

9

BRANCH, MESH, AND NODE ANALYSIS

- 9–1 Branch Current Method
- 9–2 Determinants
- 9–3 Solving Simultaneous Equations
- Using a Calculator
- 9–4 Mesh Current Method
- 9–5 Node Voltage Method
- 9–6 Technology Theory into Practice



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■ INTRODUCTION

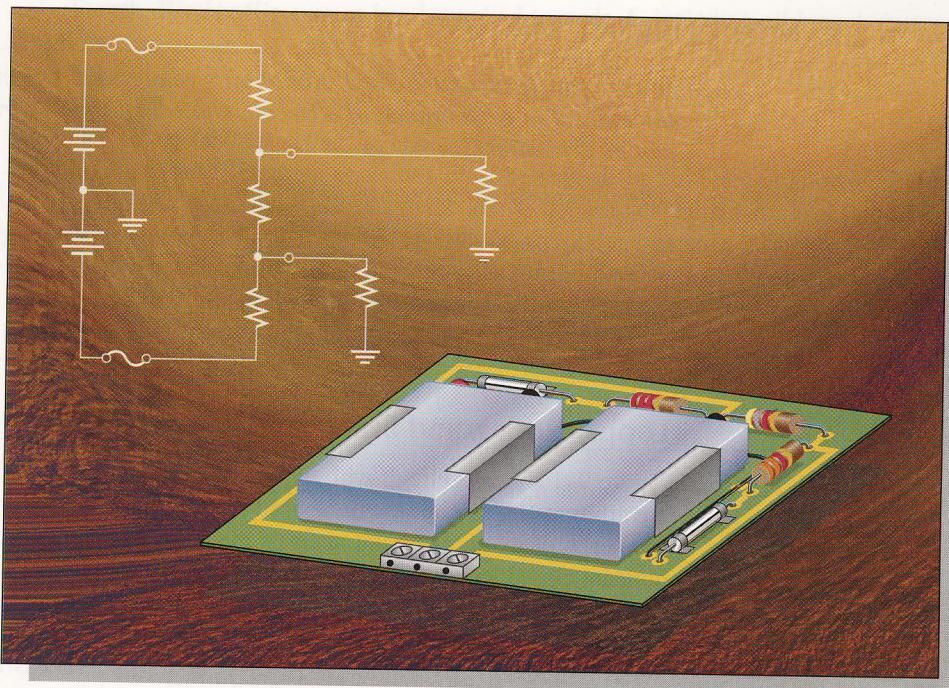
In the last chapter, you learned about the superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem, and several types of conversion methods. These theorems and conversion methods are useful in solving some types of circuit problems.

In this chapter, three more circuit analysis methods are introduced. These methods are based on Ohm's law and Kirchhoff's laws and are particularly useful in the analysis of multiple loop circuits having two or more voltage or current sources. The methods presented here can be used alone or in conjunction with the techniques covered in the previous chapters. With experience, you will learn which method is best for a particular problem or you may develop a preference for one of them.

In the branch current method, Kirchhoff's laws are applied to solve for current in various branches of a multiple-loop circuit. A loop is a complete current path within a circuit. The method of determinants is useful in solving simultaneous equations that occur in multiple-loop analysis. Using a calculator to solve simultaneous equations is also thoroughly covered. In the mesh current method, you will solve for loop currents rather than branch currents. In the node voltage method, the voltages at the independent nodes in a circuit are found. A node is the junction of two or more components.

For this chapter, the TECH TIP assignment in Section 9–6 is to analyze a dual-polarity loaded voltage divider to determine if certain voltage measurements are correct. One of the methods covered in this chapter will be used for the analysis.

■ TECHnology Theory Into Practice



■ CHAPTER OBJECTIVES

- Use the branch current method to find unknown quantities in a circuit
- Use determinants to solve simultaneous equations
- Use a calculator to solve simultaneous equations for unknown quantities
- Use mesh analysis to find unknown quantities in a circuit
- Use node analysis to find unknown quantities in a circuit

9–1 ■ BRANCH CURRENT METHOD

In the branch current method, Kirchhoff's voltage and current laws are used to find the current in each branch of a circuit. Once the branch currents are known, voltages can be determined.

After completing this section, you should be able to

- Use the branch current method to find unknown quantities in a circuit
 - Identify loops and nodes in a circuit
 - Develop a set of branch current equations
 - Solve simultaneous equations by the substitution method

Loops, Nodes, and Branches

Figure 9–1 shows a circuit with two voltage sources and two loops (the arrow directions are arbitrary). This circuit will be used as the basic model throughout the chapter to illustrate each of the three circuit analysis methods. In this circuit, there are only two nonredundant closed loops, as indicated by the arrows. A **loop** is a complete current path within a circuit, and a set of nonredundant closed loops can be viewed as a set of “windowpanes,” where each windowpane represents one nonredundant loop. Also, there are four nodes as indicated by the letters *A*, *B*, *C*, and *D*. A **node** is a point where two or more components are connected. A **branch** is a path that connects two nodes.

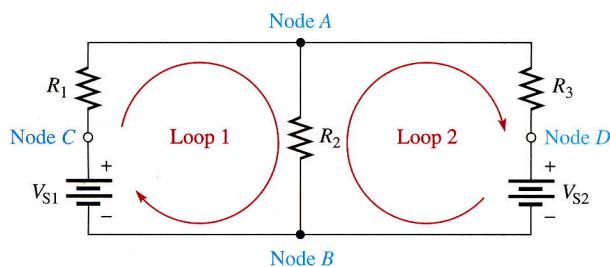


FIGURE 9–1
Two-loop circuit showing loops and nodes.

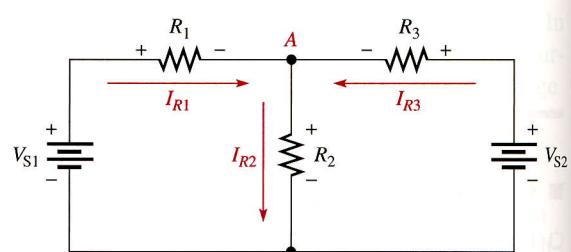


FIGURE 9–2
Circuit for demonstrating branch current analysis.

The following are the general steps used in applying the branch current method. These steps are demonstrated with the aid of Figure 9–2.

- Step 1.** Assign a current in each circuit branch in an arbitrary direction.
- Step 2.** Show the polarities of the resistor voltages according to the assigned branch current directions.
- Step 3.** Apply Kirchhoff's voltage law around each closed loop (algebraic sum of voltages is equal to zero).
- Step 4.** Apply Kirchhoff's current law at the minimum number of nodes so that all branch currents are included (algebraic sum of currents at a node equals zero).
- Step 5.** Solve the equations resulting from Steps 3 and 4 for the branch current values.

First, the **branch currents** I_{R1} , I_{R2} , and I_{R3} are assigned in the direction shown in Figure 9–2. Don't worry about the actual current directions at this point.

Second, the polarities of the voltage drops across R_1 , R_2 , and R_3 are indicated in the figure according to the assigned current directions.

Third, Kirchhoff's voltage law applied to the two loops gives the following equations where the resistance values are the coefficients for the unknown currents:

$$\text{Equation 1: } R_1 I_{R1} + R_2 I_{R2} - V_{S1} = 0 \quad \text{for loop 1}$$

$$\text{Equation 2: } R_2 I_{R2} + R_3 I_{R3} - V_{S2} = 0 \quad \text{for loop 2}$$

Fourth, Kirchhoff's current law is applied to node A, including all branch currents as follows:

$$\text{Equation 3: } I_{R1} - I_{R2} + I_{R3} = 0$$

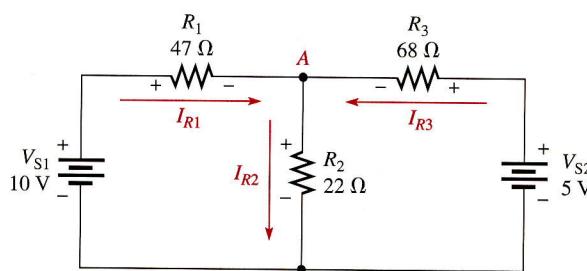
The negative sign indicates that I_{R2} is out of the junction.

Fifth and last, the three equations must be solved for the three unknown currents, I_{R1} , I_{R2} , and I_{R3} . The three equations in the above steps are called *simultaneous equations* and can be solved in two ways: by substitution or by determinants. Example 9–1 shows how to solve equations by the *substitution* method. In Section 9–2, you will study determinants and how to use them to find branch currents; in the following sections, you will use determinants in two other methods of circuit analysis.

EXAMPLE 9–1

Use the branch current method to find each branch current in Figure 9–3.

FIGURE 9–3



Solution

- Step 1.** Assign branch currents as shown in Figure 9–3. Keep in mind that you can assume any current direction at this point and that the final solution will have a negative sign if the actual current is opposite to the assigned current.
- Step 2.** Mark the polarities of the resistor voltage drops in accordance with the assigned current directions as shown in the figure.
- Step 3.** Applying Kirchhoff's voltage law around the left loop gives

$$47I_{R1} + 22I_{R2} - 10 = 0$$

Around the right loop gives

$$22I_{R2} + 68I_{R3} - 5 = 0$$

where all resistance values are in ohms and voltage values are in volts. For simplicity, the units are not shown.

- Step 4.** At node A, the current equation is

$$I_{R1} - I_{R2} + I_{R3} = 0$$

- Step 5.** The equations are solved by substitution as follows. First, find I_{R1} in terms of I_{R2} and I_{R3} .

$$I_{R1} = I_{R2} - I_{R3}$$

Now, substitute $I_{R2} - I_{R3}$ for I_{R1} in the left loop equation.

$$\begin{aligned} 47(I_{R2} - I_{R3}) + 22I_{R2} &= 10 \\ 47I_{R2} - 47I_{R3} + 22I_{R2} &= 10 \\ 69I_{R2} - 47I_{R3} &= 10 \end{aligned}$$

Next, take the right loop equation and solve for I_{R2} in terms of I_{R3} .

$$22I_{R2} = 5 - 68I_{R3}$$

$$I_{R2} = \frac{5 - 68I_{R3}}{22}$$

Substituting this expression for I_{R2} into $69I_{R2} - 47I_{R3} = 10$ yields

$$69\left(\frac{5 - 68I_{R3}}{22}\right) - 47I_{R3} = 10$$

$$\frac{345 - 4692I_{R3}}{22} - 47I_{R3} = 10$$

$$15.68 - 213.27I_{R3} - 47I_{R3} = 10$$

$$-260.27I_{R3} = -5.68$$

$$I_{R3} = \frac{5.68}{260.27} = 0.0218 \text{ A} = 21.8 \text{ mA}$$

Now, substitute the value of I_{R3} in amps into the right loop equation.

$$22I_{R2} + 68(0.0218) = 5$$

Solve for I_{R2} .

$$I_{R2} = \frac{5 - 68(0.0218)}{22} = \frac{3.52}{22} = 0.16 \text{ A} = 160 \text{ mA}$$

Substituting I_{R2} and I_{R3} values into the current equation at node A yields

$$I_{R1} - 0.16 + 0.0218 = 0$$

$$I_{R1} = 0.16 - 0.0218 = 0.138 \text{ A} = 138 \text{ mA}$$

Related Problem Determine the branch currents in Figure 9–3 with the polarity of the 5 V source reversed.

SECTION 9–1 REVIEW

- What basic circuit laws are used in the branch current method?
- When assigning branch currents, you should be careful that the assigned directions match the actual directions. (T or F)
- What is a loop?
- What is a node?

9–2 ■ DETERMINANTS

When several unknown quantities are to be found, such as the three branch currents in Example 9–1, you must have a number of equations equal to the number of unknowns. In this section, you will learn how to solve for two and three unknowns using the systematic method of determinants. This method is an alternate to the substitution method, which you used in the previous section, and will help you appreciate your calculator even more.

After completing this section, you should be able to

■ Use determinants to solve simultaneous equations

- Set up second-order determinants to solve two simultaneous equations
- Set up third-order determinants to solve three simultaneous equations
- Evaluate determinants using either the expansion method or the cofactor method

Solving Two Simultaneous Equations for Two Unknowns

To illustrate the method of second-order determinants, let's assume two loop equations as follows:

$$\begin{aligned} 10I_1 + 5I_2 &= 15 \\ 2I_1 + 4I_2 &= 8 \end{aligned}$$

We want to find the value of I_1 and I_2 . To do so, we form a **determinant** with the coefficients of the unknown currents. A **coefficient** is the number associated with an unknown. For example, 10 is the coefficient for I_1 in the first equation and is a resistance value (units are omitted for simplicity).

The first column in the determinant consists of the coefficients of I_1 , and the second column consists of the coefficients of I_2 . The resulting determinant appears as follows:

$$\begin{array}{|cc|} \hline \text{1st column} & \text{2nd column} \\ \hline 10 & 5 \\ 2 & 4 \\ \hline \end{array}$$

This is called the *characteristic determinant* for the set of equations.

Next, we form another determinant and use it in conjunction with the characteristic determinant to solve for I_1 . We form this determinant for our example by replacing the coefficients of I_1 in the first column of the characteristic determinant with the constants (fixed numbers) on the right side of the equations. Doing this, we get the following determinant:

$$\begin{array}{|cc|} \hline & \text{Replace coefficients of } I_1 \\ 15 & 5 \\ 8 & 4 \\ \hline \end{array}$$

We can now solve for I_1 by evaluating both determinants and then dividing by the characteristic determinant. To evaluate the determinants, we cross-multiply and subtract the resulting products. An evaluation of the characteristic determinant in this example is illustrated in the following two steps:

Step 1. Multiply the first number in the left column by the second number in the right column.

$$\begin{array}{|cc|} \hline 10 & 5 \\ \times & \\ 2 & 4 \\ \hline \end{array} = 10 \times 4 = 40$$

Step 2. Multiply the second number in the left column by the first number in the right column and subtract from the product in Step 1. This result is the value of the determinant (30 in this case).

$$\begin{array}{|cc|} \hline 10 & 5 \\ 2 & 4 \\ \times & \\ \hline \end{array} = 40 - (2 \times 5) = 40 - 10 = 30$$

Repeat the same procedure for the other determinant that was set up for I_1 .

$$\begin{array}{|cc|} \hline 15 & 5 \\ 8 & 4 \\ \times & \\ \hline \end{array} = 15 \times 4 = 60$$

$$\begin{array}{|cc|} \hline 15 & 5 \\ 8 & 4 \\ \times & \\ \hline \end{array} = 60 - (8 \times 5) = 60 - 40 = 20$$

The value of this determinant is 20. Now we can solve for I_1 by dividing the I_1 determinant by the characteristic determinant as follows:

$$I_1 = \frac{\begin{vmatrix} 15 & 5 \\ 8 & 4 \end{vmatrix}}{\begin{vmatrix} 10 & 5 \\ 2 & 4 \end{vmatrix}} = \frac{20}{30} = 0.667 \text{ A}$$

To find I_2 , we form another determinant by substituting the constants on the right side of the equations for the coefficients of I_2 in the second column of the characteristic determinant.

$$\begin{vmatrix} 10 & 15 \\ 2 & 8 \end{vmatrix} \quad \begin{array}{l} \text{Replace coefficients of } I_2 \\ \text{with constants from} \\ \text{right sides of equations.} \end{array}$$

We solve for I_2 by dividing this determinant by the characteristic determinant already evaluated.

$$I_2 = \frac{\begin{vmatrix} 10 & 15 \\ 2 & 8 \end{vmatrix}}{30} = \frac{(10 \times 8) - (2 \times 15)}{30} = \frac{80 - 30}{30} = \frac{50}{30} = 1.67 \text{ A}$$

EXAMPLE 9–2

Solve the following set of equations for the unknown currents:

$$\begin{aligned} 2I_1 - 5I_2 &= 10 \\ 6I_1 + 10I_2 &= 20 \end{aligned}$$

Solution The characteristic determinant is evaluated as follows:

$$\begin{vmatrix} 2 & -5 \\ 6 & 10 \end{vmatrix} = (2)(10) - (-5)(6) = 20 - (-30) = 20 + 30 = 50$$

Solving for I_1 yields

$$I_1 = \frac{\begin{vmatrix} 10 & -5 \\ 20 & 10 \end{vmatrix}}{50} = \frac{(10)(10) - (-5)(20)}{50} = \frac{100 - (-100)}{50} = \frac{200}{50} = 4 \text{ A}$$

Solving for I_2 yields

$$I_2 = \frac{\begin{vmatrix} 2 & 10 \\ 6 & 20 \end{vmatrix}}{50} = \frac{(2)(20) - (6)(10)}{50} = \frac{40 - 60}{50} = -\frac{20}{50} = -0.4 \text{ A}$$

In a circuit problem, a result with a negative sign indicates that the direction of actual current is opposite to the assigned direction.

Note that multiplication can be expressed either by the multiplication sign such as 2×10 or by parentheses such as $(2)(10)$.

Related Problem Solve the following set of equations for I_1 :

$$\begin{aligned} 5I_1 + 3I_2 &= 4 \\ I_1 + 2I_2 &= -6 \end{aligned}$$

Solving Three Simultaneous Equations for Three Unknowns

Third-order determinants can be evaluated by either the expansion method or the cofactor method. First, we will illustrate the expansion method (which is good only for second and third order) using the following three equations:

$$1I_1 + 3I_2 - 2I_3 = 7$$

$$0I_1 + 4I_2 + 1I_3 = 8$$

$$-5I_1 + 1I_2 + 6I_3 = 9$$

The three-column characteristic determinant for this set of equations is formed in a similar way to that used earlier for the second-order determinant. The first column consists of the coefficients of I_1 , the second column consists of the coefficients of I_2 , and the third column consists of the coefficients of I_3 , as shown below.

Coefficients of I_1	Coefficients of I_2	Coefficients of I_3
1	3	-2
0	4	1
-5	1	6

The Expansion Method This third-order determinant is evaluated by the expansion method as shown in the following steps:

Step 1. Rewrite the first two columns immediately to the right of the determinant.

$$\left| \begin{array}{ccc|cc} 1 & 3 & -2 & 1 & 3 \\ 0 & 4 & 1 & 0 & 4 \\ -5 & 1 & 6 & -5 & 1 \end{array} \right|$$

Step 2. Identify the three downward diagonal groups of three coefficients each.

$$\left| \begin{array}{ccc|cc} 1 & 3 & -2 & 1 & 3 \\ 0 & 4 & 1 & 0 & 4 \\ -5 & 1 & 6 & -5 & 1 \end{array} \right|$$

Step 3. Multiply the numbers in each diagonal and add the products.

$$\left| \begin{array}{ccc|cc} 1 & 3 & -2 & 1 & 3 \\ 0 & 4 & 1 & 0 & 4 \\ -5 & 1 & 6 & -5 & 1 \end{array} \right|$$

$(1)(4)(6) + (3)(1)(-5) + (-2)(0)(1) = 24 + (-15) + 0 = 9$

Step 4. Repeat Steps 2 and 3 for the three upward diagonal groups of three coefficients.

$$\left| \begin{array}{ccc|cc} 1 & 3 & -2 & 1 & 3 \\ 0 & 4 & 1 & 0 & 4 \\ -5 & 1 & 6 & -5 & 1 \end{array} \right|$$

$(-5)(4)(-2) + (1)(1)(1) + (6)(0)(3) = 40 + 1 + 0 = 41$

Step 5. Subtract the result in Step 4 from the result in Step 3 to get the value of the characteristic determinant.

$$9 - 41 = -32$$

To solve for I_1 in the given set of three equations, a determinant is formed by substituting the constants on the right of the equations for the coefficients of I_1 in the characteristic determinant.

$$\begin{vmatrix} 7 & 3 & -2 \\ 8 & 4 & 1 \\ 9 & 1 & 6 \end{vmatrix}$$

This determinant is evaluated using the method described in the previous steps.

$$\begin{array}{c|ccc|cc} 7 & 3 & -2 & 7 & 3 \\ 8 & 4 & 1 & 8 & 4 \\ 9 & 1 & 6 & 9 & 1 \end{array}$$

$$= [(7)(4)(6) + (3)(1)(9) + (-2)(8)(1)] - [(9)(4)(-2) + (1)(1)(7) + (6)(8)(3)]$$

$$= (168 + 27 - 16) - (-72 + 7 + 144) = 179 - 79 = 100$$

I_1 is found by dividing this determinant value by the value of the characteristic determinant. The negative results indicates that the actual current is in a direction opposite to the original assumption.

$$I_1 = \frac{\begin{vmatrix} 7 & 3 & -2 \\ 8 & 4 & 1 \\ 9 & 1 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{vmatrix}} = \frac{100}{-32} = -3.125 \text{ A}$$

I_2 and I_3 are found in a similar way.

EXAMPLE 9–3

Determine the value of I_2 from the following set of equations:

$$2I_1 + 0.5I_2 + 1I_3 = 0$$

$$0.75I_1 + 0I_2 + 2I_3 = 1.5$$

$$3I_1 + 0.2I_2 + 0I_3 = -1$$

Solution The characteristic determinant is evaluated as follows:

$$\begin{vmatrix} 2 & 0.5 & 1 \\ 0.75 & 0 & 2 \\ 3 & 0.2 & 0 \end{vmatrix}$$

$$= [(2)(0)(0) + (0.5)(2)(3) + (1)(0.75)(0.2)] - [(3)(0)(1) + (0.2)(2)(2) + (0)(0.75)(0.5)]$$

$$= (0 + 3 + 0.15) - (0 + 0.8 + 0) = 3.15 - 0.8 = 2.35$$

The determinant for I_2 is evaluated as follows:

$$\begin{vmatrix} 2 & 0 & 1 \\ 0.75 & 1.5 & 2 \\ 3 & -1 & 0 \end{vmatrix}$$

$$= [(2)(1.5)(0) + (0)(2)(3) + (1)(0.75)(-1)] - [(3)(1.5)(1) + (-1)(2)(2) + (0)(0.75)(0)]$$

$$= [0 + 0 + (-0.75)] - [4.5 + (-4) + 0] = -0.75 - 0.5 = -1.25$$

Finally,

$$I_2 = \frac{-1.25}{2.35} = -0.532 \text{ A} = -532 \text{ mA}$$

Related Problem Determine the value of I_1 in the set of equations used in Example 9–3.

The Cofactor Method Unlike the expansion method, the cofactor method can be used to evaluate determinants with higher orders than three and is therefore more versatile. We will use a third-order determinant to illustrate the method, keeping in mind that fourth-, fifth-, and higher-order determinants can be evaluated in a similar way. The following specific determinant is used to demonstrate the cofactor method on a step-by-step basis.

$$\begin{vmatrix} 1 & 3 & -2 \\ 4 & 0 & -1 \\ 5 & 1.5 & 6 \end{vmatrix}$$

Step 1. Select any one column or row in the determinant. Each number in the selected column or row is used as a multiplying factor. For illustration, we will use the first column.

$$\begin{vmatrix} 1 & 3 & -2 \\ 4 & 0 & -1 \\ 5 & 1.5 & 6 \end{vmatrix}$$

Step 2. Determine the cofactor for each number in the selected column (or row). The cofactor for a given number is the determinant formed by all numbers that are *not* in the same column or row as the given number. This is illustrated as follows:

$$\begin{array}{c|ccc} 1 & 3 & -2 \\ 4 & 0 & -1 \\ 5 & 1.5 & 6 \end{array} \quad \begin{array}{c|cc} 1 & 3 & -2 \\ 4 & 0 & -1 \\ 5 & 1.5 & 6 \end{array} \quad \begin{array}{c|cc} 1 & 3 & -2 \\ 4 & 0 & -1 \\ 5 & 1.5 & 6 \end{array}$$

↑ ↑ ↑

Cofactor for 1 Cofactor for 4 Cofactor for 5

Step 3. Assign the proper sign to each multiplying factor according to the following format (note the alternating pattern):

$$\begin{array}{ccc} + & - & + \\ - & + & - \\ + & - & + \end{array}$$

Step 4. Sum all of the products of each multiplying factor and its associated cofactor using the appropriate sign.

$$\begin{aligned} 1 \times \begin{vmatrix} 0 & -1 \\ 1.5 & 6 \end{vmatrix} - 4 \times \begin{vmatrix} 3 & -2 \\ 1.5 & 6 \end{vmatrix} + 5 \times \begin{vmatrix} 3 & -2 \\ 0 & -1 \end{vmatrix} \\ = 1[(0)(6) - (1.5)(-1)] - 4[(3)(6) + (1.5)(-2)] + 5[(3)(-1) + (0)(-2)] \\ = 1(1.5) - 4(15) + 5(-3) = 1.5 - 60 - 15 = -73.5 \end{aligned}$$

EXAMPLE 9-4

Repeat Example 9-3 using the cofactor method to find I_2 . The equations are repeated below.

$$2I_1 + 0.5I_2 + 1I_3 = 0$$

$$0.75I_1 + 0I_2 + 2I_3 = 1.5$$

$$3I_1 + 0.2I_2 + 0I_3 = -1$$

Solution Evaluate the characteristic determinant as follows:

$$\begin{vmatrix} 2 & 0.5 & 1 \\ 0.75 & 0 & 2 \\ 3 & 0.2 & 0 \end{vmatrix} = 2 \times \begin{vmatrix} 0 & 2 \\ 0.2 & 0 \end{vmatrix} - 0.75 \times \begin{vmatrix} 0.5 & 1 \\ 0.2 & 0 \end{vmatrix} + 3 \times \begin{vmatrix} 0.5 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 2[(0)(0) - (0.2)(2)] - 0.75[(0.5)(0) - (0.2)(1)] + 3[(0.5)(2) - (0)(1)]$$

$$= 2(-0.4) - 0.75(-0.2) + 3(1) = -0.8 + 0.15 + 3 = 2.35$$

The determinant for I_2 is

$$\begin{vmatrix} 2 & 0 & 1 \\ 0.75 & 1.5 & 2 \\ 3 & -1 & 0 \end{vmatrix} = 2 \times \begin{vmatrix} 1.5 & 2 \\ -1 & 0 \end{vmatrix} - 0.75 \times \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} + 3 \times \begin{vmatrix} 0 & 1 \\ 1.5 & 2 \end{vmatrix}$$

$$= 2[(1.5)(0) - (-1)(2)] - 0.75[(0)(0) - (-1)(1)] + 3[(0)(2) - (1.5)(1)]$$

$$= 2(2) - 0.75(1) + 3(-1.5) = 4 - 0.75 - 4.5 = -1.25$$

Then,

$$I_2 = \frac{-1.25}{2.35} = -0.532 \text{ A} = \boxed{-532 \text{ mA}}$$

Related Problem Find I_1 using the cofactor method. Use the set of equations in Example 9-4.

**SECTION 9-2
REVIEW**

1. Evaluate the following determinants:

(a) $\begin{vmatrix} 0 & -1 \\ 4 & 8 \end{vmatrix}$

(b) $\begin{vmatrix} 0.25 & 0.33 \\ -0.5 & 1 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 3 & 7 \\ 2 & -1 & 7 \\ -4 & 0 & -2 \end{vmatrix}$

2. Set up the characteristic determinant for the following set of simultaneous equations:

$$2I_1 + 3I_2 = 0$$

$$5I_1 + 4I_2 = 1$$

3. Find I_2 in Question 2.

9-3 ■ SOLVING SIMULTANEOUS EQUATIONS USING A CALCULATOR

You have learned two ways to solve simultaneous equations, the substitution method and the determinant method. In this section, you will learn how to use a calculator to solve any number of simultaneous equations up to thirty. Your calculator and the method discussed in this section can be applied whenever simultaneous equations are part of a circuit analysis problem, regardless of method used.

After completing this section, you should be able to

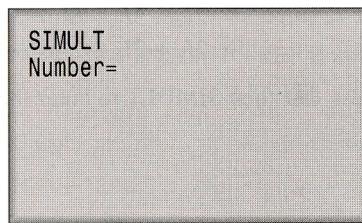
- Use a calculator to solve simultaneous equations for unknown quantities
 - Set up the calculator for any number of simultaneous equations up to thirty
 - Enter the simultaneous equations
 - Determine the values of the unknown variables
 - Edit the equations

Entering Simultaneous Equations

The following steps are used to enter simultaneous equations in the TI-85 calculator:

Step 1. Press **2nd** and then **SIMULT**; the screen in Figure 9–4 appears.

FIGURE 9–4



Step 2. Enter the number of simultaneous equations and press **ENTER**. The coefficient entry screen for the first equation appears as shown in Figure 9–5(a) for a three-equation example. The coefficient $a_{1,1}$ is the first coefficient in the first equation, $a_{1,2}$ is the second coefficient in the first equation, and so on. The equation is displayed on the top line as $a_{1,1} \times 1 + a_{1,2} \times 2 + a_{1,3} \times 3 = b_1$. Completely written out, this equation would be

$$a_{1,1} \times 1 + a_{1,2} \times 2 + a_{1,3} \times 3 = b_1$$

where $\times 1$, $\times 2$, and $\times 3$ are the unknown variables and b_1 is the constant.

Step 3. Enter the value for each of the coefficients, $a_{1,1}$ through $a_{1,3}$ and for b_1 . Press **ENTER** after each entry. If you press **ENTER** after entering the last coefficient or select **NEXT** using the **F2** key, the second equation and coefficients are displayed as shown in Figure 9–5(b). Enter the value for each of the coefficients $a_{2,1}$ through $a_{2,3}$ and for b_2 . Repeat the entry process for the third equation, which is displayed as shown in Figure 9–5(c).

$a_{1,1} \times 1 + a_{1,2} \times 2 + a_{1,3} \times 3 = b_1$				
$a_{1,1} =$	<input type="text"/>	PREV	NEXT	CLRq
$a_{1,2} =$	<input type="text"/>	SOLVE		
$a_{1,3} =$	<input type="text"/>			
$b_1 =$	<input type="text"/>			

(a)

$a_{2,1} \times 1 + a_{2,2} \times 2 + a_{2,3} \times 3 = b_2$				
$a_{2,1} =$	<input type="text"/>	PREV	NEXT	CLRq
$a_{2,2} =$	<input type="text"/>	SOLVE		
$a_{2,3} =$	<input type="text"/>			
$b_2 =$	<input type="text"/>			

(b)

$a_{3,1} \times 1 + a_{3,2} \times 2 + a_{3,3} \times 3 = b_3$				
$a_{3,1} =$	<input type="text"/>	PREV	NEXT	CLRq
$a_{3,2} =$	<input type="text"/>	SOLVE		
$a_{3,3} =$	<input type="text"/>			
$b_3 =$	<input type="text"/>			

(c)

FIGURE 9–5

Solving the Equations

After entering the coefficients in steps 1 through 3, select **SOLVE** by pressing **F5**. The values for $\times 1$, $\times 2$, and $\times 3$ are displayed.

Editing the Equations

You can change the coefficients of any of the equations by selecting COEFS using the **F1** key. This will return you to the first equation entry screen. To get to the next equation entry screen, simply step through the coefficients in the current screen until the next screen comes up.

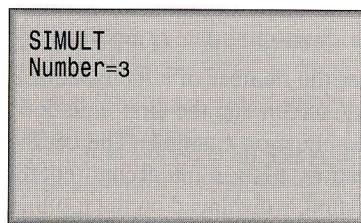
EXAMPLE 9–5

Use the TI-85 calculator to solve the following three simultaneous equations for the three unknowns.

$$\begin{aligned} 8I_1 + 4I_2 + 1I_3 &= 7 \\ 2I_1 - 5I_2 + 6I_3 &= 3 \\ 3I_1 + 3I_2 - 2I_3 &= -5 \end{aligned}$$

Solution Press **2nd** then **SIMULT** to enter the number of equations as shown in Figure 9–6.

FIGURE 9–6



After you enter 3 and press **ENTER**, the first equation screen comes up. The coefficients 8, 4, 1 and the constant 7 are entered by pressing each number key followed by the **ENTER** key, which results in the screen shown in Figure 9–7(a). After you enter the last number and press **ENTER**, the second equation screen appears. Enter the coefficients 2, -5, 6 and the constant 3 as shown in Figure 9–7(b) (A negative value is entered by first pressing the **(–)** key.). Finally, enter the coefficients of the third equation (3, 3, -2) and the constant -5 as shown in Figure 9–7(c).

a1,1X1...a1,3X3=b1
a1,1=8
a1,2=4
a1,3=1
b1=7
PREV NEXT CLRq SOLVE

(a)

a2,1X1...a2,3X3=b2
a2,1=2
a2,2=-5
a2,3=6
b2=3
PREV NEXT CLRq SOLVE

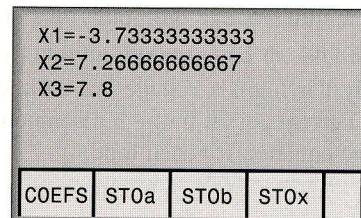
(b)

a3,1X1...a3,3X3=b3
a3,1=3
a3,2=3
a3,3=-2
b3=-5
PREV NEXT CLRq SOLVE

(c)

FIGURE 9–7

Selecting SOLVE, which is **F5**, produces the results displayed in Figure 9–8. $\times 1$ is I_1 , $\times 2$ is I_2 , and $\times 3$ is I_3 .

FIGURE 9–8

Related Problem Edit the equations to change $a_{1,2}$ from 4 to -3 , $a_{2,3}$ from 6 to 2.5, and b_3 from -5 to 8 and solve the modified equations.

SECTION 9–3 REVIEW

1. Use your calculator to solve the following set of simultaneous equations for I_1 , I_2 , I_3 , and I_4 .

$$\begin{aligned}100I_1 + 220I_2 + 180I_3 + 330I_4 &= 0 \\470I_1 + 390I_2 + 100I_3 + 100I_4 &= 12 \\120I_1 - 270I_2 + 150I_3 - 180I_4 &= -9 \\560I_1 + 680I_2 - 220I_3 + 390I_4 &= 0\end{aligned}$$

2. Modify the equations in Question 1 by changing the constant in the first equation to 8.5, the coefficient of I_3 in the second equation to 220, and the coefficient of I_1 in the fourth equation to 330. Solve the new set of equations.

9–4 ■ MESH CURRENT METHOD

In the mesh current method, you will work with loop currents instead of branch currents. A branch current is the actual current through a branch. An ammeter placed in a given branch will measure the branch current. Loop currents are different because they are mathematical quantities that are used to make circuit analysis somewhat easier than with the branch current method. The term mesh comes from the fact that a multiple-loop circuit, when drawn, can be imagined to resemble a wire mesh.

After completing this section, you should be able to

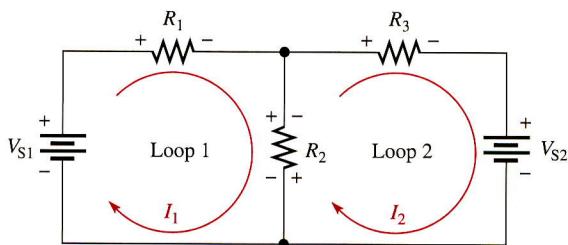
- Use mesh analysis to find unknown quantities in a circuit
 - Assign loop currents
 - Apply Kirchhoff's voltage law around each loop
 - Develop the loop (mesh) equations
 - Solve the loop equations

A systematic method of mesh analysis is given in the following steps and is illustrated in Figure 9–9, which is essentially the same circuit configuration as in Figure 9–1 used in the branch current analysis. It demonstrates the basic principles well.

- Step 1.** Although direction of an assigned loop current is arbitrary, we will assign a current in the clockwise (CW) direction around each nonredundant closed loop, for consistency. This may not be the actual current direction, but it does not matter. The number of loop-current assignments must be sufficient to include current through all components in the circuit.

- Step 2.** Indicate the voltage drop polarities in each loop based on the assigned current directions.
- Step 3.** Apply Kirchhoff's voltage law around each closed loop. When more than one loop current passes through a component, include its voltage drop. This results in one equation for each loop.
- Step 4.** Using substitution or determinants, solve the resulting equations for the loop currents.

FIGURE 9–9
Circuit for mesh analysis.



First, the loop currents \$I_1\$ and \$I_2\$ are assigned in the CW direction as shown in Figure 9–9. A loop current could be assigned around the outer perimeter of the circuit, but this would be redundant since \$I_1\$ and \$I_2\$ already pass through all of the components.

Second, the polarities of the voltage drops across \$R_1\$, \$R_2\$, and \$R_3\$ are shown based on the loop-current directions. Notice that \$I_1\$ and \$I_2\$ are in opposite directions through \$R_2\$ because \$R_2\$ is common to both loops. Therefore, two voltage polarities are indicated. In reality, the \$R_2\$ current cannot be separated into two parts, but remember that the loop currents are basically mathematical quantities used for analysis purposes. The polarities of the voltage sources are fixed and are not affected by the current assignments.

Third, Kirchhoff's voltage law applied to the two loops results in the following two equations:

$$\begin{aligned} R_1 I_1 + R_2 (I_1 - I_2) &= V_{S1} && \text{for loop 1} \\ R_3 I_2 + R_2 (I_2 - I_1) &= -V_{S2} && \text{for loop 2} \end{aligned}$$

Notice that \$I_1\$ is positive in loop 1 and \$I_2\$ is positive in loop 2.

Fourth, the like terms in the equations are combined and rearranged into a form for convenient solution so that they have the same position in each equation, that is, the \$I_1\$ term is first and the \$I_2\$ term is second. The equations are rearranged into the following form. Once the loop currents are evaluated, all of the branch currents can be determined.

$$\begin{aligned} (R_1 + R_2) I_1 - R_2 I_2 &= V_{S1} && \text{for loop 1} \\ -R_2 I_1 + (R_2 + R_3) I_2 &= -V_{S2} && \text{for loop 2} \end{aligned}$$

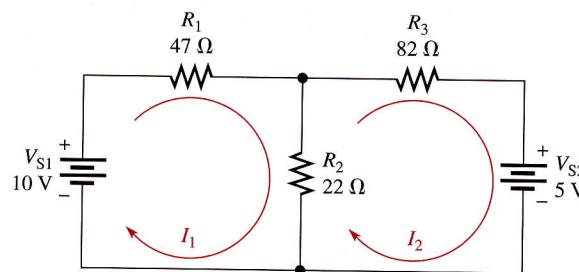
Notice that in the mesh current method only two equations are required for the same circuit that required three equations in the branch current method. The last two equations (developed in the fourth step) follow a form to make mesh analysis easier. Referring to these last two equations, notice that for loop 1, the total resistance in the loop, \$R_1 + R_2\$, is multiplied by \$I_1\$ (its loop current). Also in the loop 1 equation, the resistance common to both loops, \$R_2\$, is multiplied by the other loop current, \$I_2\$, and subtracted from the first term. The same form is seen in the loop 2 equation except that the terms have been rearranged. From these observations, a concise rule for applying steps 1 to 4 is as follows:

(Sum of resistors in loop) times (loop current) minus (each common resistor) times (associated adjacent loop current) equals (source voltage in the loop).

Example 9–6 illustrates the application of this rule to the mesh current analysis of a circuit.

EXAMPLE 9-6

Using the mesh current method, find the branch currents in Figure 9-10.

FIGURE 9-10

Solution Assign the loop currents (I_1 and I_2) as shown in Figure 9-10; resistance values are in ohms and voltage values are in volts. Use the rule described to set up the two loop equations.

$$\begin{aligned}(47 + 22)I_1 - 22I_2 &= 10 \\ 69I_1 - 22I_2 &= 10 \quad \text{for loop 1}\end{aligned}$$

$$\begin{aligned}-22I_1 + (22 + 82)I_2 &= -5 \\ -22I_1 + 104I_2 &= -5 \quad \text{for loop 2}\end{aligned}$$

Use determinants to find I_1 .

$$I_1 = \frac{\begin{vmatrix} 10 & -22 \\ -5 & 104 \end{vmatrix}}{\begin{vmatrix} 69 & -22 \\ -22 & 104 \end{vmatrix}} = \frac{(10)(104) - (-5)(-22)}{(69)(104) - (-22)(-22)} = \frac{1040 - 110}{7176 - 484} = 139 \text{ mA}$$

Solving for I_2 yields

$$I_2 = \frac{\begin{vmatrix} 69 & 10 \\ -22 & -5 \end{vmatrix}}{6692} = \frac{(69)(-5) - (-22)(10)}{6692} = \frac{-345 - (-220)}{6692} = -18.7 \text{ mA}$$

The negative sign on I_2 means that its assigned direction is opposite to the actual current.

Now find the actual branch currents. Since I_1 is the only current through R_1 , it is also the branch current I_{R1} .

$$I_{R1} = I_1 = 139 \text{ mA}$$

Since I_2 is the only current through R_3 , it is also the branch current I_{R3} .

$$I_{R3} = I_2 = -18.7 \text{ mA} \quad (\text{opposite direction of that originally assigned to } I_2)$$

Both loop currents I_1 and I_2 are through R_2 in the same direction. Remember, the negative I_2 value told you to reverse its assigned direction.

$$I_{R2} = I_1 - I_2 = 139 \text{ mA} - (-18.7 \text{ mA}) = 158 \text{ mA}$$

Keep in mind that once you know the branch currents, you can find the voltages by using Ohm's law.

Related Problem Solve for the two loop currents using your calculator.

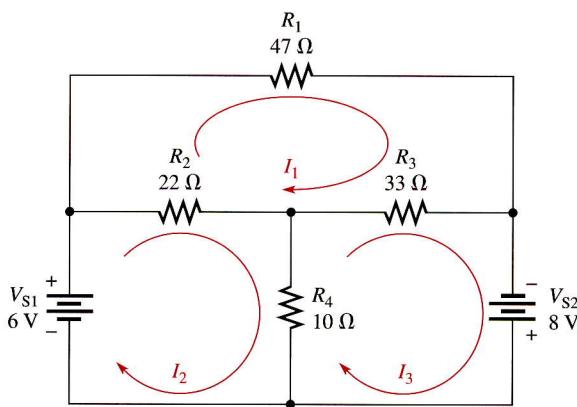
Circuits with More Than Two Loops

The mesh method also can be systematically applied to circuits with any number of loops. Of course, the more loops there are, the more difficult is the solution, but calculators have greatly simplified the problem. However, the basic procedure still applies. For example, for a three-loop circuit, three simultaneous equations are required. Example 9–7 illustrates the analysis of a three-loop circuit.

EXAMPLE 9–7

Find I_3 in Figure 9–11.

FIGURE 9–11



Solution Assign three CW loop currents (I_1 , I_2 , and I_3) as shown in Figure 9–11. Then use the rule to set up each loop equation. The polarity of a voltage source is positive when the assigned mesh current is out of the positive terminal. The loop equations are

$$\begin{aligned} 102I_1 - 22I_2 - 33I_3 &= 0 && \text{for loop 1} \\ -22I_1 + 32I_2 - 10I_3 &= 6 && \text{for loop 2} \\ -33I_1 - 10I_2 + 43I_3 &= 8 && \text{for loop 3} \end{aligned}$$

You can solve these three equations for the loop currents by substitution or, more easily, with third-order determinants.

Determine I_3 using determinants as follows. First, evaluate the characteristic determinant.

$$\begin{aligned} \begin{vmatrix} 102 & -22 & -33 \\ -22 & 32 & -10 \\ -33 & -10 & 43 \end{vmatrix} &= 102 \times \begin{vmatrix} 32 & -10 \\ -10 & 43 \end{vmatrix} - (-22) \times \begin{vmatrix} -22 & -33 \\ -10 & 43 \end{vmatrix} + (-33) \times \begin{vmatrix} -22 & -33 \\ 32 & -10 \end{vmatrix} \\ &= 102[(32)(43) - (-10)(-10)] - (-22)[(-22)(43) - (-10)(-33)] + (-33)[(-22)(-10) - (32)(-33)] \\ &= 102(1276) + 22(-1276) - 33(1276) = 130,152 - 28,072 - 42,108 = 59,972 \end{aligned}$$

Next, evaluate the I_3 determinant.

$$\begin{aligned} \begin{vmatrix} 102 & -22 & 0 \\ -22 & 32 & 6 \\ -33 & -10 & 8 \end{vmatrix} &= 102 \times \begin{vmatrix} 32 & 6 \\ -10 & 8 \end{vmatrix} - (-22) \times \begin{vmatrix} -22 & 0 \\ -10 & 8 \end{vmatrix} + (-33) \times \begin{vmatrix} -22 & 0 \\ 32 & 6 \end{vmatrix} \\ &= 102[(33)(8) - (-10)(6)] + 22[(-22)(8) - (-10)(0)] - 33[(-22)(6) - (32)(0)] \\ &= 102(316) + 22(-176) - 33(-132) = 32,232 - 3872 + 4356 = 32,716 \end{aligned}$$

Determine I_3 by dividing the value of the I_3 determinant by the value of the characteristic determinant.

$$I_3 = \frac{32,716}{59,972} = 546 \text{ mA}$$

The other loop currents are found similarly. The actual branch currents and voltages can be determined once you know the loop currents.

Related Problem Solve for the three loop currents using your calculator.

SECTION 9-4 REVIEW

1. Do the loop currents necessarily represent the actual currents in the branches?
2. When you solve for a loop current and get a negative value, what does it mean?
3. What circuit law is used in the mesh current method?

9-5 ■ NODE VOLTAGE METHOD

Another method of analysis of multiple-loop circuits is called the node voltage method. It is based on finding the voltages at each node in the circuit using Kirchhoff's current law. A node is the junction of two or more components.

After completing this section, you should be able to

- Use node analysis to find unknown quantities in a circuit
 - Select the nodes at which the voltage is unknown and assign currents
 - Apply Kirchhoff's current law at each node
 - Develop the node equations
 - Solve the node equations

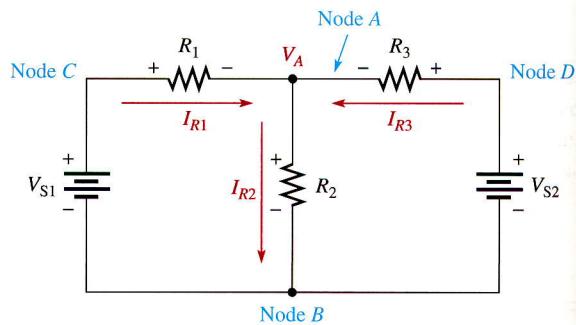
The general steps for the node voltage method of circuit analysis are as follows:

- Step 1.** Determine the number of nodes.
- Step 2.** Select one node as a reference. All voltages will be relative to the reference node. Assign voltage designations to each node where the voltage is unknown.
- Step 3.** Assign currents at each node where the voltage is unknown, except at the reference node. The directions are arbitrary.
- Step 4.** Apply Kirchhoff's current law to each node where currents are assigned.
- Step 5.** Express the current equations in terms of voltages, and solve the equations for the unknown node voltages using Ohm's law.

We will use Figure 9-12 to illustrate the general approach to node voltage analysis. First, establish the nodes. In this case, there are four nodes, as indicated in the figure. Second, let's use node B as the reference. Think of it as the circuit's reference ground. Node voltages C and D are already known to be the source voltages. The voltage at node A is the only unknown; it is designated as V_A . Third, arbitrarily assign the branch currents at node A as indicated in the figure. Fourth, the Kirchhoff current equation at node A is

$$I_{R1} - I_{R2} + I_{R3} = 0$$

FIGURE 9–12
Circuit for node voltage analysis.



Fifth, express the currents in terms of circuit voltages using Ohm's law.

$$I_{R1} = \frac{V_1}{R_1} = \frac{V_{S1} - V_A}{R_1}$$

$$I_{R2} = \frac{V_2}{R_2} = \frac{V_A}{R_2}$$

$$I_{R3} = \frac{V_3}{R_3} = \frac{V_{S2} - V_A}{R_3}$$

Substituting these terms into the current equation yields

$$\frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0$$

The only unknown is V_A ; so solve the single equation by combining and rearranging terms. Once the voltage is known, all branch currents can be calculated. Example 9–8 illustrates this method further.

EXAMPLE 9–8 Find the node voltage V_A in Figure 9–13.

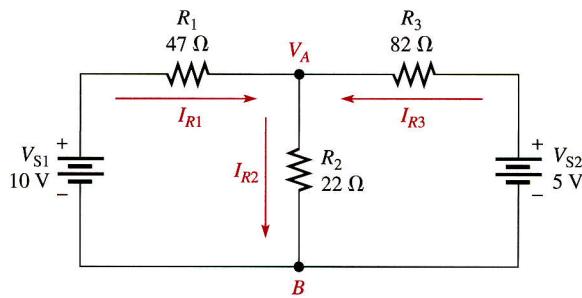


FIGURE 9–13

Solution The reference node is chosen at B . The unknown node voltage is V_A , as indicated in Figure 9–13. This is the only unknown voltage. Branch currents are assigned at node A as shown. The current equation is

$$I_{R1} - I_{R2} + I_{R3} = 0$$

Substitution for currents using Ohm's law gives the equation in terms of voltages.

$$\frac{10 - V_A}{47} - \frac{V_A}{22} + \frac{5 - V_A}{82} = 0$$

Rearranging the terms yields

$$\begin{aligned}\frac{10}{47} - \frac{V_A}{47} - \frac{V_A}{22} + \frac{5}{82} - \frac{V_A}{82} &= 0 \\ -\frac{V_A}{47} - \frac{V_A}{22} - \frac{V_A}{82} &= -\frac{10}{47} - \frac{5}{82}\end{aligned}$$

To solve for V_A , combine the terms on each side of the equation and find the common denominator.

$$\begin{aligned}\frac{1804V_A + 3854V_A + 1034V_A}{84,788} &= \frac{820 + 235}{3854} \\ \frac{6692V_A}{84,788} &= \frac{1055}{3854} \\ V_A &= \frac{(1055)(84,788)}{(6692)(3854)} = 3.47 \text{ V}\end{aligned}$$

Related Problem Find V_A in Figure 9–13 if the 5 V source is reversed.

Using the same basic procedure, you can analyze circuits with more than one unknown node voltage. Example 9–9 illustrates this calculation for two unknown node voltages.

EXAMPLE 9–9 Using the node analysis method, calculate V_A and V_B in the circuit of Figure 9–14.

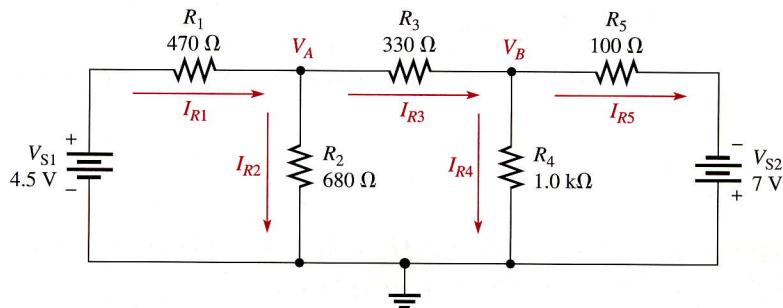


FIGURE 9–14

Solution First, assign the branch currents as shown in Figure 9–14. Next, apply Kirchhoff's current law at each node. At node A,

$$I_{R1} - I_{R2} - I_{R3} = 0$$

Using Ohm's law substitution for the currents yields

$$\begin{aligned}\left(\frac{4.5 - V_A}{470}\right) - \left(\frac{V_A}{680}\right) - \left(\frac{V_A - V_B}{330}\right) &= 0 \\ \frac{4.5}{470} - \frac{V_A}{470} - \frac{V_A}{680} - \frac{V_A}{330} + \frac{V_B}{330} &= 0 \\ \left(\frac{1}{470} + \frac{1}{680} + \frac{1}{330}\right)V_A - \left(\frac{1}{330}\right)V_B &= \frac{4.5}{470}\end{aligned}$$

Next, using your calculator, evaluate the coefficients and constant in the last equation. The resulting equation for node A is

$$0.00663V_A - 0.00303V_B = 0.00957$$

At node B,

$$I_{R3} - I_{R4} - I_{R5} = 0$$

Again use Ohm's law substitution.

$$\begin{aligned}\left(\frac{V_A - V_B}{330}\right) - \left(\frac{V_B}{1000}\right) - \left(\frac{V_B - (-7)}{100}\right) &= 0 \\ \frac{V_A}{330} - \frac{V_B}{330} - \frac{V_B}{1000} - \frac{V_B}{100} + \frac{7}{100} &= 0 \\ \left(\frac{1}{330}\right)V_A - \left(\frac{1}{330} + \frac{1}{1000} + \frac{1}{100}\right)V_B &= \frac{7}{100}\end{aligned}$$

Evaluate the coefficients and constant. The equation for node B is

$$0.00303V_A - 0.01403V_B = 0.07$$

Next, these two node equations must be solved for V_A and V_B . Use determinants to get the solutions.

$$V_A = \frac{\begin{vmatrix} 0.00957 & -0.00303 \\ 0.07 & -0.01403 \end{vmatrix}}{\begin{vmatrix} 0.00663 & -0.00303 \\ 0.00303 & -0.01403 \end{vmatrix}} = \frac{(0.00957)(-0.01403) - (0.07)(-0.00303)}{(0.00663)(-0.01403) - (0.00303)(-0.00303)} = -928 \text{ mV}$$

$$V_B = \frac{\begin{vmatrix} 0.00663 & 0.00957 \\ 0.00303 & 0.07 \end{vmatrix}}{\begin{vmatrix} 0.00663 & -0.00303 \\ 0.00303 & -0.01403 \end{vmatrix}} = \frac{(0.00663)(0.07) - (0.00303)(0.00957)}{(0.00663)(-0.01403) - (0.00303)(-0.00303)} = -5.19 \text{ V}$$

Related Problem Solve the two node equations for V_A and V_B using your calculator.

SECTION 9–5 REVIEW

- What circuit law is the basis for the node voltage method?
- What is the reference node?

9–6 ■ TECHnology Theory Into Practice



Analysis of a dual-polarity loaded voltage divider provides an opportunity to apply one of the analysis methods covered in this chapter. The dual-polarity voltage divider circuit in this section operates from two voltage sources. One source is +9 V and the other is -9 V. These two voltage sources supply both positive and negative voltages that are divided down to produce reference voltages for two different devices.

The voltage divider that you will check out in this TECH TIP section will be used to provide reference voltages to two devices using 9 V batteries. One of the devices requires a positive reference voltage and presents a 27 kΩ load to the voltage divider. The

other device requires a negative reference voltage and presents a $15\text{ k}\Omega$ load to the voltage divider. The schematic of the loaded dual-polarity voltage divider is shown in Figure 9–15.

The self-contained dual-polarity voltage divider is constructed on the PC board shown in Figure 9–16. The two batteries are clip-mounted directly on the board and wired to the printed circuit pads as indicated. The two load devices can be connected to the terminal strip.

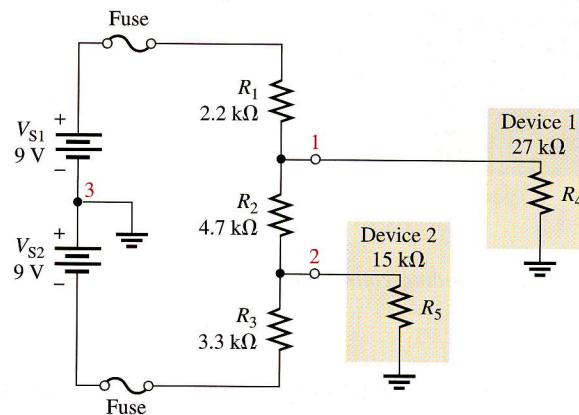


FIGURE 9–15

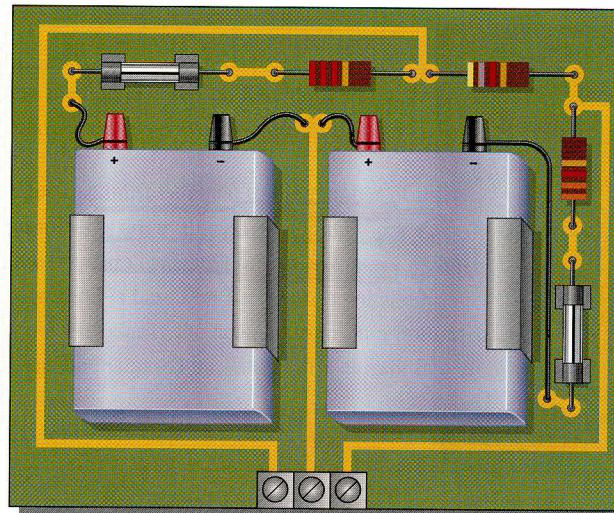


FIGURE 9–16

The Printed Circuit Board and Schematic

- Check the PC board in Figure 9–16 to make sure that it agrees with the schematic in Figure 9–15. Relate each input, output, and component on the board to the schematic.

Analysis and Troubleshooting

- Referring to Figure 9–17, determine if the voltmeter readings are correct. The output voltages are measured without the loads connected.

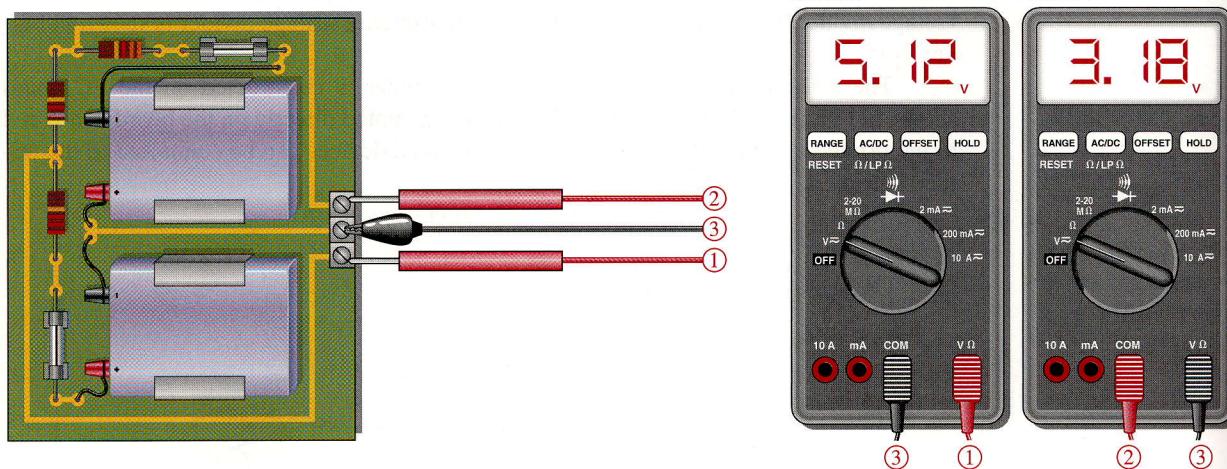


FIGURE 9–17
Circled numbers show corresponding connections.

- Referring to Figure 9–18, apply the node voltage method to determine if the voltmeter readings are correct. You may need to redraw the schematic in a more familiar form. The output voltages are measured with a $27\text{ k}\Omega$ load connected from output 1 to ground to simulate one of the devices connected to the voltage divider.

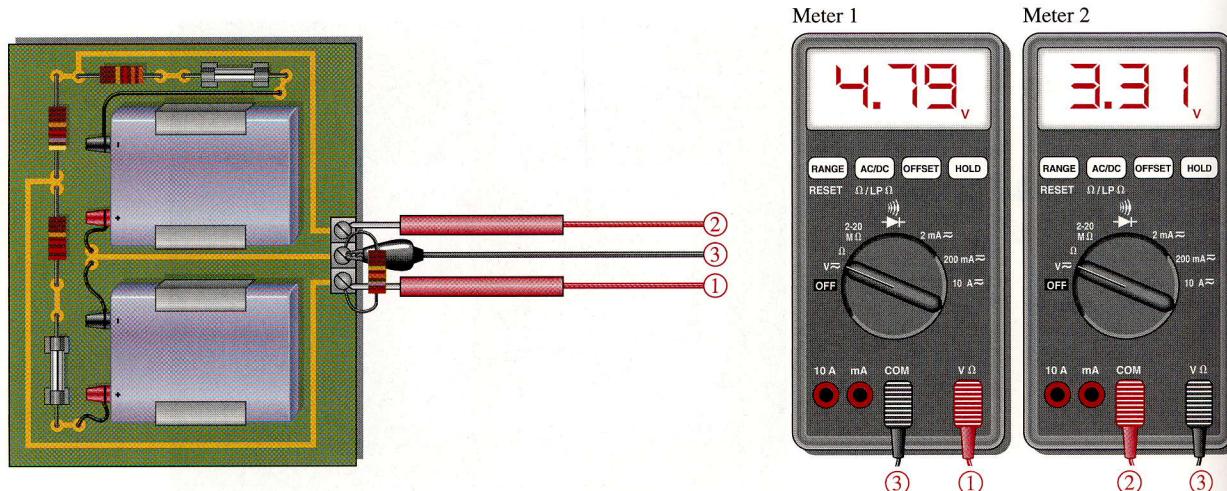


FIGURE 9–18
Circled numbers show corresponding connections.

- Referring to Figure 9–19, apply the node voltage method to determine if the voltmeter readings are correct. You may need to redraw the schematic in a more familiar form. The output voltages are measured with a $27\text{ k}\Omega$ load connected from output 1 to ground and a $15\text{ k}\Omega$ load from output 2 to ground to simulate both devices connected to the voltage divider.
- If meter 1 reads +7.50 V and meter 2 reads +5.71 V in Figure 9–19, what is the problem?
- If meter 1 reads +8.27 V and meter 2 reads -7.38 V in Figure 9–19, what is the problem?

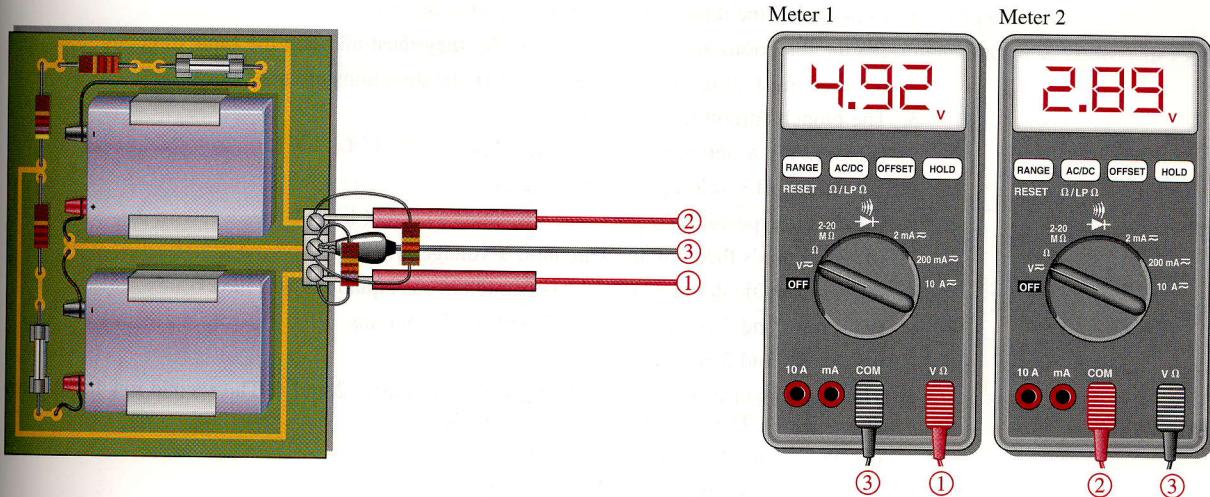


FIGURE 9-19
Circled numbers show corresponding connections.

SECTION 9-6 REVIEW

1. Determine the minimum value for the fuses in the voltage-divider circuit.
2. If the batteries are each rated at 10 Ah, how long will the voltage-divider circuit operate under unloaded conditions?

■ SUMMARY

- The branch current method is based on Kirchhoff's voltage law and Kirchhoff's current law.
- Simultaneous equations can be solved by substitution, by determinants, or by a graphics calculator.
- The number of equations must be equal to the number of unknowns.
- Second-order determinants are evaluated by adding the signed cross-products.
- Third-order determinants are evaluated by the expansion method or by the cofactor method.
- The mesh current method is based on Kirchhoff's voltage law.
- A loop current is not necessarily the actual current in a branch.
- The node voltage method is based on Kirchhoff's current law.

■ GLOSSARY

These terms are also in the end-of-book glossary.

Branch One current path that connects two nodes.

Branch current The actual current in a branch.

Coefficient The constant number that appears in front of a variable.

Determinant An array of coefficients and constants in a given set of simultaneous equations.

Loop A closed current path in a circuit.

Node The junction of two or more components.

■ SELF-TEST

1. Assuming the voltage source values in Figure 9-1 are known, there is/are
 - (a) 3 nonredundant loops
 - (b) 1 unknown node
 - (c) 2 nonredundant loops
 - (d) 2 unknown nodes
 - (e) both answers (b) and (c)

2. In assigning the direction of branch currents,
 - (a) the directions are critical
 - (b) they must all be in the same direction
 - (c) they must all point into a node
 - (d) the directions are not critical
3. The branch current method uses
 - (a) Ohm's law and Kirchhoff's voltage law
 - (b) Kirchhoff's voltage and current laws
 - (c) the superposition theorem and Kirchhoff's current law
 - (d) Thevenin's theorem and Kirchhoff's voltage law
4. A characteristic determinant for two simultaneous equations will have
 - (a) 2 rows and 1 column
 - (b) 1 row and 2 columns
 - (c) 2 rows and 2 columns
5. The first row of a certain determinant has the numbers 2 and 4. The second row has the numbers 6 and 1. The value of this determinant is
 - (a) 22
 - (b) 2
 - (c) -22
 - (d) 8
6. The expansion method for evaluating determinants is
 - (a) good only for second-order determinants
 - (b) good only for both second and third-order determinants
 - (c) good for any determinant
 - (d) better than the cofactor method
7. The mesh current method is based on
 - (a) Kirchhoff's current law
 - (b) Ohm's law
 - (c) the superposition theorem
 - (d) Kirchhoff's voltage law
8. The node voltage method is based on
 - (a) Kirchhoff's current law
 - (b) Ohm's law
 - (c) the superposition theorem
 - (d) Kirchhoff's voltage law
9. In the node voltage method,
 - (a) currents are assigned at each node
 - (b) currents are assigned at the reference node
 - (c) the current directions are arbitrary
 - (d) currents are assigned only at the nodes where the voltage is unknown
 - (e) both answers (c) and (d)
10. Generally, the node voltage method results in
 - (a) more equations than the mesh current method
 - (b) fewer equations than the mesh current method
 - (c) the same number of equations as the mesh current method

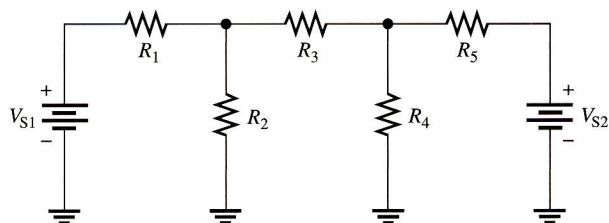
■ PROBLEMS

More difficult problems are indicated by an asterisk ().*

SECTION 9-1 Branch Current Method

1. Identify all possible nonredundant loops in Figure 9-20.

FIGURE 9-20



2. Identify all nodes in Figure 9–20. Which ones have a known voltage?
 3. Write the Kirchhoff current equation for the current assignment shown at node A in Figure 9–21.

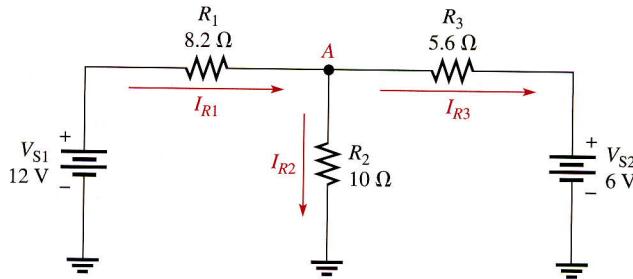
FIGURE 9–21

FIGURE 9–21

4. Solve for each of the branch currents in Figure 9–21.
 5. Find the voltage drop across each resistor in Figure 9–21 and indicate its actual polarity.
 6. Using the substitution method, solve the following set of equations for I_{R1} and I_{R2} .

$$100I_{R1} + 50I_{R2} = 30$$

$$75I_{R1} + 90I_{R2} = 15$$

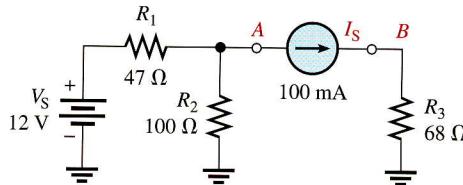
7. Using the substitution method, solve the following set of three equations for all currents:

$$5I_{R1} - 2I_{R2} + 8I_{R3} = 1$$

$$2I_{R1} + 4I_{R2} - 12I_{R3} = 5$$

$$10I_{R1} + 6I_{R2} + 9I_{R3} = 0$$

- *8. Find the current through each resistor in Figure 9–22.

FIGURE 9–22

9. In Figure 9–22, determine the voltage across the current source (points A and B).

SECTION 9–2 Determinants

10. Evaluate each determinant:

$$(a) \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix}$$

$$(b) \begin{vmatrix} 9 & -1 \\ 0 & 5 \end{vmatrix}$$

$$(c) \begin{vmatrix} 12 & 15 \\ -2 & -1 \end{vmatrix}$$

$$(d) \begin{vmatrix} 100 & 50 \\ 30 & -20 \end{vmatrix}$$

11. Using determinants, solve the following set of equations for both currents:

$$-I_1 + 2I_2 = 4$$

$$7I_1 + 3I_2 = 6$$

12. Evaluate each of the determinants using the expansion method:

$$(a) \begin{vmatrix} 1 & 0 & -2 \\ 5 & 4 & 1 \\ 2 & 10 & 0 \end{vmatrix}$$

$$(b) \begin{vmatrix} 0.5 & 1 & -0.8 \\ 0.1 & 1.2 & 1.5 \\ -0.1 & -0.3 & 5 \end{vmatrix}$$

13. Evaluate each of the determinants using the cofactor method:

$$(a) \begin{vmatrix} 25 & 0 & -20 \\ 10 & 12 & 5 \\ -8 & 30 & -16 \end{vmatrix}$$

$$(b) \begin{vmatrix} 1.08 & 1.75 & 0.55 \\ 0 & 2.12 & -0.98 \\ 1 & 3.49 & -1.05 \end{vmatrix}$$

14. Find I_1 and I_3 in Example 9–3.
 15. Solve for I_1 , I_2 , I_3 in the following set of equations:

$$2I_1 - 6I_2 + 10I_3 = 9$$

$$3I_1 + 7I_2 - 8I_3 = 3$$

$$10I_1 + 5I_2 - 12I_3 = 0$$

- *16. Find V_1 , V_2 , V_3 , and V_4 from the following set of equations:

$$16V_1 + 10V_2 - 8V_3 - 3V_4 = 15$$

$$2V_1 + 0V_2 + 5V_3 + 2V_4 = 0$$

$$-7V_1 - 12V_2 + 0V_3 + 0V_4 = 9$$

$$-1V_1 + 20V_2 - 18V_3 + 0V_4 = 10$$

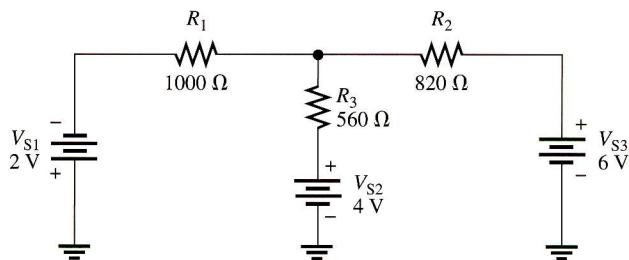
SECTION 9–3 Solving Simultaneous Equations Using a Calculator

17. Solve the two simultaneous equations in Problem 6 using your calculator.
 18. Solve the three simultaneous equations in Problem 7 using your calculator.
 19. Solve the three simultaneous equations in Problem 15 using your calculator.
 20. Solve the four simultaneous equations in Problem 16 using your calculator.

SECTION 9–4 Mesh Current Method

21. Using the mesh current method, find the loop currents in Figure 9–23.
 22. Find the branch currents in Figure 9–23.
 23. Determine the voltages and their proper polarities for each resistor in Figure 9–23.
 24. Write the loop equations for the circuit in Figure 9–24.

FIGURE 9–23



25. Solve for the loop currents in Figure 9–24 using your calculator.
 26. Find the current through each resistor in Figure 9–24.

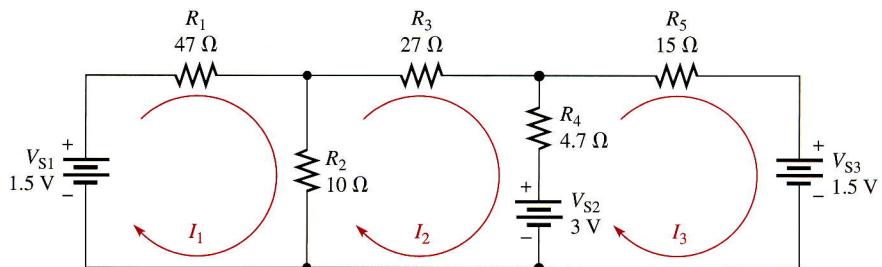


FIGURE 9–24

27. Determine the voltage across the open bridge terminals, A and B , in Figure 9–25.
 28. When a 10Ω resistor is connected from point A to point B in Figure 9–25, what is the current through it?
 29. Find the current through R_1 in Figure 9–26.

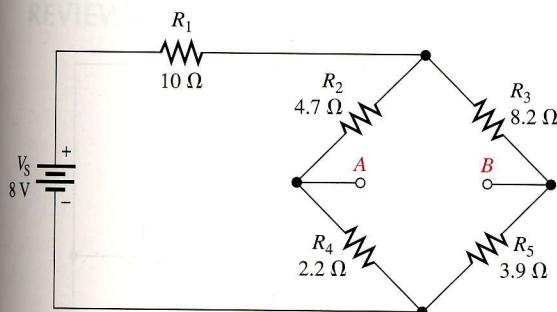


FIGURE 9-25

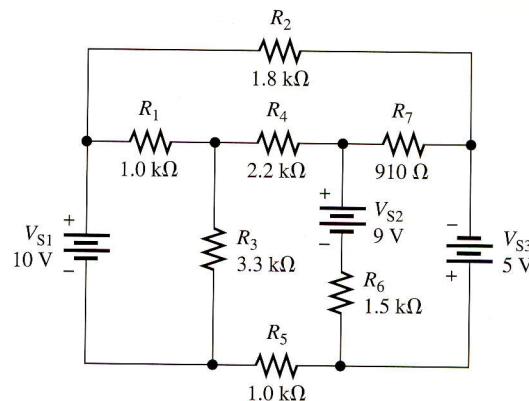


FIGURE 9-26

SECTION 9-5 Node Voltage Method

30. In Figure 9-27, use the node voltage method to find the voltage at point A with respect to ground.
31. What are the branch current values in Figure 9-27? Show the actual direction of current in each branch.
32. Write the node voltage equations for Figure 9-24. Use your calculator to find the node voltages.

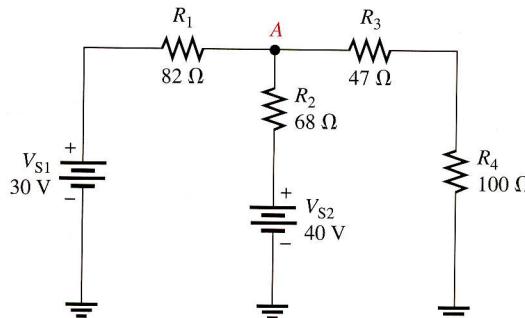


FIGURE 9-27

33. Use node analysis to determine the voltage at points A and B with respect to ground in Figure 9-28.

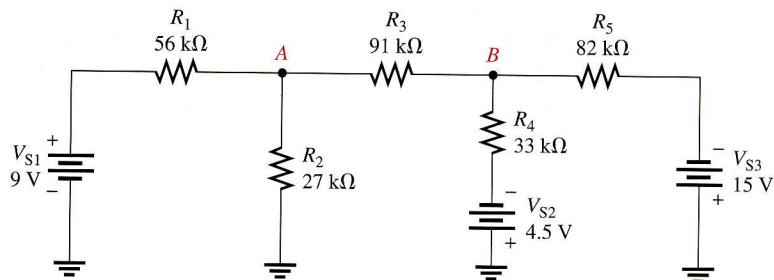
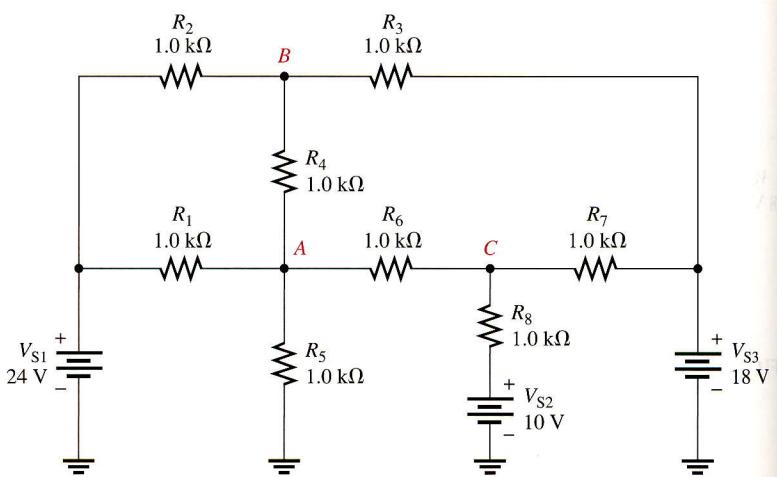


FIGURE 9-28

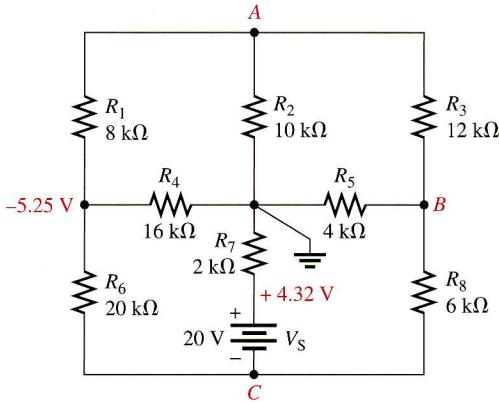
*34. Find the voltage at points A, B, and C in Figure 9–29.

FIGURE 9–29



*35. Use node analysis, mesh analysis, or any other procedure to find all currents and the voltages at each unknown node in Figure 9–30.

FIGURE 9–30



EWB Troubleshooting and Analysis

These problems require your EWB compact disk.



36. Open file PRO09-36.EWB and measure the current through each resistor.
37. Open file PRO09-37.EWB and measure the current through each resistor.
38. Open file PRO09-38.EWB and measure the voltages with respect to ground at nodes A and B.
39. Open file PRO09-39.EWB. Determine if there is a fault and, if so, specify the fault.
40. Open file PRO09-40.EWB and measure the voltages with respect to ground at output terminals 1 and 2.
41. Open file PRO09-41.EWB and determine what the fault is.
42. Open file PRO09-42.EWB and determine what the fault is.
43. Open file PRO09-43.EWB and determine what the fault is.

■ ANSWERS TO SECTION REVIEWS

Section 9–1

- Kirchhoff's voltage law and Kirchhoff's current law are used in the branch current method.
- False, but write the equations so that they are consistent with your assigned directions.
- A loop is a closed path within a circuit.
- A node is a junction of two or more components.

Section 9–2

- (a) 4 (b) 0.415 (c) -98
- $$\begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix}$$
- $-0.286 \text{ A} = -286 \text{ mA}$

Section 9–3

- $I_1 = -0.038893513289$
 $I_2 = .084110232475$
 $I_3 = .041925798204$
 $I_4 = -.067156192401$
- $I_1 = -.056363148617$
 $I_2 = .07218287729$
 $I_3 = .065684612774$
 $I_4 = -.041112571034$

Section 9–4

- No, loop currents are not necessarily the same as branch currents.
- A negative value means the direction should be reversed.
- Kirchhoff's voltage law is used in mesh analysis.

Section 9–5

- Kirchhoff's current law is the basis for node analysis.
- A reference node is the junction to which all circuit voltages are referenced.

Section 9–6

- $I_T = 1.9 \text{ mA}$; use smallest available fuse with a rating greater than 1.9 mA.
- $10 \text{ Ah}/1.76 \text{ mA} = 5682 \text{ h}$

■ ANSWERS TO RELATED PROBLEMS FOR EXAMPLES

9–1 $I_{R1} = 176 \text{ mA}$; $I_{R2} = 77.8 \text{ mA}$; $I_{R3} = -98.7 \text{ mA}$

9–2 3.71 A

9–3 -298 mA

9–4 -298 mA

9–5 $X_1 = -1.76923076923$; $X_2 = -18.5384615385$; $X_3 = -34.4615384615$

9–6 $I_1 = X_1 = .138971906754$ ($\approx 139 \text{ mA}$); $I_2 = X_2 = -.018679019725$ ($\approx -18.7 \text{ mA}$)

9–7 $I_1 = X_1 = .297872340426$ ($\approx 298 \text{ mA}$); $I_2 = X_2 = .562762622557$ ($\approx 563 \text{ mA}$);
 $I_3 = X_3 = .545521243247$ ($\approx 546 \text{ mA}$)

9–8 1.92 V

9–9 $V_A = X_1 = -.928372575682$ ($\approx -928 \text{ mV}$); $V_B = X_2 = -5.18980533887$ ($\approx -5.19 \text{ V}$)

■ ANSWERS TO SELF-TEST

- | | | | | | | | |
|---------------|----------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1. (e) | 2. (d) | 3. (b) | 4. (c) | 5. (c) | 6. (b) | 7. (d) | 8. (a) |
| 9. (e) | 10. (b) | | | | | | |