Check if Theorem: $U_3 = (\forall x)(\forall y)\, P(x,y) \leftrightarrow (\exists x)(\forall y)\, P(x,y)$

$\vdash U_3$ if and only if $(\neg U_3)^C \vdash Pr_{Res}\ \square$

So, we study $\neg U_3$ and also replace $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$$\neg U_3 = \neg\Big((\forall x)(\forall y)\, P(x,y) \wedge (\exists x)(\forall y)\, P(x,y) \Big) \wedge \neg\big(\neg (\forall x)(\forall y)\, P(x,y)$$
$$\wedge \neg (\exists x)(\forall y)\, P(x,y) \big)$$

Apply De Morgan's infinitary law: $\big(\neg (\forall x) A(x) = \exists (x)\, \neg A(x) \big)$

$$\neg U_3 = \big((\exists x)(\exists y) \neg P(x,y) \vee (\forall x)(\exists y) \neg P(y,y) \big) \wedge \big((\forall x)(\forall y)\, P(x,y)$$
$$\wedge (\exists x)(\forall y)\, P(x,y) \big)$$

Replace the bound variables:

$$\neg U_3 = \big((\exists x)(\exists y) \neg P(x,y) \vee (\forall a)(\exists b) \neg P(a,b) \big) \wedge$$
$$\wedge \big((\forall c)(\forall d)\, P(c,d) \wedge (\exists z)(\forall t)\, P(z,t) \big)$$

Extract variables:

$$\neg U_3 = (\exists x)(\exists y)(\exists b)(\exists z)(\forall a)(\forall c)(\forall d)(\forall t)\left((\neg P(x,y) \vee \neg P(a,b)) \wedge (P(c,d) \wedge P(z,t))\right)$$

Prenex Form:

$$(\neg U_3)^P = (\exists x)(\exists y)(\exists b)(\exists z)(\forall a)(\forall c)(\forall d)(\forall t)$$
$$\left((\neg P(x,y) \vee \neg P(a,b)) \wedge (P(c,d) \wedge P(z,t))\right)$$

Skolem constants: m, n, r, p

$[x \leftarrow m]$, $[y \leftarrow n]$, $[b \leftarrow r]$, $[z \leftarrow p]$

Skolem form:

$$(\neg U_3)^S = (\forall a)(\forall c)(\forall d)(\forall t)\big((\neg P(m,n) \vee \neg P(a,r)) \wedge$$
$$\wedge (P(c,d) \wedge P(p,t))\big)$$

Clausal Normal Form:

$$(\neg U_3)^C = (\neg P(m,n) \vee \neg P(a,r)) \wedge (P(c,d) \wedge P(p,t))$$

Set of clauses:

$$S_3 = \{ C_1 = \neg P(m,n) \vee \neg P(a,r);\ C_2 = P(c,d);\ C_3 = P(p,t) \}$$

Resolvents:

$$C_4 = Res^{Pr}_{[c,d \leftarrow m,n]}(C_1, C_2) = \neg P(a,r)$$

$$C_5 = Res^{Pr}_{[a,t \leftarrow p,r]}(C_1, C_3) = \square$$

$$\Rightarrow (\neg U_3)^C = \vdash^{Pr}_{Res} \square \text{, therefore } \vdash U_3$$