DSA – Seminar 3

1. Sort Algorithms

A. BucketSort

- We are given a sequence S, formed of *n* pairs (key, value), keys are integer numbers from an interval ϵ [0, N-1]
- We have to sort S based on the keys.

For example:

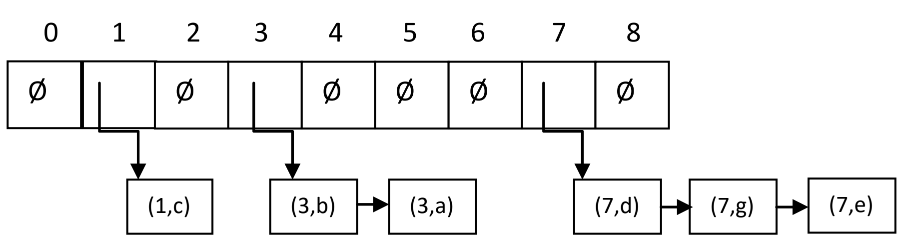
S: (7, d) (1, c) (3, b) (7, g) (3, a) (7, e) =>

(1, c) (3, b) (3, a) (7, d) (7, g) (7, e)   N = 9

Idea:

- Use an auxiliary array, B, of dimension N, in which each element is a sequence.
- Each pair will be placed in B in the position corresponding to the key (B[k]) – and will be deleted from S.
- We parse B (from 0 to N-1) and move the pairs from each sequence from each position of B to the end of S.

B:



Assume that the sequence is already implemented, and it has the following operations (we assume they all run in Θ(1) complexity):

- empty (sequence): boolean
- first (sequence): element
- remove First(sequence)
- insertLast(sequence, element)

Obs1.: element in our case will be a pair (k, v)
Obs2.: what data structure should we use if we wanted to implement *sequence* in order to get the Θ(1) complexity for the operations?

```
Subalgorithm BucketSort(S, N) is:
    //define array B of dimension N; elements of the array are sequences
    //and each element is initialized with an empty sequence
    While ¬ empty (S) execute:
        (k, v) ← first (S)
```

```
        removeFirst (S)
        insertLast (B[k], (k,v))
    end-while
    for i ← 0, N-1, execute:
        While ¬ empty (B[i]) execute:
            (k, v) ← first (B[i])
            removeFirst (B[i])
            insertLast (S, (k,v))
        end-while
    end-for
end-subalgorithm
```

Complexity: $\Theta(N + n)$

Observations:

- Keys must be natural numbers (we are using them as indexes)
- In our implementation, the relative order of the pairs that have the same key will not change -> we call such sorting algorithms *stable.*

B. Lexicographic Sort

d-tuple $(x_1, x_2, ..., x_d)$

$(x_1, x_2, ... ,x_d) < (y_1, y_2, ..., y_d) \Leftrightarrow x_1 < y_1 \vee (x_1=y_1 \wedge ((x_2,..., x_d) < (y_2, ..., y_d)))$

- We compare the first dimension, if they are equal then the 2nd and so on...

We are given a sequence S of tuples. We have to sort S in a lexicographic order.

We will use:

- R_i – a relation that can compare 2 tuples considering the i^{th} dimension.
- *stableSort(S, r)* – a stable sorting algorithm that uses a relation to compare the elements.

The lexicographic sorting algorithms will execute StableSort *d* times (once for every dimension).

```
Subalgorithm LexicographicSort(S, R, d) is:
    For i ← d, 1, -1, execute:
        stableSort(S, Ri)
    end-for
end-subalgorithm
```

Complexity: Θ (d * T(n))

where T(n) – complexity of the stableSort algorithm

Ex. (7, 4, 6) (5, 1, 5) (2, 4, 6) (2, 1, 4) (3, 2, 4)

Sort based on dimension 3: (2, 1, 4) (3, 2, 4) (5, 1, 5) (7, 4, 6) (2, 4, 6)

Sort based on dimension 2: (2, 1, 4) (5, 1, 5) (3, 2, 4) (7, 4, 6) (2, 4, 6)

Sort based on dimension 1: (2, 1, 4) (2, 4, 6) (3, 2, 4) (5, 1, 5) (7, 4, 6)

C. Radix Sort

- A variant of the lexicographic sort, which uses as a stable sorting algorithm Bucketsort → every element of the tuples has to be a natural number from some interval [0, N-1].

- Complexity: Θ (d * (n + N))

2. Write a subalgorithm to merge two sorted singly-linked lists. Analyze the complexity of the operation.

L1. 4 → 9 → 10 → 18

L2. 1 → 6 → 7 → 8 → 9

Result: 1 → 4 → 6 → 7 → 8 → 9 → 9 → 10 → 18

Representation:

<u>Node:</u>

info: TComp

next: ↑Node

<u>List:</u>

head: ↑Node

//possibly a relation, but then we have to make sure that the two lists contain the same relation.

a. We do not destroy the two existing lists: the result is a third list (we have to copy the existing nodes).

```
subalgorithm merge (L1, L2, LR) is:
    currentL1 ← L1.head
    currentL2 ← L2.head
    headLR ← NIL //the first node of the result
    tailLR ← NIL //the last node, needed because we add nodes to the end
    while currentL1 ≠ NIL and currentL2 ≠ NIL execute
        allocate(newNode)
        [newNode].next ← NIL
        if [currentL1].info  < [currentL2].info then
            [newNode].info ← [currentL1].info
            currentL1 ← [currentL1].next
        else
            [newNode].info ← [currentL2].info
             currentL2 ← [currentL2].next
        end-if
        if headLR = NIL then
            headLR ← newNode
            tailLR ← newNode
        else
            [tailLR].next ← newNode
             tailLR ← newNode
        end-if
    end-while
      //one of the currentNodes is NIL, we will keep the other one in a
      //separate variable, to write the following while loop only once
    if currentL1 ≠ NIL then
```

```
         remainingNode ← currentL1
    else
        remainingNode ← currentL2
    end-if
    while remainingNode ≠ NIL execute
        alocate (newNode)
        [newNode].next ← NIL
        [newNode].info ← [remainingNode].info
        remainingNode ← [remainingNode].next
        if headLR = NIL then
            headLR ← newNode
            tailLR ← newNode
        else
            [tailLR].next ← newNode
             tailLR ← newNode
        end-if
    end-while
    LR.head ← headLR
end-subalgorithm
```

Complexity: Θ(n + m)

n – length of L1
m – length of L2

b. We do not keep the two existing lists, the result will contain the existing nodes (but the links are changed)

```
subalgorithm merge (L1, L2, LR) is:
    currentL1 ← L1.head
    currentL2 ← L2.head
    headLR ← NIL //the first node
    tailLR ← NIL //the last node, needed because we add nodes to the end

    while currentL1 ≠ NIL and currentL2 ≠ NIL execute
      //chosenNode will be the actual node we take from a list
        if [currentL1].info  < [currentL2].info then
            chosenNode ← currentL1
            currentL1 ← [currentL1].next
        else
            chosenNode ←  currentL2
             currentL2 ← [currentL2].next
        end-if
        [chosenNode].next ← NIL
        if headLR = NIL then
            headLR ← chosenNode
            tailLR ← chosenNode
        else
            [tailLR].next ← chosenNode
             tailLR ← chosenNode
        end-if
    end-while
    if currentL1 ≠ NIL then
         remainingNode ← currentL1
    else
```

```
        remainingNode ← currentL2
    end-if
      //no need for a loop, just attach every remaining node (starting from
      //remainingNode) to the beginning/end of list. Since this is the last
      //instruction, the value of tailLR does not need to be updated.
    if headLR = NIL then
            headLR ← remainingNode
    else
            [tailLR].next ← remainingNode
    end-if
    LR.head ← headLR
    L1.head ← NIL //make sure you have no nodes left in the lists
    L2.head ← NIL
end-subalgorithm
```

Complexity: WC: $\Theta(n + m)$

BC: $\Theta(\min(n , m))$

n – length of L1

m – length of L2