

Cajocariu Robert

Resolution - 1.2.

Prove $V_2 = (B \rightarrow A) \wedge (C \rightarrow A) \rightarrow (B \wedge C \rightarrow A)$ - theorem

$$\neg V_2 = \neg((B \rightarrow A) \wedge (C \rightarrow A) \rightarrow (B \wedge C \rightarrow A))$$

- Use $X \rightarrow Y \equiv \neg X \vee Y$ for 1, 2, 4

$$\neg V_2 = \neg((\neg B \vee A) \wedge (\neg C \vee A) \rightarrow (\neg B \vee \neg C \vee A))$$

- Again for 3

$$\neg V_2 = \neg(\neg((\neg B \vee A) \wedge (\neg C \vee A)) \vee (\neg B \vee \neg C \vee A))$$

- Apply de Morgan's law

$$\neg V_2 = (\neg B \vee A) \wedge (\neg C \vee A) \wedge B \wedge C \wedge \neg A \quad \text{- CNF, 5 clauses}$$

- Take $S = \{ \underbrace{\neg B \vee A}_{C_1}, \underbrace{\neg C \vee A}_{C_2}, \underbrace{B}_{C_3}, \underbrace{C}_{C_4}, \underbrace{\neg A}_{C_5} \}$

- Apply resolution alg.

$$C_1 = \neg B \vee A$$

$$C_5 = \neg A$$

$$C_6 = \text{Res}_A(C_1, C_5) = \neg B$$

$$C_3 = B$$

$$C_7 = \text{Res}_B(C_3, C_6) = \square$$

The empty clause (\square) has been derived from $\text{CNF}(\neg V_2)$

$\Rightarrow V_2$ is a theorem