$$U_2 = (\exists x)(\forall y)P(x,y) \longleftrightarrow (\forall y)(\exists x)P(x,y)$$

$$U_2 \equiv \underbrace{((\exists x)(\forall y)P(x,y) \rightarrow (\forall y)(\exists x)P(x,y))}_{U_a} \wedge \underbrace{((\forall y)(\exists x)P(x,y) \rightarrow (\exists x)(\forall y)P(x,y))}_{U_b}$$

$$U_a = (\exists x)(\forall y)P(x,y) \rightarrow (\forall y)(\exists x)P(x,y)$$

$$\neg U_a = \neg((\exists x)(\forall y)P(x,y) \rightarrow (\forall y)(\exists x)P(x,y)) \quad (1)$$

$|$ α-rule for (1)

$$(\exists x)(\forall y)P(x,y) \quad (2)$$

$|$

$$\neg((\forall y)(\exists x)P(x,y)) \quad (3)$$

$|$ δ rule for (3), C_0-new constant

$$\neg((\exists x)P(x,C_0)) \quad (4)$$

$|$ δ rule for (2), C_1-new constant

$$(\forall y)P(C_1,y) \quad (5)$$

$|$ $\wedge$ rule for (5), C_0 used for instantiation

$$P(C_1,C_0)$$

$|$

$(\forall y)P(C_1,y)$ copy of (5)

$|$ $\wedge$ rule for (4), C_1 used for instantiation

$$\neg P(C_1,C_0)$$

$|$

$\neg((\exists x)P(x,C_0))$ copy of (4)

$|$

$\otimes$ closed branch

$\neg U_b = \neg((\forall y)(\exists x) P(x,y) \rightarrow (\exists x)(\forall y) P(x,y))$ (1)

| α rule for (1)

$(\forall y)(\exists x) P(x,y)$ (2)

$\neg((\exists x)(\forall y) P(x,y))$ (3)

| α rule for (2), C_0-new constant

$(\exists x) P(x, C_0)$ (4)

$(\forall y)(\exists x) P(x,y)$ (5) copy of formula (2)

| δ-rule for (4), C_1-a new constant

$P(C_1, C_0)$

| γ rule for (3), C_1 used for instantiation

$\neg((\forall y) P(C_1, y))$ (6)

$\neg((\exists x)(\forall y) P(x,y))$ (7) copy of formula (3)

| γ-rule for (6), C_2-a new constant

$\neg P(C_1, C_2)$

| γ-rule for (5), C_2 used for instantiation

$(\exists x) P(x, C_2)$ (8)

$(\forall y)(\exists x) P(x,y)$ (9) copy of formula (2)

| δ-rule for (9), C_3 new constant

$P(C_3, C_2)$

| γ rule for (7), C_3 used for instantiation

$\neg((\forall y) P(C_3, y))$ (9)

$\neg((\exists x)(\forall y) P(x,y))$ (10) copy of formula (3)

| δ rule for (9), C_4-a new constant

$\neg P(C_3, C_4)$

. . .