

Ts, 2n EE 730

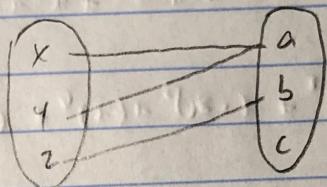
Quiz 2

- 1a) Let  $A = \{x, y, z\}$  and  $B = \{a, b, c\}$ . Give an example for each of the following

a) A relation from  $A$  to  $B$  that is not a function.

$$\{(x, a)\}$$

b) A function from  $A$  to  $B$  which is NOT onto.



c) A function from  $A$  to  $B$  which is one-to-one.

$$f = \{(x, a), (y, a), (z, b)\}$$

d) How many functions are there from  $A$  to  $B$ ?

$$|B|^{|A|} = 3^3 = 27$$

e) How many one-to-one functions are there from  $A$  to  $B$ ?

$$\frac{|B|!}{(|B| - |A|)!} = \frac{3!}{0!} = 3! = 6$$

Find the generating function (in closed form) for the following sequence:

$$0, 0, 2, -2, 2, -2, 2, -2 \dots$$

$$= 0x^0 + 0x^1 + 2x^2 - 2x^3 + 2x^4 - 2x^5 + 2x^6 - 2x^7$$

Factor  $2x^2$

$$2x^2(1 - x + x^2 - x^3 + x^4 - x^5)$$

$$\text{Since we know that } \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 \dots$$

$$\text{Let } x = -2$$

$$\begin{aligned} \frac{1}{1-(-x)} &= 1 - x + (-x)^2 + (-x)^3 + (-x)^4 + (-x)^5 \dots \\ &= 1 - x + x^2 - x^3 + x^4 - x^5 \dots \end{aligned}$$

$$\therefore \text{generating function} = \frac{1}{1+x}$$

Determine the sequence generated by  $f(x) = \frac{5}{1+2x}$ . What is the general term?

$$f(x) = \frac{5}{1+2x}$$

$$\text{Since we know that } \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5$$

$$\text{let } x = -2x$$

$$= 5 \left( \frac{1}{1+2x} \right)$$

$$\frac{1}{1-(-2x)} = 1 - 2x + (-2x)^2 + (-2x)^3 + (-2x)^4 + (-2x)^5 \dots$$

$$= 5 \left( \frac{1}{1-(-2x)} \right)$$

$$\begin{aligned} 5 \left( \frac{1}{1+2x} \right) &= 5 - 10x + 5 \cdot 2^2 x^2 - 5 \cdot 2^3 x^3 + 5 \cdot 2^4 x^4 - 5 \cdot 2^5 x^5 \dots \\ &= 5 - 10x + 2^2 x^2 - 2^3 x^3 + 2^4 x^4 - 2^5 x^5 \dots \end{aligned}$$

$$\therefore \text{Sequence is: } 5, -10, 20, -40, 80, -160 \dots$$

$$\therefore \text{General term is: } (-1)^n \cdot 5 \cdot 2^n$$