Damiaico George-Julian - 912.

Individual Homework

Semantic Tableux Method

ex. 2.2

Prove that the following formula is a tautology using semantic tableux method:

$$(p \land q \to r) \to (p \to (q \to r))$$

- A branch of a semantic tableau is called closed if it contains a formula and its negation, otherwise the branch is called open.
- A propositional formula is called a tautology <u>if and only if</u> $\neg V$ ~~has~~ is a closed semantic tableau.

- A semantic tableau is called closed if all the branches are closed.

Decomposition rules

α rule:

$$\begin{array}{c} A \land B \\ | \\ A \\ | \\ B \end{array} \qquad \begin{array}{c} \neg(A \to B) \equiv A \land \neg B \\ | \\ A \\ | \\ \neg B \end{array}$$

β rules:

$$\begin{array}{ccc} & A \lor B & \\ / & & \backslash \\ A & & B \end{array}$$

Solution

initial formula U.

$= (p \wedge q \to r) \to (p \to (q \to r))$

$\neg U = \neg((p \wedge q \to r) \to (p \to (q \to r)))$ ①

| α-rule (1)

$p \wedge q \to r$ ②

|

$\neg p \to (q \to r))$ ③

| α-rule (3)

$\neg(q \to r)$ ④

|

p

| α-rule (4)

q

|

$\neg r$

/ \ β-rule ②

$\neg(p \wedge q)$ ⑤ $\quad\quad r$

$\otimes$

/ \

$\neg p$ $\quad\quad \neg q$

$\otimes$ $\quad\quad \otimes$

α rule

$\neg(A \to B)$

|

A

|

$\neg B$

β-rule

$A \to B$

/ \

$\neg A$ $\quad B$

β rule

$\neg(A \wedge B)$

/ \

$\neg A$ $\quad \neg B$.

All Branches of the semantic tableau are closed, containing pairs of opposite literals: $(\neg q, q)$, $(\neg p, p)$, $(\neg r, r)$. $\neg U$ has no models, thus it is an inconsistent formula $\Rightarrow U$ is a tautology.