Chapter 15

Seasonal $ARIMA(p,d,q)(P,D,Q)_s$

Time series having a trend and/or a seasonal pattern are not stationary in mean. We extend ARMA(p,q) models in section 14.2 to allow removing a trend before fitting an ARMA model. Section 15.1 extends further these new models to allow seasonal pattern to be modelled.

15.1 Seasonal $ARIMA(p,d,q)(P,D,Q)_s$

As things stand, ARIMA models cannot really cope with seasonal behaviour; we see that, compared to ARMA models, ARIMA(p,d,q) only models time series with trends. We will incorporate now seasonal behaviour and present a general definition of the Seasonal ARIMA models.

1.1 Definition (Seasonal Autoregressive integrated moving average : $ARIMA(p,d,q)(P,D,Q)_s$) Seasonal ARIMA models are defined by 7 parameters $ARIMA(p,d,q)(P,D,Q)_s$

$$\underbrace{(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)}_{AR(p)}\ \underbrace{(1-\beta_1 B^s-\beta_2 B^{2s}-\cdots-\beta_P B^{Ps})}_{AR_s(P)}\ \underbrace{(1-B)^d}_{I(d)}\underbrace{(1-B^s)^D}_{I_s(D)}\ y_t = \\ c+\underbrace{(1-\psi_1 B-\psi_2 B^2-\cdots-\psi_q B^q)}_{MA(q)}\ \underbrace{(1-\theta_1 B^s-\theta_2 B^{2s}-\cdots-\theta_Q B^{Qs})}_{MA_s(Q)}\ \epsilon_t \qquad (15.1)$$

where

- $AR(p)$ Autoregressive part of order p
- $MA(q)$ Moving average part of order q
- $I(d)$ differencing of order d
- $AR_s(P)$ Seasonal Autoregressive part of order P
- $MA_s(Q)$ Seasonal Moving average part of order Q
- $I_s(D)$ seasonal differencing of order D
- s is the period of the seasonal pattern appearing i.e. $s=12$ months in the Australian beer production data.

The idea behind the seasonal ARIMA is to look at what are the best explanatory variables to model a seasonal pattern. For instance lets consider the Australian beer production that shows a seasonal pattern of period 12 months. Then to predict the production at time t, y_t, the explanatory variables to consider are:

$$y_{t-12}, y_{t-24}, \cdots, \text{and / or } \epsilon_{t-12},\ \epsilon_{t-24}, \cdots$$

For seasonal data, it might also make sense to take differences between observations at the same point in the seasonal cycle i.e. for monthly data with annual cycle, define differences (D=1)

$$y_t - y_{t-12}.$$

or (D=2)

$$y_t - 2y_{t-12} + y_{t-24}.$$

15.2 Using ACF and PACF to identify seasonal ARIMAs

You can use ACF and PACF to identify P or Q:

- For $ARIMA(0,0,0)(P,0,0)_s$, you should see major peaks on the PACF at s, $2s$, ...Ps. On the ACF, the coefficients at lags s, $2s$, ...Ps, ... should form an exponential decrease, or a damped sine wave. See examples figures 15.1 and 15.2.
- $ARIMA(0,0,0)(0,0,Q)_s$, you should see major peaks on the ACF at s, $2s$, ...Qs. On the PACF, the coefficients at lags s, $2s$, ...Qs,... should form an exponential decrease, or a damped sine wave. See examples figures 15.3 and 15.4.

When trying to identify P or Q, you should ignore the ACP and PACF coefficients other than s, $2s$, ...Ps,.. or s, $2s$, ...Qs,.... In other word, look only at the coefficients computed for multiples of s.

15.3 How to select the best Seasonal ARIMA model?

It is sometimes not possible to identify the parameters p,d,q and P,D,Q using visualisation tools such as ACF and PACF. Using the BIC as the selection criterion, we select the ARIMA model with the lowest value of the BIC. Using the AIC as the selection criterion, we select the ARIMA model with the lowest value of the AIC.

15.4 Conclusion

We have now defined the full class of statistical models $ARIMA(p,d,q)(P,D,Q)_s$ studied in this course. ARMA(p,q) can only be applied to time series stationnary in mean, hence the extension to $ARIMA(p,d,q)(P,D,Q)_s$ (introducing d,D,P,Q,s) allowed us to make the time series stationary in mean. Unfortunatly, we still are not able to deal with time series that are not stationary in variance. We propose some possible solutions in the next chapter.

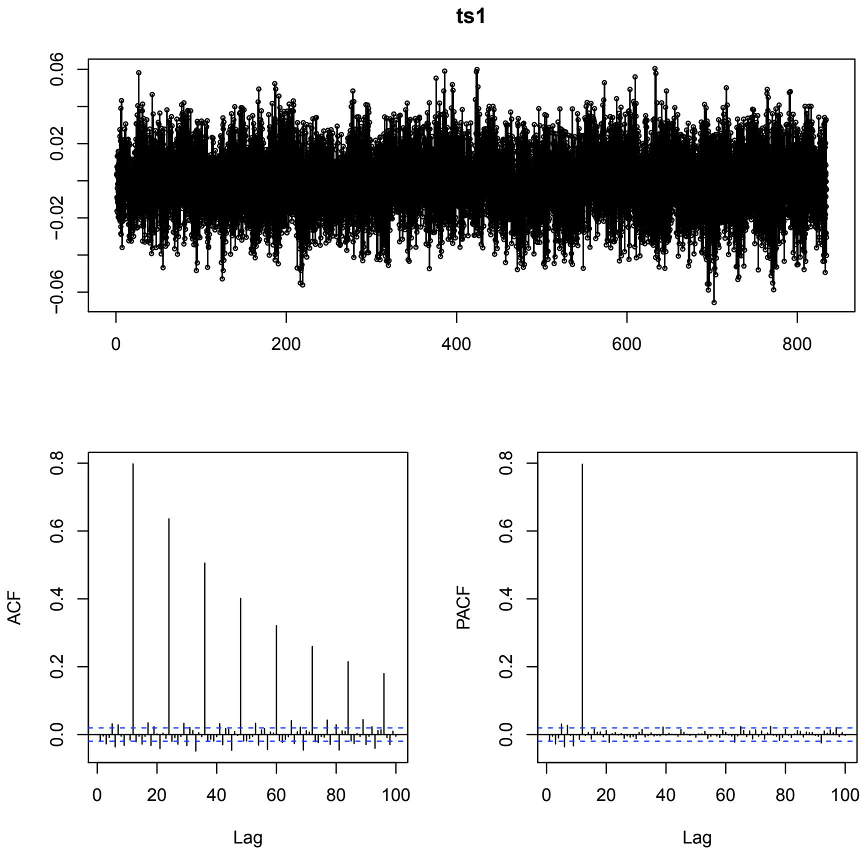

Figure 15.1: Simulation $ARIMA(0,0,0)(1,0,0)_{12}$

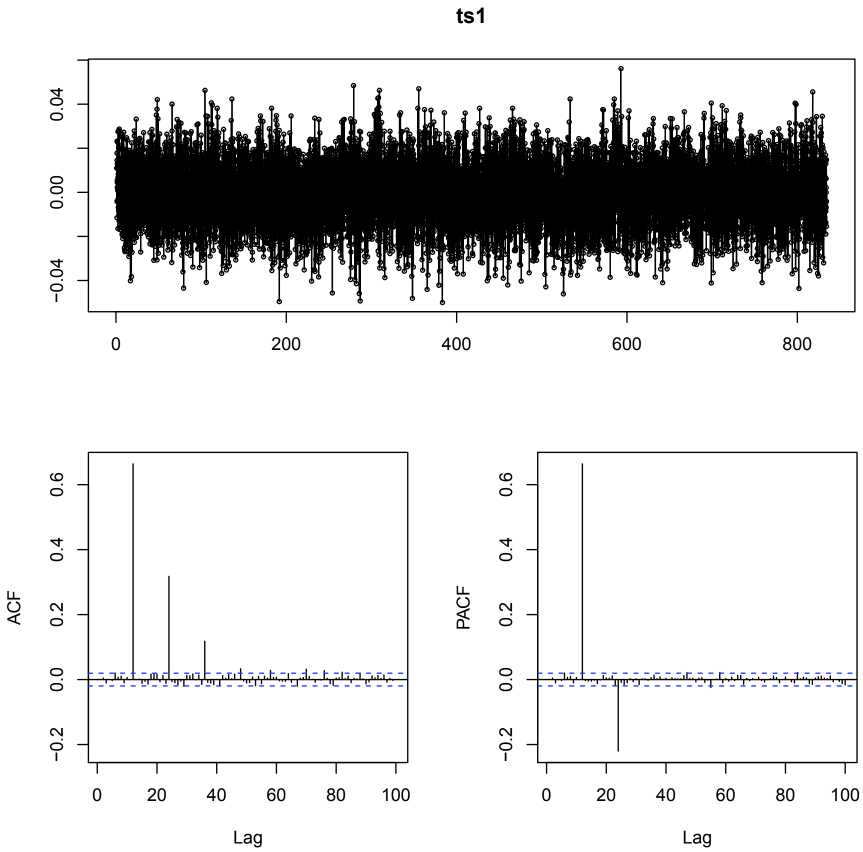

Figure 15.2: Simulation $ARIMA(0,0,0)(2,0,0)_{12}$

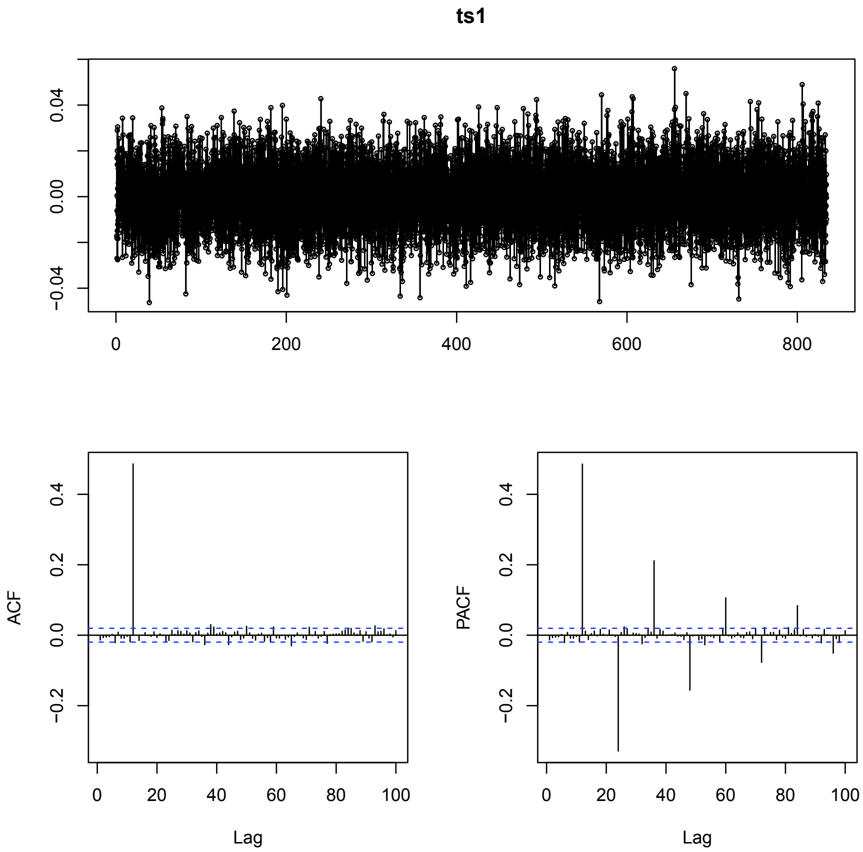

Figure 15.3: Simulation $ARIMA(0,0,0)(0,0,1)_{12}$

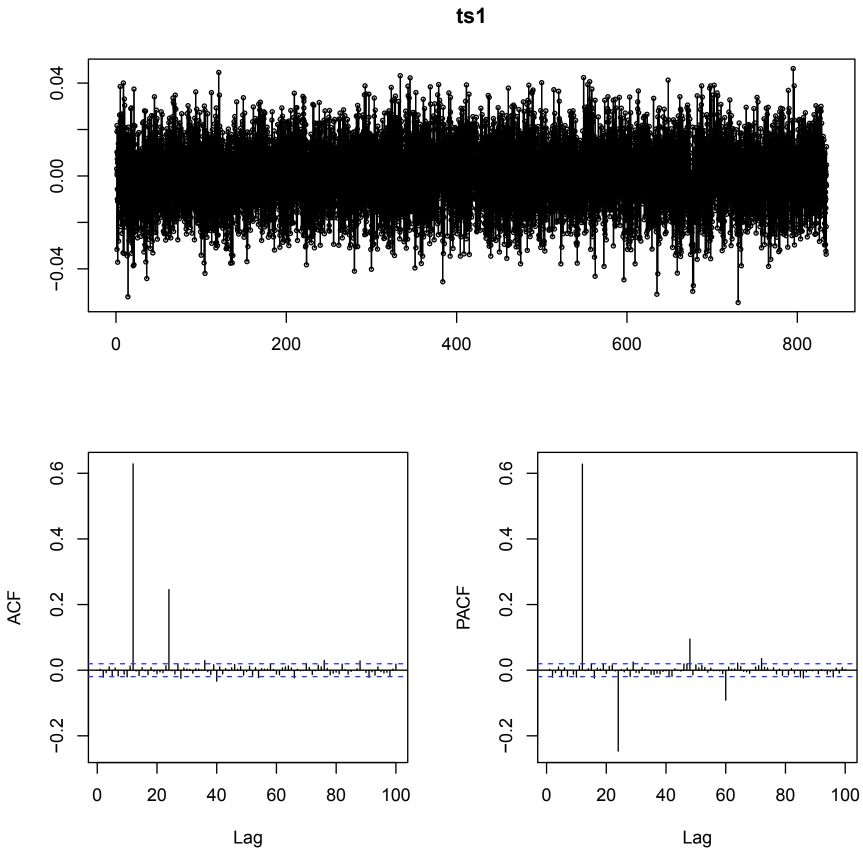

Figure 15.4: Simulation $ARIMA(0,0,0)(0,0,2)_{12}$