Robust Deformation Model Approximation for Robotic Cable Manipulation

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Related Works

- Learning from demonstration
  - Cable manipulation by Coherent Point Drift
  - Cable manipulation by tangent plane mapping
  - ...
- Deformation Model Approximation
  - Gaussian process regression
  - Fourier-LS
  - SPR-RWLS

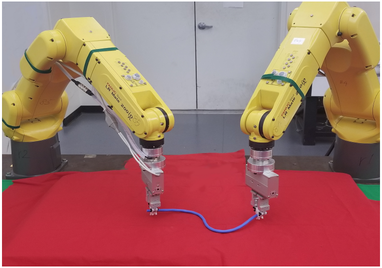


Figure 1: Cable manipulation.

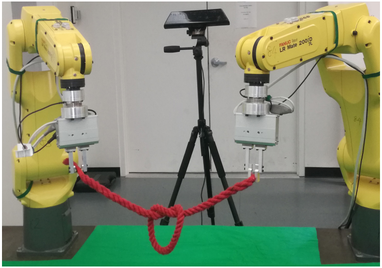


Figure 2: Rope knotting.

Variable Definitions

Variable	Definition
N_t	Tracking points
N	Feature points
D	Degree of freedom
L	Number of robots
K	DOF of robot end-effector
$c = [c_1, c_2, ..., c_N]^T \in \mathbb{R}^{N \times D}$	State of the cable
$r = [r_{11}, r_{12}, ..., r_{1K}, ..., r_{LK}] \in \mathbb{R}^{LK}$	State of robot end-effector

Table 1: Variable definitions.

Usually, feature points are a subset of tracking points. Because of the sparsity of feature points, we can assume that the position of every feature point is uncorrelated with other feature points.

Introduction

- Object tracking
  - Uniformly select tracking points along point cloud
  - Structure Preserved Registration

- Deformation Model Approximation

$$\delta c(t) = \begin{bmatrix} \delta c_{d1} \\ \vdots \\ \delta c_{dD} \end{bmatrix}^T = \frac{\delta c}{\delta r}\delta r(t) = \begin{bmatrix} \frac{\delta c_{d1}}{\delta r} \\ \vdots \\ \frac{\delta c_{dD}}{\delta r} \end{bmatrix}^T \delta r$$

Notice that deformation model is independent for each DOF.
Make use of parallel computation!

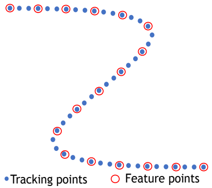


Figure 3: Tracking points and feature points.

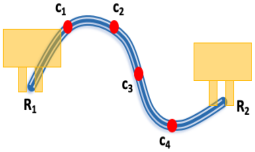


Figure 4: Deformation Model

Local Linear Model

$A(t) \in \mathbb{R}^{N \times LK}$ is a time-varying Jacobian Matrix (local linear model).

$$\delta c(t) = A(t)\delta r(t) \tag{1}$$

We define the pseudo-inverse of Jacobian matrix $G(t) = A^{\dagger}(t)$.

- $A(t) \in \mathbb{R}^{N \times LK}$ & $N >> LK \longrightarrow G(t)$ exists.

To estimate $G(t)$, we denote the current time as t_m. Using a constant sampling period δt, within the time period $(m-1)\delta t$, we collect m consecutive data of δc_i and δr_i while the robot is moving:

$$\delta C(t_m) = \begin{bmatrix} \delta c_1 & \delta c_2 & . & . & . & \delta c_m \end{bmatrix} \in \mathbb{R}^{N \times m}$$
$$\delta R(t_m) = \begin{bmatrix} \delta r_1 & \delta r_2 & . & . & . & \delta r_m \end{bmatrix} \in \mathbb{R}^{LK \times m}$$

Local Linear Model

The local linear model can be found by solving (2), which can be decomposed to a sum of several least square problems. $G_n^T(t)$ represents the *nth* column of the matrix $G^T(t)$, and similarly $\delta R_n^T(t)$ represents the *nth* column of $\delta R_n^T(t)$.

$$\begin{aligned} G(t)^* &= arg \min_{G(t)} \quad \|\delta C^T(t) G^T(t) - \delta R^T(t)\|_F^2 \\ &= \sum_{n=1}^{LK} arg \min_{G_n(t)} \quad \|\delta C^T(t) G_n^T(t) - \delta R_n^T(t)\|_2^2 \end{aligned} \tag{2}$$

Notice that the least square problem can be solved in parallel.

Robust Weighted Least Squares

- Motivation of robust optimization:
  - Noise, outliers, and occlusion
  - Tracking inaccuracy
  - ...

In SPR, we regard tracking points in the last time step as Gaussian centroids and each point in the new point cloud as a sample from Gaussian mixture model. The objective is to maximize the log-likelihood of the point cloud sampled from GMM, so it's natural to regard the variance of each Gaussian as the uncertainty of this movement.

Robust Weighted Least Squares

We take the variance of Gaussian centroids as the tracking uncertainty. In SPR, we assume each Gaussian has equal membership probability $1/N$ and consistent isotropic covariance $\sigma^2 I$. So all the selected tracking points on the object have the same variance σ_t at time step t, and all the feature points are uncorrelated.

$$\delta C^T(t) = \mu(\delta C^T(t)) + \Delta \tag{3}$$

$\mu(\delta C^T(t))$ is the estimation mean, and Δ denotes the uncertainty. From the above analysis, the *jth* column of the matrix Δ can be regarded as a sample from a Gaussian Distribution $N(0, \Sigma)$, where $\Sigma = diag(\sigma_1^2, \sigma_2^2, ..., \sigma_m^2)$.

Robust Weighted Least Squares

We rewrite (2) in the form of robust optimization in (4),

$$\sum_{n=1}^{LK} \min_{G_n(t)} \max_{\|\Delta\|_2 \leq s} \quad \|W[(\mu(\delta C^T(t)) + \Delta) G_n^T(t) - \delta R_n^T(t)]\|_2^2 \tag{4}$$

where s is the upper bound or equivalently the largest singular value of of Δ, and $W = diag(w_1, w_2, ..., w_m)$ is a weight matrix measures the weights for data from different time steps. Since W does not affect the way to solve problem (4), for ease of notation, we ignore W in the following proof.

Matrix Concentration

Theorem

Let $\Delta \in \mathbb{R}^{m \times n}$ be drawn according to the Σ-Gaussian ensemble. Then for all $\delta > 0$, the maximum singular value $\sigma_{max}(\Delta)$ satisfies the upper deviation inequality, $\gamma_{max}(\sqrt{\Sigma})$ denotes the largest eigenvalue of $\sqrt{\Sigma}$.

$$\mathbb{P}[\frac{\sigma_{max}(\Delta)}{\sqrt{n}} \leq \gamma_{max}(\sqrt{\Sigma})(1+\delta) + \sqrt{\frac{trace(\Sigma)}{n}}] \geq 1 - e^{-n\delta^2/2}$$

Theorem 1 provides a theoretically tight bound of random matrix Δ. Using this theorem, we can find an upper bound of the largest singular value of uncertainty matrix Δ given a desired probability.

[1] Wainwright, Martin J. High-dimensional statistics: A non-asymptotic viewpoint. Vol. 48. Cambridge University Press, 2019.

Robust Weighted Least Squares

Theorem

For any robust least square problem in the form:

$$\min_{x \in \mathbb{R}^n} \max_{\|\Delta\|_2 \leq s} \quad \|(A + \Delta)x - b\|_2$$

is equivalent to a SOCP problem:

$$\min_{x \in \mathbb{R}^n} \quad \|Ax - b\|_2 + s\|x\|_2$$

Robust Weighted Least Squares

Proof.

For fixed x, and using the fact that the Euclidean norm is convex, we have that

$$\|(A+\Delta)x - b\|_2 \leq \|Ax - b\|_2 + \|\Delta x\|_2$$

By the definition of the largest singular value norm, and given our bound on the size of the uncertainty, we have

$$\|\Delta x\|_2 \leq \|\Delta\|_2 \|x\|_2 \leq s\|x\|_2$$

Thus, we have a bound on the objective value of the robust least square problem:

$$\max_{\|\Delta\|_2 \leq s} \quad \|(A+\Delta)x - b\|_2 \leq \|Ax - b\|_2 + s\|x\|_2$$

The upper bound is actually attained by

$$\Delta = \frac{s}{\|Ax - b\|_2 \|x\|_2}(Ax - b)x^T$$

□

Robust Weighted Least Squares

Therefore, the robust weighted least square (RWLS) in (4) can be written in the form of a summation over several SOCPs as shown in (5). For each SOCP, we find one column of the model matrix $G^T(t)$, and these columns are independent with each other, which means that we can make use of parallel computation to solve each SOCP and greatly increase the efficiency.

$$\sum_{n=1}^{LK} \min_{G_n(t)} \quad \|\mu(\delta C^T(t)) G_n^T(t) - \delta R_n^T(t)\|_2 + s\|G_n^T(t)\|_2 \qquad (5)$$

RWLS can be solved by parallel computation!

Algorithm Overview

- Object Tracking
  - Initialize: N_t, N
  - Input: point cloud
  - Output: mean, variance
- Model Approximation
  - Initialize: δC, δR
  - Input: $\mu(\delta c(t))$, σ_t
  - Output: $G(t)$
- Trajectory Planning
  - Input: $c_{desired}$
  - Output: $r_{desired}$

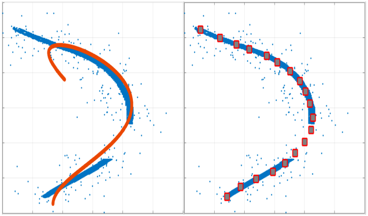
Algorithm 1: SPR-RWLS

Initialize cable, and record desired cable shape;
Using SPR to get initial and desired tracking points along the cable;
Downsample tracking points with a fixed index to get feature points;
Initialize data set D(δR, δC) by randomly executing robot δR and collecting corresponding movement of cable δC for m_0 times;
while $diff(c_{current}, c_{desired}) > \varepsilon$ **do**
  Compute weight matrix W;
  Solve Robust Weighted Optimization for local deformation Jacobian Matrix $G(t)$;
  Compute $\delta r = \lambda G(t) \delta C_{desired}$;
  Execute δr, collect new δc by SPR;
  Append δr and δc to dataset D;
end

Figure 5: Algorithm Overview.

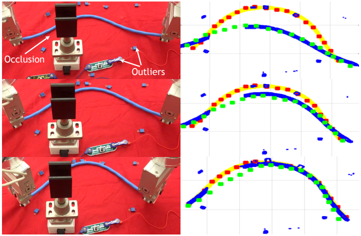
Experiment Results

- Visual Tracking Results
  - Under outliers and occlusion
  - Compare with Fourier based method



(a) Fourier-Based Estimation (b) SPR Estimation

Figure 6: Comparison of Fourier method and SPR on cable tracking.

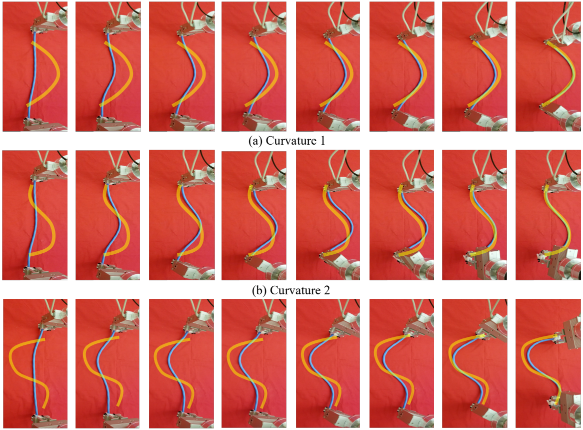


(a) RGB Image (b) Object Tracking

Figure 7: SPR cable tracking with outliers and occlusion.

Experiment Results

- Cable Manipulation
  - Under outliers and occlusion
  - Compare with Fourier based method



(a) Curvature 1

(b) Curvature 2

(c) Curvature 3

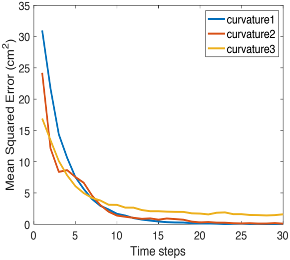
Figure 8: Snapshots of the cable manipulation experiments.

Experiment Results

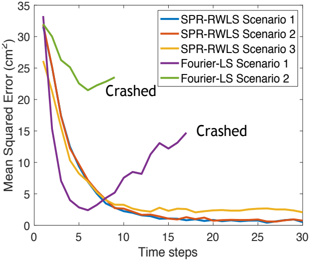
[1]https://changhaowang.github.io/IROS2019/SPRRWLS.html

Experiment Results

- Mean Squared Error for different scenarios
  - Curvature 1, 2, 3 are shown in Fig.8
  - Scenario 1: add 20% occlusion, and 5% noise with $\sigma = 10\%\delta c$.
  - Scenario 2: add 25% occlusion, and 5% noise with $\sigma = 15\%\delta c$.
  - Scenario 3: shown in video.



(a) SPR-RWLS performance



(b) Comparison of Fourier-LS and SPR-RWLS under outliers and occlusions

Figure 9: Mean Squared Error vs Timesteps.

Experiment Results

Table 2: Experiments with different cables

Cable diameters	Success rate	Mean squared error (cm^2)
$4.04mm$	9/10	0.242 ± 0.079
$8.10mm$	10/10	0.051 ± 0.014

Table 3: Comparison of mean squared error of our robust method and Fourier-LS Method

Feature	Fouriers-LS	SPR-RWLS
No outliers and occlusion	$0.045 \pm 0.026cm^2$	$0.051 \pm 0.014cm^2$
Uncertainty scenario 1	$14.712 \pm 1.5832cm^2$	$0.411 \pm 0.093cm^2$
Uncertainty scenario 2	$23.45 \pm 1.259cm^2$	$0.629 \pm 0.1623cm^2$
Uncertainty scenario 3	fail	$2.503 \pm 0.438cm^2$

Questions