Resolution in Predicate Logic

Exercise 3:

Prove the inconsistency of the following set of clauses using lock resolution

2. $S_2 = \{ P(x) \vee \neg Q(x), \neg P(a) \vee R(x), Q(x), W(z), \neg R(y) \vee \neg W(y) \}$

Clauses:

$C_1 = {}_{(2)} P(x) \vee {}_{(1)} \neg Q(x)$

$C_2 = {}_{(3)} \neg P(a) \vee {}_{(4)} R(x)$

$C_3 = {}_{(5)} Q(x)$

$C_4 = {}_{(6)} W(z)$

$C_5 = {}_{(8)} \neg R(y) \vee {}_{(7)} \neg W(y)$

The following resolvents are obtained:

$C_6 = Res^{Pr}(C_1, C_3) = {}_{(2)} P(x)$

$C_7 = Res^{Pr}_{\theta_1 = [z \leftarrow y]}(C_4, C_5) = {}_{(8)} \neg R(y)$

$C_8 = Res^{Pr}_{\theta_2 = [x \leftarrow a]}(C_2, C_6) = {}_{(4)} R(a)$

$C_9 = \mathrm{Res}^{P_r}_{\theta_3 = [y \leftarrow a]}(C_7, C_8) = \square$

Conclusion:

$S \vdash^{lock, P_r}_{Res} \square$ and thus S is inconsistent