Exercise 7.2. Resolution Proposition

We denote the propositional clauses as follows and indexing of the literals:

$$C_1 = _{(1)}P \lor _{(2)}\lnot r \; ; \; C_2 = _{(3)}q \lor _{(4)}r \; ; \; C_3 = _{(5)}\lnot P \lor _{(6)}\lnot r \; ; \; C_4 = _{(7)}\lnot q \lor \lnot r$$

we use the level saturation strategy:

$$S^0 = S = \{C_1, C_2, C_3, C_4\}$$

$$S^1 = \{Res^{lock}(C_i, C_j) \mid C_i, C_j \in S^0\}$$

$$C_5 = Resp^{lock}(C_1, C_3) = _{(2)}\lnot r$$

$$S^1 = \{C_5\} \; ; \; S^2 = \{Res^{lock}(C_i, C_j) \mid C_i \in S^1, C_j \in S^1, C_j \in S^0 \cup S^1 \}$$

$$S^2 = \varnothing$$

No block resolvents can be generated, the empty clause cannot be derived from S and we conclude that the set is consistent.