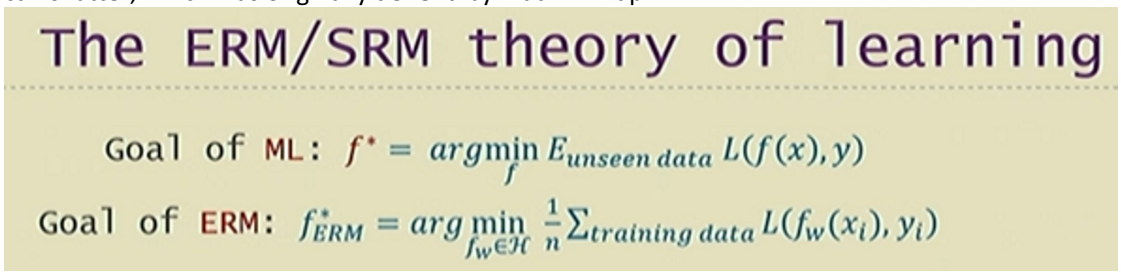
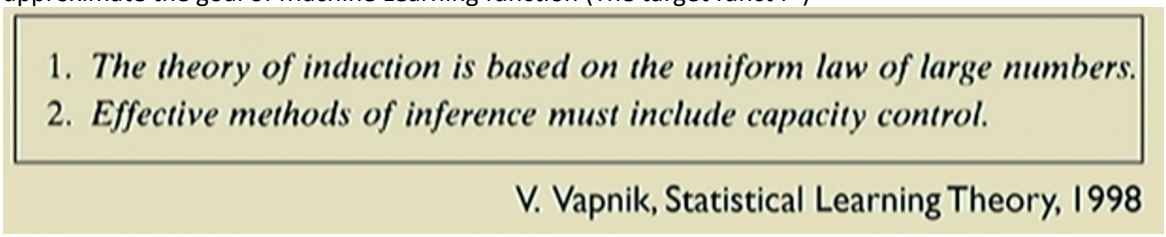
Why it's not overfitting

Thursday, March 26, 2020 2:54 PM

3/25
Why it's not overfitting? -- From Classical Statistics to Modern ML: the Lessons of Deep Learning - Mikhail Belkin, From Classical Statistics to Modern ML: the Lessons of Deep Learning, From Classical Statistics to Modern ML: the Lessons of Deep Learning, https://www.youtube.com/watch?v=5-Kqb80h9rk&feature=youtu.be

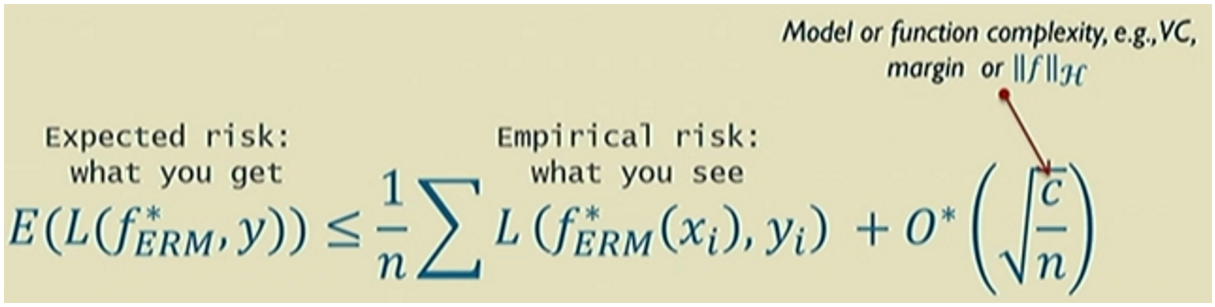
Mikhail (Misha) Belkin, https://scholar.google.com/citations?hl=en&user=Iwd9DdkAAAAJ&view_op=list_works&sortby=pubdate

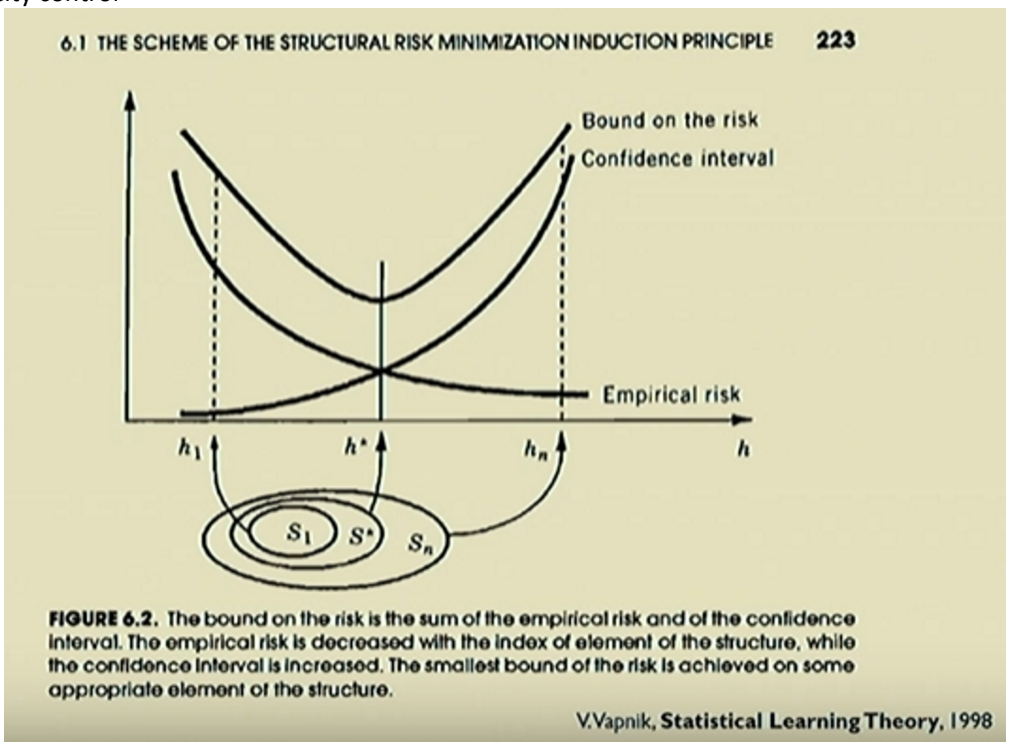
- VC dimension: A measure of the capacity of a space of function that can be learned by a statistical classification algorithm. It's defined as the cardinality of the largest set of points that the algorithm can shatter, which was originally defiend by Vladimir Vapnik.
- 

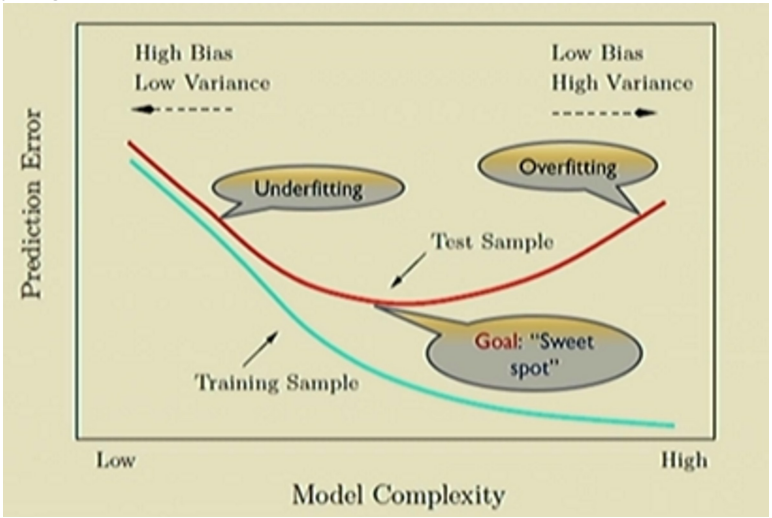
  - ML: the goal of Machine Learning is to find a function that minimize the loss in the future (Or the expected loss over prob dist under some assumption).
  - ERM: the goal of Empirical risk minimization is to find a fucntion over a class of function that minimize the loss over the training data
- How should we compare them in the classical approach?
  - According to Vapnik: 1) Uniform Law of Large number. 2)Capacity control
    i. **uniform Law of large number** says that the ERM is approximately equal to the expected loss, so it allow us to compare the empirical loss to the expected loss
    ii. **Capacity control:** say there are H (A set of function) contrains a funciton that approximate the goal of Machine Learning function (The target funct f*)
    - 

  - You can see your ERM solution is nearly optimal (Close to expected loss)
    - 

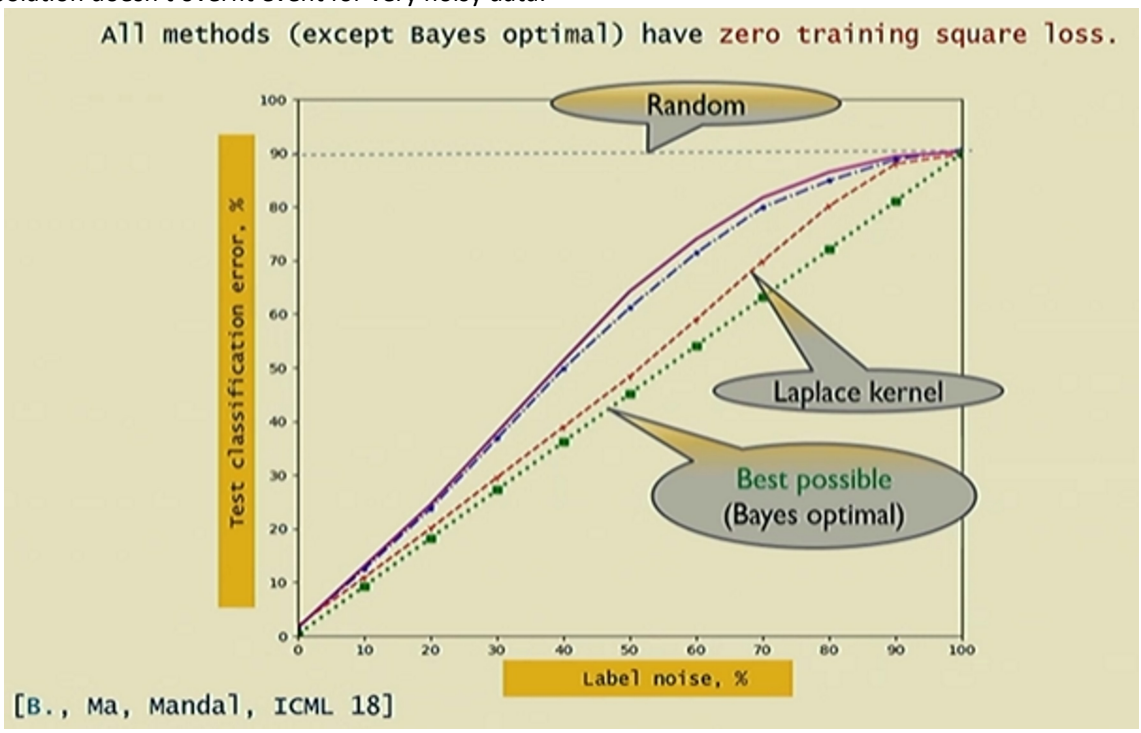
- Uniform laws of large numbers (Aka WYSIWG bounds), e.g. vc-dim, fat shattering, rademacher, covering number, margin…

- 

  - In the left: the expected risk, what you get, the future
  - In the right: the Empirical risk (What you see when in your training data) + capacity term (e.g. VC, margin)
- Capacity control
  - 

  - U-shaped generalization curve
    - 

      - In the left side is underfitting, and right side is overfitting, and our goal is to find this "sweet spot"
      - However, a model with **zero training error** is overfitting to the training data and will typically generalize poorly.
      - zero training error is also known as Interpolation, in mathmatical term!
- Does interpolation overfit?

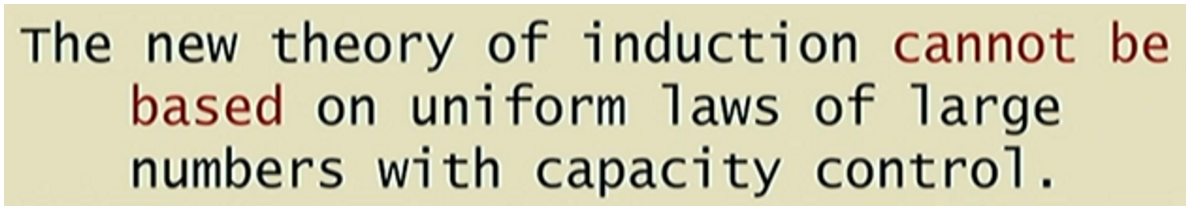
    - This is a paper, prove that you can interpolation and still have a good test result!
- Interpolation doesn't overfit event for very noisy data:
    - 

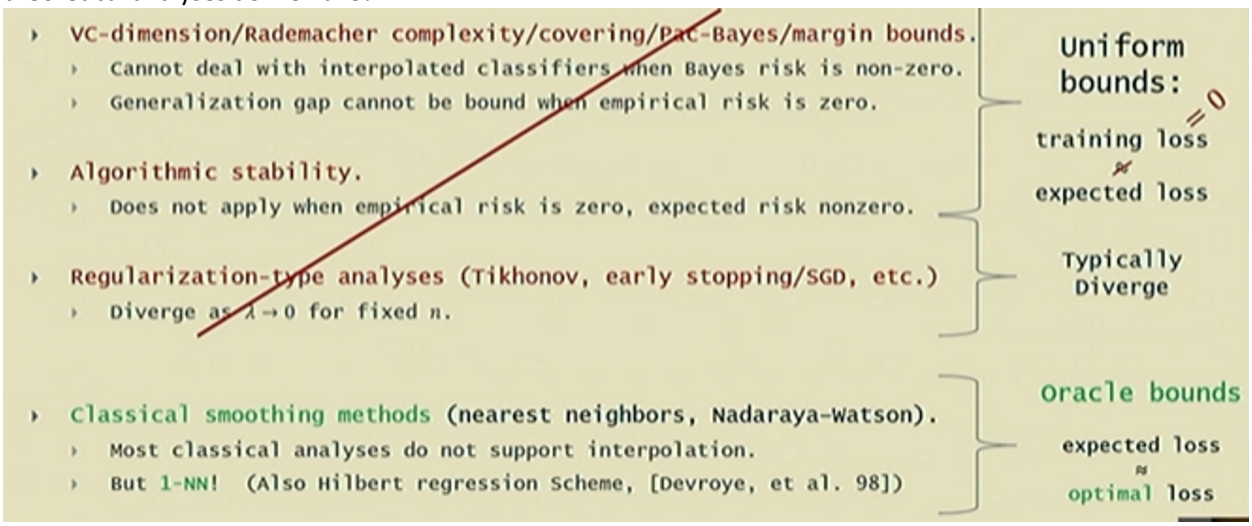
        - Randomization: take 10% of the data and assign random labels
        - Green line: 50%, the best you can possibly do, based on Bayes optimal classifier.
        - Random guess: is 90%, because the dataset has 10 classes.
        - Red line: a kernel machine train to have zero loss on the noisy data. However, the result doesn't seem to overfit.
- why bounds fail?
    - $$\text{correct} \quad 0.7 < O^*\left(\sqrt{\frac{c(n)}{n}}\right) < 0.9 \quad \text{nontrivial} \qquad n \to \infty$$
        - There are two problem:
            - 1. The constant in O^* needs to be exact. There are no known bounds like that.

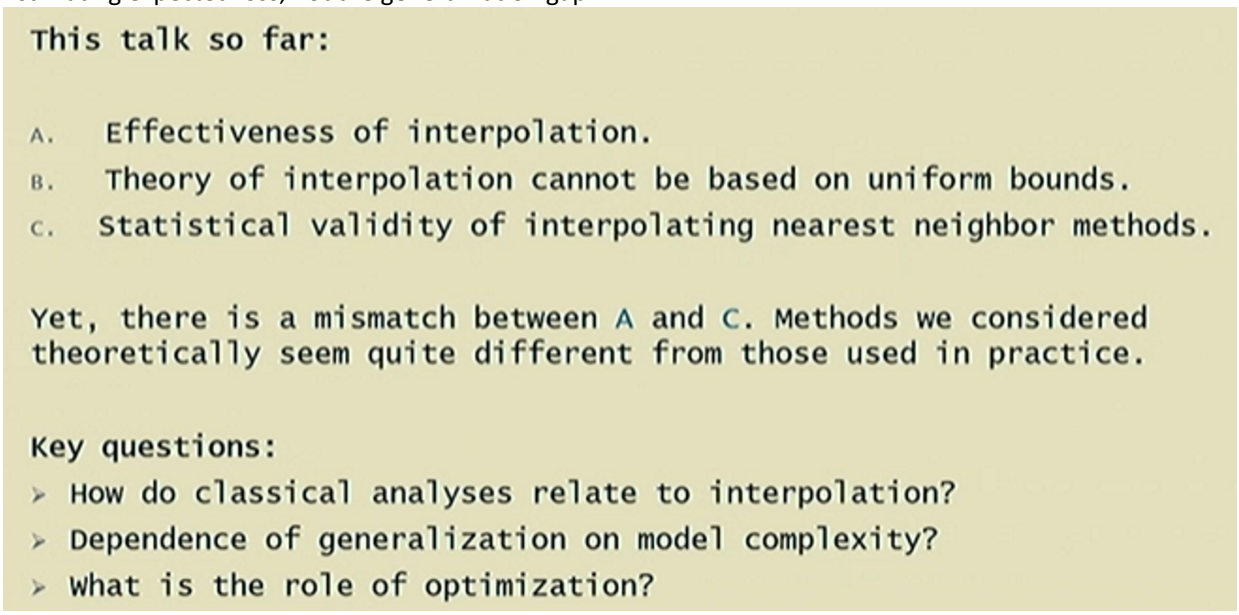
              2. Conceptually, how would the quantity $c(n)$ "know" about the Bayes risk?
- Interpolation is best practice for deep learning:
    - The best way to solve the problem from practical standpoint is you build a very big system …. Basically you want to make sure you hit the zero training error (Interpolation)
        - Yann Lecun (IPAM talk, 2018):

          Deep learning breaks some basic rules of statistics.

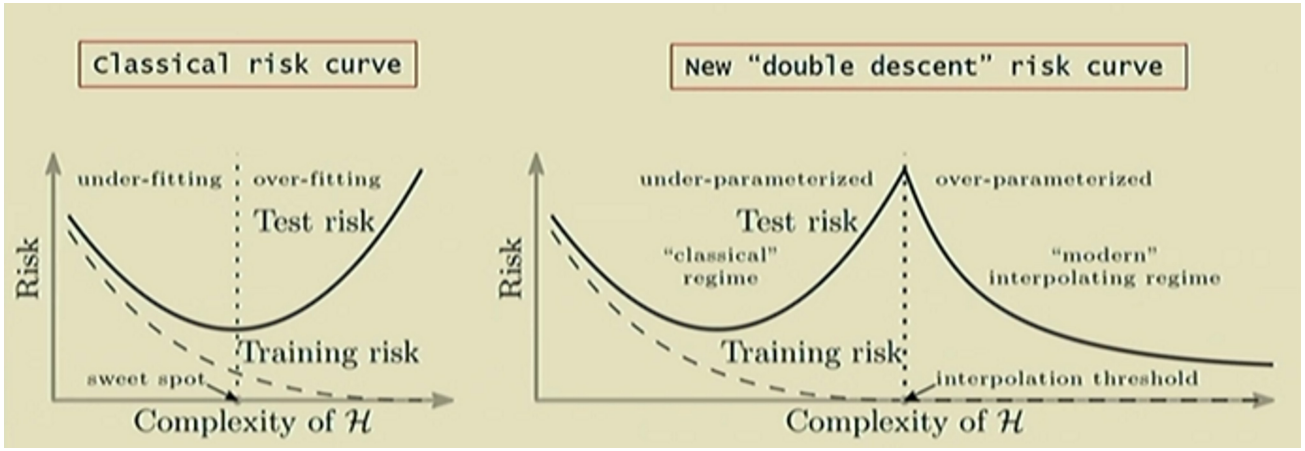
- **The modern ML: The key lesson**

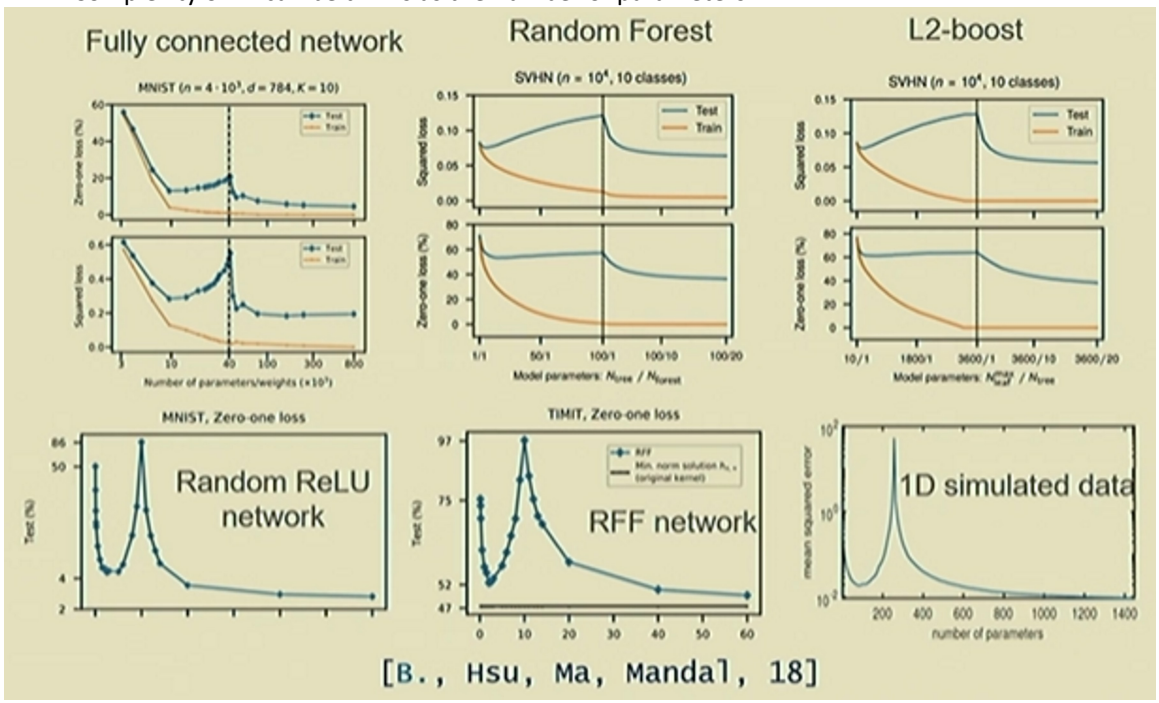
- 

- So the generalization theory will be helpful for understanding interpolation?
  - What theoretical analyses do we have?
    - 

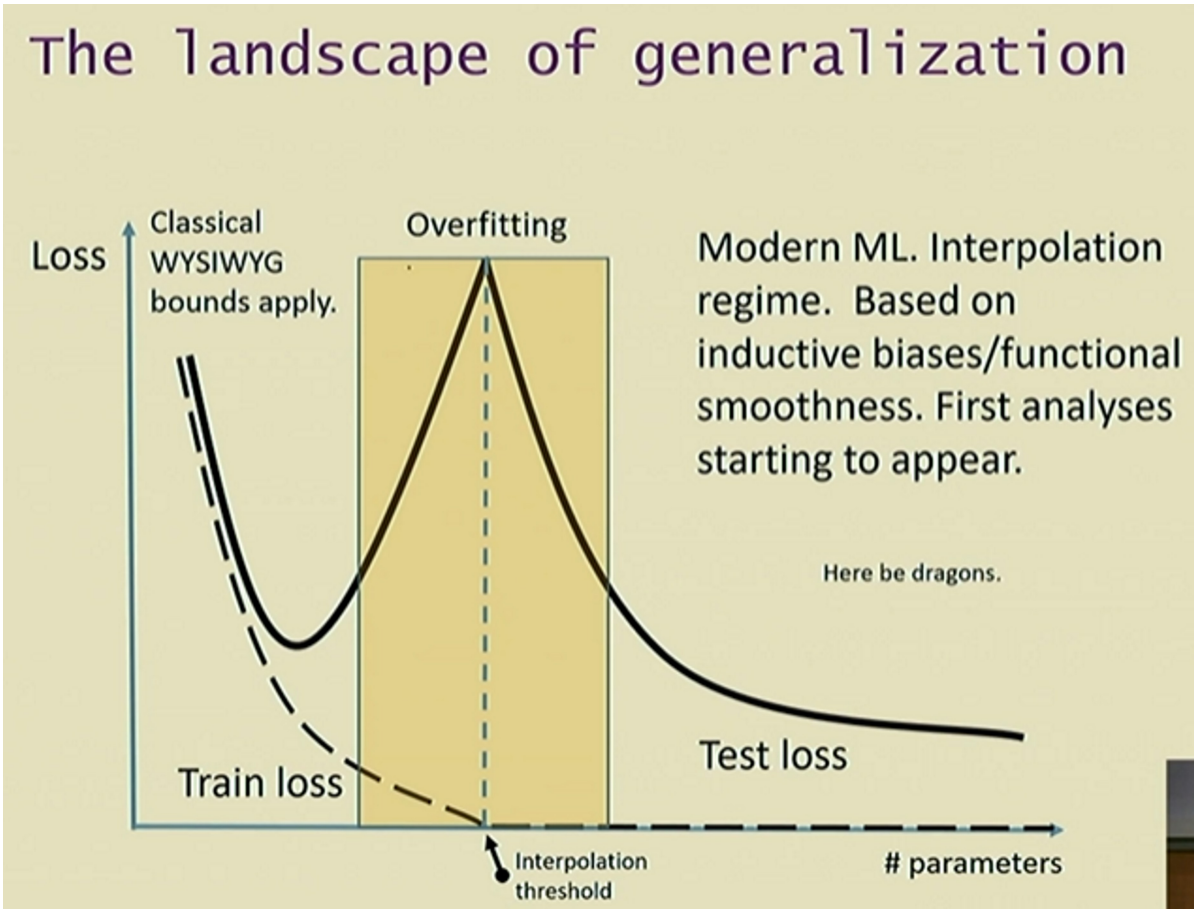
  - 1-NN
    - Analysis doesn't based on complexity bounds
    - Estimating expected loss, not the generalization gap
    - 

- "Double descent" risk curve: Reconciling modern machine-learning practice and the classical bias–variance trade-off, https://www.pnas.org/content/pnas/116/32/15849.full.pdf

- 

  - Complexity of H: can be thinks as the number of parameters
- 

- More parameters are better: an example
- Random Fourier network
- Why is training better as I am increasing the number of features?
  - the space of solution, the interpolated data
- New understanding of overfitting
  - Previous: while the training loss is low, we must be overfitting, and therefore we should decrease the number of parameter, such introduce regularization,
  - Now: there are two way to avoid the overfitting. 1)The classical way is to reduce the number of parameter. 2)The modern ML is to increase the number of parameter, moving to the right. It's counterintuitive, but

- 

- Classical Optimization(Under-parametrized)
  - Many local minima
  - SGD(Fixed step size) doesn't converge
- Modern Optimization(**Interpolation**/over-parametrized)
  - 1. Every local minimum is global (for networks wide enough)
    [Li, Ding, Sun, 18], [Yu, Chen, 95]

    2. Local methods converge to global optima
    [Kawaguchi, 16] [Soheil, et al, 16] [Bartlett, et al, 17]
    [Soltanolkotabi, et al, 17, 18] [Du, et al, 19] …

    3. Small batch SGD (fixed step size) converges as fast as GD per iteration.
    [Ma, Bassily, B., ICML 18] [Bassily, Ma, B., 18]