Rec 9 Solu

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Rec 9

1. Given a $n \times n$ matrix A, how to find the eigenvalues and eigenvectors?

$\det|A-\lambda I| = 0$ solve λ for eigenvalues.

For each eigenvalue λ, find a solution for $(A-\lambda I)u=0$ $\to u$ eigenvector.

2. Definition of Markov matrix and make an example.

a positive square matrix with each column entries summing up to 1.

$U \Sigma U^{-1}$

3. Solve differential equation system
$$\frac{du}{dt} = Au$$
for $u(t)$ being a n dimensional vector fuction.

$$\begin{pmatrix} \frac{du_1}{dt} \\ \frac{du_2}{dt} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$u(t) = e^{At}u(0)$,

$$e^{At} = I + At + \frac{A^2t^2}{2} + \dots + \frac{A^n t^n}{n!} + \dots$$

Problems

1. If A has two eigenvalues being λ_1 and λ_2 and their eigenvalues being u_1 and u_2 respectively.

 (a) Then what is $A(xu_1 + yu_2)$ where x, y are numbers?

 $\lambda_1 u_1 x + \lambda_2 u_2 y$

 (b) What is $e^A(xu_1 + yu_2)$?

 $e^{\lambda_1} u_1 x + e^{\lambda_2} u_2 y$.

2. Find the eigenvalues and eigenvectors for the following matrices.

 (a) $\begin{pmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{pmatrix}$ eigenvalues 1 and 0.4.

eigenvector for 1 $\begin{pmatrix} -0.2 & 0.4 \\ 0.2 & -0.4 \end{pmatrix} |u_1| = 0$ $u_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

eigenvector for 0.4 $\begin{pmatrix} 0.4 & 0.4 \\ 0.2 & 0.2 \end{pmatrix} u_2 = 0$ $u_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

 (b) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$A - \lambda I = \begin{pmatrix} -\lambda & 1 \\ -1 & -\lambda \end{pmatrix}$ $\det(A-\lambda I) = 0 \Rightarrow \lambda^2 + 1 = 0$

$\lambda = \pm i$

eigenvector for i $\begin{pmatrix} -i & 1 \\ -1 & -i \end{pmatrix} u_1 = 0$ $u_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}$

eigenvector for $-i$ $\begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix} u_2 = 0$ $u_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

3. What can you say about $\lim_{n\to\infty} A^n$ for A of the two cases above?

a) $\lim_{n\to\infty} A^n = \lim_{n\to\infty} (U \Sigma U^{-1})^n = \lim_{n\to\infty} U \Sigma^n U^{-1}$

$= \lim_{n\to\infty} U \begin{pmatrix} 1^n & \\ & 0.4^n \end{pmatrix} U^{-1} = U \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} U^{-1} = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \frac{1}{3}$

$= \frac{1}{3} \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$

b) $\lim_{n\to\infty} A^n = \lim_{n\to\infty} U \begin{pmatrix} i^n & \\ & (-i)^n \end{pmatrix} U^{-1}$ does not have a limit.