

SEMINAR 3

GRAPHS

Week 8 – Seminar 3

Before you attempt this seminar, please make sure you have watch this week's videos on Graphs.

Exercise 1:

Draw the adjacency matrix for each graph shown in Figure 1.

Exercise 2:

Let's suppose we have $N=5$ peoples $\{A, B, C, D, E\}$ and the **following** adjacency matrix is as follows:

$$\text{following} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The convention used is rows are the source nodes, and columns are the target nodes.

Draw the corresponding directed graph (also known as digraph).

Exercise 3:

Draw the adjacency list for each graph shown in Figure 1.

Exercise 4:

An influencer is a person who doesn't follow anybody but is followed by everybody. Given N peoples and an adjacency matrix for the graph "following", where the edge (A,B) means A follows B, find the influencer among the group of N peoples if it exists.

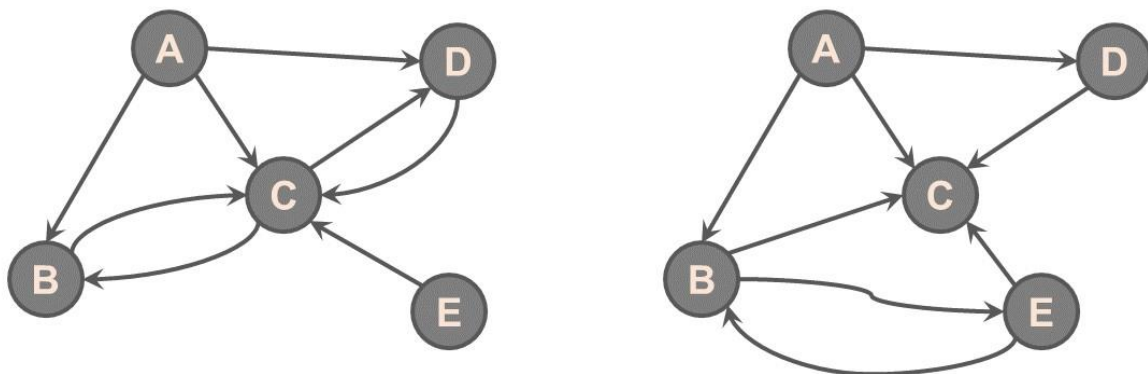


Figure 1: Example of 2 social networks. On the left-hand side, the network does not have an influencer, however on the right-hand side, C is an influencer.

In this exercise, you must use the adjacency matrix representation. Given a matrix **following** that represents users following other users within a community of N users (with ids from 0 to $N-1$):

- $following[i][j] = 1$ if and only if user i is following user j . However, it doesn't imply $following[j][i] = 1$.
- Let's also agree that $followingMatrix[i][i] = 0$.

Find an Influencer find the influencer in this community network (that is its index), or return -1 if there is no Influencer in this group.

Brute force

For each user U , you can check that the two properties below hold:

1. Everybody follows U ,
2. U doesn't follow anybody.

A better approach

We can find the influencer in $O(n)$ time by testing each person for the above properties. We can select a person U as a candidate influencer and then test if U is indeed a influencer by testing with next person. If the next person U_k doesn't follow the current candidate or the candidate follows U_k then U_k may be the influencer. We can choose such a candidate and then test if all other person follows the candidate U_k and the candidate U_k doesn't follow anybody. If any of these properties fail, we say there is no such influencer.

Write the pseudo-code to implement this approach.

Exercise 5:

Before attempting this exercise, you must watch the videos on DFS (Depth first Search) and BFS (Breadth First Search).

If you have been using LinkedIn you may have seen that whenever you open your connections, you find 1st, 2nd or 3rd written. It means the following:

- 1st-degree – People you're directly connected to because you have accepted their invitation to connect, or they have accepted your invitation.
- 2nd-degree – People who are connected to your 1st-degree connections but are not 1st degree connections.
- 3rd-degree – People who are connected to your 2nd-degree connections, but are not 1st or 2nd degree connection
- Out of Network – LinkedIn members who fall outside of the categories listed above.

Write the pseudo-code for the function:

```
connexionDegree(network:Graph, source:Node, target:Node): int
```

Given a connexion network like LinkedIn, which is undirected and unweighted, a source user, and a target user, returns in which of the four categories describe above the target user is (0,1,2, or3, where 0 means "out of network").