

## Homework 3

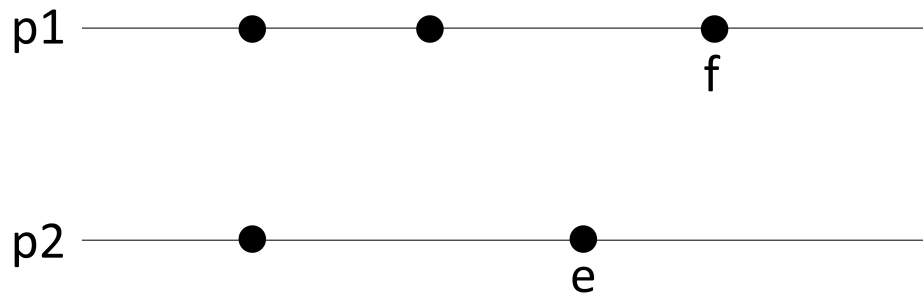
### Problem 3.1

**Solution:**

- a) (i)  $\Theta_L(e) < \Theta_L(f) \implies e \prec f$   
 (ii)  $\Theta_L(e) < \Theta_L(f) \implies f \prec e$

Both of the statements are false, because it is impossible to determine which event happened first according to the clock value.

From the following picture we can see that although  $\Theta_L(e) < \Theta_L(f)$ , nothing can be inferred about which event happened first.



- b) (i)  $\Theta_V(e) < \Theta_V(f) \implies e \prec f$   
 (ii)  $\Theta_V(e) < \Theta_V(f) \implies f \prec e$

The first statement is true, whereas the second one is false. In order to prove that, I am going to prove the right side of

$$e \prec f \iff \Theta_V(e) < \Theta_V(f)$$

This is equivalent to showing

$$\neg(e \prec f) \rightarrow \neg(\Theta_V(e) < \Theta_V(f))$$

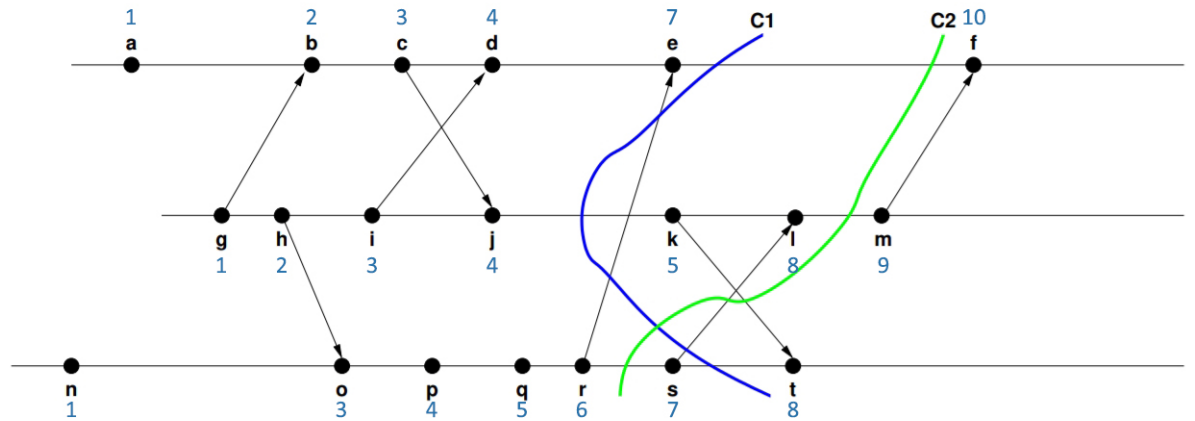
Let's suppose  $e$  occurs at  $p_i$  and  $f$  at  $p_j$ , and  $e$  doesn't happen before  $b$ . Let  $\Theta_V(e)_i = k$ . Since  $e$  doesn't happen before  $f$  there is no chain of messages from  $p_i$  to  $p_j$  originating at  $p_i$ 's  $k$ th step or later and ending at  $p_j$  before  $f$ . Thus  $\Theta_V(f)_j < k$ . Thus  $\neg(\Theta_V(e) < \Theta_V(f))$ .

- c) In order to find out whether  $e$  and  $f$  are concurrent we must do an element by element comparison of the corresponding timestamps; if some elements of  $\Theta_V(e)$  are smaller than their  $\Theta_V(f)$  counterparts and some are greater than their matches, then we know that the events are concurrent.

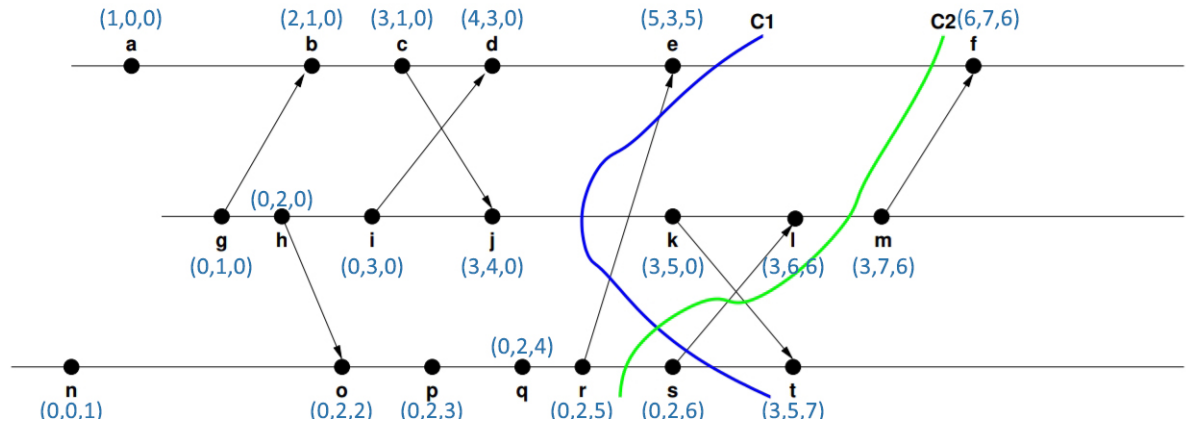
### Problem 3.2

Solution:

a) Lamport clock values for all events.



b) Vector clock values for all events.



c) The cut  $C_1$  is consistent as there is no step outside the cut that happens before steps  $e$ ,  $j$  and  $s$ . On the other hand, cut  $C_2$  is not consistent as we can see that  $s$  who happens outside the cut, happens before  $l$  that is inside the cut.

### Problem 3.3

Solution:

Referenced from [here](#)

```

upon event < Init > do
  for all  $p_i \in S$ :  $VC[p_i] := 0$ ;
  pending :=  $\emptyset$ 

upon event < rcoBroadcast, m > do
  trigger < rcoDeliver, self, m >;
  trigger < rbBroadcast, [Data, VC, m] >;
   $VC[self] := VC[self] + 1$ ;

upon event < rbDeliver, pj, [Data, VCm, m] > do
  if  $p_j \neq self$  then
    pending := pending  $\cup$  ( $p_j$ , [Data, VCm, m]);
    deliver-pending.

procedure deliver-pending is
  While ( $s$ , [Data, VCm, m])  $\in$  pending s.t.
    for all  $p_k$ : ( $VC[p_k] \geq VCm[p_k]$ ) do
      pending := pending - ( $s$ , [Data, VCm, m]);
      trigger < rcoDeliver, self, m >;
       $VC[s] := VC[s] + 1$ .

```