boolean functions

exercise 6.8. Using Quine's method, simplify the following Boolean functions given in DCF (disjunction of minterms):

$$f_8(x_1,x_2,x_3) = m_0 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_6$$

$f_8(x_1,x_2,x_3) = m_0 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_6$

$f_8(x_1,x_2,x_3) = x_1^0x_2^0x_3^0 \vee x_1^0x_2^1x_3^0 \vee x_1^0x_2^1x_3^1 \vee x_1^1x_2^0x_3^0 \vee x_1^1x_2^0x_3^1 \vee x_1^1x_2^1x_3^0$

$f_8(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2\bar{x}_3 \vee \bar{x}_1x_2\bar{x}_3 \vee \bar{x}_1x_2x_3 \vee x_1\bar{x}_2\bar{x}_3 \vee x_1\bar{x}_2x_3 \vee x_1x_2\bar{x}_3$

$S_f = \{(0,0,0),(0,1,0),(0,1,1),(1,0,0),(1,0,1),(1,1,0)\}$

$S_f = \{(0,0,0),(0,1,0),(1,0,0),(0,1,1),(1,0,1),(1,1,0)\}$ - the support set of f sorted in ascending order (with respect to the number of "1")

group		x_1	x_2	x_3	representation / factorization
I	✓	0	0	0	m_0
II	✓	0	1	0	m_2
	✓	1	0	0	m_4
III	✓	0	1	1	m_3
	✓	1	0	1	m_5
	✓	1	1	0	m_6
IV = I + II	✓	0	–	0	$m_0 \vee m_2$
	✓	–	0	0	$m_0 \vee m_4$
V = II + III		0	1	–	$m_2 \vee m_3 = \bar{x}_1x_2 = max_1$
	✓	–	1	0	$m_2 \vee m_6$
		1	0	–	$m_4 \vee m_6 = x_1\bar{x}_2 = max_2$
	✓	1	–	0	$m_4 \vee m_6$
VI = IV + V		–	–	0	$m_0 \vee m_2 \vee m_4 \vee m_6 = \bar{x}_3 = max_3$

simple factorization (IV, V)

double factorization (VI)

all those which are unchecked will be in the set of maximal monoms

$M(f) = \{max_1, max_2, max_3\} = \{\bar{x}_1x_2, x_1\bar{x}_2, \bar{x}_3\}$ - 2 simple characterizations, 1 double factorization

minterms \ max. monoms	max_1	max_2	max_3
m_0			*
m_2	*		*
m_3	*		
m_4		*	*
m_5		*	
m_6			*

$M(f) = \{max_1, max_2, max_3\}$

$C(f) = \{max_1, max_2, max_3\}$

all the columns have a ⊛, thus all the maximal monoms are central monoms $\Rightarrow M(f) = C(f)$

by the first case of simplification algorithm, we have an unique disjunctive simplified form:

$$f_8^S(x_1, x_2, x_3) = max_1 \vee max_2 \vee max_3 = \overline{x_1} x_2 \vee x_1 \overline{x_2} \vee \overline{x_3}$$