Exercise

8.8. Using Moisil's method, simplify the following Boolean function of 3 variables

$$f_8(x_1, x_2, x_3) = \underset{\bar{x}_1\bar{x}_2\bar{x}_3}{m_0} \vee \underset{\bar{x}_1 x_2 x_3}{m_3} \vee \underset{x_1\bar{x}_2\bar{x}_3}{m_4} \vee \underset{x_1 x_2\bar{x}_3}{m_6} \vee \underset{x_1 x_2 x_3}{m_7}$$

$S_f = \{(0,0,0), (0,1,1), (1,0,0), (1,1,0), (1,1,1)\}$ – support of f

First we order the support set in ascending order

$S_f = \{(0,0,0), (1,0,0), (1,1,0), (0,1,1), (1,1,1)\}$

The minterms from the function's expression are represented using the powers of variables, in a tableau, each minterm on a line, in ascending or descending order w.r.t the number of "1" values in the 3-tuples of the support of the function

		x_1	x_2	x_3	
I	✓	0	0	0	m_0
II	✓	1	0	0	m_4
III	✓	1	1	0	m_6
	✓	0	1	1	m_3
IV	✓	1	1	1	m_7
V = I + II		–	0	0	$m_0 \vee m_4 = \bar{x}_2\bar{x}_3 = max_1$
VI = II + III		1	–	0	$m_4 \vee m_6 = x_1\bar{x}_3 = max_2$
VII = III + IV		1	1	–	$m_6 \vee m_7 = x_1 x_2 = max_3$
		–	1	1	$m_3 \vee m_7 = x_2 x_3 = max_4$

The result of the factorization of the neighbor monoms from the neighbor groups is a new monom represented as a row at the end of the tableau.

The row contains the same values in the columns corresponding to the common variables of the adjacent monoms and the symbol "–" for the variable which is eliminated.

The rows corresponding to the neighboring monoms used in factorization are marked on the left side, meaning they are not maximal monoms

The set of maximal monoms obtained:

$$M(f) = \{\bar{x}_2\bar{x}_3, x_1\bar{x}_3, x_1x_2, x_2x_3\} = \{max_1, max_2, max_3, max_4\}$$

Because the simplified forms of a function contain only maximal monoms we consider the following propositional sentences:

"p_i: "max_i belongs to the simplified form of f", $i = 1, 2, .., 4$

Each minterm from the function's expression must be covered by a maximal monom in a simplified form, therefore according to the result of the factorization process we have the following true sentences:

"m_0 is covered by max_1" $\overset{\text{translated}}{\underset{\text{as}}{\Longrightarrow}}$ $p_1 \equiv T$

"m_4 is covered by max_1 or by max_2" $\Rightarrow$ $p_1 \vee p_2 \equiv T$

"m_6 is covered by max_2 or by max_3" $\Rightarrow$ $p_2 \vee p_3 \equiv T$

"m_3 is covered by max_4" translated as $p_4 \equiv T$

"m_7 is covered by max_3 or by max_4" $\Rightarrow$ $p_3 \vee p_4 \equiv T$

We model the following propositional formula, obtained as a conjunction of the previous true sentences:

$$p_1 \wedge (p_1 \vee p_2) \wedge (p_2 \vee p_3) \wedge p_4 \wedge (p_3 \vee p_4) \equiv T$$ (CNF with 5 clauses)

Through repeated use of distributive laws we get to:

$$T \equiv (p_1 \wedge p_1 \wedge p_2 \wedge p_4) \vee (p_1 \wedge p_2 \wedge p_2 \wedge p_4) \vee (p_1 \wedge p_1 \wedge p_3 \wedge p_4) \vee (p_1 \wedge p_2 \wedge p_3 \wedge p_4)$$

Apply idempotency:

$$T \equiv (p_1 \wedge p_2 \wedge p_4) \vee (p_1 \wedge p_2 \wedge p_4) \vee (p_1 \wedge p_3 \wedge p_4) \vee (p_1 \wedge p_2 \wedge p_3 \wedge p_4)$$

Apply absorbtion laws:

$$T \equiv (p_1 \wedge p_2 \wedge p_4) \vee (p_1 \wedge p_3 \wedge p_4)$$ (DNF with 2 cubes)

From the DNF we obtain the following simplified forms of f:

For the cube $(p_1 \wedge p_2 \wedge p_4) \equiv T$ the corresponding simplified form is:

$$f_1^s(x_1, x_2, x_3) = max_1 \vee max_2 \vee max_4 = \bar{x}_2\bar{x}_3 \vee x_1\bar{x}_3 \vee x_2x_3$$

For the cube $(p_1 \wedge p_3 \wedge p_4) \equiv T$ the corresponding simplified form is:

$$f_2^s(x_1, x_2, x_3) = max_1 \vee max_3 \vee max_4 = \bar{x}_2\bar{x}_3 \vee x_1x_2 \vee x_2x_3$$