42. Predicate logic

Choose an arbitrary interpretation with a finite domain for the formula U_2 and prove that it's a model of U_2

$$U_2 - (\forall x)(A(x) \to B(x)) \to ((\forall x) A(x) \to (\forall x) B(x))$$

Theoretical results:

→ A formula A is true under the interpretation i if for any assignment function $a \in AS(i)$, $v_a^i(A) = T$, notation: $\models_i A$, i is called model of A.

→ $v_a^i((\forall x)\ A(x)) = T$ if and only if $v_{a'}^i(A(x)) = T$, for any function $a' \in [a]_x$

→ $v_a^i(A \to B) = v_a^i(A) \to v_a^i(B)$

Let us consider the interpretation $i = \langle D, m \rangle$, where:

- $D = \{2, 5\}$ is the finite domain of the interpretation
- $m(A): \{2,5\} \to \{T, F\}$, $m(A)(x) = "x > 4"$
- $m(B): \{2,5\} \to \{T, F\}$, $m(B)(x) = "x > 1"$

$$v^i(U_2) = v^i((\forall x)(A(x) \to B(x))) \to v^i((\forall x) A(x) \to (\forall x) B(x))$$

$$= v^i((\forall x)(A(x) \to B(x))) \to v^i((\forall x) A(x)) \to v^i((\forall x) B(x))$$

To evaluate U_2 under the interpretation i, with $D = \{2, 5\}$, the universally quantified subformulas are replaced by the conjunction of their instances for $x = 2$ and $x = 5$.

$$= (2>4 \to 2>1) \land (5>4 \to 5>1) \to (2>4 \land 5>4) \to (2>1 \land 5>1)$$

$$= (F \to T) \land (T \to T) \to (F \land T) \to (T \land T)$$

$$= T \land T \to F \to T$$

$$= T \to T = T$$

So, i evaluates U_2 as true ⇒ i is a model of U_2

→ U_2 is consistent