Question 5

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proposition: for any integer n at least one of the integers n, $n+2$, $n+4$ is divisible by 3.

proof: we prove by analysis the remainder of $n, n+2, n+4$ as divisor is 3. We show that there must be one of these 3 number has a remainder 0.

1. by division theorem, any integer divided by 3 has a remainder of $0, 1, 2$ (note that $0 \leq r < b$ in the theorem)

2. for any integer n, divided by 3, it has 3 possible remainders $0, 1, 2$

3.1 if the remainder is 0 ($n = 3q + 0$), n is divisible by 3.

3.2 if the remainder is $1(n = 3q + 1)$, n is not divisible by 3. however $n + 2$ has a remainder $0(n+2 = 3q+3 = 3(q+1))$. (Equally, $n = 3*q+r \Rightarrow n+2 = 3*(q+1)-1+r$, when $r = 1$ $n + 2 = 3 * (q + 1)$)

3.3 if the remainder is $2(n = 3q + 2)$, n is not divisible by 3. $n + 2$ has a remainder 1 ($n + 2 = 3q + 4 = 3(q + 1) + 1$),however $n + 4$ has a remainder $0(n + 4 = 3q + 6 = 3(q+2))$.(Equally, $n = 3*q+r \Rightarrow n+4 = 3*(q+2)-2+r$, when $r = 2$ $n+4 = 3*(q+2)$)

4. in all the cases of n's remainder, we find one of n, $n + 2$,$n + 4$ is divisible by 3.

conclusion: at least one of the integers $n, n + 2, n + 4$ is divisible by 3.