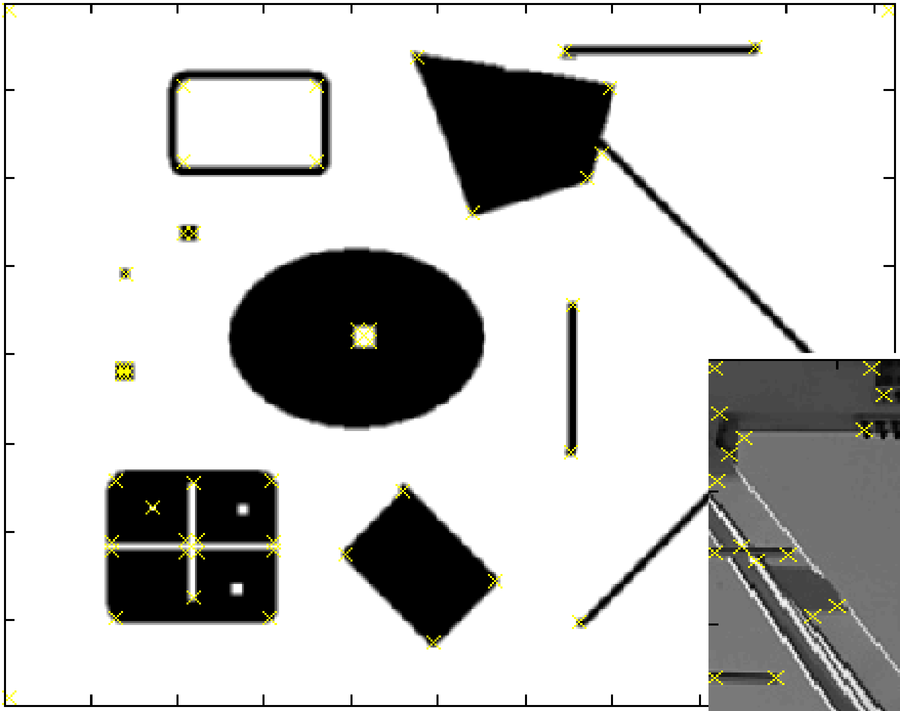
4. Cornerness function - two strong eigenvalues

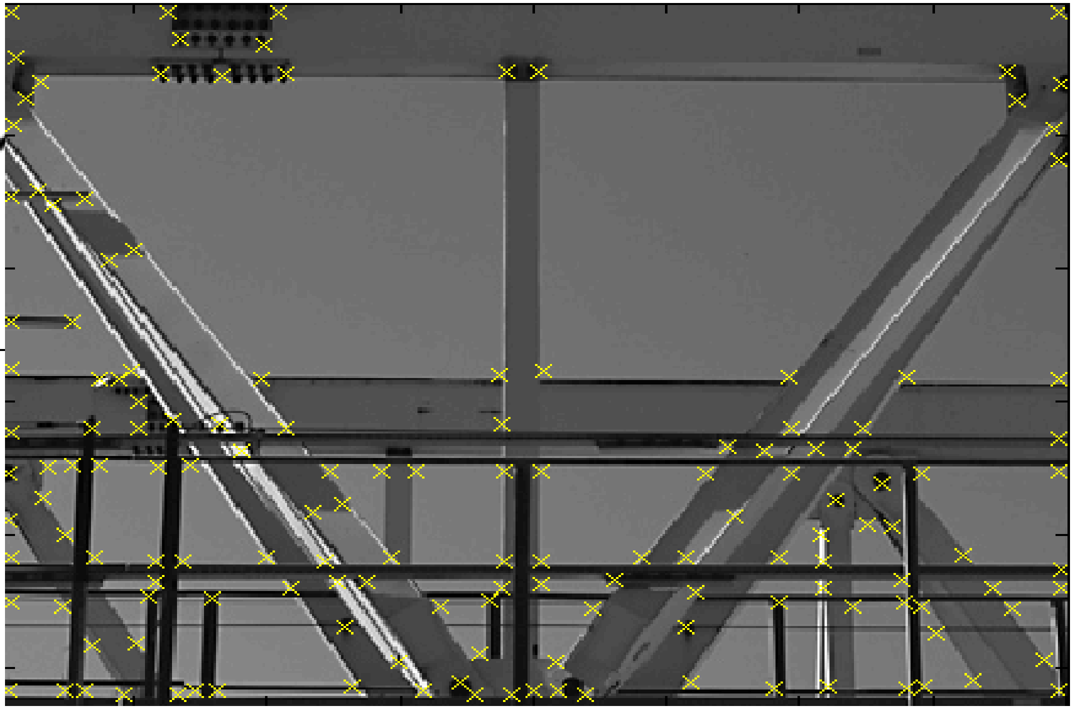
$$q=\det[M(s_I,s_D)]-a[\mathrm{trace}(M(s_I,s_D))]^2$$

$$=g(I_x^2)g(I_y^2)-[g(I_xI_y)]^2-\alpha[g(I_x^2)+g(I_y^2)]^2$$

5. Perform non-maximum suppression

HARRIS DETECTOR - RESPONSES





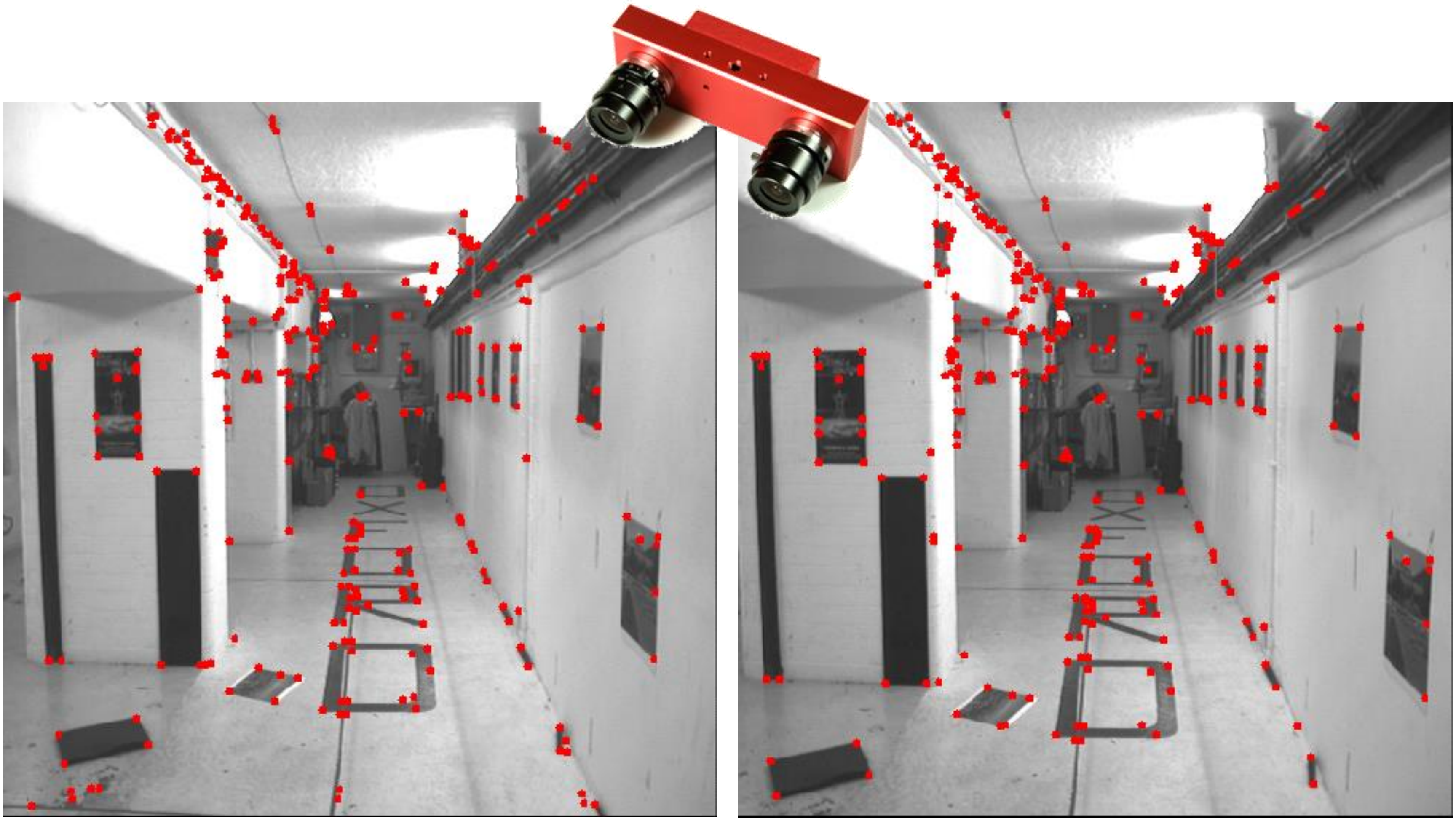
Effect: **A very precise corner detector.**

Slide credit: Krystian Mikolajczyk

HARRIS DETECTOR - RESPONSES

Slide credit: Krystian Mikolajczyk

HARRIS DETECTOR - RESPONSES



1. Results are well suited for finding stereo correspondences

Slide credit: Kristen Grauman

HARRIS DETECTOR - PROPERTIES

1. Translation invariance?

Slide credit: Kristen Grauman

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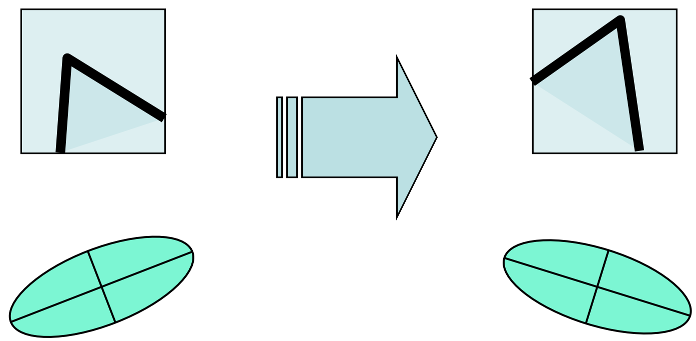
1. Translation invariance
2. Rotation invariance?



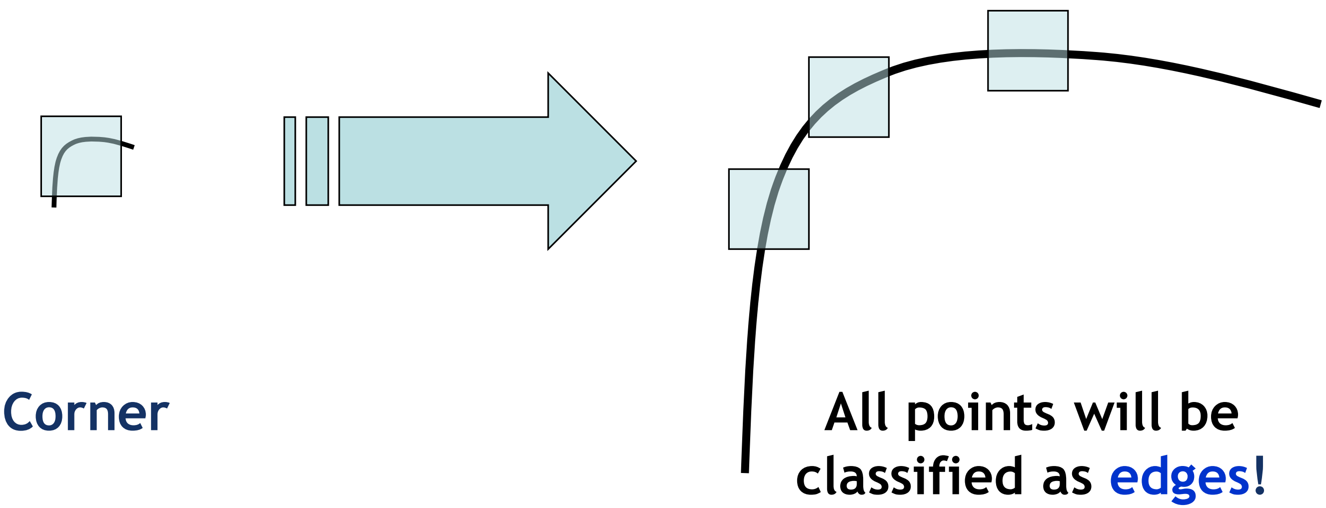
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response θ **is invariant to image rotation**

Slide credit: Kristen Grauman

HARRIS DETECTOR - PROPERTIES

1. Translation invariance
2. Rotation invariance
3. Scale invariance?



Not invariant to image scale!

Slide credit: Kristen Grauman