Teoretical requirements:

1. Logical Equivalence - $A \to B \equiv \neg A \vee B$
2. Morgan's Law - $\neg(A \vee B) \equiv \neg A \wedge \neg \beta$, $\neg(A \wedge B) \equiv \neg A \vee \neg B$
3. Distributive Law - $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$ (here only this)

Write all the anti-models of "U = (q∨r→p) →(p→r)∧q" using CNF

1. Eliminate arrows usig their logical equivalent:

$$U = (q \vee r \xrightarrow{1} p) \xrightarrow{2} (p \xrightarrow{3} r) \wedge q \quad \text{apply } (a \to b \equiv \neg a \vee b) \text{ to 1}$$

$$U = (\neg(q \vee r) \vee p) \xrightarrow{2} (p \xrightarrow{3} r) \wedge q \quad \text{apply } (a \to b \equiv \neg a \vee b) \text{ to 2}$$

$$U = \neg(\neg(q \vee r) \vee p) \vee (p \xrightarrow{3} r) \wedge q \quad \text{apply } (a \to b \equiv \neg a \vee b) \text{ to 3}$$

$$U = \neg(\neg(q \vee r) \vee p) \vee (\neg p \vee r) \wedge q$$

2. Apply De Morgan's law:

$$U = ((q \vee r) \wedge \neg p) \vee ((\neg p \vee r) \wedge q)$$

3. Apply Distributive law:

$$U = \overset{A}{((q \vee r) \wedge \neg p)} \overset{\vee}{\vee} \overset{(\beta}{((\neg p \vee r)} \overset{\wedge C\,)}{\wedge q)}$$

$$U = \{[(q \vee r) \wedge \neg p] \vee (\neg p \vee r)\} \wedge \{[(q \vee r) \wedge \neg p] \vee q\}$$

Write all the anti-models of "$U = (q \vee r \rightarrow p) \rightarrow (p \rightarrow r) \wedge q$" using CNF

3. Apply Distributive law:

$U = \{[(q \vee r) \wedge \neg p] \vee (\neg p \vee r)\} \wedge \{[(q \vee r) \wedge \neg p] \vee q\}$

For convenience, we take $U_1=\{[(q \vee r) \wedge \neg p] \vee (\neg p \vee r)\}$ and $U_2=\{[(q \vee r) \wedge \neg p] \vee q\}$, so that $U = U_1 \wedge U_2$

(A ∧ B) ∨ C

$U_1=\{[(q \vee r) \wedge \neg p] \vee (\neg p \vee r)\}$

$U_1=[(q \vee r) \vee (\neg p \vee r)] \wedge [\neg p \vee (\neg p \vee r)]$

$U_1=(q \vee r \vee \neg p) \wedge (\neg p \vee r)$

(A ∧ B) ∨ C

$U_2=\{[(q \vee r) \wedge \neg p] \vee q\}$

$U_2=[(q \vee r) \vee q] \wedge [\neg p \vee q]$

$U_2=(q \vee r) \wedge (\neg p \vee q)$

Write all the anti-models of "$U = (q \vee r \rightarrow p) \rightarrow (p \rightarrow r) \wedge q$" using CNF

3. Apply Distributive law:

$U_1 = (q \vee r \vee \neg p) \wedge (\neg p \vee r)$

$U_2 = (q \vee r) \wedge (\neg p \vee q)$

$U = U_1 \wedge U_2$

$U = (q \vee r \vee \neg p) \wedge (\neg p \vee r) \wedge (q \vee r) \wedge (\neg p \vee q)$ here we apply absorption law

$U = (\neg p \vee r) \wedge (q \vee r) \wedge (\neg p \vee q)$

4. Find any models from each clause

Clause: $(\neg p \vee r)$ provides two anti-model

$i_1: \{p,q,r\} \to \{T,F\}$, $i_1(p)=T$ $i_1(q)=F$ $i_1(r)=F$ $i_2: \{p,q,r\} \to \{T,F\}$, $i_2(p)=T$ $i_2(q)=T$ $i_2(r)=F$

Write all the anti-models of "$U = (q \vee r \to p) \to (p \to r) \wedge q$" using CNF

4. Find any models from each clause

Clause: $(q \vee r)$ provides two anti-model

i_3:{p,q,r}->{T,F}, $i_3(p)=T$ $i_3(q)=F$ $i_3(r)=F$ i_4:{p,q,r}->{T,F}, $i_4(p)=F$ $i_4(q)=F$ $i_4(r)=F$

Clause: $(\neg p \vee q)$ provides two anti-model

i_5:{p,q,r}->{T,F}, $i_5(p)=T$ $i_5(q)=F$ $i_5(r)=T$ i_6:{p,q,r}->{T,F}, $i_6(p)=T$ $i_6(q)=F$ $i_6(r)=F$

Only, i_2, i_3, i_5, i_6 are unique so they are antimodels of U with:

$i_5(U)=i_2(U)=i_4(U)=i_5(U)=F$

Conclusion:

We can "easily" find the anti-models of a formula by following these steps:

1. Eliminate single and double arrows using their definitions
2. Drive in negations using De Morgan's Law
3. Distribute **or** over **and**
4. Converting to **CNF**
5. Finding the anti-model of each clause