

18.06 Recitation April 7

Kai Huang

Projection

Consider the projection of b onto the column space of A .

1. If A is a matrix with linearly independent columns, the projection is _____.
2. If $A = QR$, then the projection is _____.
3. If $A = U_1 \Sigma_r V_1^T$, then the projection is _____.

Determinant

1. Formula of the determinate of a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. What is the geometric meaning?
2. Formula of the determinate of a 3×3 matrix $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$. What is the geometric meaning?
3. Axiomatic Approach:
 - The det is a linear function in each row.
 - A matrix with two equal rows has determinant 0.
 - The determinant of the identity matrix is 1.
4. Properties:
 - $|AB| =$
 - $|A^T| =$
 - $|A^{-1}| =$

Problems

1. What is the projection of $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ on

(a) the column space of $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$?

(b) the column space of $B = \begin{pmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{pmatrix}$?

2. Suppose the determinate of $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is a .

(a) What is the determinant of $2A$?

(b) What is the determinant of $B = \begin{pmatrix} a_{21} & a_{22} & a_{23} + a_{21} \\ a_{11} & a_{12} & a_{13} + a_{11} \\ a_{31} & a_{32} & a_{33} + a_{31} \end{pmatrix}$?

(c) Given a random 3×3 matrix D , how is the $|A + D|$ relate to $|D|$?

(d) Suppose further that $a_{11} = a_{12} = a_{13}$. What is the determinate of

$$C = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} + 1 & a_{22} + 1 & a_{23} + 1 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}?$$