A RST DESIGN APPROACH FOR THE LAUNCHERS FLIGHT CONTROL SYSTEM

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ABSTRACT

This article is aiming to introduce a new design methodology for the automatic flight control system of launch vehicles, using discrete-time RST controllers. Regardless the employed controller, the structure of such a system has three degrees of freedom (roll, yaw, pitch). They are determined such that the closed-loop dynamics of the launch vehicle tracks the output of a desired reference model. The RST control technique focuses on the pitch angle, as the roll and yaw angles are tracking their references much easier.

***Index Terms* —** Launch vehicles**,** automatic flight control, RST approach.

1. INTRODUCTION

The increase and the diversification of the space missions require launch vehicles capable of planning a wide variety of solitary or formations of satellites into prescribed Earth orbits. The goals of making the access to space cheaper and more accurately generated a growing interest for investigating alternative solutions for the launchers automatic flight control systems design. This interest is also encouraged by the new control techniques developed over the last few decades, which seemingly are able to accomplish a wide variety of control requirements. Among them, the control engineer must take into account the inherent instability of the launch vehicle, the fast change of its dynamics during its evolution towards the upper layers of the atmosphere, the presence of structural flexible effects, modeling uncertainties and the atmospheric disturbances influence. The automatic flight control system is design to ensure the launcher robust stability and tracking performances of the desired trajectory during all flight phases. Many modern design approaches have been used and tested from the perspective of the above mentioned particularities. Among them one mentions the optimal techniques based on the systems norms minimization (e.g. [1], [2], [3]), robust control ([4], [5], [6], [7]), nonlinear control ([8], [9], [10]), neural and fuzzy control ([11], [12]). The complexity of resulting control system is an important aspect to be taken into account when choosing a design technique. A moderate complexity is highly recommended, in order to alleviate the control laws implementation.

The goal of this article is to describe a design method for the automatic flight control system of a launcher based on the RST control configuration. The RST controllers have been introduced in the late '80s [13] as a generalization of the well known PID controllers. Their synthesis is performed in discrete time, by several techniques. Although the one employed within this article is quite simple and effective, the resulted RST controller (<u>RSTc</u>) is able to meet stability robustness and tracking requirements. One of the RST method advantages consists in its capability to get a low order solution for the automatic flight control system.

The paper is organized as follows: after the introductory part, the second section presents the design models (after linearization) describing the dynamics and kinematics of the launcher, together with the design objectives concerning the control system. The design methodology of the RSTc is presented in the third section. The theoretical developments are demonstrated and analyzed through numerical examples in Section 4. Some concluding remarks and future developments of the proposed approach complete the article.

2. LAUNCHER MODEL AND CONTROL OBJECTIVES

The design methodology described in this paper addresses to the automatic flight control system of a VEGA-like launcher, controlled by a Thrust Vector Control (<u>TVC</u>) system. The linearized model of the pitch motion, in absence of flexible modes, can be expressed in state space representation (<u>SSR</u>) form below (with usual notations):

$$\begin{cases} \dot{\mathbf{x}} \equiv \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \ddot{z} \end{bmatrix} \equiv \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ a_6 & 0 & \dfrac{a_6}{V} \\ -a_1 & 0 & -a_2 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{z} \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 0 \\ -k_1 \\ -a_3 \end{bmatrix}}_{\mathbf{b}} \underbrace{\delta}_{u} + \underbrace{\begin{bmatrix} 0 \\ -a_6 \\ a_2 V \end{bmatrix}}_{\mathbf{e}} \underbrace{\alpha_w}_{w} \\ y \equiv x_1 \equiv \theta \end{cases}, \quad (1)$$

where θ is the pitch angle, $\dot{z}$ is the lateral drift speed, V is the launcher velocity, the control δ stands for the gimbals deflection angle of the TVC system, while α_w is the wind incidence (perturbation). The coefficients a_1, a_2, a_3, a_6, k_1 and V are assumed to be piecewise constant during the ascent of the first three stages of launcher.

Of course, the model (1) has been derived starting from the analytic equations describing both the dynamics and the kinematics of the VEGA-like launcher. Similar models were derived for yaw and roll angles.

The design objectives of the automatic flight control are the following (for each angle): (a) tracking some prescribed trajectory of the pitch angle (θ); (b) rejecting step-like perturbations for reasonable step heights (no more than few hundreds); (c) rejecting stochastic perturbations for reasonable values of signal-to-noise (SNR) ratio (at least 30 dB); (d) ensuring a reasonable steady-state response delay (no more than 3 s). The robustness of RSTc usually results from the above objectives, although the design method does not focus on such a property.

3. RST DESIGN THROUGH BÉZOUT IDENTITY

The RSTc is mostly employed in case an input-output (I/O) plant model is available. Nevertheless, the I/O model can be adapted to SSR, as it will be shown later. The basic diagram of RSTc looks like in Figure 1.

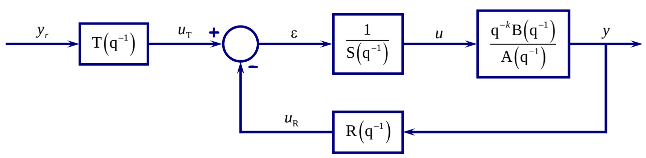

Figure 1. *RST controller structure.*

Thus, three blocks are composing the command u:

- the *regulation* FIR filter:

$$\mathrm{R}(q^{-1}) = r_0 + r_1 q^{-1} + r_2 q^{-2} + \cdots + r_{nr} q^{-nr}, \tag{2}$$

- the *sensitivity* IIR filter:

$$\frac{1}{\mathrm{S}(q^{-1})} = \frac{1}{s_0 + s_1 q^{-1} + s_2 q^{-2} + \cdots + s_{ns} q^{-ns}}, \tag{3}$$

- the *tracking* FIR filter:

$$\mathrm{T}(q^{-1}) = t_0 + t_1 q^{-1} + t_2 q^{-2} + \cdots + t_{nt} q^{-nt}, \tag{4}$$

where q^{-1} is the one step delay operator.

The plant model is expressed in rational form:

$$\mathrm{H}(q^{-1}) = \frac{q^{-k}\mathrm{B}(q^{-1})}{\mathrm{A}(q^{-1})}, \tag{5}$$

where $k \in \mathbb{N}^*$ is the intrinsic delay, whilst:

$$\begin{cases} \mathrm{B}(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \cdots + b_{nb} q^{-nb} \quad (b_0 \neq 0) \\ \mathrm{A}(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_{na} q^{-na} \end{cases}. \tag{6}$$

In order to design the RSTc, a desired behavior of closed loop (CL) system has to be set. Usually, a second order continuous system plays the role of dominant model. Some parasite model might complete the dominant one, in order to match the plant structure. More specifically, the dominant transfer function can be expressed as:

$$\mathrm{H_c}(s) = \frac{\omega_0(s+\omega_0)}{s^2 + 2\varsigma\omega_0 s + \omega_0^2}, \tag{7}$$

where $\omega_0 > 0$ is the natural (oscillation) pulsation and $\varsigma > 0$ is the damping factor. The two parameters allow the user to specify some desired performances in terms of system overshoot and steady-state delay of step response.

Since the RSTc works in discrete time, the desired model (7) has to be discretized, for example by means of bilinear (Tustin's) method:

$$s = \frac{2}{T_s}\frac{1 - z^{-1}}{1 + z^{-1}}, \tag{8}$$

where $T_s > 0$ is the sampling period. When inserting the conform transformation (8) into definition (7), one obtains:

$$\mathrm{H}_{T_s}(z^{-1}) = \frac{\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2}}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}. \tag{9}$$

The coefficients in (9) can straightforwardly be computed from the coefficients of system (7).

Now, the problem is to design the RSTc such that the CL system mainly behaves like a delayed second order filter, with the system function given by:

$$\mathrm{H}_d(q^{-1}) = q^{-k}\mathrm{H}_{T_s}(q^{-1})\mathrm{H}_p(q^{-1}) = \frac{q^{-k}\mathrm{B}_d(q^{-1})}{\mathrm{A}_d(q^{-1})}, \tag{10}$$

where H_p is a possible parasite transfer function, including zeros and poles that modify to some extent the dominant behavior (H_p could be unit). The final term introduces polynomials A_d and B_d of degrees $n\alpha \geq 2$ and $n\beta \geq 2$, respectively, to be determined. Usually, A_d and B_d are coprime polynomials ($(\mathrm{A}_d, \mathrm{B}_d) = 1$).

When looking again at Figure 1, one derives that the CL behavior is modeled by the following system function:

$$\mathrm{H_{CL}}(q^{-1}) = \frac{q^{-k}\mathrm{B}(q^{-1})\mathrm{T}(q^{-1})}{\mathrm{A}(q^{-1})\mathrm{S}(q^{-1}) + q^{-k}\mathrm{B}(q^{-1})\mathrm{R}(q^{-1})}, \tag{11}$$

whereas the RSTc is described by:

$$\mathrm{H_{RST}}(q^{-1}) = \frac{\mathrm{A}(q^{-1})\mathrm{T}(q^{-1})}{\mathrm{A}(q^{-1})\mathrm{S}(q^{-1}) + q^{-k}\mathrm{B}(q^{-1})\mathrm{R}(q^{-1})}. \tag{12}$$

Let Q and P be the numerator and the denominator of system function (11), respectively (where Q does not include the delay q^{-k}). Conspicuously, $\mathrm{H_{CL}}$ and $\mathrm{H_{RST}}$ have the same poles (given by the roots of P). Then the CL system (11) has to match the desired system (10):

$$\frac{q^{-k}Q\left(q^{-1}\right)}{P\left(q^{-1}\right)}=\frac{q^{-k}B_d\left(q^{-1}\right)}{A_d\left(q^{-1}\right)} \Leftrightarrow \frac{Q\left(q^{-1}\right)}{P\left(q^{-1}\right)}=\frac{B_d\left(q^{-1}\right)}{A_d\left(q^{-1}\right)}. \quad (13)$$

The P polynomial (of degree np) plays the major role in the RSTc design, starting from equation (13). Since $n\alpha$ and $n\beta$ are bounded by the degrees of corresponding polynomials in the left side of (13), the polynomials P and Q are not coprime and some couples {zero, pole} have to be canceled from the CL transfer function. Obviously, one aims to work parsimoniously, i.e. with as less polynomial degrees as possible. Beside the design equation (13), one wants that the RSTc is stable and physically realizable, after conversion to continuous time. This involves that all the poles of RSTc (12) have to be inside the unit disk, whilst the numerator degree is at most equal to the denominator degree. All the requirements above lead to:

$$\begin{cases} ns = nb + k + p - 1 \\ nr = na + p - 1 \end{cases}, \quad (14)$$

where $p \in \mathbb{N}$ is an offset to be determined such that the RSTc becomes physically realizable.

As a direct consequence of equation (13), one can set:

$$T\left(q^{-1}\right)=\frac{P\left(q^{-1}\right)}{A_d\left(q^{-1}\right)}\cdot\frac{B_d\left(q^{-1}\right)}{B\left(q^{-1}\right)}. \quad (15)$$

Since T is a polynomial, all poles of expression (15) have to be cancelled. At a first sight, the only polynomial that can perform the cancellation is P, as the other ones are already specified. Thus, P can be expressed as:

$$P\left(q^{-1}\right)=B\left(q^{-1}\right)\cdot A_d\left(q^{-1}\right)\cdot\hat{P}\left(q^{-1}\right), \quad (16)$$

where $\hat{\mathrm{P}}$ is another unknown polynomial, of degree $na - n\alpha + k + p - 1 \geq 0$ (see equations (14)).

Since P contributes to the denominator $\mathrm{H}_{\mathrm{RST}}$ (see equation (12)), it can only include stable roots. Moreover, all poles of CL system, given by P as well (see equation (11)), are required to be stable. However, unlike A_d, the B polynomial is already set and cannot be modified. Or, B could have one or more unstable roots (perhaps all of them are unstable). Therefore, P cannot have the whole B as divisor, but only its stable part. From the RSTc point of view, the unstable zeros of the plant are uncontrollable. Fortunately, such zeros are not affecting the CL stability, but they could lower the CL system performances.

Assume B is expressed as $\mathrm{B} \equiv \mathrm{B}_s\mathrm{B}_u$, where B_s is stable and B_u is unstable. Then, the expression (15) of tracking polynomial becomes:

$$T\left(q^{-1}\right)=\frac{P\left(q^{-1}\right)}{A_d\left(q^{-1}\right)B_s\left(q^{-1}\right)}\cdot\frac{B_d\left(q^{-1}\right)}{B_u\left(q^{-1}\right)}, \quad (17)$$

whereas the equation (16) is replaced by:

$$P\left(q^{-1}\right)=B_s\left(q^{-1}\right)\cdot A_d\left(q^{-1}\right)\cdot\hat{P}\left(q^{-1}\right). \quad (18)$$

While P cancels the denominator in (17), the B_d polynomial has to cancel the B_u polynomial. Thus, the unstable zeros of plant have to be included into the desired behavior of the CL system, as the RSTc is unable to cancel them. Hereafter, for the sake of simplicity, one considers that $n\beta$ is the degree of $\mathrm{B}_d/\mathrm{B}_u$ polynomial (re-noted by B_d). It follows that:

$$T\left(q^{-1}\right)=\hat{P}\left(q^{-1}\right)B_d\left(q^{-1}\right), \quad (19)$$

with $nt = na - n\alpha + n\beta + k + p - 1$. As already mentioned before, the RSTc has to be physically realizable, so that, from (12), it results: $na + nt \leq na + nb + k + p - 1$. In order to work with quality tracking filter, one can raise the degree of T polynomial to the maximum (i.e. $nt = nb + k + p - 1 = ns$, according to equations (14)), which, together with definition (19), leads to an interesting requirement: $na - nb = n\alpha - n\beta$, which actually is crucial for solving the design problem. Thus:

a. if $n\beta \leq nb + k - 1$, then $p = 0$, while B_d and (maybe) A_d are augmented with parasite roots, so that $\hat{\mathrm{P}}$ becomes unit and the requirement is met; also, in this case, it is easy to derive that: $nr = na - 1$ and $ns = nt = nb + k - 1$;
b. otherwise, one can choose $p = n\beta - nb - k + 1$, so that, again, $\hat{\mathrm{P}}$ becomes unit , while B_d and/or A_d might need augmentation, in order to meet the requirement; now: $nr = na + n\beta - nb - k = n\alpha - k$ and $ns = nt = n\beta$.

Augmentation of a polynomial, say C, with m parasite roots (while keeping constant the gain $\mathrm{C}(1)$) is performed by a simple technique. One single real pole, say $z_a \in \mathbb{R}$ with $0 \cong |z_a| < 1$ (around the origin of complex plane), but of multiplicity equal to m, can be set. Then:

$$C\left(q^{-1}\right) \leftarrow C\left(q^{-1}\right)\frac{\left(1 - z_a z^{-1}\right)^m}{\left(1 - z_a\right)^m}. \quad (20)$$

Anyway, the equation (16) becomes:

$$\begin{aligned} P\left(q^{-1}\right) &= A\left(q^{-1}\right)S\left(q^{-1}\right) + q^{-k}B\left(q^{-1}\right)R\left(q^{-1}\right) = \\ &= A_d\left(q^{-1}\right)B_s\left(q^{-1}\right) \end{aligned} \quad (21)$$

and $np = na + nb + k - 1$. It has to be outlined that the genuine polynomial B contributes to the definition of P on the left side of equation (21), whilst the polynomial B_s appears on the right side of equation (21). They are not the same, unless all the plant zeros are stable.

The unknown polynomials R and S can now be determined from Bézout identity (21), where the right side term is completely known. The equation can be expressed in compact form, if the unknown coefficients of R and S are packed into a vector: $\boldsymbol{\xi} = \left[s_0 \cdots s_{ns} \,\middle|\, r_0 \cdots r_{nr} \right]^T$. After some elementary manipulations, the equation (21) can be expressed as a linear system:

$$\mathbf{S}_{\mathrm{A,B},k} \boldsymbol{\xi} = \boldsymbol{\psi} , \tag{22}$$

where $\mathbf{S}_{\mathrm{A,B},k} \in \mathbb{R}^{(np+1)\times(np+1)}$ is the Sylvester matrix of polynomials A and $\mathrm{q}^{-k}\mathrm{B}$, whilst $\boldsymbol{\psi} \in \mathbb{R}^{np+1}$ is the vector of polynomial coefficients computed from the product $\mathrm{A}_d \mathrm{B}_s$.

In order to express both the Sylvester matrix and the polynomial product, it is useful to introduce the following *step descending matrix*, constructed for a polynomial C of degree nc and a given number of columns, m:

$$\mathbf{S}_{\mathrm{C},m} = \underbrace{\begin{bmatrix} c_0 & 0 & \cdots & 0 & 0 \\ c_1 & c_0 & \cdots & 0 & 0 \\ c_2 & c_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & c_{nc-1} & c_{nc-2} \\ 0 & 0 & \cdots & c_{nc} & c_{nc-1} \\ 0 & 0 & \cdots & 0 & c_{nc} \end{bmatrix}}_{m} \in \mathbb{R}^{(nc+m)\times m} . \tag{23}$$

It is easy to prove that the product of two polynomials, say C of degree nc and D of degree nd, leads to a polynomial given by $\mathbf{S}_{\mathrm{C},nd+1}\mathbf{d}$ (where $\mathbf{d}$ is the vector of D coefficients).

The Sylvester matrix of system (22) is then:

$$\mathbf{S}_{\mathrm{A,B},k} = \left[\mathbf{S}_{\mathrm{A},nb+k} \;\middle|\; \begin{matrix} \mathbf{0}_{k\times na} \\ \mathbf{S}_{\mathrm{B},na} \end{matrix} \right] \in \mathbb{R}^{(na+nb+k)\times(na+nb+k)} , \tag{24}$$

whereas $\boldsymbol{\psi} = \mathbf{S}_{\mathrm{A}_d, nb_s+1} \cdot \mathbf{b}_s \in \mathbb{R}^{n\alpha + nb_s + 1}$ (with natural notations). Obviously, A_d has to be adjusted such that the size of $\boldsymbol{\psi}$ (i.e. $n\alpha + nb_s + 1$) equals $np + 1 = na + nb + k$.

James Joseph Sylvester proved in 1851 [14] that the matrix $\mathbf{S}_{\mathrm{A,B},k}$ is invertible if and only if the polynomials A and $\mathrm{q}^{-k}\mathrm{B}$ are coprime ($\left(\mathrm{A}, \mathrm{q}^{-k}\mathrm{B}\right) = 1$). Therefore, before constructing the Sylvester matrix, all mutual zeros and poles of the two polynomials have to be cancelled. Then:

$$\boldsymbol{\xi} = \mathbf{S}_{\mathrm{A,B},k}^{-1} \boldsymbol{\psi} . \tag{25}$$

The coefficients of the polynomials R and S are straightforwardly recovered from solution (25):

$$s_j = \xi_{j+1} ,\ \forall\, j \in \overline{0, ns} \quad \& \quad r_i = \xi_{ns+2+i} ,\ \forall\, i \in \overline{0, nr} . \tag{26}$$

Finally, according to equation (19), it follows:

$$\mathrm{T}\left(\mathrm{q}^{-1}\right) = \mathrm{B}_d\left(\mathrm{q}^{-1}\right) . \tag{27}$$

The RSTc transfer function to implement is then the following (with two inputs (y_r , y) and one output (u)):

$$\mathrm{H}_{\mathrm{RST}}\left(\mathrm{q}^{-1}\right) = \begin{bmatrix} \dfrac{\mathrm{T}\left(\mathrm{q}^{-1}\right)}{\mathrm{S}\left(\mathrm{q}^{-1}\right)} & \dfrac{-\mathrm{R}\left(\mathrm{q}^{-1}\right)}{\mathrm{S}\left(\mathrm{q}^{-1}\right)} \end{bmatrix} . \tag{28}$$

In the end, the RSTc transfer function (28) has two features: (a) it could exhibit some redundancy degree; (b) it could include some parasite zeros and poles. Therefore, it is suitable to cancel all mutual zeros and poles, in order to keep the system size as small as possible. Moreover, some of the parasite zeros can be removed, in order to smooth the command signal. The described RSTc design method (also known as *the poles placement method*) is not unique, but probably is the simplest.

To conclude this section, the problem of accommodation between VEGA-like models and RSTc is shortly addressed. The following strategy was adopted to solve this problem:

1. Convert the continuous time SSR of VEGA-like model to continuous time transfer function.
2. Discretize the transfer function by setting the sampling period to $T_s = 0.01$ s.
3. Extract the numerator and denominator of the resulted transfer function, and set the intrinsic delay to the difference between the denominator number of roots and the numerator number of roots. If the difference is null, set the delay to unit.
4. Set the natural pulsation ω_c and the dumping factor ς for the second order continuous system to match.
5. Construct the numerator and the denominator of the desired continuous time transfer function.
6. Construct the desired continuous time transfer function.
7. Discretize the transfer function by setting the sampling period to $T_s = 0.01$ s.
8. Extract the numerator and denominator of the resulted transfer function.
9. Preset the intrinsic delay of the desired system to the plant intrinsic delay.
10. Design the RSTc.
11. Convert both the discrete time transfer function and the SSR of RSTc to continuous time objects.
12. Remove all mutual zeros and poles from the transfer function and reduce the size of SSR, if necessary.

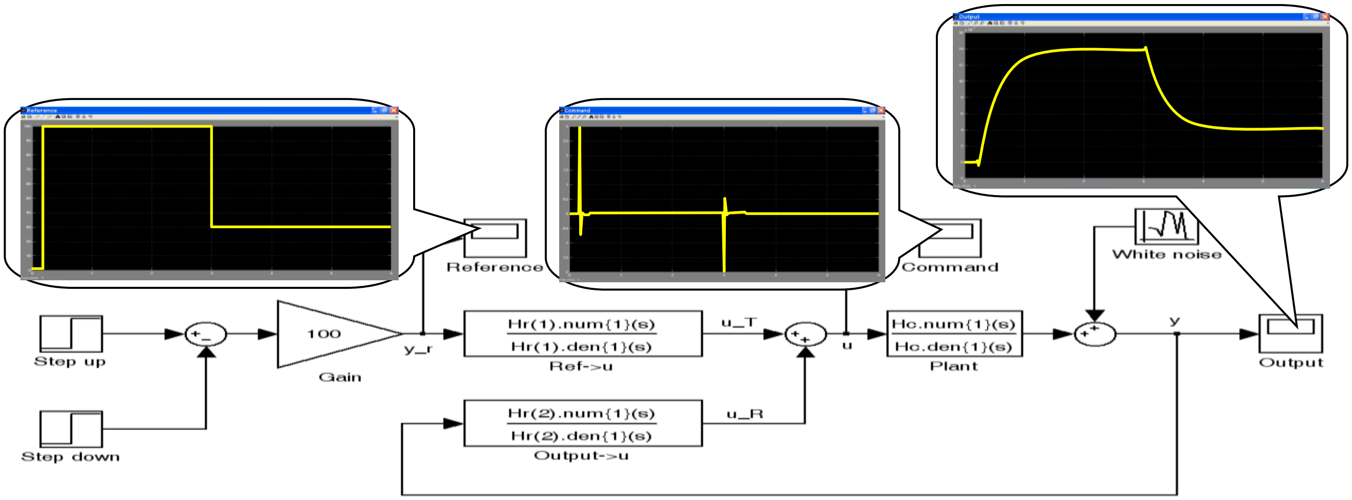

Figure 2. *SIMULINK CL model to test RST controller robustness in case of stochastic perturbations corrupting the output.*

4. ANALYSIS OF SIMULATION RESULTS

The reference signal y_r actually is a prescribed trajectory of launcher pitch angle θ, from the launching instant to the moment the launcher shuts off its engines (outside the terrestrial atmosphere). The continuous time VEGA-like models are associated to segments of that trajectory, not necessarily equal in duration.

Before testing the RSTc in a real simulation, the robustness of the CL system to stochastic perturbations has been experimentally determined with the help of SIMULINK model depicted in Figure 2. Note that the RSCc design procedure in this article did not refer to robustness at all. This seemingly is an intrinsic feature of RSTc.

The insets into the Figure 2 display the reference signal (on the left), the command signal (in the middle) and the CL output signal (on the right), in absence of stochastic noise. Such variations are typical and repeat (more or less) for each trajectory segment.

Focus now on a specific segment in the launcher dynamics, e.g. the one starting at 55 s into the flight. In this case, the SSR model (1) is determined by:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 3.2297 & 0 & 0.0058 \\ -37.87 & 0 & -0.0273 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ -7.0738 \\ -25.54 \end{bmatrix}, \quad (29)$$

which corresponds to the discrete time transfer function:

$$H\left(q^{-1}\right) = 10^{-6}\,\frac{-176.9 - 177q^{-1} + 176.7q^{-2} + 176.8q^{-3}}{1 - 3q^{-1} + 3q^{-2} - 0.9997q^{-3}}, \quad (30)$$

as computed by using Tustin method and $T_s = 0.01$ s. The dominant desired CL system is defined by $\omega_0 = 1.2$ rad and $\varsigma = 0.5$. Then, the RSTc design procedure returns:

$$\begin{cases} R\left(q^{-1}\right) = 10^{3}\left(675.4 - 1349.6q^{-1} + 673.9q^{-2}\right) \\ S\left(q^{-1}\right) = 119.4469 + 239.1438q^{-1} + 120.1939q^{-2} \\ T\left(q^{-1}\right) = -419.4736 \end{cases}. \quad (31)$$

Obviously, when comparing the numerical values of coefficients above to the sampling period, one can notice that high accuracy of coefficients representation is needed.

While running the model of Figure 2 in this framework, the noise energy is increased, in order to reach for the lowest SNR bound still allowing the RSTc to work properly. The simulation results in Figure 3 reveal the RSTc behavior in case several white noise stochastic perturbations corrupt the CL trajectory.

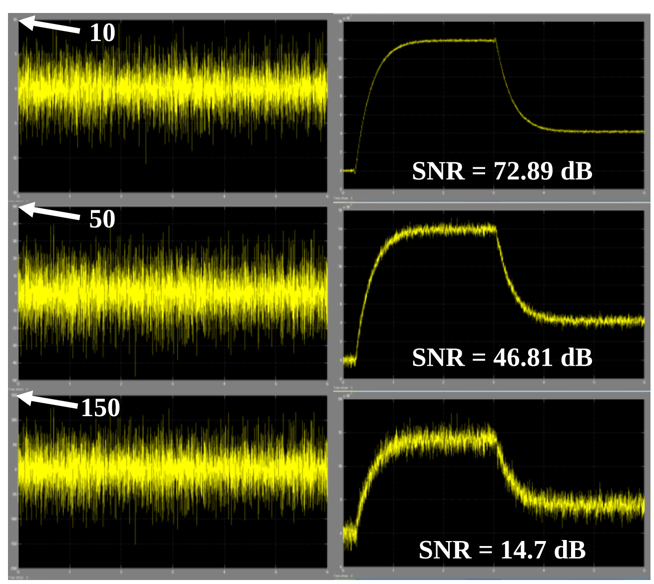

Figure 3. *Noisy command (left) and output (right) signals.*

According to the above simulations, a minimum of 50 dB SNR attitude sensor is required, in order to obtain good enough tracking performance.

Finally, the design method was tested on the non-linear model of the VEGA-like launcher, in a real simulation. To cover the powered flight corresponding to solid fuel stages, the evolution was split into 20 non-equal segments. One RSTc was designed for each nominal points between successive segments. At any time, the current controller is computed by interpolating between nominal adjacent controllers. The desired trajectory corresponds to a launch in a 550 km circular orbit with 45^o inclination. Evolution of the attitude angle and generated commands are given in Figures 4 and 5, respectively. One can see that the RSTc is able to track the trajectory, even when changing, by sending reasonable commands.

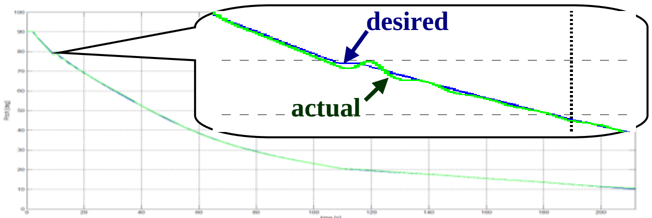

Figure 4. *Desired versus actual pitch angle trajectories.*

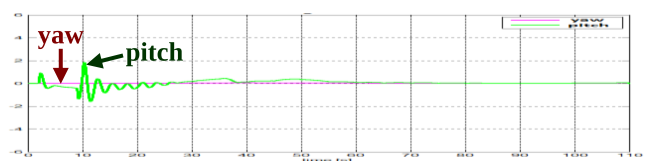

Figure 5. *First stage pitch and yaw TVC commands.*

5. CONCLUDING REMARKS AND FUTURE WORK

The control technique introduced in this article is based upon the RST structure. Although the RST controller is not meant to be optimal (according to some cost function), it behaves quite well, both in terms of robustness and trajectory tracking. The main advantage of the described RST design method is the simplicity. Nevertheless, one assumes that better controllers (like e.g. from Guardian Maps or LQR-LQG classes) could overtake the performance of this controller, although at the expense of design method complexity increase. Moreover, the RST design itself can be performed by more sophisticated methods, which take into account robustness and tracking as important requirements.

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