2.

a) $l_1 = P(a, x, f(g(y)))$ and $l_2 = P(y, f(z), f(z))$

$\theta := \varepsilon$

$\theta(l_1) = P(\underline{a}, x, f(g(y)))$

$\theta(l_2) = P(\underline{y}, f(z), f(z))$

$\boxed{\text{it } 2}$

$\lambda := [x \leftarrow f(z)]$; as a unifier for x and $f(z)$

$\theta := \theta\lambda = [y \leftarrow a][x \leftarrow f(z)] = [y \leftarrow a, x \leftarrow f(z)]$

$\theta(l_1) = P(a, f(z), \underline{f(g(a))})$

$\theta(l_2) = P(a, f(z), \underline{f(z)})$

it 1.

$\lambda : [y \leftarrow a]$; unifier for y and a

$\theta := \theta\lambda = [y \leftarrow a]$;

$\theta(l_1) = P(a, \underline{x}, f(g(a)))$

$\theta(l_2) = P(a, \underline{f(z)}, f(z))$

it 3

$\lambda := [z \leftarrow g(a)]$ unifier of the terms

z and $g(a)$

$\theta := \theta\lambda = [y \leftarrow a, x \leftarrow f(z)][z \leftarrow g(a)]$

$= [y \leftarrow a, x \leftarrow f(g(a)), z \leftarrow g(a)] = mgu(l_1, l_2)$

Therefore $\theta(l_1) = \theta(l_2) = P(a, f(g(a)), f(g(a)))$

is the common instance of l_1 and l_2

h) $P(x, g(f(a)), f(h))$ and $P(f(y), z, z)$

$\theta := \varepsilon$

$\theta(l_1) = P(\underline{x}, g(f(a)), f(h))$

$\theta(l_2) = P(\underline{f}(y), z, z)$

it 1

$\lambda := [x \leftarrow f(y)]$ unifier for x and $f(y)$

$\theta := \theta\lambda = [x \leftarrow f(y)]$

$\theta(l_1) = P(f(y), \underline{g(f(a))}, f(h))$

$\theta(l_2) = P(f(y), z, z)$

it 2

$\lambda := [z \leftarrow g(f(a))]$ unifier for term z and $g(f(a))$

$\theta : \theta\lambda = [x \leftarrow f(y)][z \leftarrow g(f(a))]$

$= [x \leftarrow f(y), z \leftarrow g(f(a))]$

$\theta(l_1) = P(f(y), g(f(a)), f(h))$

$\theta(l_2) = P(f(y), g(f(a)), g(f(a)))$

it 3

The terms $f(h)$ and $g(f(a))$ are not unifiable and we conclude that the literals l_1 and l_2 are not unifiable.

c) $P(a, x, f(g(y)))$ and $P(z, h(z, w), f(h))$

$\theta := \varepsilon$

$\theta(l_1) = P(a, x, f(g(y)))$

$\theta(l_2) = P(z, h(z, w), f(h))$

it 1

$\lambda := [z \leftarrow a]$ unifier for z and a

$\theta = \theta\lambda = [z \leftarrow a]$

$\theta(l_1) = P(a, \underline{x}, f(g(y)))$

$\theta(l_2) = P(a, \underline{h(a,u)}, f(l))$

it 2

$\lambda : [x \leftarrow h(a, u)]$

$\theta : \theta\lambda = [z \leftarrow a][x \leftarrow h(a,u)]$

$= [z \leftarrow a, x \leftarrow h(a,u)]$

$\theta(l_1) = P(a, h(a,u), \underline{f(g(y))})$

$\theta(l_2) = P(a, h(a,u), \underline{f(l)})$

it 3

The terms l and $g(y)$ are not unifiable therefore we conclude that the literals l_1 and l_2 are not unifiable.