1. Let A be a $n \times n$ symmetric matrix.
   (a) A is diagonalizable and the eigenvalues of A are ____________________________.
   (b) A can be decomposed as ____.

2. Equivalent conditions for positive definite (semi-positive definite.)

Problems

1. Is the set of positive semi-definite $n \times n$ matrices a vector space?

2. Let A be a 2×2 symmetric matrix with two different eigenvalues λ_1 and λ_2. The corresponding eigenvectors are u_1 and u_2.

   (a) Prove that u_1 and u_2 are perpendicular to each other.

   (b) If $\lambda_1 = 0$, $\lambda_2 = 1$, interpret Ab using projection of a vector b.

   (c) If $\lambda_1 = 1$, $\lambda_2 = -1$, interpret Ab geometrically.

3. Given an invertible matrix A, can $A^T A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$? How about $A^T A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$?