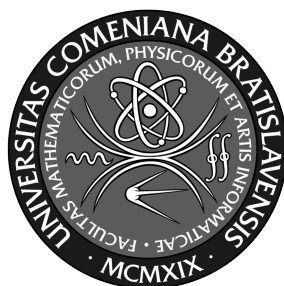


Comenius university in bratislava
Faculty of Mathematics, Physics and Informatics

**ABDUCTION SOLVER BASED ON HIGHLY
EFFICIENT C++ DL REASONER**

Master thesis

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Master thesis

Study programme: Applied informatics
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todo:zadanie

Čestne prehlasujem, že túto diplomovú prácu som
vypracoval samostatne len s použitím uvedenej literatúry
a za pomoci konzultácií u môjho školiteľa.

Bratislava, 2017

.....

Bc. Drahomír Mrózek

Acknowledgments

TODO

Abstract

This work will start from an existing solution of an abduction solver based on Pellet reasoner for description logics, implemented in Java. The implementation in C++ opens different ways for improvement and effectivization, starting with the utilization of a more effective C++ inference system for description logics.

Keywords: abduction, description logic

Abstrakt

Práca bude vychádzať z existujúceho riešenia abduktívneho systému založeného na reasoneri pre deskripčné logiky Pellet implementovaného v Jave. Pri implementácii v C++ sa otvárajú možnosti na zlepšenie a zefektívnenie existujúceho návrhu abduktívneho solvera, počnúc využitím efektívnejšieho C++ inferenčného systému pre deskripčné logiky.

Kľúčové slová: abdukcia, deskripčná logika

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Chapter 1

Introduction

Story No. 1 TODO Úvod, troska kontextu, asi na 1,5 strany Cieľom práce je návrh a vývoj abduktívneho systému pre deskripčné logiky založený na existujúcom reasoneri s dôrazom na optimalizačné techniky a efektívnosť.

Chapter 2

Description logic

2.1 Introduction to description logic

This chapter is based on Handbook on ontologies [6] and Handbook of knowledge representation [7].

The word “ontology” is used with different meanings in different communities. In philosophy, Aristotle in his *Metaphysics* defined **Ontology** as the study of attributes that belong to things because of their very nature.

Ontology focuses on the nature and structure of things independently of any further considerations, and even independently of their actual existence.

For example, it makes perfect sense to study the Ontology of unicorns and other fictitious entities: although they do not have actual existence, their nature and structure can be described in terms of general categories and relations.

in Computer Science, we refer to an **ontology** as a special kind of information object or computational artifact. Computational ontologies are a means to

formally model the structure of a system, that is the relevant entities and relations that emerge from its observation, and which are useful to our purposes. An example of such a system can be a company with all its employees and their interrelationships. The ontology engineer analyzes relevant entities and organizes them into concepts and relations, being represented, respectively, by unary and binary predicates.

Description logics (DLs) are a family of knowledge representation languages that can be used to represent an ontology in a structured and formally well-understood way. The "description" part of their name is based on how the important notions of the domain are described by concept descriptions (unary predicates) and atomic roles (binary predicates). The "logic" part comes from their formal, logic-based semantics, unlike some other methods of representation of ontologies, for example semantic networks.

Knowledge base (a set of facts) in description logics typically comes in two parts: a terminological part (**TBox**) and an assertional part (**ABox**).

TBox consists of general statements about concepts. Some examples,:

Example 2.1.1 [7] $HappyMan \equiv Human \sqcap \neg Female \sqcap (\exists married.Doctor) \sqcap (\forall hasChild.(Doctor \sqcup Professor))$.

This example defines a concept, 'HappyMan', as a human who is not female, is married to a doctor and his every child is a doctor or a professor.

Example 2.1.2 [7] $\exists hasChild.Human \sqsubseteq Human$

Or, in natural language, if someone has a child that is human, then they are human.

ABox consists of specific statements about individuals.

Example 2.1.3 [7] *bob : HappyMan*

bob,mary : hasChild

mary : \neg Doctor

This is an **ABox** of 3 statements: Bob is a happy man, Bob has a child - Mary, and Mary is not a doctor. You may notice that if we had a knowledge base consisting of TBox 2.1.1 and ABox 2.1.3, we may deduce that Mary must be a professor.

Interpretation of description logics is done using sets. We will formally define interpretations with specific description logics, but to informally make sense of previous examples:

Concepts can be interpreted as sets of constants,

individuals can be interpreted as constants,

relations as a set of pairs of constants,

\sqcap as set conjunction \cap ,

\sqcup as set disjunction \cup ,

\neg as set complement,

\sqsubseteq as subset symbol \subseteq ,

existential restriction $\exists \mathbf{r}.\mathbf{C}$ as a set of constants that are in relation \mathbf{r} with at least one individual in concept \mathbf{C} ,

and **universal restriction** $\forall \mathbf{r}.\mathbf{C}$ as a set of constants that are not in relation \mathbf{r} with any constant in complement of \mathbf{C} .

Also, $A \equiv B$ means " $A \sqsubseteq B$ and $B \sqsubseteq A$ ".

2.2 Selected description logics

2.2.1 \mathcal{ALC}

In this thesis, we will be using a widely used description logic \mathcal{ALC} and its extensions. \mathcal{ALC} stands for "Attributive concept Language with Complements". It's one of the less expressive languages, for example, it can't express the concept "someone who has 2 children". You can see examples of statements in \mathcal{ALC} in the previous section.

Definition 2.2.1 [7] (*Syntax of \mathcal{ALC} concepts and roles*). Let N_C be a set of concept names and N_R be a set of role names. The set of Concepts is the smallest set such that

1. \top, \perp , and every concept name $A \in N_C$ is an Concept,
2. If C and D are Concepts and $r \in N_R$, then $C \sqcap D$,
 $C \sqcup D$, $\neg C$, $\forall r.C$, and $\exists r.C$ are Concepts.

\top and \perp are special concepts 'everything' and 'nothing'. Every individual belongs to concept \top and no individuals belong to concept \perp .

The semantics of \mathcal{ALC} (and of DLs in general) are given in as interpretations.

Definition 2.2.2 [7] (*\mathcal{ALC} semantics*). An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a nonempty set $\Delta^{\mathcal{I}}$, called the domain of \mathcal{I} , and a function $\cdot^{\mathcal{I}}$ that maps every \mathcal{ALC} Concept to a subset of $\Delta^{\mathcal{I}}$, and every \mathcal{ALC} role to a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ such that, for all \mathcal{ALC} Concepts C, D and all role names r :

$$\begin{aligned}
\top^{\mathcal{I}} &= \Delta^{\mathcal{I}} & \perp^{\mathcal{I}} &= \emptyset, \\
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}}, \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
\neg C^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}, \\
(\exists r.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}} \text{ with } \langle x, y \rangle \in r^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\}, \\
(\forall r.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \forall y \in \Delta^{\mathcal{I}}, \text{ if } \langle x, y \rangle \in r^{\mathcal{I}}, \text{ then } y \in C^{\mathcal{I}}\}.
\end{aligned}$$

You may notice that nothing in the definition of interpretation says that an ontology must be 'true'. An interpretation which can intuitively be called 'true' for an ontology is called a model. We will now formally define it. An ontology in description logic is often called 'knowledge base'. It consists of various statements, in \mathcal{ALC} it consists of general concept inclusions (GCI), and assertional axioms. A set of GCIs are usually called a TBox (example ??) and a set of assertional axioms ABox (example 2.1.3).

Definition 2.2.3 [7] (*\mathcal{ALC} TBox model*)

A general concept inclusion (GCI) axiom is of the form $C \sqsubseteq D$, where C, D are \mathcal{ALC} Concepts. An interpretation \mathcal{I} is a model of a GCI $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.

\mathcal{I} is a model of a TBox T if it is a model of every GCI in T .

Definition 2.2.4 [7] (*\mathcal{ALC} ABox model*)

An assertional axiom is of the form $x : C$ or $(x, y) : r$, where C is an \mathcal{ALC} Concept, r is a role name, and x and y are individual names. An interpretation \mathcal{I} is a model of an assertional axiom $x : C$ if $x^{\mathcal{I}} \in C^{\mathcal{I}}$, and \mathcal{I} is a model of an assertional axiom $(x, y) : r$ if $(x^{\mathcal{I}}, y^{\mathcal{I}}) \in r^{\mathcal{I}}$.

\mathcal{I} is a model of an ABox A if it is a model of every assertional axiom in A .

Definition 2.2.5 [7] (*Consistency*)

\mathcal{I} is a model of a knowledge base $\mathcal{K}=(Tbox \mathcal{T}, Abox \mathcal{A})$ if it's a model of \mathcal{A} and \mathcal{T} .

If a model of \mathcal{K} exists, we say that \mathcal{K} is consistent.

An ontology can have multiple models, some less intuitive than other.

Example 2.2.1 (*model*)

Knowledge base $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$, $\mathcal{T} = \{$

$$A \sqsubseteq B$$

$$A \sqsubseteq \exists r.C$$

$$C \sqsubseteq \forall r.D$$

$\}$

$\mathcal{A} = \{$

$$a : A$$

$$c, d : r$$

$\}$

One possible model of \mathcal{K} , \mathcal{M}_1 , would be: $\{$

$$\Delta^{\mathcal{I}} = \{ a_x, c_x, d_x \}$$

$$a^{\mathcal{I}} = a_x, c^{\mathcal{I}} = c_x, d^{\mathcal{I}} = d_x$$

$$A^{\mathcal{I}} = \{a_x\}, B^{\mathcal{I}} = \{a_x\}, C^{\mathcal{I}} = \{c_x\}, D^{\mathcal{I}} = \{d_x\}$$

$$r^{\mathcal{I}} = \{ \langle a_x, b_x \rangle, \langle a_x, c_x \rangle, \langle c_x, d_x \rangle \}$$

$\}$

But other models also exist, for example \mathcal{M}_2 and \mathcal{M}_3 . $\mathcal{M}_2 = \{$

$$\Delta^{\mathcal{I}} = \{ a_x, c_x, d_x, c_n \}$$

$$a^{\mathcal{I}} = a_x, c^{\mathcal{I}} = c_x, d^{\mathcal{I}} = d_x$$

$$A^{\mathcal{I}} = \{a_x\}, B^{\mathcal{I}} = \{a_x\}, C^{\mathcal{I}} = \{c_n\}, D^{\mathcal{I}} = \{\}$$

$$\begin{aligned}
r^{\mathcal{I}} &= \{ \langle a_x, b_x \rangle, \langle a_x, c_x \rangle, \langle c_x, d_x \rangle \} \\
\mathcal{M}_3 &= \{ \\
&\Delta^{\mathcal{I}} = \{ i_x \} \\
&a^{\mathcal{I}} = i_x, c^{\mathcal{I}} = i_x, d^{\mathcal{I}} = i_x \\
&A^{\mathcal{I}} = B^{\mathcal{I}} = C^{\mathcal{I}} = D^{\mathcal{I}} = \{ i_x \} \\
&r^{\mathcal{I}} = \{ \langle i_x, i_x \rangle \} \\
&\}
\end{aligned}$$

2.2.2 \mathcal{SHIQ}

\mathcal{SHIQ} is one of the most expressive description logics. **S** - abbreviation of \mathcal{ALC} with transitive roles.

H - Role hierarchy (role r_1 can be subrole of role r_2)

I - Inverse properties (if $a, b : r$, then $b, a : r^-$)

Q - Quantified cardinality restrictions (for example $\leq 2hasChild$)

Examples of Concepts in \mathcal{SHIQ} :

Example 2.2.2 [6] $Human \sqcap \neg Female \sqcap \exists married.Doctor$

$\sqcap (\geq 5hasChild) \sqcap \forall hasChild.Professor$.

"A man that is married to a doctor and has at least five children, all of whom are professors".

Example 2.2.3 [6] $Human \sqsubseteq \forall hasParent.Human \sqcap (\geq 2hasParent.\top)$

$\sqcap (\leq 2hasParent.\top) \sqcap \forall hasParent^-.Human$

"If someone is a human, all their parents are human. they have exactly two parents, and everything that has them as a parent (i.e. is their child) is a human."

Example 2.2.4 [6]

$hasParent \sqsubseteq hasAncestor$.

"hasParent is a subrole of hasAncestor (i. e. If A hasparent.B , then A hasAncestor.B)."

Example 2.2.5 $Trans(hasAncestor)$

"The role hasAncestor is transitive (i.e. if A hasAncestor.B and B hasAncestor.C then A hasAncestor.C)."

The definitions of syntax and semantics of \mathcal{SHIQ} are similar to those of \mathcal{ALC} .

Definition 2.2.6 [6] (\mathcal{SHIQ} concept and role syntax) Let R be a set of role names, which is partitioned into a set R_+ of transitive roles and a set R_p of normal roles. The set of all \mathcal{SHIQ} roles is $R \cup \{r^- | r \in R\}$, where r^- is called the inverse of the role r .

Let C be a set of concept names. The set of \mathcal{SHIQ} concepts is the smallest set such that:

1. every concept $A \in C$ is a \mathcal{SHIQ} concept.
2. if A and B are \mathcal{SHIQ} concepts and r is a \mathcal{SHIQ} role, then $A \sqcap B, A \sqcup B, \neg A, \forall r.A$, and $\exists r.A$ are \mathcal{SHIQ} concepts.
3. if A is a \mathcal{SHIQ} concept and r is a simple \mathcal{SHIQ} role (simple role is neither transitive nor has a transitive subrole), and $n \in \mathbb{N}$, then $(\leq nr.A)$ and $(\geq nr.A)$ are \mathcal{SHIQ} concepts.

\mathcal{SHIQ} semantics can be described as \mathcal{ALC} semantics with same additions.

Definition 2.2.7 (*SHIQ semantics*) [6] in addition to definition ??, for all $p \in R$ and $r \in R_+$:

$$\langle x, y \rangle \in p^{\mathcal{I}} \text{ iff } \langle y, x \rangle \in (p^-)^{\mathcal{I}}.$$

$$\text{if } \langle x, y \rangle \in r^{\mathcal{I}} \text{ and } \langle y, z \rangle \in r^{\mathcal{I}} \text{ then } \langle x, z \rangle \in r^{\mathcal{I}}.$$

$$(\leq nr.C)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} \mid \#r^{\mathcal{I}}(x, C) \leq n\},$$

$$(\geq nr.C)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} \mid \#r^{\mathcal{I}}(x, C) \geq n\},$$

where $\#M$ denotes the cardinality of the set M , and $r^{\mathcal{I}}(x, C) := \{y \mid \langle x, y \rangle \in r^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\}$.

SHIQ ABox and its model are the same as in *ALC* (definition 2.2.4). For the TBox, we have to add role subsumption axioms. Some authors define TBox as only containing GCIs, and use a new structure, RBox, to contain role inclusions. For this thesis, role inclusion axioms are a part of TBox.

Definition 2.2.8 [6] (*SHIQ TBox*)

A role inclusion axiom is of the form $r \sqsubseteq s$, where r, s are roles. An interpretation \mathcal{I} is a model of a role inclusion axiom $r \sqsubseteq s$ if $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$.

\mathcal{I} is a model of a TBox \mathcal{T} if it is a model of every role inclusion axiom and GCI axiom (definition ??) in TBox.

2.2.3 *ALCHO*

ALC with role hierarchy and Nominals.

Nominals are concepts with exactly one specific instance. For example, $\{\text{john}\}$ is a concept with its only instance being the individual 'john'.

Nominals can be used to express enumerations, for example [2]:

$$\text{Beatle} \equiv \{\text{john}\} \sqcup \{\text{paul}\} \sqcup \{\text{george}\} \sqcup \{\text{ringo}\}$$

Role hierarchy was already described in section 2.2.2 (example 2.2.4).

Definition 2.2.9 (*Syntax and semantics of \mathcal{ALCHO}*)

Syntax of \mathcal{ALCHO} is the syntax of \mathcal{ALC} (definition 2.2.1), with $\{a\}$ added to the set of concepts C for each individual 'a'.

Similarly, semantics of \mathcal{ALCHO} are the semantics of \mathcal{ALC} (definition 2.2.2) with the addition of $\{a\}^{\mathcal{I}} = \{a^{\mathcal{I}}\}$, where 'a' is an individual and \mathcal{I} is the interpretation.

In \mathcal{ALCHO} , the definition of an ABox is the same for \mathcal{ALC} (definition 2.2.4) and the definition of a TBox is the same as for \mathcal{SHIQ} (definition 2.2.8).

2.2.4 $\mathcal{SROIQV}(\mathcal{D})$

Jazyk koncludu, TODO asi ked bude hotova implementacia.

2.3 tableau algorithm

In our algorithm, we heavily make use of tableau algorithm. Tableau algorithm is a method of constructing a model of a knowledge base \mathcal{K} if \mathcal{K} is consistent, and stops if no model of \mathcal{K} exists and therefore \mathcal{K} is inconsistent.

Tableau algorithm uses knowledge base in negation normal form (NNF), that is, every concept complement \neg applies only to a concept name [7]. Any \mathcal{ALC} concept can be transformed to an equivalent concept in NNF by using de Morgan's laws and the duality between existential and universal restrictions ($\neg\exists r.C \equiv \forall r.\neg C$). For example, the concept $\neg(\exists r.A \sqcap \forall s.B)$, where A, B are concept names, can be transformed using de Morgan's laws

to $\neg\exists r.C \sqcup \neg\forall s.B$, and this can then be transformed using the existential-universal duality into $(\forall r.\neg A) \sqcup (\exists s.\neg B)$.

The idea behind the tableau algorithm for $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ is to start with the concrete situation described in \mathcal{A} and expand based on what can be inferred from \mathcal{T} and currently known ABox statements. This is done using something called completion graph, which is a graph where nodes represent individuals, directed edges represent relations between individuals, each node has a label containing concepts the individual belongs to, and each edge has a label consisting of the names of its roles.

Definition 2.3.1 (*Completion graph*)

A completion graph is a pair (G, \mathcal{L}) , where G is a directed finite graph and \mathcal{L} is a labeling function mapping each node from G to a set of concepts, and each edge to a set of roles.

Tableau algorithm for $\mathcal{K}(\mathcal{T}, \mathcal{A})$ first creates a completion graph based on \mathcal{A} , and then expands it using tableau expansion rules.

Definition 2.3.2 [7] (*\mathcal{ALC} tableau expansion rules*)

- \sqcap -rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$, x is not blocked (definition 2.3.4), and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$ then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$
- \sqcup -rule: if 1. $C_1 \sqcup C_2 \in \mathcal{L}(x)$, x is not blocked, and $\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$, then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C\}$, for some $C \in \{C_1, C_2\}$
- \exists -rule: if $\exists r.C \in \mathcal{L}(x)$, x is not blocked, and x has no r -successor y with $C \in \mathcal{L}(y)$, then create a new node y with $\mathcal{L}(\langle x, y \rangle) = \{r\}$ and $\mathcal{L}(y) = \{C\}$.

- \forall -rule: if $\forall r.C \in \mathcal{L}(x)$, x is not blocked, and there is an r -successor y of x with $C \notin \mathcal{L}(y)$ then set $\mathcal{L}(y) = \mathcal{L}(y) \cup \{C\}$.
- \sqsubseteq -rule: if $C_1 \sqsubseteq C_2 \in \mathcal{T}$, x is not blocked, and $C_2 \sqcup \text{NNF}(\neg C_1) \notin \mathcal{L}(x)$, then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_2 \sqcup \text{NNF}(\neg C_1)\}$

Where C, C_1, C_2 are concepts, r is role, \mathcal{T} is TBox, and NNF is normal negation form.

Why are we checking whether nodes are blocked? And what are blocked nodes?

If we removed "x is not blocked" from all expansion rules, then the algorithm could generate an infinite graph using the \exists rule, for example for $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, $\mathcal{T} = \{C \sqsubseteq \exists r.C\}$, $\mathcal{A} = \{a : C\}$ the algorithm would create a node a_2 with $\langle a, a_2 \rangle : r$, then a node a_3 with $\langle a_2, a_3 \rangle : r$ and so on. To guarantee termination, we define node blocking.

Definition 2.3.3 (*ancestor*) In completion graph $\mathcal{CG} = (G, \mathcal{L})$, $G = (N, E)$, nodes $x, y \in N$, and edge $\langle x, y \rangle \in E$, then y is a successor of x and x is an ancestor of y .

If $\mathcal{L}(\langle x, y \rangle) = r$, then y is an r -successor of x .

Ancestry is transitive (if node x is an ancestor of node y and y is an ancestor of node z , then x is an ancestor of z).

Definition 2.3.4 (*blocking*) A node x is blocked if it has an ancestor y that is either blocked, or $\mathcal{L}(x) \subseteq \mathcal{L}(y)$.

And now we can show the pseudocode of (nondeterministic) tableau algorithm.

```

1: function TABLEAUALG( Knowledge base  $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ )
2:   Convert  $\mathcal{T}$  to NNF.
3:   Completion graph  $\mathcal{CG} = (G, \mathcal{L})$ ,  $G = (N, E)$ ,  $N = E = \mathcal{L} = \emptyset$ 
4:   for Each individual  $i \in \mathcal{A}$  do
5:      $N = N \cup n_i$ 
6:     for Each assertional axiom  $(i : C) \in \mathcal{A}$  do
7:        $\mathcal{L}(n_i) = \mathcal{L}(n_i) \cup C$ 
8:     end for
9:     for Each assertional axiom  $(i, x : r) \in \mathcal{A}$  do
10:       $E = E \cup \langle n_i, n_x \rangle$ 
11:       $\mathcal{L}(\langle n_i, n_x \rangle) = \mathcal{L}(\langle n_i, n_x \rangle) \cup r$ 
12:    end for
13:  end for
14:  while A tableau expansion rule can be applied on  $\mathcal{CG}$  do
15:    Apply a rule on  $\mathcal{CG}$ 
16:    if Clash exists in  $\mathcal{CG}$  then
17:      return " $\mathcal{K}$  is inconsistent" (assuming algorithm
undeterministically always picks the 'correct' decision in  $\sqcup$  rule if
one exists)
18:    end if
19:  end while
20:  return " $\mathcal{K}$  is consistent"
21: end function

```

Definition 2.3.5 (*clash*)

In completion graph $\mathcal{CG} = ((N, E), \mathcal{L})$, if there is a node $n \in N$ and a concept C where $\{C, \neg C\} \in \mathcal{L}(n)$, then \mathcal{CG} has a clash.

If the algorithm cannot apply any expansion rules, then \mathcal{K} is consistent and \mathcal{CG} is its model, or a finite part of an infinite model due to blocking. Our abduction algorithm only needs this finite part.

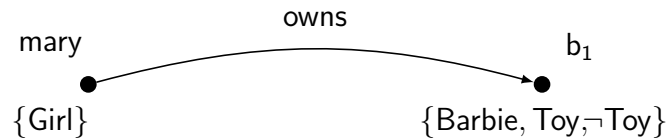
The tableau algorithm has 2 sources of nondeterminism : picking which rule to apply, and how to apply the \sqcup rule. The order of rule applications doesn't matter for the sake of determining consistency, but the application of the \sqcup rule does, as one choice can lead to a clash while the other not.

If a clash happens in a deterministic implementation, we can backtrack to

a previous \sqcup decision with an unexplored choice and continue from there. If there are no unexplored \sqcup decisions and we find a clash, \mathcal{K} is inconsistent (no model exists). If there is no decision with an unexplored possibility we can backtrack to, we can declare \mathcal{K} to be inconsistent. I think it would be useful to show an example of tableau completion graph:

Example 2.3.1

$$\mathcal{K} = \left\{ \begin{array}{l} \text{Girl} \sqsubseteq \exists \text{owns.Barbie} \\ \text{Girl} \sqsubseteq \forall \text{owns.}\neg \text{Toy} \\ \text{Barbie} \sqsubseteq \text{Toy} \\ \text{mary: Girl} \end{array} \right\}$$



In this case, the tableau algorithm stops on a contradiction - mary must own a barbie and no toys, but barbie is a toy, so b1 is supposed to both be a toy and not be a toy.

Note that this picture shows only partial labels, the full label for mary according to expansion rules could be $\{\neg \text{Girl} \sqcup \exists \text{owns.Barbie}, \exists \text{owns.Barbie}, \neg \text{Girl} \sqcup \forall \text{owns.}\neg \text{Toy}, \forall \text{owns.}\neg \text{Toy}, \neg \text{Barbie} \sqcup \text{Toy}, \neg \text{Barbie}\}$

Chapter 3

Abduction

Logical thinking can be divided into 3 categories: **Deduction**, **Induction** and **Abduction**.

Deduction is the process of using known causes and rules to infer the results of a rule. For example, if we know the fact that a floor is wet, and the rule that this particular floor is slippery when wet, we can arrive at a conclusion that the floor must be slippery. Deduction is the only one of the 3 types of logical reasoning where if the premises and rules used are true, then the conclusion must also be true.

Induction is the process of knowing the cause and effect and creating a plausible rule. For example, we know that whenever we've seen this floor wet, it was also slippery, therefore, the floor is slippery when wet. Unlike deduction, induction the result of induction isn't necessarily correct.

Abduction, also known as hypothesis or diagnosis, is when we know the rules and observe an effect, and try to find a plausible cause. For example a doctor may observe someone's symptoms, know which diseases cause which symptoms, and based on those rules guess which disease the patient may

have. Like with induction, there is no certainty that the explanation we reach is correct.

3.1 ABox Abduction in description logic

In description logic, we define inference through models. For any sets of ABox and TBox axioms \mathcal{A} and \mathcal{B} , $\mathcal{A} \models \mathcal{B}$ if there is no model of $\mathcal{A} \cup \neg\mathcal{B}$ ($\mathcal{A} \cup \neg\mathcal{B}$ is inconsistent).

In our algorithm, we do Abox abduction - that means both the observation and explanation can contain only ABox axioms - only the knowledge base can contain TBox axioms. When using abduction in description logic, we are looking for a set of explanations:

Definition 3.1.1 (*explanation*) For a knowledge base \mathcal{K} and an observation O , a set of axioms \mathcal{E} is an explanation when all of the following are true:

- $\mathcal{K} \cup \mathcal{E} \models O$.
- $\mathcal{K} \cup \mathcal{E}$ is consistent.
- \mathcal{E} is relevant - $\mathcal{E} \not\models O$.
- \mathcal{E} is explanatory - $\mathcal{K} \not\models O$.

For example, for a simple ontology where the Abox of knowledge base is empty and the Tbox= $\{\mathcal{A} \sqsubseteq \mathcal{C}; \mathcal{B} \sqsubseteq \mathcal{C}\}$, and an observation Abox= $\{a : \mathcal{C}\}$, these are some explanations: $\{\{a : \mathcal{A}\}, \{a : \mathcal{B}\}\}$. $\{a : \mathcal{C}\}$ is not an explanation, since it's irrelevant.

However, you may see that there are also other possible explanations, for example $\{a : \mathcal{A}, a : \mathcal{B}\}$, $\{a : \mathcal{A}, a : \neg\mathcal{B}\}$, or even $\{a : \mathcal{A}, a : \mathcal{D}, a : \neg\mathcal{G}, \dots\}$, and many others, where \mathcal{D}, \mathcal{G} are concepts not used for any rule in the knowledge

base. For this reason, it also makes sense to only look for explanations that are syntactically minimal, which means they are not a superset of any other explanation. This would leave only the first two explanations, $\{\{a : \mathcal{A}\}$ and $\{\{a : \mathcal{B}\}$.

TODO: Abduction as set cover, abduction as probability

3.2 Uses

TODO: Medical, automatic code checking, automatic planning, any type of diagnosis

Chapter 4

Previous abduction solvers

4.1 theoretical approaches

TODO: abdukcia v FOL TODO: bol nejaký pokus o abdukciu v deskriptnej logike nezaložený na hitting setoch? - ten čo som našiel na mobile by sa možno dal spomenúť, nepoužíva termín "hitting set", používa substitúcie na uzatváranie tableaux s dvojítmymi labelmi ‘

4.2 implemented solutions

TODO: Racer

Chapter 5

Our approach

The approach most often used to perform abductive reasoning in description logic without translating to other formal logics is based on the following thought: Let there be a knowledge base \mathcal{K} , observation O , and a set of axioms \mathcal{S} . If $\mathcal{K} \sqcup \mathcal{S}$ is consistent, then by definition there is at least one model of $\mathcal{K} \sqcup \mathcal{S}$. If $\mathcal{K} \sqcup \mathcal{S} \sqcup \neg O$ is inconsistent, then there is no model for $\mathcal{K} \sqcup \mathcal{S}$ where $\neg O$ is not true, that is every model of $\mathcal{K} \sqcup \mathcal{S}$ contains O , so $\mathcal{K} \sqcup \mathcal{S} \models O$.

We can find such a set \mathcal{S} by generating every model of $\mathcal{K} \sqcup \neg O$, and picking a set of complements of axiom in these models so that every model has at least one axiom complement in \mathcal{S} . This can be formulated as the hitting set (defined below 5.0.1) problem (which is equivalent to the set cover problem) - for each model in the set of models M of $\mathcal{K} \sqcup \neg O$, we create an **antimodel** (5.0.2) consisting of negations of every axiom in the model, the set of these antimodels we call M' . Our goal is finding a minimal (inclusion-wise) set \mathcal{S} containing at least one axiom from each antimodel in M' - \mathcal{S} a hitting set 5.0.1 for M' .

Additionally, if \mathcal{S} is relevant and explanatory (3.1.1) and $\mathcal{K} \sqcup \mathcal{S}$ is consistent, it's an explanation for observation O .

Definition 5.0.1 (*Hitting set (TODO: cituj paper)*)

A hitting set for a collection of sets F (in our case, F is the collection of antmodels) is a set H s.t. $H \cap S \neq \{\}$ for every $S \in F$. A hitting set H for F is minimal if there is no other hitting set H' for F s.t. $H' \subsetneq H$.

Definition 5.0.2 *To better explain our algorithm, it will be useful to define the term 'antmodel'. Let M be a model of knowledge base \mathcal{K} . Then M' is an antmodel of \mathcal{K} iff it contains negations of every axiom from M and nothing else - for every axiom of form $a : C$ in M , M' has $a : \neg C$, and for every axiom $a, b : R$ in M it has $a, b : \neg R$ (a and b are not in role R). (TODO: toto asi pre niektore deskripčne logiky nieje možné vyjadriť, (role complements))*

This idea was first introduced by Raymond Reiter in "A Theory of Diagnosis from First Principles"[4] as a general method for abductive reasoning in any formal logic with binary semantics (every statement is either true or false) and operands \wedge, \vee, \neg with their usual semantic meaning, including first order logic. Additionally, Reiter proposes using a hitting set tree, which we will describe later.

Ken Halland and Katarina Britz in "ABox abduction in ALC using a DL tableau"[1] proposed an algorithm using the idea of hitting sets for abduction in description logic ALC, using a modified tableaux algorithm - their algorithm first develops all possible completion graphs (multiple graphs resulting from the use of \sqcup rule), based on the knowledge base, and once there are no more rules to apply, they add an axiom from the observation complement $\neg O$ to the knowledge base. If there is no rule to apply or unused

observation, the model is added to the list of models from which minimal hitting set is generated.

This algorithm generates every model reachable by tableaux algorithm for $\mathcal{K} \sqcup O$, which as we will show, is not necessary.

Our work is mainly based on the algorithm by Martin Homola and Júlia Pukancová, using the idea of a hitting set tree from Reiter [4], which make generating every model of $\mathcal{K} \sqcup \neg O$ not necessary.

Definition 5.0.3 (*Hitting set tree (TODO: reference)*) A hitting set tree (HS-tree) for a collection of sets F is $T = (V, E, L, H)$, where (V, E) is the smallest tree in which the labelling function L labels the nodes of V by elements of F , the edges of E by elements of sets in F , and $H(n)$ is the set of edge-labels from the root node to $n \in V$, s.t.:

- (a) for the root $r \in V$: $L(r) = S$ for some $S \in F$, if $F \neq \{\}$, otherwise $L(r) = \{\}$;
- (b) for each $n \in V$: $L(n) = S$ for some $S \in F$ s.t. $S \cap H(n) = \{\}$, if such $S \in F$ exists, otherwise $L(n) = \{\}$;
- (c) each $n \in V$ has a successor n_σ for each $\sigma \in L(n)$ with $L(n, n_\sigma) = \sigma$.

We can generate the HS-tree in the following way: let F be the list of antimodels for $\mathcal{K} \cap \neg O$. The label of each vertex n is either an antimodel of $\mathcal{K} \cap H(n) \cap \neg O$ (which is also an antimodel of $\mathcal{K} \cap \neg O$) or empty, if no such model exists - in this case $H(n)$ is a hitting set of F , and $\mathcal{K} \cap H(n) \models O$ - $H(n)$ is an explanation of $H(n)$ if it also fulfills the other conditions (3.1.1). If the vertex n is labeled by an antimodel, we create a child vertex for each axiom of antimodel $L(n)$ - we label the edge by that axiom.

By using a HS-tree, we can find a hitting set that hits every antimodel of $\mathcal{K} \cap \neg O$ while having to generate only a few of these models - each choice of edge label usually also eliminates many models that were not generated. The H and P (TODO ref) algorithm uses this algorithm, combined with the pruning methods (for example, eliminate a vertex n if $H(n)$ contains axioms $\{A, \neg A\}$ (inconsistent with itself) or if there is a vertex n' with empty label (either due to pruning or being a hitting set) and $H(n') \subseteq H(n)$, or if there is a vertex n' where $H(n) = H(n')$).

Chapter 6

Implementation

```

1: function ABDUCTION( Knowledge base  $\mathcal{K}$ , observation  $O$ , maximum
   Depth  $\text{maxD}$ )
2:   Output: Set of minimum explanations  $\mathcal{S}$ 
3:   if  $\mathcal{K} \cup O$  is inconsistent then
4:     return  $\emptyset$  //observation not consistent with knowledge base
5:   end if
6:   if  $\mathcal{K} \cup \neg O$  is inconsistent then
7:     return  $\{\{\}\}$  //observation can be inferred from knowledge base
8:   end if
9:    $\mathcal{C} = \{\{\}\}$  //hitting set candidates for this iteration (one empty set)
10:   $\mathcal{NC} = \emptyset$  //hitting set candidates for next iteration
11:   $\mathcal{S} = \emptyset$ 
12:   $D = 1$  //depth
13:  while  $\mathcal{C} \neq \emptyset \wedge D \leq \text{maxD}$  do
14:    for each candidate  $c \in \mathcal{C}$  //TODO: malo by c tiez byt v mathcal?
15:    do
16:      for each hitting set  $s \in \mathcal{S}$  do
17:        if  $s \in c$  then
18:          Continue to next candidate //expalanation wouldn't
19:          be minimal
20:        end if
21:      end for
22:      if  $\mathcal{K} \cup \{\neg O\} \cup c$  is inconsistent then
23:        if  $\mathcal{K} \cup c$  is consistent then
24:           $\mathcal{S} = \mathcal{S} \cup c$  //c is a hitting set (explanation)
25:        end if
26:        Continue to next candidate
27:      end if
28:      if  $D = \text{maxD}$  then
29:        Continue to next candidate // no need to create
30:        candidates for next while iteration
31:      end if
32:      Axioms  $\mathcal{AX} = \text{getAntiModel}(\mathcal{K}, O, c)$ 
33:      for each axiom  $ax \in \mathcal{AX}$  do
34:        if  $ax \in c$  then
35:          Continue to next axiom //c  $\cup ax \equiv c$ 
36:        end if
37:         $nc = c \cup ax$ 
38:        if  $O \in nc$  then
39:          Continue to next axiom //explanation would be trivial
40:        end if
41:         $\mathcal{NC} = \mathcal{NC} \cup nc$ 
42:      end for
43:    end for
44:     $\mathcal{C} = \mathcal{NC}$ 
45:     $\mathcal{NC} = \emptyset$ 
46:  end while
47: end function

```

```

1: function GETANTIMODEL( Knowledge base  $\mathcal{K}$ , observation  $O$ , set of
   axioms  $\mathcal{AX}$ )
2:   Output: Antimodel  $\mathcal{AM}$  of a model  $\mathcal{M}$  of  $\mathcal{K} \cup \{\neg O\} \cup \mathcal{AX}$ 
3:    $\mathcal{AM} = \emptyset$ 
4:    $\mathcal{M} = \text{model of } \mathcal{K} \cup \{\neg O\} \cup \mathcal{AX}$ 
5:   for each individual  $\mathcal{I} \in \mathcal{K} \cup O \cup \mathcal{AX}$  : do
6:      $\mathcal{C}_k = \text{set of concepts } \{\forall C | \mathcal{I} : C \in \mathcal{K}\}$  //known concepts
7:      $\mathcal{C}_a = \text{set of all concepts } \in \mathcal{K}$  //all concepts
8:      $\mathcal{C}_i = \text{set of concepts } \{\forall C | \mathcal{I} : C \in \mathcal{M}\}$  //inferred concepts
9:     for each concept  $C \in \mathcal{C}_i$  do
10:      if not  $C \in \mathcal{C}_k$  then
11:         $\mathcal{AM} = \mathcal{AM} \cup (\mathcal{I} : C)$ 
12:      end if
13:    end for
14:    for each concept  $C \in \mathcal{C}_a$  do
15:      if not  $C \in \mathcal{C}_i$  then
16:         $\mathcal{AM} = \mathcal{AM} \cup (\mathcal{I} : C)$ 
17:      end if
18:    end for
19:  end for
20:  return  $\mathcal{AM}$ 
21: end function

```

Chapter 7

Results

Chapter 8

Conclusion

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Zoznam obrázkov