FILIP CRIȘAN, gr. 912

Individual homework

Exercise 11.2

Using the theorem of deduction and its reverse prove that:

$$\vdash (p \to q) \to ((\neg r \lor p) \to (r \to q)).$$

Solution:

Step 1: We apply the reverse of the theorem of deduction to obtain the initial deduction.

if $\vdash (p \to q) \to ((\neg r \lor p) \to (r \to q))$ then

$p \to q \vdash (\neg r \lor p) \to (r \to q)$ then

$p \to q, \neg r \lor p \vdash r \to q$ then

$p \to q, \neg r \lor p, r \vdash q$.

Step 2: We prove the deduction obtained at Step 1:

$p \to q, \neg r \lor p, r \vdash q$, building the sequence of formulas: $(F1, F2, F3, F4, F5)$.

$F1: \quad r \qquad$ (premise)

$F2: \quad \neg r \vee p \equiv r \to p \qquad$ (premise)

$F1, F2 \vdash_{mp} p$

$F3: \quad p$

$F4: \quad p \to q \qquad$ (premise)

$F3, F4 \vdash_{mp} q$

$F5: \quad q$

The sequence $(F_1, F_2, F_3, F_4, F_5)$ is the deduction of q from the premises $p \to q$, $\neg r \vee p$, r.

Step 3: We begin with the deduction

$p \to q, \neg r \vee p, r \vdash q$ proved at Step 2 and we apply 3 times the theorem of deduction. There are 6 possible theorems.

if $p \to q, \neg r \vee p, r \vdash q$ then

$p \to q, \neg r \vee p \vdash r \to q$ then

$p \to q \vdash (\neg r \vee p) \to (r \to q)$ then

$\vdash T1 = (p \to q) \to ((\neg r \vee p) \to (r \to q))$, the theorem to be proved