

Linear Resolution

Theoretical Result

• Soundness and completeness theorem:

- the set S of clauses is inconsistent if and only if
 $S \vdash_{\text{Res}} \square$

• The backtracking algorithm is the strategy provided at the implementation level

- the algorithm stops in 2 cases:

1. The empty clause was derived $\Rightarrow S$ is inconsistent
2. For the top clause all possible side clauses were used, but the empty clause was not derived
 $\Rightarrow S$ is consistent

Exercise 5.2: Prove the consistency of the following set of clauses using linear resolution: $\{p \vee q \vee r, \neg p \vee q, \neg p \vee \neg q\}$

$$C_1 = p \vee q \vee r$$

$$C_2 = \neg p \vee q$$

$$C_3 = \neg p \vee \neg q$$

Take C_1 as the top clause

$$C_1 = p \vee q \vee r$$

$$C_2 = \neg p \vee q$$

$$\text{Res}_p(C_1, C_2) = C_4 = q \vee r$$

~~$$C_4 = \text{Res}_p(C_1, C_2) = q \vee r$$~~

$$C_3 = \neg p \vee \neg q$$

$$\text{Res}_q(C_4, C_3) = C_5 = r \vee \neg p$$

$$C_1 = p \vee q \vee r$$

$$\text{Res}_p(C_5, C_1) = C_6 = q \vee r = C_4$$

\Rightarrow Process blocked

$$C_1 = p \vee q \vee r$$

$$C_3 = \neg p \vee \neg q$$

$$\text{Res}_q(C_1, C_3) = C_4 = q \vee \neg q \vee r - \text{tautology, process blocked}$$

$$C_1 = p \vee q \vee r$$

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$$\text{Res}_p(C_1, C_3) = C_4 = p \vee \neg p \vee r - \text{tautology, process blocked}$$

Result : A complete linear derivation search was performed but \square was not derived \Rightarrow Σ is a consistent set of clauses