

## Exercise 5.2. - Semantic Tableaux Method

Using the semantic tableaux method, prove the following properties in predicate logic:

- ' $\forall$ ' is semi-distributive over ' $\vee$ ':

$$\cdot \models ((\forall x)A(x) \vee (\forall x)B(x)) \rightarrow (\forall x)(A(x) \vee B(x)) \text{ and}$$

$$\cdot \not\models (\forall x)(A(x) \vee B(x)) \rightarrow ((\forall x)A(x) \vee (\forall x)B(x))$$

- We take  $U_1$  and  $U_2$  as it follows:

$$U_1 = ((\forall x)A(x) \vee (\forall x)B(x)) \rightarrow (\forall x)(A(x) \vee B(x))$$

$$U_2 = (\forall x)(A(x) \vee B(x)) \rightarrow ((\forall x)A(x) \vee (\forall x)B(x))$$

- We will build the semantic tableaux of  $\neg U_1$  and  $\neg U_2$ .

- The following theoretical result is used:

$$\cdot \models U \text{ if and only if } \neg U \text{ has a closed semantic tableaux.}$$

- If  $\models U_1$  and  $\not\models U_2$  then ' $\forall$ ' is semi-distributive over ' $\vee$ '