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Review

A review on belt and chain continuously variable transmissions (CVT): Dynamics and control

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ABSTRACT

Over the last two decades, significant research effort has been directed towards developing vehicle transmissions that reduce the energy consumption of an automobile. This effort has been a direct consequence of the growing environmental concern imposing the directives of reduced exhaust emissions and increased vehicle efficiency on current vehicle manufacturers and users. A continuously variable transmission (CVT) offers a continuum of gear ratios between desired limits, which consequently enhances the fuel economy and dynamic performance of a vehicle by better matching the engine operating conditions to the variable driving scenarios. Although a CVT plays a crucial role in the plan to improve vehicle fuel economy, its complete potential has not been realized in a mass-production vehicle. The current paper reviews the state-of-the-art research on dynamic modeling and control of friction-limited continuously variable transmissions. The basic concepts, mathematical models, and computational schemes are extensively discussed. Challenges and critical issues for future research on modeling and control of such CVTs are also discussed.

Contents

1. Introduction

With growing socioeconomic and environmental concern, automobile energy consumption has become a key element in the current debate on global warming. Over the past few decades, vehicles have been increasingly facing stringent

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performance, emissions, and fuel economy standards driven by regulatory and market forces (e.g. CAFE standards in the US, ACEA standards in Europe, emissions trading policies, etc.). Emissions of carbon dioxide (CO_2), the principal greenhouse gas produced by transportation sector, have steadily increased along with travel, energy use, and oil imports. Vehicle fuel economy plays a crucial role in determining the emission of greenhouse gases from an automobile. There are three fundamental ways to reduce greenhouse gas emissions from the transportation sector [1]: (a) increase the energy efficiency of transportation vehicles, (b) substitute energy sources that are low in carbon for carbon-intensive sources (i.e. the use of alternative fuel technologies), and (c) reduce transportation activity. With tremendous growth in consumerism and urbanization, there is little scope for emissions reduction to occur through a decrease in the amount of vehicle use. Although alternative energy technologies like fuel cells, electric motors, batteries, solar power, wind power, etc. could significantly lower greenhouse gas emissions, they currently face major challenges in terms of implementation strategies, infrastructure, cost, and drivability [1]. On the other hand, significant research opportunity lies in refining the current engine and transmission technologies to meet the early wave of emissions regulations while the development of alternative fuel technologies continues. Lately, continuously variable transmissions (CVT) have aroused a great deal of interest in the automotive sector due to the potential of lower emissions and better performance. A CVT is an emerging automotive transmission technology that offers a continuum of gear ratios between high and low extremes with fewer moving parts. This consequently enhances the fuel economy and acceleration performance of a vehicle by allowing better matching of the engine operating conditions to the variable driving scenarios. Today, CVTs are aggressively competing with automatic transmissions and several vehicle manufacturers such as Honda, Toyota, Ford, Nissan, etc., are already keen on exploiting the various advantages of a CVT in a production vehicle. A continuously variable transmission is also a promising powertrain technology for future hybrid vehicles. However, in spite of the several advantages proposed by a CVT system, the goals of higher fuel economy and better performance have not been realized significantly in a real production vehicle. In order to achieve lower emissions and better performance, it is necessary to capture and understand the detailed dynamic interactions in a CVT system so that efficient controllers could be designed to overcome the existing losses and enhance the fuel economy of a vehicle. There are many kinds of CVTs, each having their own characteristics, e.g. Spherical CVT [2], Hydrostatic CVT [3,4], E-CVT [5,6], Toroidal CVT [7–9], Power-split CVT [10–12], Belt CVT, Chain CVT, Ball-type toroidal CVT [13], Milner CVT [14], etc. However, belt and chain types are the most commonly used CVTs, among all, in automotive applications. Thus, the current work reviews the state-of-the-art research, in the context of dynamics and controls, of belt and chain CVTs for achieving the targets of increased fuel economy and enhanced vehicle performance. The underlying theories, mathematical models, and challenging future research directions are subsequently discussed. This paper will not only give profound insight into dynamic modeling of such CVT systems, but also evaluate current and future research strategies needed to design efficient CVT controllers, identify possible loss mechanisms, characterize operating regimes, and perform design optimization studies. Since it is difficult to report a comprehensive (i.e. all-inclusive) survey on the literature of this rapidly emerging field, we hope the publications referenced here are at least representative of the literature and would provide a good research platform to people interested in dynamics and control of friction-limited continuously variable transmissions. The authors sincerely apologize for not being able to report results from other papers which could have been included.

The basic configuration of a CVT comprises two variable diameter pulleys kept at a fixed distance apart and connected by a power-transmitting device like belt or chain. One of the sheaves on each pulley is movable. The belt/chain can undergo both radial and tangential motions depending on the torque loading conditions and the axial forces on the pulleys. This con-

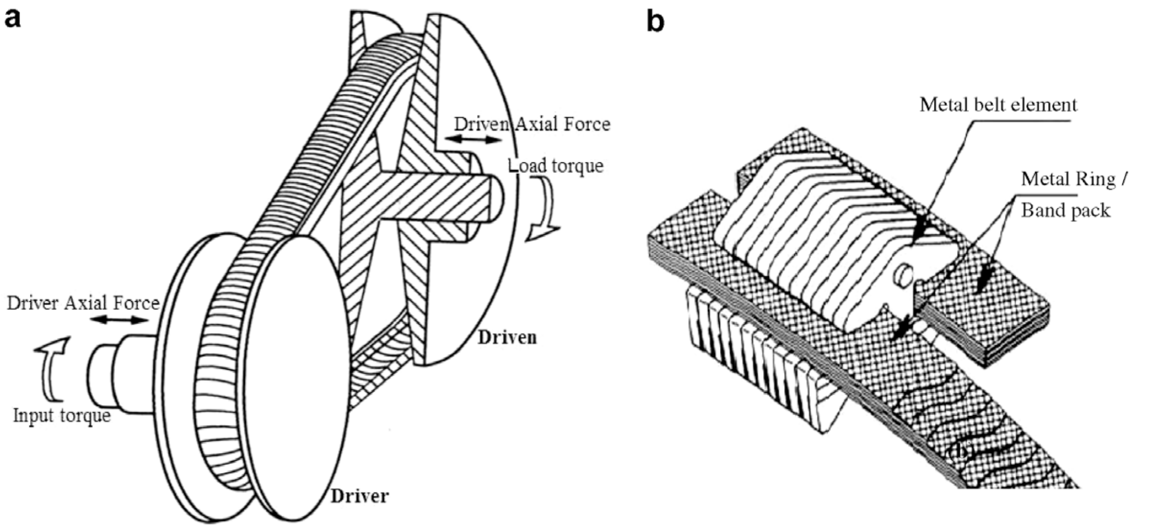

Fig. 1. Metal V-belt CVT drive: (a) basic configuration; (b) belt structure [15].

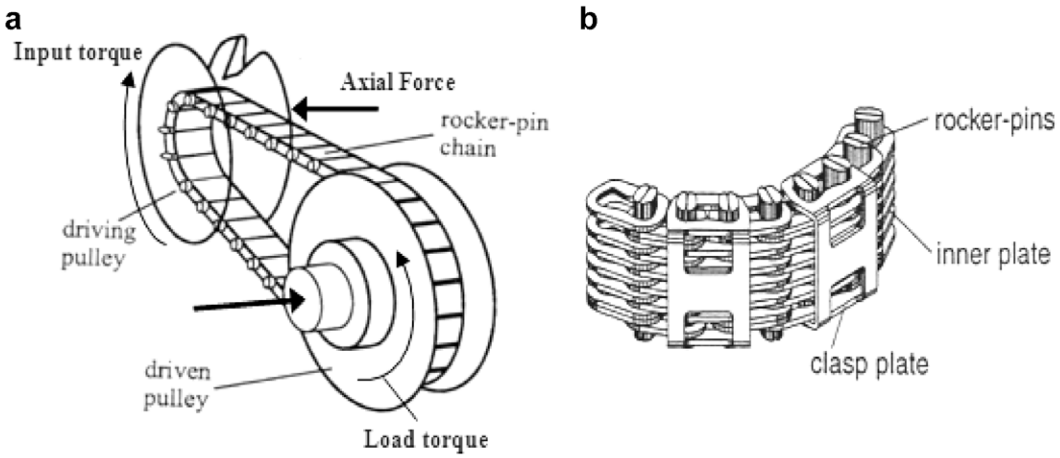

Fig. 2. Chain CVT drive: (a) basic configuration; (b) chain structure [16,72].

sequently causes continuous variations in the transmission ratio. The pulley on the engine side is called the driver pulley and the one on the final drive side is called the driven pulley. Fig. 1 [15] and Fig. 2 [16,72] depict the basic layout of a metal V-belt CVT and a chain CVT. In a metal V-belt CVT, torque is transmitted from the driver to the driven pulley by the pushing action of belt elements. Since there is friction between bands and belt elements, the bands, like flat rubber belts, also participate in torque transmission. Hence, there is a combined push–pull action in the belt that enables torque transmission in a metal V-belt CVT system.

On the other hand, in a chain CVT system, the plates and rocker pins, as depicted in Fig. 2b, transmit tractive power from the driver pulley to the driven pulley. Unlike a belt CVT, the contact forces between the chain and the pulleys are discretely distributed in a chain CVT drive. This leads to impacts as the chain links enter and leave the pulley groove. Hence, excitation mechanisms exist, which are strongly connected to the polygonal action of chain links. This causes vibrations in the entire chain CVT system, which further affects its dynamic performance. Both belt and chain CVT systems fall into the category of friction-limited drives as their dynamic performance and torque capacity rely significantly on the friction characteristic of the contact patch between the belt/chain and the pulley. A sundry of research has been conducted on different aspects of CVTs, e.g. performance, slip behavior, efficiency, configuration design, loss mechanisms, vibrations, operating regime, etc. For better comprehensibility, the literature reviewed has been categorized into different sections.

2. Dynamic modeling of belt CVT

Many papers have been published over the last two decades to describe dynamic interactions in a belt CVT system. The most commonly used power-transmitting device in a belt-type CVT is either a steel V-belt or a rubber V-belt. A lot of relevant work on metal and rubber V-belts is subsequently cited as such CVTs are extensively researched by today's automobile manufacturers and scientists. Most existing models of belt CVTs, with a few exceptions, are steady-state models that are based on the principles of quasi-static equilibrium. Gerbert [17,18] extensively work on understanding the mechanics of traction belts, especially metal pushing V-belts and rubber V-belts. The author used quasi-static equilibrium analysis to develop a set of equations that capture the dynamic interactions between the belt and the pulley. Since the belt is capable of moving both radially and tangentially, variable sliding angle approach was implemented to describe friction between the belt and the

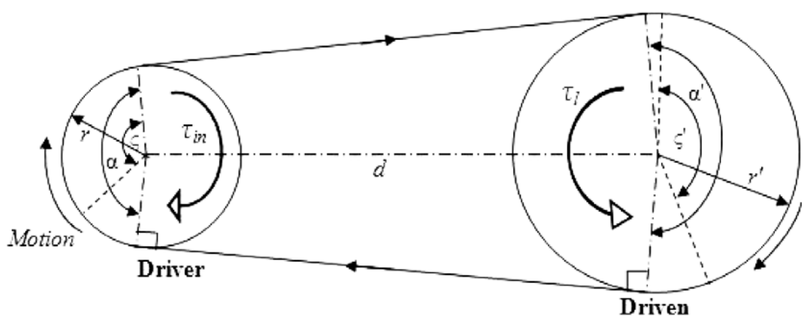

Fig. 3. Geometrical description of a belt CVT drive.

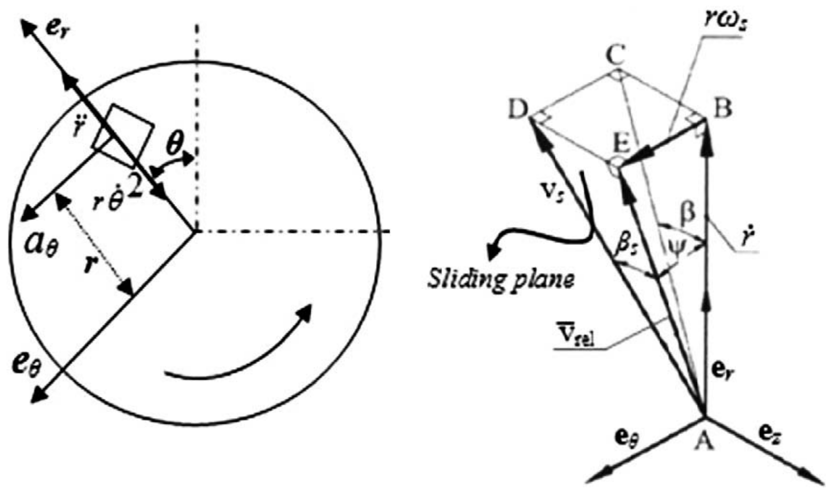

Fig. 4. Kinematic description of a belt element on the pulley wrap.

pulley. Previously, many researchers like Kimmich [19], Gutyar [20], Amijima [21], etc., assumed constant sliding angle over the pulley wrap to derive the equations of motion of belt. The variable sliding angle approach required that the equilibrium, compatibility, and constitutive equations be solved simultaneously for predicting the dynamics of a belt–pulley system. Only centripetal effects were modeled to account for the influence of belt inertia on system dynamics. Figs. 3 and 4 illustrate the geometric configuration, sliding plane, and simplified kinematics of a V-belt CVT drive with negligible belt flexural effects. Only the sliding or active arcs, represented in Fig. 3 as (ς, ς'), contribute actively to torque transmission. (α, α') represent the wrap angles of the belt–pulley contacting arcs, whereas (τ_{in}, τ_l) represent the input and load torques on the driver and driven pulleys, respectively.

Gerbert [22] also analyzed the slip behavior of a rubber belt CVT. Slip was classified on the basis of creep, compliance, shear deflection, and flexural rigidity of the belt. The author also discussed slip during wedging due to poor fit between the belt and the pulley. Finite element analysis was used to calculate shear deflections in the belt and to determine stick–slip conditions for the belt. However, the work did not account for the influence of belt inertia, loading conditions, and belt radial variations (due to pulley axial forces) on the slip behavior. Gerbert [17] also studied the influence of flexural rigidity and inertia on the dynamics of a rubber V-belt pulley system. Again, only centripetal effects were modeled to account for belt inertia. Owing to flexure, the belt no longer follows a straight-line motion after exiting the pulleys. As a result of the flexural effects, the contact arc becomes smaller than the nominal one and the traction capacity of the belt drive increases. The author used ordinary beam theory to predict the entrance and exit slopes of the belt. The flexural rigidity has tremendous influence on the seating and unseating behavior of the belt. Rapid variation of curvature may change the direction of frictional forces, which consequently affects the torque capacity of the belt–CVT drive. Gerbert [23] also studied the influence of pulley skewness and flexure on the mechanics of V-belt drives. Deflection of pulley sheaves in the axial direction widens

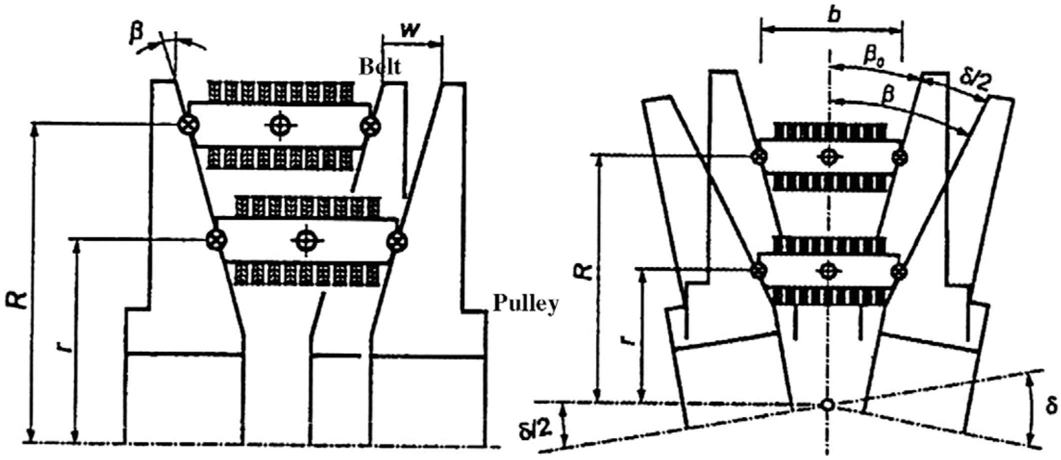

Fig. 5. Pulley deformation model: (a) axial deformation; (b) pulley skewness [44].

or shortens the groove width, thereby influencing the motion of belt in the pulley groove. The displacement of pulley sheaves can be attributed to the phenomena of local deflection, plate deflection, and pulley skewness. Fig. 5 [44] illustrates the variation in pulley groove width due to elastic deformation in the axial direction and due to skewness of the pulley halves. The local deflection depends on the local axial pressure. Plate theory was used to obtain the global deflection of pulley halves. Later, the plate equations and the belt equations were solved simultaneously to obtain the dynamic performance indicators of the belt–pulley system. Skewness of the pulley may be caused by clearances in the guides of the movable pulley half. The author reported the existence of singular solutions (also known as orthogonal points, "*orthogonalpunkts*") where the frictional forces are radially directed and all the derivatives in the differential equations vanish. The character of a solution at an orthogonal point to a great extent determines the character of the solution at other locations and conditions. The work concluded that the axial forces for pulleys with small to medium skewness did not differ much from the ones in ideal case (i.e. with parallel pulley halves). On the other hand, large skewness considerably increased the pulley axial forces for low to medium torque applications.

Worley and Dolan [24] extended Gerbert's work [17] to formulate closed-form solutions using approximate mathematical functions to describe the numerical results of both driver and driven pulleys. Hyperbolic and trigonometric forms were developed to describe the tension distribution over the main active arc of contact (excluding the seating and unseating regions). The authors were able to get results in close agreement to Gerbert's numerical results; however, conformity to the driver pulley system dynamics was not obtained. Sorge [25] studied transient mechanics of rubber V-belt variators in order to understand CVT behavior during speed ratio shift. The results obtained differed significantly from those at steady state. For moderate shift speed and high angular speed, partial adhesive regions appeared where the belt wound along the spirals of Archimedes. Simple closed-form approximations were proposed to facilitate the calculation of variator operative characteristics. Sorge and Gerbert [26] developed a third order rubber V-belt model considering no inertial and flexural effects. They introduced the concept of "adhesive-like" contact in V-belt mechanics. They proposed that V-belts do not stick to pulley walls along the entire contact arc, but pass through an idle-like region where sliding occurs at extremely low relative velocity. The trajectory is nearly circular in this region and the tension is almost constant. Sorge [27] also analyzed the mechanics of a metal V-belt CVT under the influence of pulley bending. The belt was considered rigid for the purpose of model development. The author used the concepts of virtual displacement to obtain approximations to belt trajectories, tension distributions, axial thrusts, and slip. The author reported that the influence of pulley deflection is important when belt trajectory is closer to the outer radius. The influence decreases near the middle radius and becomes negligible near the inner clamped radius. Therefore, the use of a mixed model was suggested at smaller radii in order to aptly capture the dynamics due to elastic deformations of both belt and pulley. Sorge [28] also used the concepts of deformation and elastic wedging to analyze rubber V-belt mechanics. The author's primary objective was to analyze belt trajectory. The study involved the theoretical problem of determining tension distribution and radial penetration of a rubber V-belt. Flexural and inertial effects were neglected. The belt deformation was correlated only to the axial forces, and Poisson effects were neglected. The author incorporated transverse elasticity of the belt and used force-deformation equations to obtain sliding conditions and approximate closed-form solutions to the belt–pulley system.

Micklem et al. [15] incorporated elastohydrodynamic lubrication theory to model friction between a metal V-belt and the pulley sheaves, and studied transmission losses due to wedging action of the belt. The authors also discussed the influence of elemental gaps on the slip behavior of a belt CVT system. However, certain unrealistic assumptions, such as uniform band tension, uniform loading and unloading arcs over the pulley wraps, negligible inertial effects, etc., were incorporated during the course of model development. Speed variation in the belt due to deformation and compressive loads was also neglected. Karam and Play [29] did a discrete analysis of a metal V-belt drive. They used quasi-static equilibrium analysis and numerical techniques to derive global equilibrium equations from elemental part equations. The belt elements were always assumed to be in compression, while the bands were always in tension. The bands and pulleys were also assumed to be rigid. They also observed that only part of the contact arc (i.e. the active or sliding arc) contributed effectively to torque transmission. The steel bands aided power transmission at low transmission ratios, but acted against it at high ratios. They later included an approximate model of pulley deformations in their CVT model and observed an increase in pulley axial forces for the same torque loading conditions.

Asayama et al. [30] developed a theoretical model-based on quasi-static equilibrium analysis to describe variations in band tension and segment compression force and to predict clamping pressures necessary to prevent gross belt slip. They also included relative motion between bands and segments (since they have different radii of rotation) in their model to predict changes in band tension. The bands were treated as rigid. However, microslip behavior of the CVT system was predicted using the elastic deformations of belt segments. Later, the authors also incorporated an approximate pulley deformation model to study the influence of pulley flexibility on the dynamic performance of the CVT system. Although the results from the numerical model were able to conform to the experimental observations for band tensile force, conformity with measured compressive force was not attained. Kim and co-workers [31,32] investigated the metal belt behavior analytically and experimentally. They proposed a speed ratio–torque load–axial force relationship to calculate belt slip. They obtained the equations of motion using quasi-static equilibrium conditions and reported that the gross slip points depend on the torque transmitting capacity of the driven side. Numerical results showed that the belt radial displacement increased in the radial inward direction for the driven pulley, while that of the driver increased slightly and decreased with the increasing torque load. The effects of inertia and flexure were neglected and the band tension was assumed to be constant. Massouros [33] investigated elastic creep velocity of a rubber V-belt analytically and experimentally. The belt creep velocity depends not only on the structural characteristics of the belt, but also on the operational characteristics of the CVT. It also affects the torque transmitting capacity of a belt drive. During power transmission with a belt–pulley system, the velocity of the driving side of the belt is larger than the driven side. The gradual change in velocity from the larger value to the lower value, and inversely, occurs due to the elastic creep of the belt on the pulley wrap arcs. The belt creep occurs in the direction of motion on the driven pulley and opposite to the moving direction on the driving pulley. It was reported that the belt creep velocity at each point of the arc of creep is a linear function of transmitted power and varies exponentially along the arc of creep. The belt dynamics was again modeled using quasi-static equilibrium concepts.

Kobayashi et al. [34] investigated the torque transmitting capacity of a metal pushing V-belt CVT under no driven-load condition. A simulation procedure was outlined to predict the slip ratio and slip-limit torque of a metal V-belt CVT system under steady-state quasi-static equilibrium conditions. Their research focused on the microslip characteristic of a V-belt CVT, which is due to the redistribution of elemental gaps in the belt. The slip hypothesis was based on the assumption that slip only occurred on the pulley where the gaps were present and those gaps were distributed evenly in an idle sector at the entrance of the loading pulley. The authors observed the existence of active and idle arcs during different phases of transmission, as depicted in Fig. 6 [34]. The active arcs are those regions of belt–pulley wrap that contribute effectively to torque transmission, whereas the idle arcs do not participate in torque transmission. The band tension remains nearly constant in these idle arcs. Since the compressive force in belt elements decreases in the idle arc region, there is redistribution of elemental gaps, which consequently causes microslip phenomenon in a metal belt CVT system.

Sun [35] did performance-based analysis of a metal V-belt drive, using quasi-static equilibrium concepts, to obtain a set of equations to describe belt behavior at steady state. Friction between individual bands in a band pack was also taken into consideration during the course of model development. Bonsen et al. [36] analyzed slip and efficiency in a metal pushing V-belt CVT. High clamping force levels reduce the efficiency of a CVT. However, high clamping forces are necessary to avoid excess slip between the belt and the pulley. If a small amount of slip is allowed, the clamping force level can be reduced. The authors investigated the variation of transmitted torque with slip. Radial slip was attributed to CVT shifting and spiral running of the belt. It was proposed that the amount of radial slip depends on the pulley deformation effects, shifting speed, and driver pulley angular speed. Tangential slip was defined based on the redistribution of elemental gaps (as in [34]). It was reported that the traction coefficient, a measure of the torque capacity of a CVT, increases with slip in the microslip regime. However, once the maximum torque capacity of a CVT is attained, the slip rises dramatically and the traction coefficient begins to decrease. The friction between the belt and the pulley was modeled according to Stribeck's friction law. The force distributions were obtained using Asayama's [30] model. The authors also concluded that the traction coefficient is largely dependent on CVT ratio and is not much affected by pulley speed and clamping pressure. However, the efficiency of a CVT depends not only on the clamping pressure but also on the CVT ratio. Amijima et al. [37] analyzed the axial force distribution on a block-type CVT analytically as well as experimentally. The authors proposed a unique relation to relate the angular position of a belt element on the pulley wrap to its sliding angle. The equations of motion were derived using quasi-static equilibrium concepts and a relationship was developed to relate the axial forces to belt tension, belt material properties and groove angle. The experiments indicate that the thrust force profile varies over the pulley wrap; it is the maximum at the exit point and increases as the transmitted load increases. The authors reported that even when only initial tension is applied, the thrust pressure is not constant through the contact arc in spite of there being no power transmission. The thrust force increases as the transmitted load increases in any speed ratio. It was also reported that the thrust force on the smaller pulley is usually higher than that on the larger pulley.

Sferra et al. [38] developed a unique model of a metal V-belt CVT in order to simulate its transient behavior. The model included inertial and pulley deformation effects. Discrete and continuous shifting behaviors were simulated in order to analyze efficiency and power losses due to friction between the belt and the pulley halves. The results showed high loss of effi-

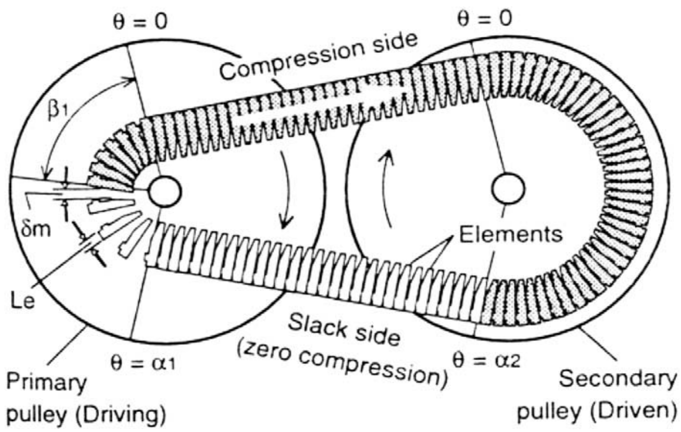

Fig. 6. Slip mechanism due to elemental gap redistribution in a metal belt CVT [34].

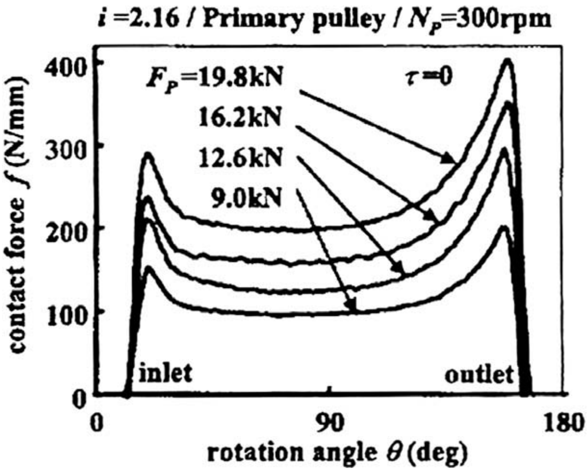

Fig. 7. Belt–pulley contact force distribution at no load for different clamping (axial) forces [39].

ciency during shifting transients. Power losses due to other parasitic effects were not included in the model. Ide and Tanaka [39] experimentally measured the contact forces between a metal pushing V-belt and the pulley sheaves using ultrasonic waves. The relative change in the shape of belt wrapping arc due to variations in the clamping force and transmitted torque was detected. The results showed that the shape of belt wrapping arc was not co-axial with the pulley axis, and this was attributed to asymmetrical pulley deformation. In driven condition and driving with a small torque, the contact force distribution exhibited peaks at the inlet and outlet of the pulleys, as depicted by a representative example plot in Fig. 7 [39]. The peak on the outlet was higher, especially on the pulley with larger pitch radius. It was proposed that asymmetrical pulley deformation and self-locking of the belt on the pulley were responsible for the high peak at the outlet. The authors also observed that the high peak of the contact force at the outlet decreases as the driving torque increases. This was attributed to the increase in belt compressive force and the elimination of self-locking phenomenon of the belt.

Ide et al. [40] also used simplified dynamic models of a metal V-belt CVT to analyze the response of a CVT-equipped vehicle to a rapid speed ratio change. They also experimentally investigated [41] the shift speed characteristics of a metal belt CVT. They observed that there were two different phases during ratio change. One was the creep mode when the belt clamping force was not small, and the other was slip mode when the belt clamping force was small. They also noted that the shift speed in creep mode increased in proportion to the pulley rotating speed, whereas it was independent of the pulley speed under slip mode. Also, a large radial slip occurred over the entire contacting arc of the belt on the pulley under slip mode conditions. Bullinger and Pfeiffer [42,43] developed a detailed elastic model of metal V-belt CVT system to determine its power transmission characteristics at steady state. Pulley, shaft, and belt deformations were taken into account. The frictional constraints were modeled using the theory of unilateral constraints. The belt dynamics was specified by separate longitudinal and transverse approaches. The transversal dynamics was modeled using the Ritz-approach based on B-splines. The longitudinal dynamics was described by using the Lagrange coordinate formulation. Sattler [44] analyzed the mechanics of a metal chain and V-belt considering longitudinal and transverse stiffness of the chain/belt, and pulley misalignment and deformations. The pulley deformation was modeled using a standard finite element analysis. The pulley was assumed to deform in two ways, pure axial deformation and a skew deformation. The model was primarily used to study efficiency aspects of belt and chain CVTs. Ye and Yi [45] also developed a multibody dynamic model of a metal V-belt CVT to find the metal block trajectories and to calculate the forces acting on the block and the ring/band by changing the speed and torque ratios. The ring/band was modeled using spring-damper elements. However, in their study, the simulation was conducted under steady-state conditions and the effects due to transient conditions were not considered.

Carbone et al. [46] developed a theoretical model of a metal pushing V-belt to understand the transient dynamics of a belt CVT drive during rapid speed ratio variations. The belt was modeled as a one-dimensional continuous body with zero radial thickness and infinite axial stiffness. Non-dimensional equations were defined to encompass different loading scenarios; however, the inertial coupling between the belt and the pulley was not modeled in detail. Carbone et al. [47] also studied the influence of clearance between belt elements on the rapid shifting dynamics of a metal pushing V-belt CVT. The effects of band tension and belt inertia were neglected in the analysis. The clearance between belt elements was modeled as a kinematic strain. The authors reported that power transmission is assured only if an active arc exists where the elements are pressed against each other and where compressive forces among the steel elements grow. The idle arc, where the steel elements are separated and no compression exists, does not contribute to torque transmission. On the driver pulley, a "shock" section was observed which separated the idle arc from the active arc. On the driven pulley, there was no shock section, but an "expansion wave" kind of phenomenon occurred as the clearances among belt elements grew on the idle arc. Carbone et al. [48] used two friction models, namely a Coulomb friction model and a visco-plastic friction model, to model friction between the belt and the pulley for accurately predicting CVT shifting dynamics during slow and fast maneuvers. The authors reported that the Coulomb friction model is unable to correctly predict the shift dynamics of a CVT during slow maneuvers (i.e. creep-mode phases), but it could well predict the limiting traction capacity and dynamic behavior of a CVT in slip-mode (rapid shifting) phases. However, a visco-plastic model is not only able to accurately predict CVT behavior in creep-mode phases, but also able to detect the transition from creep mode to slip mode. The authors also proposed simple relations to correlate shift speed to clamping force ratio, driver pulley speed, and torque load. Later, Carbone et al. [49,50] extended their previous work [46] to investigate the influence of pulley deformation on the shifting mechanism of a metal V-belt CVT. Coulomb friction hypothesis was used to model friction between different surfaces. Flexural effects of the belt were neglected; however, pulley bending was considered based on Sattler's model [44]. The authors assumed equal pulley deformations and also that the belt–pulley wrap angles did not deviate considerably from 180°. The only inertial effects taken into account were due to the centripetal acceleration of the belt. Moreover, the free span dynamics of the belt were not modeled. The authors predicted that the Coulomb friction model was able to accurately describe shifting behavior of a CVT in creep and slip modes if pulley bending effects were taken into account. The authors suggested that in steady state, the pressure and tension distributions were unaffected by pulley bending and depended only on the thrust ratio. However, pulley bending played a significant role in determining the transient response of the variator. It was shown that in creep mode, the rate of change of speed ratio continuously increases as the stiffness of pulley decreases. The model was also able to predict the influence of pulley angular speeds on the rate of change of speed ratio. The authors reported that during creep-mode phases, the shift speed is almost linearly related to the logarithm of pulley clamping force ratio.

Experiments done by a number of researchers, especially those done by Fujii and Kanehara, have shown that both tensile force in the band pack and compressive force in the belt elements aid in torque transmission. Their work suggests that these forces vary non-uniformly over the pulley wrap, which invalidates the assumption of constant band tension made by other researchers. Fujii et al. [51] experimentally investigated the tensile force in bands and the compressive force in belt elements. The authors observed the existence of active and idle arcs on the driving pulley and suggested that under some operating conditions, it is plausible for the bands to contribute up to 40–45% of the total transmitted torque. The authors also reported that the band tension distributions exist due to relative slip velocities between the bands and the segments. The band pack tensile force was observed to decrease around the smaller of the two pulleys in the direction of belt travel. Consequently, the band tension impeded torque transfer in high ratio and facilitated the transfer of torque in low ratio. This could happen to such an extent that in low ratio, for a range of lower torque levels, the entire torque load was observed to be transferred by the band tension alone. Fujii and co-workers [52] used a number of strain gauged belt elements to measure the forces acting in various directions on the belt element. The experiments were conducted on a constant transmission ratio CVT at low belt speeds and clamping pressures. Time histories of pulley normal force, belt compressive force, transmitting force (i.e. the tangential friction force between a belt element and the pulley), radial friction force, element shoulder force, etc. were recorded during the experiment. Since it was difficult to measure compression in the free span of the belt CVT system, a straight-line fit between the entry and exit conditions on each pulley was assumed. The pulley normal force was reported to be unevenly distributed on the contact arcs. The element normal force exhibited peaks at the entry and exit of the pulleys (similar to the observations of Ide and Tanaka [39]). Also, it was observed that the normal force per element was lower on the pulley with the larger wrap angle. The authors also reported the existence of idle and active arcs at higher transmission ratios. The radial friction force restricted the penetration of a belt element into the pulley, except at the pulley exits where it acted to retain the belt element in the pulley wedge. Fushimi et al. [53] developed a numerical model to calculate the steady-state force distributions and compared them to the previously published experimental results. The metal V-belt is modeled using a lumped-parameter approach with three kinds of springs and two kinds of interfacial elements for the block and the ring,

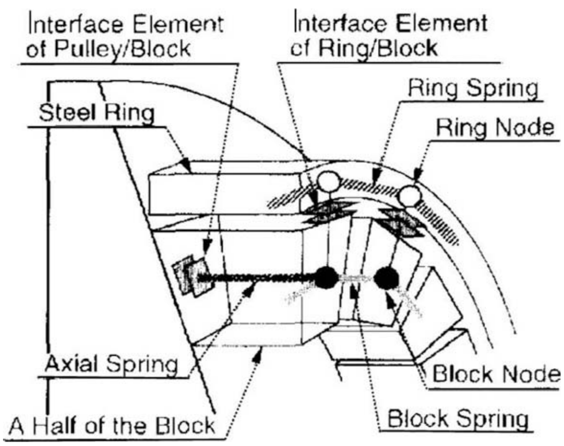

Fig. 8. Lumped-parameter discrete model of a metal V-belt CVT [53].

as shown in Fig. 8 [53]. The interfacial elements were used to capture friction effects between the block and the pulley, and between the ring and the block. Fujii and co-workers [54] later extended their previous experimental setup to record the forces in a metal belt CVT system under the conditions of varying transmission ratio. Time histories of various forces acting in a belt–pulley system were recorded. However, none of the above-mentioned papers of Fujii et al. included any kind of theoretical modeling of the metal V-belt CVT system. Moreover, since the experiments were conducted at low speeds and low pressures, inertial and deformation effects could not be captured in detail.

Later, Fujii and Kanehara co-authored a paper with Kuwabara [55] where they proposed an advanced numerical model to analyze power-transmitting mechanism in a metal pushing V-belt CVT. The forces acting in the system were estimated not only at steady states but also during transitional states where the speed ratio was changing. The band tension is not assumed to be constant. The model takes into account the various dynamic interactions occurring among bands, elements and pulleys. It was observed that bands transmitted negative power under over-drive conditions (i.e. when the pitch radius on driven pulley is smaller than the driver pulley radius). So, in order to meet load, the blocks transmitted more power than the nominal. Much greater thrust was needed to shift the speed ratio during transitional state than in steady state. The authors also observed that the thrust ratio (ratio of the driver axial force to the driven axial force) slightly increased with increasing the coefficient of friction between the belt and the pulley. Coulomb friction was used to model friction between all the surfaces. The influence of flexure and belt inertia was neglected during the course of model development. Fig. 9 [55] depicts some steady-state results from Fujii and co-workers [55] that highlight the combined role of band pack and element forces in successful torque transmission. Here, the authors (i.e. Fujii et al.) defined speed ratio as the ratio of belt pitch radius on the driven pulley to that on the driver pulley.

Fujii and Kanehara also co-authored a paper with Fujimura [56,57] where they used mean coefficients of friction and shifting gradients to reveal the shifting mechanism of a metal V-belt CVT. Shifting gradient is a non-dimensional parameter which is defined as the radial increment of a block sitting on a pulley in radial direction per unit block path. The experimental results showed that thrust force and sliding speed influenced the mean coefficients of friction. The authors also observed that the shifting gradient was influenced neither by pulley speed nor by torque ratio. Torque ratio is defined as the ratio of transmitted torque to the maximum transmittable torque. An increase in shifting gradient was observed with increasing the thrust of a pulley in which the belt pitch radius was increasing, whereas it decreased with increasing the thrust of a pulley in which the belt pitch radius was decreasing. However, the authors did not include the influence of belt tensile and compressive forces on the shifting gradient of the variator. Fujii and Kanehara later co-authored a paper with Kataoka [58] where they analyzed shift mechanisms of the variator and characterized friction between the blocks and the pulleys. The authors concluded that the shifting gradient is governed not only by block elastic deformations (due to the pulley thrusts), but also by ring tension and block compression. The authors developed quasi-static equilibrium equations to estimate target pulley thrusts at steady and transitional states. Both driving and driven pulley thrusts were calculated by considering the forces on the blocks at pulley entrance. Experimental conformity was also reported for not only steady state, but also transitional state conditions. The frictional performance of CVT fluids and the frictional characteristics of block–pulley contact interfaces were evaluated by applying the mean coefficient of friction as a friction parameter. It was found from experiments that the estimated coefficient of friction of CVT fluids was not constant with respect to the operating conditions. It varied with the relative sliding speed of the blocks with respect to the pulleys, sliding direction, and normal pressure acting on the V-surface of the block.

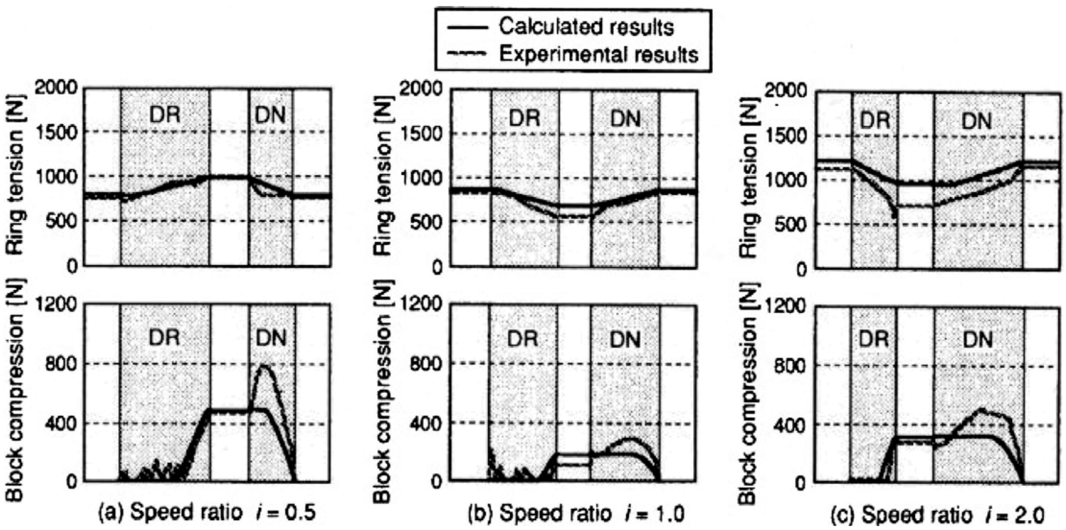

Fig. 9. Time histories of belt tensile and compressive forces at steady state for different speed ratios [55].

Srivastava and Haque [59–61] developed a detailed transient dynamic model of a metal pushing V-belt block-type CVT. The goal was to understand the transient dynamic behavior of a belt element as it traveled from the inlet to the exit of driver and driven pulleys. The inertial coupling due to radial and tangential motions of the belt was modeled in detail. The interaction between the band pack and the belt element (which has been neglected in a lot of previous models) was also taken into account during the course of model development. Flexural effects due to belt motion were neglected; however, pulley flexibility was taken into account using Sferra's correlations [38]. Also, the contact between the belt and the pulleys was modeled not only by classical continuous Coulomb friction law, but also by certain mathematical/analytical models of friction that capture different loading/operating conditions (e.g. stiction, lubrication, etc.). It is evident from their work that not only the configuration and loading conditions, but also the inertial forces, influence the dynamic performance of a CVT, especially the slip behavior and torque capacity. The authors also reported that the inactive arc region of a band pack is different from that of a belt element. This consequently led to the formation of shock sections at the element–band interface separating the inactive region of pulley wrap from the active region. However, the results reported were valid for one cycle i.e. till the belt moved past the exit of either of the two pulleys. Fig. 10 [61] depicts the free-body diagrams of the metal V-belt CVT drive reported by the authors to capture the various transient dynamic performance indices of the CVT system. Srivastava et al. [59,62] also highlighted the significance of a feasible set of initial operating conditions required to initiate torque transmission in a CVT system. The authors proposed that a CVT, being a highly nonlinear system, needs a specific set of operating conditions, which can be found using an efficient search mechanism, in order to successfully meet the load requirements. The authors used genetic algorithms (GA) to capture this feasible set and also highlighted its efficiency in capturing this set by comparing it to the results generated from the design of experiments (DOE). The optimization objective function was aptly chosen to maximize the torque transmission capacity of the CVT system.

Srivastava and Haque [63] developed a metal pushing V-belt CVT model at steady state to study its microslip behavior and to define its operating regime. They discussed the influence of torques and axial forces on belt slip. Slip was based on the redistribution of elemental gaps and formation of inactive arcs (as proposed by Kobayashi et al. [34]). The model is able

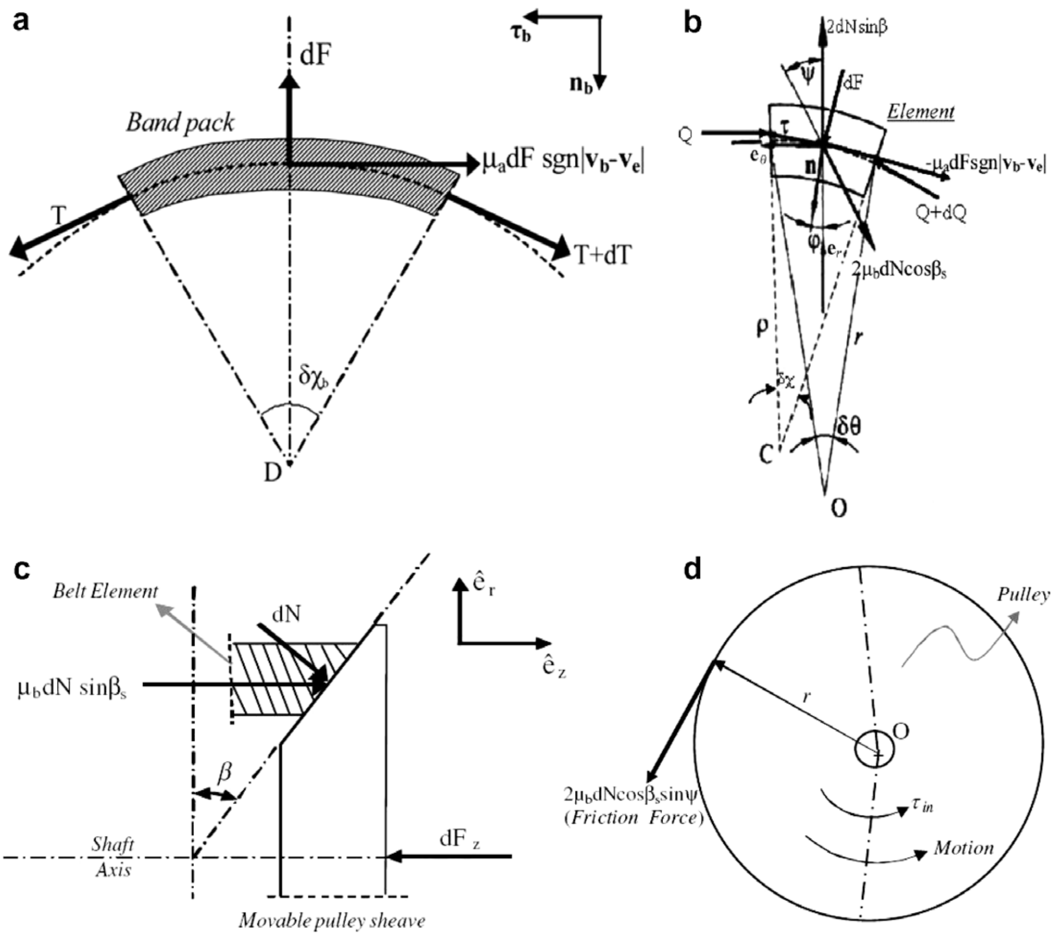

Fig. 10. Free-body diagrams of a metal V-belt CVT system: (a) band pack forces; (b) belt element forces; (c) forces on the pulley sheave; (d) torques on the driver pulley [61].

to predict the maximum transmittable torque before the belt undergoes gross slip. The authors again observed that the CVT operated in a definite regime of axial forces and torques for successfully meeting the load requirements on driven pulley. They predicted the minimum axial force necessary to initiate torque transmission and the maximum axial force that the CVT could sustain (based on slip behavior and not on stress, wear, or fatigue effects). The operating regimes for axial forces and torques were obtained by running numerous simulations with varying transmission ratios. Fig. 11 [61] illustrates representative time histories from the authors' belt CVT model for an operating condition of 200 N m input torque and 100 N m load torque at a constant driver pulley speed of 2000 rpm under stiction and lubricated contact conditions. The results emphasize the influence of inertial effects on the dynamic performance and torque capacity of the CVT system (refer to [61] for details).

Akehurst [64] theoretically and experimentally investigated the loss mechanisms associated with an automotive metal V-belt CVT. The author developed mathematical models to describe torque loss mechanisms in a belt CVT drive. Radial friction effects were neglected in the analysis except in the entry and exit regions of the pulleys. The author also assumed that all the bands shared the tension load evenly. The main torque loss event is proposed to occur due to bands sliding relative to each other and the segment, and the segments sliding relative to the pulley. Two further loss models were developed to describe losses as the segments traversed the pulley wrap arcs. These losses were due to pulley deflections causing the segments to contact the pulley past ideal exit and entry points (wedge loss) and to penetrate further into the pulley than the ideal belt pitch radius (penetration loss). The wedge loss proposed was similar to the previously developed empirical model of Micklem et al. [65] and was based on the radial deflection of belt elements at the entry and exit regions of the pulleys. Tangential belt slip was again investigated on the basis of redistribution of gaps between the belt elements (as in [34]). In addition to the torque loss associated with the belt, the author studied other parasitic losses in the transmission system, like seal and bearing drag losses, pump losses, clutch drag losses, meshing losses, churning and windage losses. In steady state, it was observed that the losses in high ratio at low speeds were lower than low ratio losses, due in part to the reduced pump losses. However, as speeds increased, the high ratio losses exceeded those in low ratio. This was attributed to increased belt torque losses and

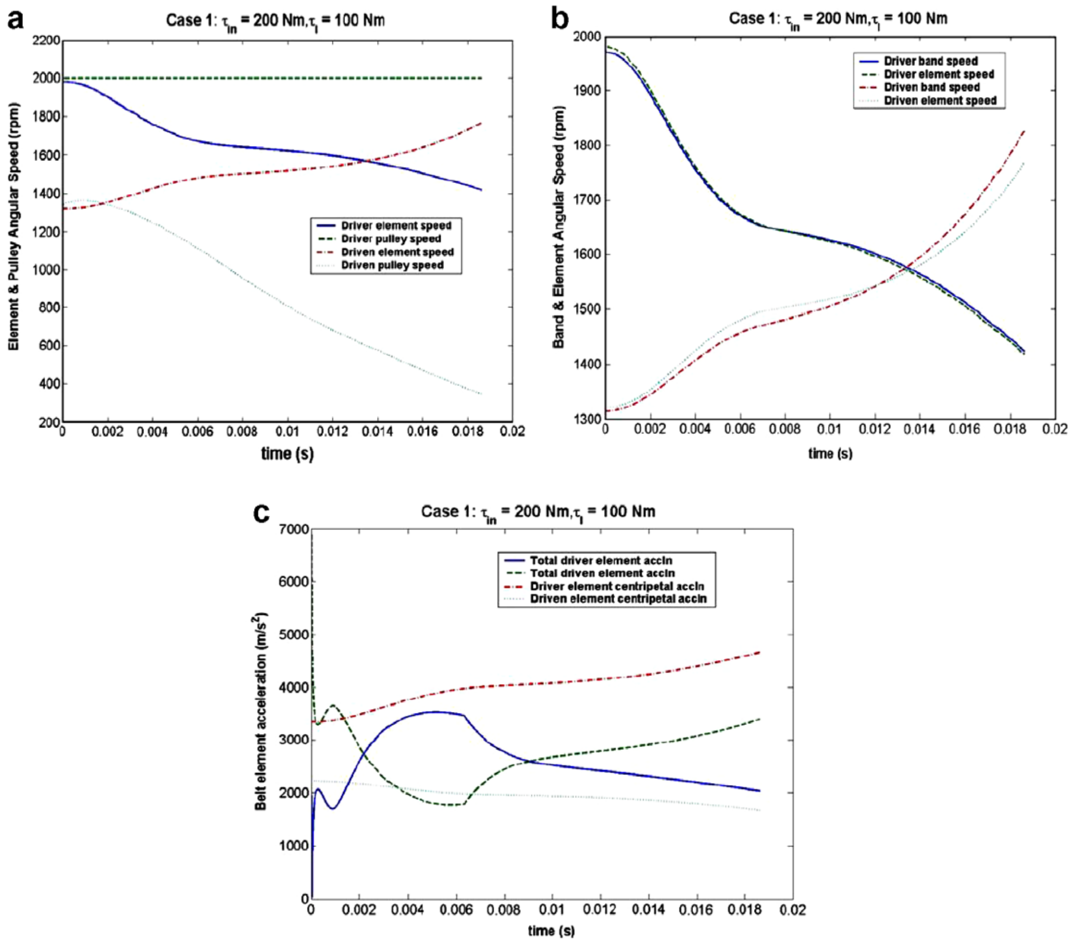

Fig. 11. Time histories from belt CVT model: (a) element and pulley speeds; (b) element and band speeds; (c) belt element acceleration [61].

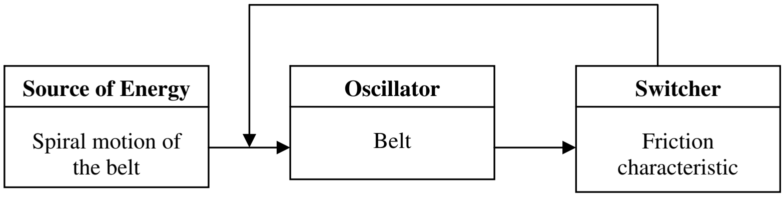

Fig. 12. Proposed self-excitation mechanism in a metal belt CVT [70].

final drive losses. Also, it was noted that in both ratios the torque loss increased with output torque loading. However, in low ratio conditions, the torque loss under no load conditions was higher than when the transmission was lightly loaded.

Kluger and Long [66] presented an overview of current manual, automatic, and continuously variable transmission efficiencies. The authors qualitatively discussed loss mechanisms in these transmissions and suggested design improvements to enhance transmission efficiency. As the belt-type CVTs require pressures of up to 3 MPa, the pumping power requirements represent a very large portion of the overall CVT power loss. The authors suggested the use of a radial ball pump or a radial piston pump to reduce the pumping losses during sheave actuation. The authors reported that the manual transmission is the most efficient transmission among all three transmission technologies. However, since their technology is quite mature, there is very little room for improvement in terms of efficiency and fuel economy. Kluger and Fussner [67] also discussed the mechanisms, forces, and efficiencies of different types of CVT, especially push belt CVT, elastomer belt-type CVT, toroidal CVT, nutating type CVT, and epicyclic CVT. The amount of power which can be transmitted by a push belt-type CVT is dependent on either the tensile strength of the bands or the transverse buckling strength of the belt elements. The dynamics within the steel bands play a crucial role in determining the maximum torque that can be transmitted. The authors suggested that the torque transmission in a metal belt CVT system is due to both pushing forces in the belt elements and the friction forces between the elements and the bands. Singh and Nair [68] developed mathematical models of different CVT designs by using existing literature and later normalized the information to allow comparison of different CVT concepts (i.e. toroidal, chain, belt, hydrostatic types) on an equitable basis. The models were used to compute efficiencies of individual CVTs at selected points of the entire operating envelope. The authors reported that the rubber belt CVT, in general, is the most efficient among all. The PIV chain CVT is inefficient for low torques and the traction CVT is less efficient than the chain and belt CVTs for under-drive ratios.

Chung and Sung [69] analytically and experimentally studied vibration of a rubber V-belt CVT during speed ratio change. On the basis of motion characteristics of the CVT, the major components were modeled as an instantaneous flexible four-bar linkage for studying the belt vibration behavior. The free span of the axially moving belt was modeled as a flexible coupler link, while the variable diameter driving and driven sheaves were modeled as the crank and rocker links, respectively. The authors used the mixed-variational principle to derive the equations of motion of the vibratory CVT system. The boundary and initial conditions were determined from the physical configuration of a practical CVT system. The free span length of the belt was obtained from contact analysis. A parametric study on natural frequencies of the belt was performed using assumed-modes method. The authors observed that the natural frequencies of both tight- and slack-sides of the belt decreased as the CVT accelerated. Moreover, in steady state, higher belt tensions led to higher natural frequencies. Also, the natural frequency of the belt was observed to decrease with increasing transport velocity. Pfeiffer and co-workers [70] analyzed self-induced vibrations in a pushing V-belt CVT using highly simplified dynamic models. The free span length of the belt was assumed to be constant in the analysis. It was shown that certain friction characteristics, especially those having negative gradient with respect to relative velocity, could induce self-excited vibrations in the belt. The friction characteristic and the elasticity of the pulley sheaves determined the working area where vibrations occurred. The authors also observed that increasing pulley stiffness decreased belt vibration, but failed to eliminate them completely. Fig. 12 [70] depicts the excitation mechanism proposed by the authors.

3. Dynamic modeling of chain CVT

In addition to belt CVTs, considerable research effort is also being directed towards understanding the dynamics and power transmission characteristics of chain CVTs. A chain drive CVT consists of two variable diameter pulleys connected to each other by means of a rocker-pin chain. Fawcett [71] extensively reviewed the existing literature on belt and chain drives. The publications from different sources were grouped into three sections: dynamics of axially moving materials, chain drive dynamics, and belt-drive dynamics. Most of the literature discussed by the author was related to the dynamics of roller chain drives, rubber V- and flat belts, and toothed belts. Unlike a roller chain drive, a CVT chain drive transmits power exclusively through frictional forces in the contact zones between the bolts of the chain and the cone sheaves of the pulleys. Every contact has two possible states, stick or slip, depending on the relatively velocity of contacting surfaces. The possible transitions between these states make CVT a non-smooth time-varying mechanical system. Sauer [72] developed a finite element method (FEM) based static model of a CVT chain drive with elastic pulley-sets and a chain described link by link. The model gives a good theoretical description of the torque transfer behavior of CVT chain drives.

Srnik and Pfeiffer [72–74] studied dynamic behavior of CVT chain drives for high torque applications. They developed a planar model of chain CVT with three-dimensional contact between a chain link and the pulley. Their work dealt with multi-body formalisms and finite element modeling. In order to account for the polygonal excitations due to the discrete nature of chain, the chain was modeled link by link. The chain links were thus modeled as kinematically decoupled rigid bodies inter-connected by spring-damper force elements, as proposed by Fritz and Pfeiffer [75]. The pulley was modeled as an elastic multibody system. Neglecting its own internal dynamics, the pair of rocker pins was modeled as one stiff massless spring acting along the direction of shaft axis. The model was simulated under the conditions of a constant driver pulley speed and a constant driven pulley load torque. Fig. 13 [74] depicts representative time histories from their chain CVT model. The authors used classical Coulomb friction law as well as its time efficient continuous approximation to calculate the contact forces between the links and the pulleys. The computational cost to determine the exact contact transitions between the

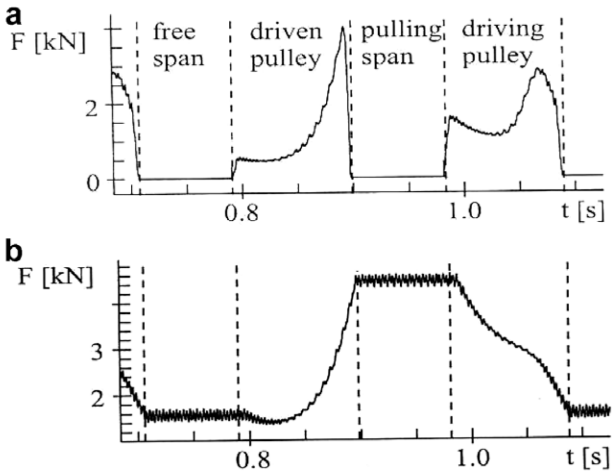

Fig. 13. Time histories from chain CVT model: (a) pulley normal force: (b) chain tensile force [74].

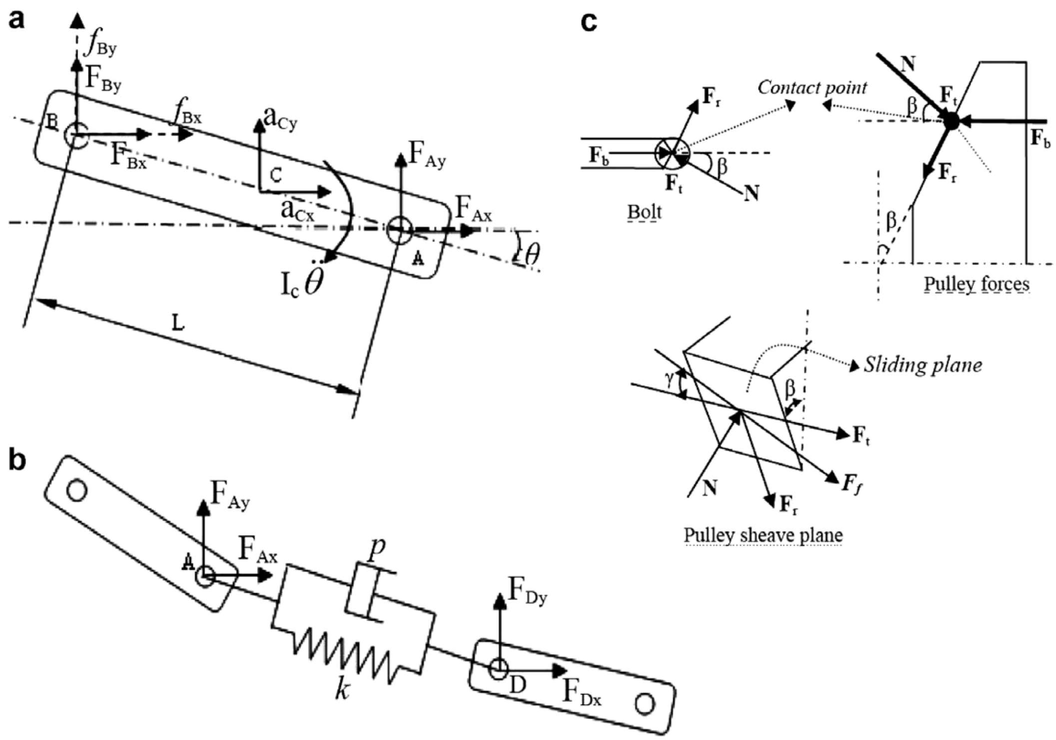

Fig. 14. Free-body diagrams for chain CVT model: (a) forces on a chain link; (b) chain link interaction; (c) link-pulley contact description [16].

links and the pulley was reported to be higher for the case of a discontinuous Coulomb friction model. The authors used the Theory of Unilateral Contacts (developed by Pfeiffer and Glocker [76]) to formulate the system equations as a linear complementarity problem (LCP) and solved it to obtain the time histories of the chain CVT model. The simulation results showed the influence of pulley deformation on the power transmission characteristics of a chain CVT. The authors reported that pulley deformation has a decisive influence on the relative contact dynamics between a chain link and the pulley sheave. It was observed that the efficiency of a chain CVT drive with flexible sheaves was lower than with rigid sheaves. This was attributed to substantial radial movements of the chain links in the pulley grooves for the case with higher pulley flexibility. The authors also discussed the influence of chain's pitch on the vibratory behavior of transmission. It was proposed that an unintegrated chain pitch reduced vibration amplitudes but also included a wider excitation band. Fig. 14[16] depicts representative free-body diagrams of a planar multibody chain CVT model.

Sedlmayr and Pfeiffer [77–80] studied spatial dynamics of CVT chain drives. The authors modeled the links and pulleys as elastic bodies and also included pulley misalignment effects. Since even a small pulley misalignment could yield significant tensile forces in the chain, the authors proposed a spatial chain CVT model. The pulley deformation was modeled using a static finite element approximation and the reciprocal theorem of elasticity. Both pulleys had one rotational, two translational (in-plane), and one axial (out-of-plane) degrees of freedom. The pulleys also had other degrees of freedom related to elastic deformation. The pulley misalignment was dependent on the CVT speed ratio and the length of the chain. Each chain link was modeled as an elastic body with three translational rigid body degrees of freedom. The links also had a few more degrees of freedom to describe the orientation and elastic deformation of pins. The chain links were kinematically connected between pairs of rocker pins. The elasticity and the translational damping of the joint were taken into account by the link force element, whereas the rotational damping and the axial friction between the pair of pins were embedded in the model of the joint force element. In the link force element, each plate was modeled as a spring element. The effect of moving contacts relative to the plate spring reference points between a rocker pin and an adjoining plate was modeled as an exponentially varying contact torque. Edge bearing effects between the pin ends and the sheaves were observed due to the influence of shear, torsional, and bending deformations of rocker pins. The authors reported that owing to bending effects, the pulley misalignment induced large gradients in the chain tensile forces. On entering the pulley, the shape of a pin changed abruptly because of the sudden growth of contact forces. The authors also suggested that a CVT chain without clasp plates is more flexible in the shear direction and thus have lower tensile forces. It was proposed that the tensile forces in chain clasp plates could be reduced by using longer rocker pins and more plates. However, this was not a good solution because with the same cross section of pins, the load splitting on plates became worse under pin bending conditions. However, the tensile forces in such plates could be effectively reduced by modifying the design of a plate. Tensile forces were much lower for softer and lighter plates. The authors also did a comparative study on the efficiencies of metal V-belts and chains, and concluded that CVT chain drives have higher efficiencies especially with curved pulleys.

A comparative study on the dynamics and performance of a metal V-belt CVT and a chain CVT was done by Lebrecht and co-workers [81]. The authors observed that for the same torque-run-up conditions, the clamping force ratio of a belt CVT system was higher than that for a chain CVT drive. Moreover, since belts tend to slip more than chains, the efficiency of a belt CVT system was observed to be lower than that of a chain CVT especially at high transmission ratios. It was also reported that the belt CVT drive exhibits lower contact forces at the pulleys and smaller polygonal vibrations in comparison to a chain CVT drive. Pausch and Pfeiffer [82] analyzed the nonlinear dynamics of a chain drive CVT in an automotive drive train system. Maier and Pfeiffer [83] developed a method to assess noise emission characteristics of the driveline components of a hybrid vehicle. Multibody models of gearbox, chain CVT, and other driveline components were used. The authors used complex nonlinear stiffness functions in the analysis to capture excitation mechanisms related to both impact and friction forces in a chain CVT and tooth forces in a gearbox. A method to calculate the A-rated equivalent continuous force level of the bearing forces, which is proportional to the sound, was also discussed. Stepanenko and Sankar [84] investigated the limit cycle behavior induced by clearances in kinematic chains. They incorporated a nonlinear dynamic model of clearance and analyzed its influence on the dynamics of power kinematic chains. Tenberge [85] developed a fast computational algorithm to compute dynamic indicators of a chain CVT from a mathematical model which included deformations, loadings, and other inertial effects at constant and variable speed ratios.

Although the friction characteristic of contacting surfaces inevitably plays a crucial role in CVT's performance, literature pertaining to the influence of friction on CVT dynamics is scarce [48,57,58,60,61,70,86–89]. Almost all the models, except a few, mentioned in literature use Coulomb friction theory to model friction between the contacting surfaces of a CVT. However, depending on different operating (or loading) conditions and design configurations, the friction characteristic of contacting surfaces may vary. For instance, in a fully lubricated CVT, the friction characteristic of the contact zone may bear resemblance to the Stribeck curve [90] rather than to the classical continuous Coulomb characteristic. Moreover, very high forces in the contact zone further lead to the conditions of elasto-plastic-hydrodynamic lubrication, which may yield a different friction characteristic. Lebrecht et al. [70] analyzed self-induced vibrations in a metal pushing V-belt CVT by using highly simplified dynamic models. It was reported that certain friction characteristics, especially those having negative gradient with respect to relative velocity, could induce self-excited vibrations in the CVT system. The friction characteristic and the elasticity of the sheaves determined the working area where vibrations occurred. However, it is not clear whether such phenomenon is an artifact of the friction model or the real behavior of a CVT system. It is thus necessary to study the influence of different friction characteristics on the performance of a CVT in order to better identify various loss mechanisms and design efficient controllers. Srivastava and Haque [16,91–94] developed a planar multibody model of chain CVT and investigated the influence of clearance and friction on its dynamic performance indices for high torque applications. They reported that clearance and friction parameters drastically influence the performance of a CVT system, reduce its torque carrying capacity, and may even render the operation perilous by inducing chaos or irregular behavior in the system. However, their work did not include any mathematical analysis needed to capture such friction-induced chaos in a CVT system. Figs. 15 and 16 [16] illustrate representative time histories from the authors' chain CVT model emphasizing the influence of contact-zone friction conditions and clearances between chain links on the torque capacity and dynamic performance of the chain CVT system (refer to [16] for details). The authors reported that certain contact-zone friction characteristics (e.g. a mix of lubrication and stiction) could cause reduction in the torque capacity of the CVT system and also induce hard-to-be-controlled irregular or chaotic dynamics in the system. Srivastava and Haque [60,61] developed a detailed transient dynamic model of a metal pushing V-belt CVT to evaluate its dynamic performance indices under the influence of pulley flexibility and varying friction characteristics of the belt–pulley contact zone. Two different mathematical models of friction were incorporated to describe friction between the belt and the pulley. These friction models were able to capture effects due to kinetic friction, stiction, and lubricated contact conditions. The authors performed a comparative analysis on the dynamic performance of a metal V-belt CVT under the influence of these friction models. However, chaos related to friction-induced nonlinear phenomena was not reported in their work. Later, Srivastava and Haque [95] exploited their previous planar multibody chain CVT model [16] to identify interesting friction-induced chaotic phenomena in a chain CVT drive under high input-low load con-

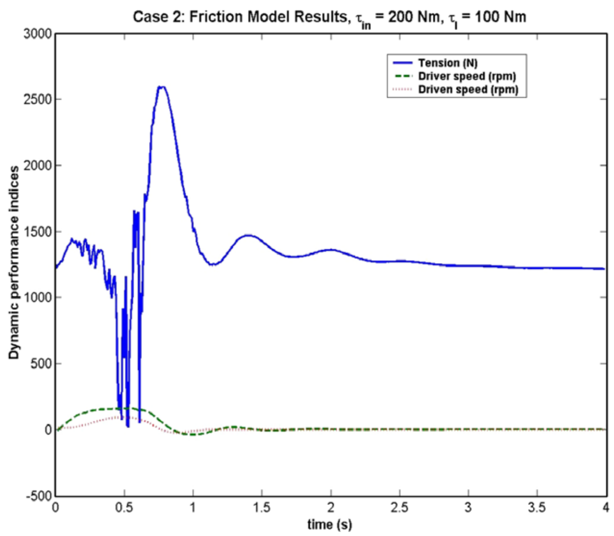

Fig. 15. Dynamic performance indices of chain CVT emphasizing reduced torque carrying capacity [16].

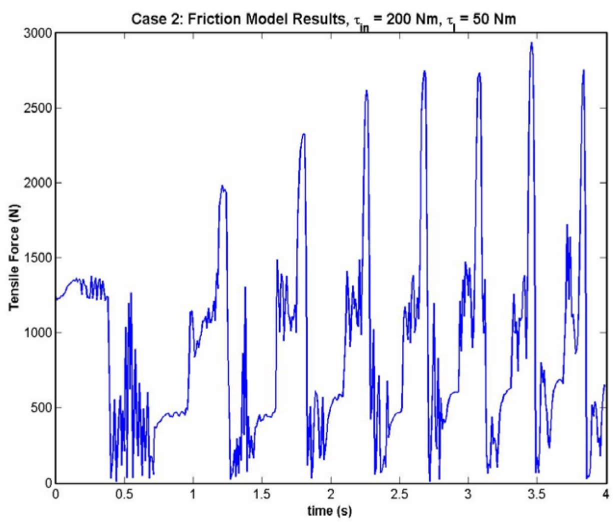

Fig. 16. Dynamic performance indices of chain CVT: tensile force in links showing irregular periodicity [16].

ditions. Tools from nonlinear dynamics such as Lyapunov exponents, recurrence plotting, phase-space reconstruction, etc. were employed to characterize the observed chaos in the CVT system for varying friction characteristics of the contact zone. The goal was not only to understand friction-induced nonlinear dynamic behavior of a chain CVT drive as the chain links traversed the contacting arcs of driver and driven pulleys, but also to evaluate system performance under the influence of different contact-zone friction characteristics. The authors reported that certain contact-zone friction conditions not only reduce torque transmitting capacity of a CVT, but also induce early wear in the system through the mechanism of chaos or irregular behavior.

4. CVT control

The control aspect of achieving a desired gear ratio profile by pulley actuation forces has also been an inevitable part of CVT research over the last two decades. The development of an optimum CVT control strategy is not an easy task owing to two partially opposite features that have to be satisfied: the reduction of fuel consumption and the requirement of appropriate drivability/acceleration performance (which is dependent on the torque capacity of the CVT system). An accurate and fast control of the rate of change of speed ratio is a prerequisite for meeting these goals. The advanced control strategy must implement an accurate model of transmission shifting dynamics in order to foresee the actual clamping forces needed to change the CVT speed ratio and the axial position of pulley sheaves. So, the challenges for an efficient CVT controller primarily are to increase the torque capacity of a CVT system, minimize belt slip losses, and maximize vehicle fuel economy and acceleration performance.

In order to achieve minimum fuel consumption relative to the various levels of desired drive torque, both engine speed and engine torque needs to be controlled simultaneously, thereby requiring the implementation of an integrated engine-transmission control. The purpose of an integrated CVT-engine control is to achieve optimal engine operation for minimum fuel consumption while satisfying the driver's demand. For optimal engine operation, the engine should be operated on optimal operating line (OOL). In Fig. 17 [96], an OOL for minimum fuel consumption is shown on the engine characteristic map. The OOL for minimum fuel consumption can be obtained from the specific fuel consumption contours and iso-power curves. The optimal engine operation point is defined as the point where the optimal engine power curve intersects with the OOL. Minimum fuel consumption can be achieved by operating the engine at the optimal operation point by simultaneous TVO (throttle valve opening) and CVT ratio control, i.e. an integrated control.

An engine-CVT integrated control was suggested by Takiyama and Morita [97]. The authors developed an algorithm to simultaneously control the vehicle velocity from the difference between the desired and actual velocity and the CVT ratio from the difference between the desired and actual engine speed. However, the authors neglected the transient characteristics of powertrain, which resulted in poor vehicle/powertrain performance during acceleration transients. Later, Takiyama [98] modified the previous control algorithm [97] by incorporating air–fuel ratio effects in the model, and investigated the improvements in vehicle fuel economy. Vehicle speed control subsystem, fuel optimizing control subsystem, and A/F control subsystem were treated as single input–output subsystems (SISO) independently using the decoupling control theory. Sakaguchi et al. [99] developed an integrated engine-CVT control from the viewpoint of minimizing powertrain losses. The authors suggested that in an integrated engine-CVT control, it is necessary to consider powertrain losses in order to determine the target engine torque. Yasuoka et al. [100] developed an integrated engine-CVT control algorithm to obtain the demanded drive torque for optimum fuel economy. The authors used the engine torque to compensate for the drive torque response delay caused by the CVT response lag. The authors calculated the target torque by assuming that the accelerator pedal travel represents the demanded drive torque and used the target gear ratio as the CVT ratio.

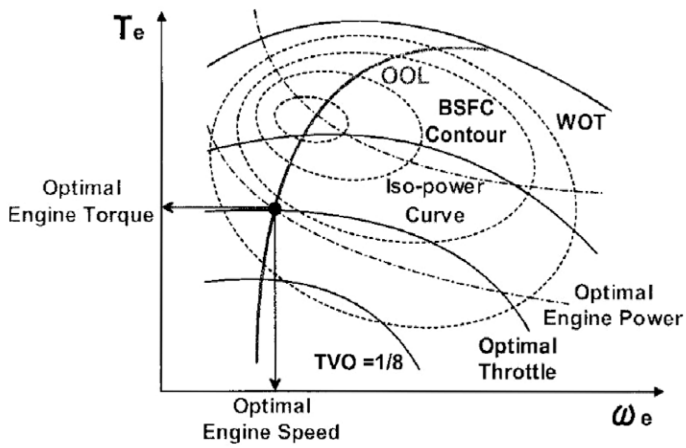

Fig. 17. Typical operating characteristics of a CVT system [96].

Kim and Kim [96] developed an integrated engine-CVT control algorithm by considering the powertrain loss and inertia torque due to CVT ratio change during transient states. They also proposed compensation algorithms to reduce the effect of CVT ratio response lag on the drive torque. The authors conducted experiments to conclude that the optimal engine speed compensation algorithm gives better engine operation around the OOL, compared to the optimal torque compensation algorithm, while showing nearly the same acceleration response. Kim and co-workers [101] experimentally investigated the response characteristics of a parallel hybrid electric vehicle (HEV) CVT and developed a ratio control algorithm integrated into a hydraulic control system. Based on the ratio control algorithm, the effect of CVT system dynamics on the HEV engine operation is investigated by a "hardware-in-the-loop" simulation. The simulation showed that the engine performance improved by using a closed loop control, where variable control gains were used depending on the shift direction and the CVT speed ratio, considering the nonlinear characteristics of ratio-control hydraulic valves and CVT belt–pulley dynamics. In order to maintain a steady-state speed ratio, an optimal pulley thrust is required. Decreasing clamping forces in the variator improves the efficiency of a CVT; however, it also increases the risk of belt wear due to excessive slippage. Bonsen et al. [102,103] developed a robust gain-scheduling PI controller based on a linearized slip model to measure and control slip in a CVT while minimizing clamping forces and preventing destructive belt slip. The gains were scheduled based on primary speed, ratio, and slip. Slip was used to determine whether the system underwent microslip or macroslip. The set point also varied with the ratio, since the maximum traction coefficient could be attained for different slip values depending on the ratio. However, the slip control system was designed for quasi-static ratio control, which does not hold in dynamic driving situations. Saito and Lewis [104] developed a simulation technique for a metal pushing V-belt CVT with feedback thrust controllers. Multi-body formalisms were used to model the belt, and a modified PI controller was used to adjust the pulley thrusts to obtain a desired speed ratio.

Liu and Stefanopoulou [105] considered the wheel speed problem of an automotive powertrain with a conventional SI-engine directly connected to a continuously variable transmission. They developed a two-input two-output control structure

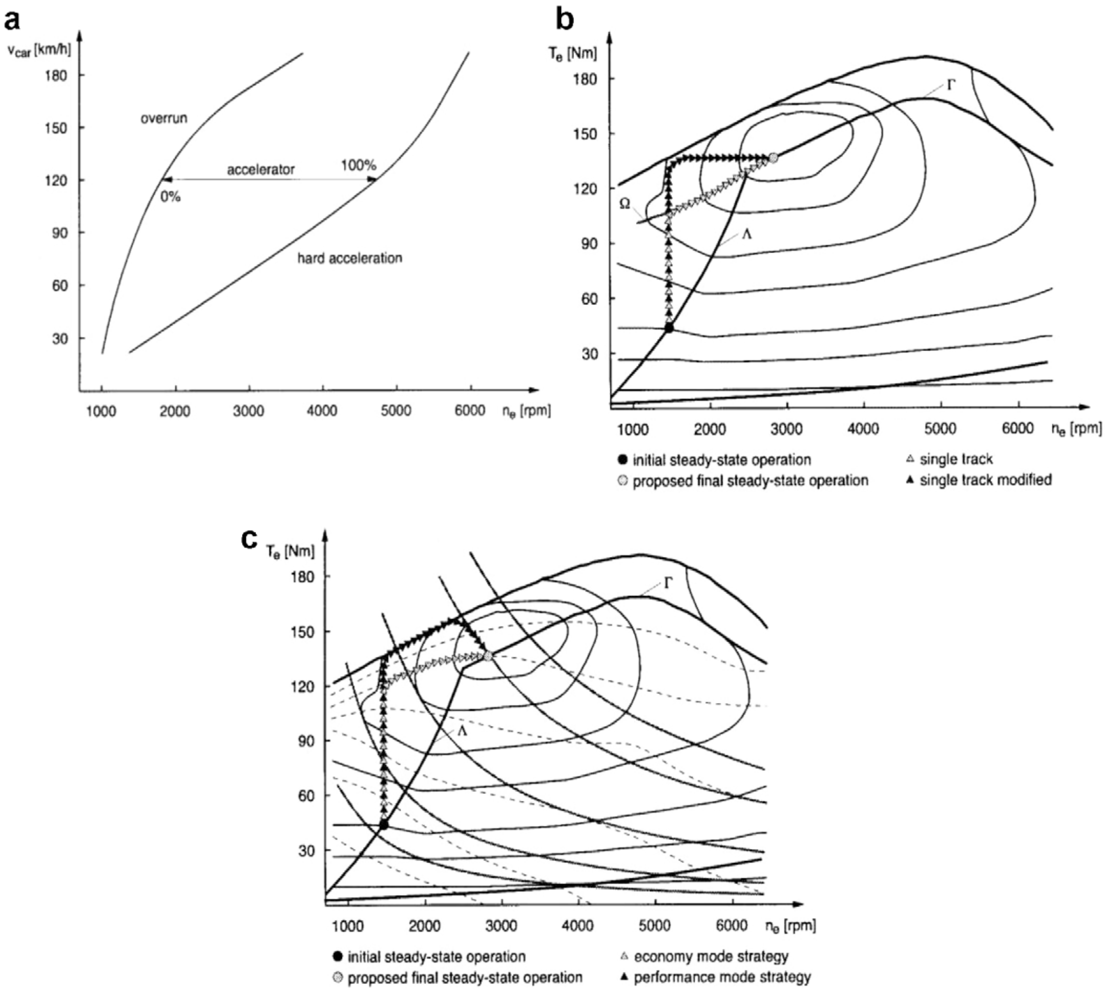

Fig. 18. CVT control strategies: (a) speed envelope strategy; (b) single track strategy; (c) off the beaten track strategy [108].

of the powertrain using simplified CVT models to improve the vehicle/powertrain performance without significant transient deviations from the optimal fuel economy operation. Guzzella and Schmid [106] used robust state-feedback linearization approach to keep the engine-CVT system on the optimal fuel-efficiency curve for a continuously running engine (i.e. when the vehicle/car is in the high power regime). Liu and Paden [107] and Pfiffner and Guzzella [108] surveyed the basic shift control strategies of a CVT system for a vehicle which is already launched and operating in forward driving conditions. They categorized shift control strategies into three broad philosophies: (a) "Speed envelope" strategy where the desired operating area of the CVT system is formed by two curves in the engine speed versus vehicle speed plane, as shown in Fig. 18a [108]. The improvements in the fuel economy of a vehicle are realized by simply choosing a relatively low desired engine speed at cruising conditions; (b) "Single track" strategy where the engine torque is brought as quickly as possible to the 'quasi-static' peak efficiency curve Ω, as shown in Fig. 18b [108]. Concurrently, the gear ratio is adjusted to correspond to the proposed final steady-state operating point; (c) "Off-the-beaten track" strategy where two or more trajectories that represent different driving modes (the economy and the performance mode) are used to continuously adjust the gear ratio to reach the final steady-state operating point based on varying throttle input conditions. However, since these three strategies are based on heuristics philosophy, Pfiffner et al. [109] later extended their previous work to explore optimal and suboptimal control strategies for fuel-efficient CVT powertrains by using simplified CVT dynamic models. Lee and Kim [110] developed an algorithm to improve the fuel economy of a vehicle by shift speed control of a CVT system. In a metal belt CVT, line pressure is required to generate clamping forces between the belt and the pulleys. Insufficient line pressure causes gross slippage of the belt, which results in a loss of power transmission capacity. On the other hand, excessive line pressure causes a hydraulic loss as well as a short life of the belt. Consequently, line pressure control is considered to be an integral part of CVT design. In addition, since the maximum primary pressure (a quantity that influences shift speed) is limited by the line pressure, the metal belt CVT can also possibly suffer from the drawback of a relatively slow response to the speed ratio variations. The authors first created a CVT shift speed map using simplified dynamic models of CVT shifting. Based on the CVT shift speed map, an algorithm to calculate the line pressure required to achieve the target shift speed was suggested. Using the shift speed control algorithm and simplified dynamic models of the line pressure control valve and the ratio control valve, performance simulations of a CVT-equipped vehicle were carried out to evaluate the optimal engine operation.

Setlur et al. [111] developed an adaptive nonlinear control algorithm for the asymptotic tracking of the desired wheel speed by ensuring that the CVT ratio tracks a desired gear ratio profile. The CVT used in their model to capture the gear ratio dynamics was assumed as a pure first-order integrator system. Foley et al. [112] developed a continuous-time model and controller for the drive train dynamics and the hydraulics associated with the ratio shifting mechanism of a metal belt CVT. The author introduced separate notions of geometric ratio and speed ratio and their relation to each other through the output power efficiency curve of the CVT. A data driven model was then created to represent the relationship between ratio dynamics and control hydraulics. The controller was designed using backstepping strategies to effectively track a desired engine speed. Adachi et al. [113] modeled the CVT system as a first-order lag system with an uncertain time constant and time delay, and applied μ-synthesis, a robust control method, to design the controller. In order to further improve the control performance, they coupled their robust controller to a feedforward controller. Thus, their CVT control model was implemented as a two-degree-of-freedom control system. Kim et al. [114] suggested a fuzzy logic based ratio control algorithm for the metal belt CVT system considering the on–off characteristics of the ratio control valve and the nonlinear characteristics of CVT dynamics. Experimental results showed that a desired speed ratio could be achieved at steady state by fuzzy logic in spite of the fluctuating primary pressure. In addition, it was found that faster response and better robustness characteristics could be obtained by fuzzy logic control than with a standard PID control. Ryu et al. [115] developed a model-based control algorithm for the pressure-control type CVT using the steady state characteristics of the ratio control valve. In a pressure-control CVT system, the desired speed ratio is obtained by controlling the primary actuator pressure. The authors proposed that linear control algorithms such as PID type control could be used for the pressure-control type CVT whereas nonlinear or adaptive control logic should be implemented for the flow-control type CVT. Pesgens et al. [116] developed a new ratio controller for a metal pushing V-belt CVT with a hydraulic clamping system. Using the dynamic models of the variator and hydraulics, and compensator constraints, a set point feedforward and a linearizing feedback controller were implemented. The feedback controller was a PID controller with a conditional anti-windup protection. The total ratio controller guaranteed that at least one of the pressure set points was always minimal with respect to its constraints, while the other was raised above the minimum level to enable shifting. Other interesting CVT modeling and control strategies proposed in the literature include Kolmanovsky et al. [117], Pfiffner et al. [118], Shafai et al. [119], Laan et al. [120], Sorge [121,122], Kong and Parker [123], Frank and co-workers [124,125], Saito [126], Tani et al. [127], etc.

5. Conclusions

Although a continuously variable transmission plays a crucial role in the plan to improve the fuel economy and dynamic performance of a vehicle, its complete potential has not been realized so far in a mass-production vehicle. The expected increase in fuel economy and acceleration performance has not been achieved in many cases. The control logic has not been accurate enough to deliver a desired shifting behavior. Since belt and chain type CVTs are the most commonly used drive transmissions, this paper reviews the state-of-the-art research that has been accomplished to understand the dynamics and control of such CVT systems. The models discussed in literature vary in their level of complexity, their mode of analysis, and their research scope. Further, the literature reviewed reveals significant opportunities of research that could be necessary to gain better insight into the dynamics of such CVT systems to maximize the dynamic performance and fuel economy of a CVT-equipped vehicle, design better/efficient controllers, identify loss mechanisms, and characterize operating regimes for maximum torque transmissibility or efficiency. Based on the literature surveyed, the following conclusions could be drawn:

- Since belt and chain CVT systems fall under the category of friction-limited drives, it is crucial to understand the influence of varying contact-zone friction characteristic on the dynamics of such CVT systems. Although active research efforts have been made [16,48,58,60,61,86–88,94] in this area over the past few years, there is still ample scope to research the nonlinear dynamics of such non-smooth mechanical transmissions, especially in the context of chaos identification and control. Also, CVT, being a complex nonlinear vibro-impact system, offers tremendous research opportunities in the areas of active vibration control [69,70], chaos [16,95], characterization of stable and successful operating regimes [62,63], etc.
- Almost all models (especially of belt CVTs) mentioned in the literature are assumed to be under the conditions of quasi-static equilibrium or steady state. However, recent research developments [25,42,59–62,121,122] over the past few years have shown that inertial/acceleration terms significantly influence the dynamics of torque transmission, especially during rapid speed ratio/load changing phases. Thus, although the assumption of steady-state quasi-equilibrium conditions may yield a simplistic CVT model, the dynamics captured by such model may not be accurate enough for control development, slip analysis, optimization and efficiency studies.
- Certain nonlinearities like pulley flexibility and clearance significantly influence the thrust ratio, torque capacity, and slip behavior of a CVT system. Although a lot of work on the influence of pulley flexibility and clearance on the dynamic behavior of CVTs [16,27,44,47,49,58,60,61,74,80] has been actively pursued, there is still scope to research and understand the nonlinear dynamics associated with such nonlinearities in a CVT system from a physics point of view. It has been reported that clearance could reduce torque capacity of a CVT system by not only fostering microslip losses due to redistribution of gaps among the belt elements/chain links, but also inducing irregular/chaotic behavior. Also, it has been commonly assumed in the analysis that the elastic deformations in the belt/chain are negligible in comparison to the elastic deformations of the pulleys. So, it is reasonable to assume the power-transmitting device (i.e. the belt or chain) to be rigid during model development.
- Certain authors [16,42,45,53,73,78] have also developed detailed multibody models of belt and chain CVTs using finite element methods and lumped parameters to account for the elastic deformations in both pulleys and the belt/chain. Although such models give a profound insight into the torque transmission dynamics of a CVT system, they are not useful for the development of fast and efficient controllers. Although it is feasible to model the belt in a belt–CVT system as a one-dimensional continuous rigid body, it is inevitable that one models chain CVT drive as a multibody system to account for the polygonal excitations due to the discrete structure of the chain. The challenge then is to develop a system-level model that could not only accurately represent the detailed underlying multibody dynamic interactions in such CVT systems, but also facilitate the development of fast and reliable CVT shift-ratio controllers.
- Also, the literature reviewed suggested that there is considerable disparity in the type of CVT models that have been used for control development. Almost all models for CVT control design assume CVT to be a pure integrator, which is not the best approach for capturing and controlling the dynamics associated with the various components of a CVT system. Moreover, such models would also be inadequate (or rather inaccurate) for the purpose of optimizing fuel economy and acceleration performance of a vehicle. Since CVTs are designed inherently to maximize the fuel economy of a vehicle, the authors believe that there lies tremendous research opportunity to develop accurate ratio shift controllers that could track a desired gear ratio profile with minimum slip losses, maximize the torque capacity of the CVT, and maximize the fuel economy of a vehicle. Such a task would entail development of simple yet accurate system-level models of CVTs (as depicted in Fig. 19) that can ably capture the transmission ratio (i.e. the geometric radius ratio and the pulley speed ratio) variation in accordance with the pulley loading conditions (i.e. axial forces and input and load torques). The model represented in Fig. 19 though simplistic at first sight is challenging in terms of accurately describing the transients in shift

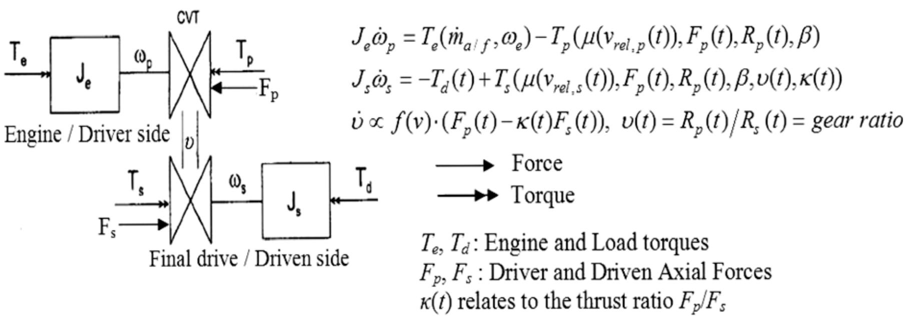

Fig. 19. System-level model for CVT ratio control.

ratio dynamics (i.e. gear ratio change and thrust ratio change). The model should also be able to accurately predict how much the belt slips (both macro- and micro-slip) in the pulley groove, given the conditions of axial forces and torques on the pulleys. Also, it has been reported that CVT, being a highly nonlinear system, needs a specific set of operating conditions to meet the load torque requirements. Not every combination of radius ratio and pulley axial force would be able to transmit torques high enough to meet the load requirements. Thus, considerable effort could also be directed towards characterizing the operating regime of a CVT system for successful torque transmission, high torque capacity, and minimum transmission losses.

- Further, new research frontiers should be investigated in the context of CVT design and configuration. Certain new configurations of CVT designs have been reported (for instance [2,5,8,13,14]) to achieve continuous variations in transmission ratio with lower losses, however, the range of applicability of such CVTs for high torque applications is yet to be analyzed/verified.

A continuously variable transmission is a promising automotive transmission technology that can provide higher fuel economy, reduced emissions, and better vehicle performance. The current paper not only addresses the state-of-the-art research accomplished towards understanding CVT dynamics and control, but also hopefully highlights the challenges/directions for future research, which could foster better understanding and designing of such systems and their controllers.

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