Exercise 4.2 Boolean Functions

Simplify the following Boolean function using Veitch diagrams.

$f2(x_1,x_2,x_3,x_4) = x_1 x_2 \vee x_1 \overline{x_2}\,\overline{x_3} x_4 \vee \overline{x_1}\,\overline{x_2} x_4 \vee \overline{x_1} x_3 \vee x_2 x_3 =$
$= x_1 x_2 x_3 x_4 \vee x_1 x_2 \overline{x_3}\,\overline{x_4} \vee x_1 x_2 \overline{x_3} x_4 \vee x_1 x_2 x_3 \overline{x_4} \vee x_1 \overline{x_2}\,\overline{x_3} x_4 \vee$
$\vee \overline{x_1}\,\overline{x_2} x_3 x_4 \vee \overline{x_1}\,\overline{x_2}\,\overline{x_3} x_4 \vee \overline{x_1} x_2 x_3 x_4 \vee \overline{x_1}\,\overline{x_2} x_3 \overline{x_4} \vee \overline{x_1} x_2 x_3 \overline{x_4} =$
$= m_{15} \vee m_{12} \vee m_{13} \vee m_{14} \vee m_9 \vee m_3 \vee m_1 \vee m_7 \vee m_2 \vee m_6.$

Veitch diagram

	x_1	x_1	$\overline{x_1}$	$\overline{x_1}$	
x_2	m_{15}	m_{13}		m_7	x_4
x_2	m_{14}	m_{12}		m_6	$\overline{x_4}$
$\overline{x_2}$				m_2	$\overline{x_4}$
$\overline{x_2}$		m_9	m_1	m_3	x_4
	x_3	$\overline{x_3}$	$\overline{x_3}$	x_3	

$max_1 = m_{15} \vee m_{14} \vee m_{13} \vee m_{12} = x_1 x_2$
$max_2 = m_7 \vee m_6 \vee m_3 \vee m_2 = \overline{x_1} x_3$
$max_3 = m_{15} \vee m_{14} \vee m_7 \vee m_6 = x_2 x_3$
$max_4 = m_9 \vee m_1 = \overline{x_2}\,\overline{x_3} x_4$
$max_5 = m_1 \vee m_3 = \overline{x_2} x_4 \overline{x_1}$

$M(f) = \{ max_1, max_2, max_3, max_4, max_5 \} =$
$= \{ x_1 x_2, \overline{x_1} x_3, x_2 x_3, \overline{x_2}\,\overline{x_3} x_4, \overline{x_1}\,\overline{x_2} x_4 \}.$

$M(f)$ - the set of maximal monoms.

Then we select from $M(f)$ the central monoms.

$C(f) = \{ max_1, max_2, max_4 \}.$

$M(f) \neq C(f)$ and $C(f) \neq \varnothing$ - the second case of simplification.

We denote $g = max_1 \vee max_2 \vee max_4 = x_1 x_2 \vee \overline{x_1} x_3 \vee \overline{x_2}\,\overline{x_3} x_4$.

Then we shade all the minterms that are corresponding to the central monoms.

	x_1	x_1	$\overline{x_1}$	$\overline{x_1}$	
x_2	m_{15}	m_{13}		m_7	x_4
x_2	m_{14}	m_{12}		m_6	$\overline{x_4}$
$\overline{x_2}$				m_2	$\overline{x_4}$
$\overline{x_2}$		m_9	m_1	m_3	x_4
	x_3	$\overline{x_3}$	$\overline{x_3}$	x_3	

We see that all the minterms are covered => there exist just one simplification

$f^s(x_1,x_2,x_3,x_4) = max_1 \vee max_2 \vee max_4 =$
$= x_1 x_2 \vee \overline{x_1} x_3 \vee \overline{x_1}\,\overline{x_2} x_4$