Exercise 4.2

Using a refinement of predicate resolution prove:

· the semi-distributivity of 1 = 1 over 1 .

U1 = - (3x)(P(K)) (XE) <- ((x)) P(K) A (3x) Q(K)

UZ = H (3K) P(K) A (3K)Q(K) -> (3K)(P(K) A Q(K))

TUI = 7(EX)(LEX)(P(W)AQ(X)) -> (ZX)P(X) A (DX)Q(X))

= 7(7(3K)(P(K))Q(K))V(3K)P(K) A (3K)Q(K))

= (3x)(P(x)) A ((x)) / ((3x) P(x) A (3x) Q(x))

= (3x) (P(x) 1 Q(x)) 1 (7 (3x) P(x) V 7 (3x) Q(x))

= (Zx)(P(K) A Q(K)) N((VX) 7 P(K) V (YX) 7 Q(K))

= (3K)(P(K) / Q(K)) / ((+4)) P(y) V (+2) 7 Q(2))

= (3x)(4y)(42)(P(K) A Q(K) A (7P(y) V 7Q(2))

(7U1)P=(3K)(4Y)(4Z)(P(K)AQ(K)A(7P(y)V7Q(Z))

L the prenex form [x1-a], a- Skalam accustont

(7U1)6=(4y)(42)(P(a) A Q(a) A (7P(y) V 7 Q(2))

4 the Skalem form

(TUI) = P(a) A Q(a) A(7Ply) V 7 Q(Z)) - the clousof normal form

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5,= / C1=P(a), C2=Q(a), C3= 7P(y) V7Q(2)4
  C4 = Res Pr (C1, C3) = 7Q(2)
 C5 = Res Pr
[2+07 (C2, C4) = 17
  (TUI) CHAM B => HUI
2. TUZ = 7 ((FK)P(K) A (FK)Q(K) -> (FK)(P(K) A Q(K)))
          = 7( 1(3x) P(x) 1 (3x) Q(x)) V (3x) (P(x) 1 Q(x)))
         = BNYP(K) NBNQ(K) N 7 (3K)(P(K) NQ(K))
         = (JK)P(K) N(JK)Q(K) N(YK)7(P(K))AQ(K))
         = (JN)PN A (JN)Q(N) A (FN)(7P(N) V 7Q(N))
         = (3K)P(K) N(3y)Q(y)N (42)(7P(2) V7Q(2))
        = (FW(Fy)(42)(PW)(Q(y))(7P(2))7Q(2))
  (102)P= (3x)(3y)(42)(P(N)AQ(y))(7P(2) V7Q(2))
                                [x <a], [y + b], a1b - 5Kalem constants
 (702)5=(42)(P(a)AQ(b)A(7R2)V7Q(2))-+he skolem form
 (7 Uz) = P(a) A Q(b) A (7 P(z) VA(2)) - the cloude normal form
  52 = 4 C1 = P(a), C2 = Q(6), C3 = 7 P(2) V 7 Q(2) 4
 C_{4} = \text{Res} \quad C_{1}, C_{3} = \neg Q(a)
C_{5} = \text{Res} \quad C_{2} \leftarrow s_{3} \quad (C_{2}, c_{6}) = \neg P(b)
Q(a) \text{ and } Q(b) \text{ are not unifiable because a and b are constants,}
60 the clauses Cz and Cy do not resolve Likewise for Pla) oud 76)
   (702) Bos D => HUZ
                                                    ( c1' and C5')
    HUI and HUZ => 17 is only semi-distributive over 11
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