Non-Intrusive Load Curve Disaggregation using Sparse Decomposition with a Translation-Invariant Boxcar Dictionary

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***Abstract*—We present a non-intrusive load monitoring (NILM) method based on sparse decomposition techniques in order to extract the individual appliance signals from the aggregated load curve of an household. The propoed method is generic and does not need to be adapted for each home. It is based on a translation-invariant boxcar dictionary of atoms, each of them modeling a complete on-off appliance activity event. We evaluated our algorithm on synthetic and real load curve dataset. Experiments showed that we can extract the individual appliance signals with more than 20 dB of signal-to-distortion ratio (SDR), and estimate the energy consumption of the main consumers with a relative error less than 1%. We think that this ability to automatically provide accurate feedback information on the user appliances consumption through a single point of measurement open new perspective in demand-side management.**

***Index Terms*—demand side management, energy disaggregation, load management, nonintrusive load monitoring, sparse signal approximation**

I. INTRODUCTION

Today, there is a growing interest of reducing domestic electrical consumption due to both economic and ecological reasons. The energy demand is increasing exponentially with negative implications on the environment (e.g. carbon dioxide emissions) while at the same time the availability of energy resources such as fossil fuels is rapidly decreasing.

The residential sector alone accounts for 30% of electricity usage in the European Union (EU) [1], [2] and 38% in USA [3], while 25% [2] (respectively 44%) of the residential energy consumption is electrical in EU (respectively in USA) and this rate has constantly increased over the past 30 years. Moreover, most of the electrical energy (67% in USA [3]) is lost during the processes of generation and distribution. If we include the electricity losses in our computation, the percentage of electrical energy consumption in the residential sector of the USA is 70% [4]. This is a growing concern as it is predicted that global energy demands will double by the end of 2030 [1], [5].

New green building policies and energy-efficiency appliance standards started to address this problem and contribute in reducing these growing demands. According to some studies [6], when combined with demand-side management (DSM) programs, i.e. modification of consumer habits in order to use less energy during peak hours, these approaches have the potential to reduce electrical usage by as much as 30% over the next decade [6]. Other studies show that providing relevant information about the home consumption could lead to a reduction of electricity consumption by up to 10 and 15 % [7]. Also, a detailed review [8] of more than 60 feedback studies suggest that maximum energy saving can be achieved using direct feedback mechanisms (i.e., real-time appliance level consumption information) as opposed to indirect feedback mechanisms (i.e., monthly bills, weekly advice on energy usage).

Non-Intrusive Load Monitoring (NILM) is a technique that determines the load composition of a household, i.e. the activity of the individual appliances which constitute the load, through a single point of measurement, e.g. the main electrical service entry point of the home. This technique has the great advantage to not require installing meters on each individual appliance. NILM is likely to be a hot topics in the coming years for multiple reasons. Indeed the transition from fixed to time-dependent pricing in Europe and North America should increase the demand in energy monitoring and management solutions. Moreover, it is expected that around 80 M smart meters, which can be used by NILM methods, will be installed in USA by 2013 thanks to the development on Smart Grid.

Several NILM methods have been proposed in the last two decades. Despite few works based on hidden Markov models (HMM) [9], [10], or non-negative matrix factorization (NMF) [19], most of the methods of the literature are based on the analysis of each transition signature, i.e. any set of features that describes a *transition event* (e.g., a turn `on` or a turn `off`), given by the real and reactive power of the appliance. These approaches can be based on installations with a low sampling rate of 1-10 Hz as in [11], or on high frequency analysis so as to extract spectral features and make the appliance more easily distinguishable. These high frequency installations typically require a high sampling rate (e.g., 40 kHz in [12]) which also implies high computational and economical costs.

One important limitation of the signature based approaches is that they often require a relatively high sampling frequency and thus costly installations. Moreover, as they have to pair each `on` transition with its `off` transition, which is an NP-hard problem, heuristic approaches, which are likely to produce wrong pairing, have to be used. Also, if an error is made on one `on-off` pairing, it implies that there is an error on at least one other `on-off` pairing. It would be more convenient to have a global approach that can directly identify each *complete activity event* (i.e. the event corresponding to an appliance being turned `on` and then turned `off`) instead of a bottom-up approach where each error is propagated to the upper layer.

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In this paper we propose a new method which is generic for each appliance type (e.g. fridge, washing machine,...) and does not need to be adapted for each home. The number and types of appliances of the aggregated load do not need to be known as well, the algorithm extracting the appliance one after the other, until no more appliance is detected in the signal.

The proposed method is based on recent advances in non-linear signal processing, in order to tackle the NILM problem with a more global approach than signature based approaches. We propose to model each power consumption event with one mathematical object called *atom*. We rely on the theory of sparse decomposition to decompose the aggregated signal in a set of atoms from which each *(individual) appliance signal* can be reconstructed by a simple linear combination. The resulting decomposition depends on the set of pre-defined atoms, called *dictionary*, used in the decomposition. One of our main contribution is to use a sparse decomposition approach with a translation-invariant dictionary of boxcar atoms, i.e. rectangular atoms modelling each complete activity event. This decomposition is multichannel and exploits the correlations between the active and reactive power signals in the flavour of [13], but also between the three phases of the aggregated signal. After the decomposition, a clustering step is performed on the atoms of the decomposition in order to classify them in their respective appliance class, and linearly combined them to reconstruct each individual appliance signal. The energy of each appliance can then simply be obtained by integrating the corresponding signal.

We evaluated our algorithm on a dataset of *aggregated load signals*, obtained from a low-frequency (1 Hz) acquisition system which measures the three phases of standard households. In order to have a quantitative evaluation of the proposed algorithm, we also perform an evaluation on a dataset of load signals obtained by an appliance simulator. The advantage of this synthetic dataset is that we also have access to the individual appliance signals and thus are able to measure the performance of our algorithm in term of signal and energy estimation.

II. Sparse decomposition model

Let $\mathbf{x}(t) = (x_1(t), \dots, x_M(t))^\intercal$, where $(\cdot)^\intercal$ denotes the matrix transposition, be the aggregated load signal, i.e. a *mixture* signal of $M = 6$ channels populated with the active and reactive power signals of the three phases, and $\mathbf{s}_n(t), n = 1, \dots, N$ the individual appliance signals, i.e. the *sources* in the *blind source separation* (BSS) terminology. Then we have $\mathbf{x}(t) = \sum_{n=1}^{N} \mathbf{s}_n(t)$ or in a matrix form, $\mathbf{x} = \sum_{n=1}^{N} \mathbf{s}_n$, where $\mathbf{x}$ and $\mathbf{s}_n$ are matrices of size $M \times T$ and $N \times T$ respectively, where T is the number of samples.

Our approach relies on the decomposition of the multichannel mixture signal $\mathbf{x} \in \mathbb{R}^{M\times T}$ of entries $x_m(t)$ in a collection of K atoms $\{\varphi_k\}_{k=1}^{K}$, i.e. $x_m(t) = \sum_{k=1}^{K} c_{m,k}\varphi_k(t)$. These atoms are picked from a dictionary $\mathbf{\Phi}$, i.e. a large collection of unit norm vectors $\|\varphi_k\|_2 = 1, k = 1, \dots, K$ in $\mathbb{R}^{1\times T}$ with $K \gg T$.

We assume that each signal $x_m \in \mathbb{R}^{1\times T}$ admits a sparse approximation over the dictionary $\mathbf{\Phi}$, i.e. $x_m = c_m\mathbf{\Phi} + e_m$ where most of the coefficients $c_m = (c_{m,1}, \dots, c_{m,K})$ are equal to zero, i.e. the vectors $c_m, m = 1, \dots, M$ have few non zero entries as measured by their ℓ_0 "norm" $\|c_m\|_0 \le S$, where S is the sparsity of vector c_m.

In order to model the correlation between signals, which in our problem comes from the fact that appliances are consuming energy on multiple channels, we refine this model by imposing that all the signals share a common sparse support, i.e. $x_m = c_m\mathbf{\Phi}_\Lambda + e_m$ where $\mathbf{\Phi}_\Lambda$ is the restriction of the matrix $\mathbf{\Phi}$ to the rows listed in the set Λ. In this case, sparsity is conveyed by the size of the support set, $|\Lambda| \le S$.

The sparse decomposition algorithm Orthogonal Matching Pursuit (OMP) [14] or more precisely its multichannel extension Simultaneous-OMP (SOMP) [15] is able to perform such decomposition. Our source separation assumption is that each of these atoms belongs to one source $\mathbf{s}_n \in \mathbb{R}^{M\times T}$ of entries $[s_{n,m}(t)]_{m=1,t=1}^{M,T}$, and thus we can reconstruct each source $\mathbf{s}_n$ by a simple linear combination of the K_n atoms belonging to that source:

$$s_{n,m}(t) = \sum_{k=1}^{K_n} c_{m,n_k}\varphi_{n_k}(t), \tag{1}$$

where $\{n_1, \dots, n_{K_n}\}$ is the set of atom indices corresponding to source n. Thus, one important component of our approach consists in decomposing the signal $\mathbf{x}$ with the SOMP algorithm and then clustering the atoms of the decomposition into classes corresponding to the different appliances/sources.

III. Method

A. Simultaneous Orthogonal Matching Pursuit (SOMP)

Simultaneous Orthogonal Matching Pursuit (SOMP) [15] is an iterative algorithm for sparse approximation of a multi-channel signal. At each iteration j, an atom k_j is selected, and a residual R_j is updated. At the first iteration the residual is simply the mixture $R_1 := \mathbf{x}$. After J iterations, the set of selected atoms is $\Lambda_J = \{k_j\}_{j=1}^{J}$, and the new residual is computed as:

$$R_j = \mathbf{x} - \mathbf{c}_j\mathbf{\Phi}_{\Lambda_j}$$

where the coefficients $\mathbf{c}_j$ are estimated, usually in the least square sense with:

$$\mathbf{c}_j := \arg\min_{\mathbf{c}} \left\|\mathbf{x} - \mathbf{c}\mathbf{\Phi}_{\Lambda_j}\right\|_2 = \mathbf{x}\mathbf{\Phi}_{\Lambda_j}^{\dagger}, \tag{2}$$

where $\mathbf{\Phi}_{\Lambda_j}^{\dagger}$ denotes the Moore-Penrose pseudo-inverse of $\mathbf{\Phi}_{\Lambda_j}$.

The next selected atom k_{j+1} is the one which maximises the p-correlation with the residual R_j:

$$k_{j+1} := \arg\max_{k} \|R_j\varphi_k^\intercal\|_p \tag{3}$$

where $\|\cdot\|_p$ is the ℓ_p norm (typically $p = 2$). In the rest of the paper we use the general term OMP to denote our algorithm because the properties are true for both OMP and SOMP.

B. Design of the Dictionary

In order to model the complete activity events of the appliance signals, we designed a dictionary of translation-invariant atoms having a rectangular shape, i.e. each atom is a boxcar function $\varphi_{l,w} = \frac{1}{\sqrt{w}}\Pi_{l-w/2,l+w/2}(t)$ where $\Pi_{a,b}(t) = H(t-a) - H(t-b)$, and $H(t)$ is the Heaviside step function.

If we incorporate all the possible boxcar translations l and width w, the size of the dictionary is $K = WT$[1] where W is the number widths per boxcar atom in the dictionary.

C. Algorithm complexity

The most computational intensive step of the OMP algorithm is the atom selection step (3). This step requires K correlations, each of them costing of the order of T multiply-add. However, faster correlation computations can be achieved through FFT-based methods. This decreases the full correlation computation cost from $\mathcal{O}(KT)$ down to $\mathcal{O}(K\log_2 T)$. Hence the total complexity of J iterations of OMP with translation invariant boxcar dictionary is then $\mathcal{O}(JWT\log_2 T)$.

In order to accelerate the algorithm, we can perform OMP only on smaller segments and/or decrease the sampling rate when accurate precision is note required. Note that performing OMP on a smaller segment will not only decrease T, but also J because smaller the segment analysed, fewer the number of atoms present in that segment. If we know what kind of atoms we are looking for in a segment, we can also decrease the number W of possible widths of each boxcar atom.

D. Non-negativity constraint

The atoms of our boxcar dictionary presented in section III-B, model complete activity events of the appliances. As these appliances only consume energy, all the atom coefficients should only have non-negative values on the active power channels. Taking this constraint into account can easily be implemented in OMP. In order to incorporate this constraint, we modify the coefficient estimation step (2) for the active channels with a positive constraint: $\arg\min_{\mathbf{c}_p} \left\|\mathbf{x}_p - \mathbf{c}_p\mathbf{\Phi}_{\Lambda_j}\right\|_2$ s.t. $\mathbf{c}_p \geq 0$ which can be computed efficiently [16], e.g. in Matlab by using the "lsqnonneg" function. We also discard from the selection (3), the atoms being negatively correlated with the residual.

E. Joint estimation

Once the atoms $\varphi_{n_k}, k = 1,\ldots,K_n$ of an appliance n have been detected, it is possible to estimate the signal corresponding to this group of atoms jointly, by merging these atoms into one "molecule" $\widetilde{\varphi}_n$, which we defined as the normalized vector, constant on its support $\mathrm{supp}(\widetilde{\varphi}_n)$ and such that $\mathrm{supp}(\widetilde{\varphi}_n) := \cup_{k=1}^{K_n}\mathrm{supp}(\varphi_{n_k})$. In order to estimate the signal corresponding to this group of atoms, we replace the atoms $\varphi_{n_k}, k = 1,\ldots,K_n$ in the dictionary $\mathbf{\Phi}_\Lambda$, by the molecule $\widetilde{\varphi}_n$, and estimate the molecule coefficient by (2) or its non-negative variant (see section III-D).

[1] if we neglect the border effects.

F. Atom Clustering

Once an OMP decomposition has been performed, the next step is to identify the atoms corresponding to the appliance we want to estimate, and then reconstruct the appliance signal by summing the contribution of these atoms according to (1). Each appliance type has its specificities, specially concerning the clustering step, which are described in the next sub-section. Note that, for a given appliance, some atoms are supposed to have the same power. These atoms can by grouped into one molecule as explained in section III-E in order to perform a joint estimation which can improve the quality of the estimation.

G. Appliances detection

We defined the general components of our method in the previous sections. We now detail the algorithm specificities of each of the appliance types that our algorithm can detect and estimate.

1) Base component: The base component is simply detected by searching for the point having the smallest apparent power in each channel.

2) Fridge: The fridge signal has a relatively small amplitude which makes it a priori difficult to detect but on the other hand it has a very particular signature. It is composed of a succession of quasi-periodic cooling cycles which start with a high amplitude peak easily detectable. Thus our approach to detect the fridge consists in 1) detecting the peaks in the signal having a relatively high amplitude, 2) looking for a periodic pattern in the sequence of detected peaks, 3) performing a local OMP decomposition around each cycle localized by its peak. For step 2), we implemented two different methods. Both methods take as an input an intermediate peak signal which is obtained by convolving the detected peak sequence with a gaussian kernel, the role of the kernel being to allow a small variability in each peak location. The first method to detect the periodicity is based on the autocorrelation of the intermediate peak signal and is able to detect the periodicity pattern if the peak period is approximately constant. The second method which is based on the predominant local pulse (PLP) approach [17] (initially proposed to track the beat of music pieces) is able to track a smooth variation of the periodicity. We use this second method if a strict periodicity was not detected by the first method.

3) Heat pump: In order to not perform the OMP decomposition on the whole signal which would be particularly costly, we first segment the signal according to some properties of the heat pump signal. The properties of these segments are that the active power is higher than 500W, on the 3 phases, and on a duration of at least 15 minutes. An OMP decomposition is then done locally in each detected segment, and the heat pump signal is re-estimated jointly as explained in section III-E.

4) Washing machine: The washing machine signal is composed of high power water heating cycles and fast oscillations due to the rotating drum. In order to detect the washing machine, we first identify segments that have an active power higher than 1500W during a time period longer than 5 minutes.

We then check that in the same time, the signal contains fast oscillations (at least 2 per minutes) with a power variation by between 50W and 250W on a period of 50 minutes. Once the segments are detected, an OMP decomposition is performed on each segment. The atoms of the decomposition are then clustered, and the washing machine signal is reconstructed. Note that we are only looking for atoms corresponding to the heating cycles, which contain most of the washing machine energy. In fact we noticed that the heating cycles are responsible for 95% of the energy consumed by the washing machine. In order to have an unbiased estimator of the energy consumed, we add the 5% remaining as a post-processing step.

5) Dishwasher: The dishwasher signal is composed of few high power water heating cycles spread on a period of about one hour but as opposed to the washing machine, does not contain any oscillations because of the absence of any rotating drum. The detection of the dishwasher is then similar to the washing machine but different. We first identify segments that have an active power higher than 1700W, and discard isolated group of segments that are shorter than 35 minutes. As for the washing machine, we then perform an OMP decomposition on each segment followed by a clustering step and a signal reconstruction step. Note than in the clustering step, we discard atoms corresponding to the washing machine. We noticed that the detecting heating cycles are responsible for 96% of the energy consumed by the dishwasher, and consequently, we then add an extra 4% to estimate its energy consumption.

IV. Experiments

As discussed in the introduction, we also have a dataset of real aggregated load signals, sampled at 1 Hz, of the three phases of few households. An example of a load signal is depicted in figure 1. Unfortunately, as we don't have the corresponding individual appliance signals, which would have necessitate to install an acquisition system for each appliance, we can only asses the results visually and cannot compute quantitative results.

However, in order to evaluate the proposed algorithm, we used a simulator which is able to generate the load signals of the different electrical appliances based on a stochastic model of each appliance. The advantage of using a simulator is that, as we have access to each individual appliance signals, we are able to perform a quantitative evaluation of the algorithm. We are interested in two measures of performance: the signal-to-distortion ratio (SDR), a classical performance measure in source separation [18], and the relative error in (real) energy estimation.

A. Experimental protocol

With the appliance simulator, we generated the aggregated load signals of different artificial households of increasing complexity, for a duration of 10 days per household, each day corresponding to a signal composed of 6 channels of 86'400 samples each. We described in table I, the appliances involved in each household.

TABLE I
DESCRIPTION OF THE ARTIFICIAL HOUSEHOLDS. THE CHECK BOXES INDICATE THE PRESENCE OF THE APPLIANCES LISTED IN THE LEFT COLUMN ON THE 3 PHASES OF EACH HOUSEHOLD.

Appliances	Household #1			Household #2			Household #3		
	1	2	3	1	2	3	1	2	3
Base component	✓	✓	✓	✓	✓	✓	✓	✓	✓
Fridge #1		✓			✓			✓	
Fridge #2						✓			✓
Heat pump	✓	✓	✓	✓	✓	✓	✓	✓	✓
Washing machine	✓	✓		✓	✓			✓	✓
Dishwasher	✓			✓			✓		
Light				✓			✓		
Light Fluo				✓				✓	
Oven Microwave Grill							✓		

B. Results

Results on a real aggregated load signals are depicted in figure 2. We can notice that a fridge, a dishwasher, a heat pump, and three successive occurrences of a washing machine have been identified in addition to the base component. The estimated energy consumption of this load curve is depicted in figure 3. According to this estimation, the heat pump was responsible for about half of the total energy consumption, the washing machine and the dishwasher consumed about the same amount of energy (3.9% and 3.7% respectively), while the base component was responsible for more than twice the fridge consumption (2.4% and 1.1% respectively).

The results obtained in term of source separation and energy estimation on the synthetic dataset are depicted in figure 4 and figure 5 respectively. The results show that the source separation performance is by between 20 dB and 40 dB of SDR for all the appliance sources on each of the three households. In term of energy estimation, we can see in figure 5 that the energy estimation errors of the different appliances on the three households are by between 0.06% and 1.2%. While we only model the heating cycles of the washing machine and the dishwasher (but used an energy correction), the average (over the thee households) error is less than 1% for all the appliances and even less than 0.25% for the heat pump, the base component and the fridge.

V. Conclusion

We proposed a non-intrusive load monitoring (NILM) algorithm based on the sparse decomposition of the aggregated load curve of an household using a translation-invariant boxcar dictionary. The advantage of this particular dictionary is its ability to model the on-off behaviour of the appliances. Experiments on real and synthetic mixtures showed very promising results, with an energy estimation error less than 1% for the main contributors of the household. This algorithm can be used for demand-side management (DSM) to help the user reducing its energy consumption, as well as to predict its electrical consumption. Future work includes extending the method to other appliance types such as the boiler, the oven, stoves, and developing specific applications based on this technology. Also, as suggested in [9], [19], the method might be improve

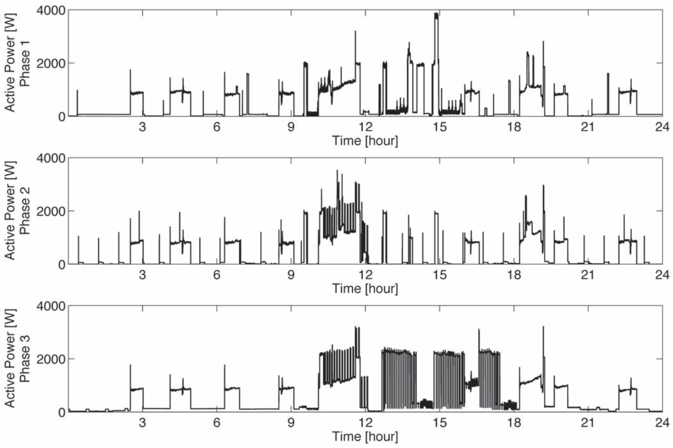

Fig. 1. Example of a 24-hour recording of the aggregated signal of a household.

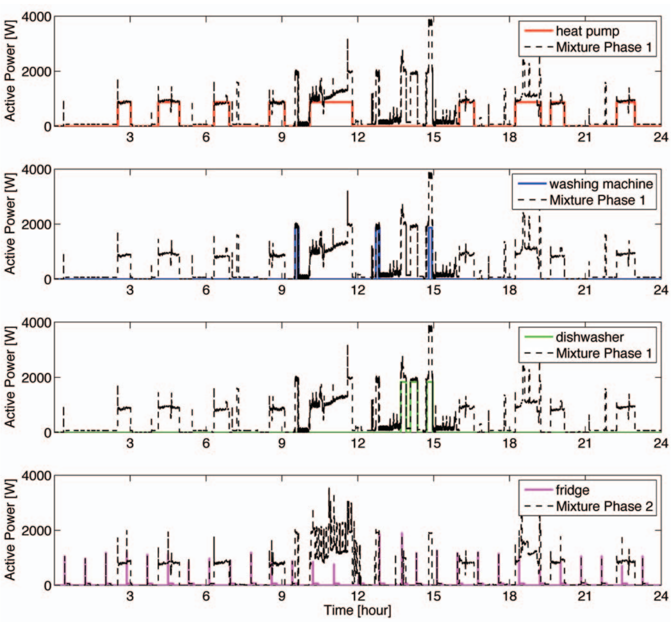

Fig. 2. Result of our disaggregating algorithm on the mixture of figure 1.

by exploiting information such as the daily and weekly activity patterns of the different appliances.

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References

[1] A. Zoha, A. Gluhak, M. A. Imran, and S. Rajasegarar, "Non-intrusive load monitoring approaches for disaggregated energy sensing: A survey," *Sensors*, vol. 12, no. 12, pp. 16 838–16 866, 2012.

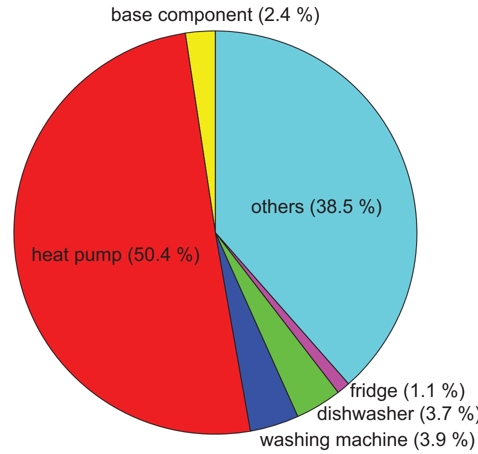

Fig. 3. Distribution of the estimated energy consumption of the detected appliances of the mixture of figure 1.

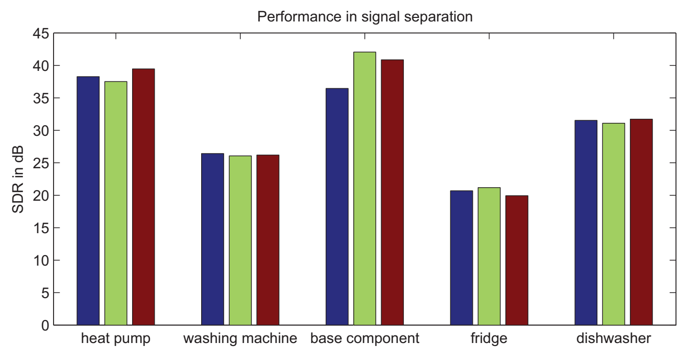

Fig. 4. Source separation performance of each appliance obtained on the three artificial households of table I. The first and blue bar representing Household #1, the second and green bar representing Household #2, and the last and brown bar representing Household #3.

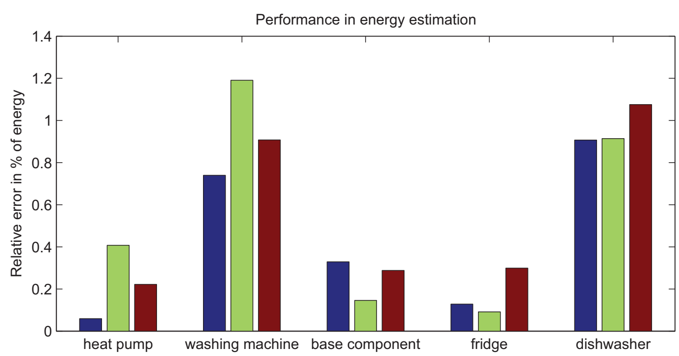

Fig. 5. Performance result in term of energy estimation for the appliances of table I. The first and blue bar representing Household #1, the second and green bar representing Household #2, and the last and brown bar representing Household #3.

[2] "Eurostat database," March 2012. [Online]. Available: http://epp.eurostat.ec.europa.eu/portal/page/portal/energy/data/main_tables

[3] L. L. N. Laboratory, "Energy flow charts in 2012," May 2013. [Online]. Available: https://flowcharts.llnl.gov/

[4] "Buildings energy data book," 2009.

[5] J. Utley and L. Shorrock, "Domestic energy fact file 2008," *Department of Energy and Climate Change, BRE, Watford*, 2008.

[6] M. Berges, E. Goldman, H. S. Matthews, L. Soibelman, and K. An-

derson, “User-centered nonintrusive electricity load monitoring for residential buildings,” *Journal of Computing in Civil Engineering*, vol. 25, no. 6, pp. 471–480, 2011.

[7] M. Zeifman and K. Roth, “Nonintrusive appliance load monitoring: Review and outlook,” *Consumer Electronics, IEEE Transactions on*, vol. 57, no. 1, pp. 76–84, 2011.

[8] K. Ehrhardt-Martinez, K. A. Donnelly, S. Laitner *et al.*, “Advanced metering initiatives and residential feedback programs: a meta-review for household electricity-saving opportunities.” American Council for an Energy-Efficient Economy Washington, DC, 2010.

[9] H. Kim, M. Marwah, M. F. Arlitt, G. Lyon, and J. Han, “Unsupervised disaggregation of low frequency power measurements.” in *SDM*, vol. 11. SIAM, 2011, pp. 747–758.

[10] O. Parson, S. Ghosh, M. Weal, and A. Rogers, “Non-intrusive load monitoring using prior models of general appliance types.” in *AAAI*, 2012.

[11] G. W. Hart, “Nonintrusive appliance load monitoring,” *Proceedings of the IEEE*, vol. 80, no. 12, pp. 1870–1891, 1992.

[12] C. Laughman, K. Lee, R. Cox, S. Shaw, S. Leeb, L. Norford, and P. Armstrong, “Power signature analysis,” *Power and Energy Magazine, IEEE*, vol. 1, no. 2, pp. 56–63, 2003.

[13] H. Gonçalves, A. Ocneanu, M. Bergés, and R. Fan, “Unsupervised disaggregation of appliances using aggregated consumption data,” in *The 1st KDD Workshop on Data Mining Applications in Sustainability (SustKDD)*, 2011.

[14] S. Mallat, *A wavelet tour of signal processing: the sparse way*. Academic press, 2008.

[15] R. Gribonval, H. Rauhut, K. Schnass, and P. Vandergheynst, “Atoms of all channels, unite! average case analysis of multi-channel sparse recovery using greedy algorithms,” *Journal of Fourier analysis and Applications*, vol. 14, no. 5-6, pp. 655–687, 2008.

[16] C. L. Lawson and R. J. Hanson, *Solving least squares problems*. SIAM, 1974, vol. 161.

[17] P. Grosche and M. Muller, “Computing predominant local periodicity information in music recordings,” in *Applications of Signal Processing to Audio and Acoustics, 2009. WASPAA’09. IEEE Workshop on*. IEEE, 2009, pp. 33–36.

[18] R. Gribonval, L. Benaroya, E. Vincent, C. Févotte *et al.*, “Proposals for performance measurement in source separation,” in *4th Int. Symp. on Independent Component Anal. and Blind Signal Separation (ICA2003)*, 2003, pp. 763–768.

[19] J. Z. Kolter, S. Batra, and A. Y. Ng, “Energy disaggregation via discriminative sparse coding,” in *Advances in Neural Information Processing Systems*, 2010, pp. 1153–1161.