Statistics, Simulation & Optimization: Assignment 3

Exercise 3.1 Single-Machine Scheduling

A single-machine has to produce 10 jobs numbered from 1 to 10. Each job has a duration, release time, and deadline as given in the following table (times are indicated in hours, the current time is 0):

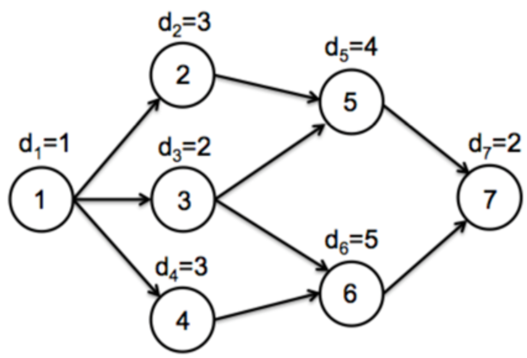
Jobs	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Duration	4	5	3	5	7	1	0	3	2	10
Release time	3	4	7	11	10	0	0	10	0	15
Due date	11	12	20	25	20	10	30	30	10	20

For example, Job 4 can only start after 11 hours, takes 5 hours and its deadline is at 25 hours. For each hour that a job exceeds its deadline a cost of 1 euro is incurred. No other costs are considered. We want to find the production schedule that minimizes the total costs.

a) (2.5) Implement an integer linear optimization model for the single-machine scheduling problem in Python using PuLP and solve it. Ensure that the data and the model are separated (so that new problem instances can be easily solved by only adjusting the data). Report on the optimal schedule, the starting times and the tardiness of all jobs. Also include the (structured) code in your report. (Hint: Use the ILO model as discussed in Lecture 9 and see the PuLP tutorial on Canvas.)

b) (0.75) Now jobs 1, 3, 4, 6, 10 require sieve type 1, whereas the other jobs require sieve type 2. Changing the sieve takes 1 hour. During this sieve changing time, the machine cannot work on any job. Modify the model from a (in a linear way) and solve it again in Python using PuLP. Report on the optimal schedule, the starting times and the tardiness of all jobs. Also report on the code modifications. Explain the differences with the schedule found in a.

Exercise 3.2 Project Planning (will be discussed in Lecture 11)
A project is composed of 7 activities labeled from 1 to 7. Each activity starts as soon as possible. The activity durations are all independent and exponentially distributed. For each activity, the expected duration in days and precedence constraints are given in the following figure:



For example, Activity 5, whose duration follows an exponential distribution with rate parameter 1/4 (and thus has an expected duration of 4 days), can only start once Activities 2 and 3 have been finished. Use Excel to answer the following questions.

a) (2) Simulate the project 10000 times and approximate the expected project finish time.
b) (0.5) Give a 95% confidence interval for the expected project finish time in days. How can you interpret this confidence interval?
c) (0.75) We are interested in the probability that the project takes more than 12 days. Simulate this probability and give a 95% confidence interval.

Exercise 3.3 Optimal Hotel Prizes (will be discussed in Lecture 12)
A hotel with 10 rooms has to set its price. There are 9 possible prices: 100, 110, ..., 180 euro. The hotel demand has a binomial distribution with parameters 20 and 0.9 - price/300. For example, if price 100 is set, the demand has a binomial distribution with parameters 20 and 0.9 - 100/300 = 17/30. We are interested in finding the price which maximizes the expected revenue. Use Excel to answer the following questions.

a) (1.75) Simulate the revenue for every price often enough (e.g., 100000 times) to determine the optimum price with a 0.05 significance level using an appropriate test.
b) (1.75) Execute "ranking and selection option 2" (from the slides of Lecture 12) with a simulation budget of 10000. Spend in total 900 simulations in Step 1 and the remaining budget of 9100 in Step 4. Use a significance level of 0.05.