

Question 1

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proposition: $\neg(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$

proof: prove by contradiction. The contradiction is, when $3m + 5n = 12$ holds, m and n cannot be natural number at the same time.

1. Assume $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$
2. $\Leftrightarrow (\exists m \in \mathbb{N})(\exists n \in \mathbb{N})m = 4 - \frac{5}{3}n$
3. if $n \in \mathbb{N}$ which means $n \geq 1$. To satisfy the equation above. we illustrate all possible n value.
 - 3.1. we have $m = \frac{7}{3}$ when $n = 1$ and
 - 3.2. $m = \frac{2}{3}$ when $n = 2$. under both situation, m is not a natural number
 - 3.3. for $n \geq 3$, we will result a negative m which was not a natural number.
4. therefore, m and n cannot be natural number at the same time. contradiction.

5. **conclusion:** the assumption $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$ is false.