Question 8

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proposition:

$\forall \varepsilon_1 > 0, \exists N_1 \in \mathbb{N} \forall n > N_1 |a_n - L| < \varepsilon_1 \Leftrightarrow \forall \varepsilon_2 > 0, \exists N_2 \in \mathbb{N} \forall n > N_2 |a_n - L| < \varepsilon_2$

proof: prove by definition of sequence's limitation.

1. $\exists N_1 \in \mathbb{N} \forall n > N_1 |a_n - L| < \varepsilon_1$
2. $\exists N_1 \in \mathbb{N} \forall n > N_1 |Ma_n - ML| = |M(a_n - L)| = M|a_n - L| < M\varepsilon_1$
3. let $\varepsilon_2 = M\varepsilon_1$ we have $\exists N_1 \in \mathbb{N} \forall n > N_1 |Ma_n - ML| = |M(a_n - L)| = M|a_n - L| < \varepsilon_2$
4. $\forall \varepsilon_2 \exists N_1 \in \mathbb{N} \forall n > N_1$ we have $|Ma_n - ML| < \varepsilon_2$.

conclusion: step 4 is the definition of $\lim_{n \to \infty} Ma_n = ML$.