

Comenius university in bratislava
Faculty of Mathematics, Physics and Informatics

**ABDUCTION SOLVER BASED ON HIGHLY
EFFICIENT C++ DL REASONER**

Master thesis

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Master thesis

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Čestne prehlasujem, že túto diplomovú prácu som
vypracoval samostatne len s použitím uvedenej literatúry
a za pomoci konzultácií u môjho školiteľa.

Bratislava, 2017

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Bc. Drahomír Mrózek

Acknowledgments

TODO

Abstract

This work will start from an existing solution of an abduction solver based on Pellet reasoner for description logics, implemented in Java. The implementation in C++ opens different ways for improvement and effectivization, starting with the utilization of a more effective C++ inference system for description logics.

Keywords: abduction, description logic

Abstrakt

Práca bude vychádzať z existujúceho riešenia abduktívneho systému založeného na reasoneri pre deskripčné logiky Pellet implementovaného v Jave. Pri implementácii v C++ sa otvárajú možnosti na zlepšenie a zefektívnenie existujúceho návrhu abduktívneho solvera, počnúc využitím efektívnejšieho C++ inferenčného systému pre deskripčné logiky.

Kľúčové slová: abdukcia, deskripčná logika

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Chapter 1

Introduction

Story No. 1 TODO Úvod, troska kontextu, asi na 1,5 strany Cieľom práce je návrh a vývoj abduktívneho systému pre deskripčné logiky založený na existujúcom reasoneri s dôrazom na optimalizačné techniky a efektívnosť.

Chapter 2

Description logic

2.1 Introduction to description logic

This chapter is based on Handbook on ontologies [5] and Handbook of knowledge representation [6].

The word “ontology” is used with different meanings in different communities. In philosophy, Aristotle in his *Metaphysics* defined **Ontology** as the study of attributes that belong to things because of their very nature.

Ontology focuses on the nature and structure of things independently of any further considerations, and even independently of their actual existence.

For example, it makes perfect sense to study the Ontology of unicorns and other fictitious entities: although they do not have actual existence, their nature and structure can be described in terms of general categories and relations.

in Computer Science, we refer to an **ontology** as a special kind of information object or computational artifact. Computational ontologies are a means to

formally model the structure of a system, that is the relevant entities and relations that emerge from its observation, and which are useful to our purposes. An example of such a system can be a company with all its employees and their interrelationships. The ontology engineer analyzes relevant entities and organizes them into concepts and relations, being represented, respectively, by unary and binary predicates.

Description logics (DLs) are a family of knowledge representation languages that can be used to represent an ontology in a structured and formally well-understood way. The "description" part of their name is based on how the important notions of the domain are described by concept descriptions (unary predicates) and atomic roles (binary predicates). The "logic" part comes from their formal, logic-based semantics, unlike some other methods of representation of ontologies, for example semantic networks.

Knowledge base (a set of facts) in description logics typically comes in two parts: a terminological part (**TBox**) and an assertional part (**ABox**).

TBox consists of general statements about concepts. Some examples,:

Example 2.1.1 [6] $HappyMan \equiv Human \sqcap \neg Female \sqcap (\exists married.Doctor) \sqcap (\forall hasChild.(Doctor \sqcup Professor))$.

This example defines a concept, 'HappyMan', as a human who is not female, is married to a doctor and his every child is a doctor or a professor.

Example 2.1.2 [6] $\exists hasChild.Human \sqsubseteq Human$

Or, in natural language, if someone has a child that is human, then they are human.

ABox consists of specific statements about individuals.

Example 2.1.3 [6] *bob : HappyMan*

bob, mary : hasChild

mary : \neg Doctor

This is an **ABox** of 3 statements: Bob is a happy man, Bob has a child - Mary, and Mary is not a doctor. You may notice that if we had a knowledge base consisting of TBox 2.1.1 and ABox 2.1.3, we may deduce that Mary must be a professor.

Interpretation is done using sets. We will formally define interpretations with specific description logics, but to informally make sense of previous examples:

Concepts can be interpreted as sets of constants,

individuals can be interpreted as constants,

relations as a set of pairs of constants,

\sqcap as set conjunction \cap ,

\sqcup as set disjunction \cup ,

\neg as set complement,

\sqsubseteq as subset symbol \subseteq ,

existential restriction $\exists \mathbf{r}.\mathbf{C}$ as a set of constants that are in relation \mathbf{r} with at least one individual in concept \mathbf{C} ,

and **universal restriction** $\forall \mathbf{r}.\mathbf{C}$ as a set of constants that are not in relation \mathbf{r} with any constant in complement of \mathbf{C} .

Also, $A \equiv B$ means " $A \sqsubseteq B$ and $B \sqsubseteq A$ ".

2.2 Types of description logics

2.2.1 \mathcal{ALC}

In this thesis, we will be using a widely used description logic \mathcal{ALC} and its extensions. \mathcal{ALC} stands for "Attributive concept Language with Complements".

Definition 2.2.1 [6] (*\mathcal{ALC} syntax*). Let N_C be a set of concept names and N_R be a set of role names. The set of Concepts is the smallest set such that

1. \top, \perp , and every concept name $A \in N_C$ is an Concept,
2. If C and D are Concepts and $r \in N_R$, then $C \sqcap D$,
 $C \sqcup D$, $\neg C$, $\forall r.C$, and $\exists r.C$ are Concepts.

The semantics of \mathcal{ALC} (and of DLs in general) are given in terms of interpretations.

Definition 2.2.2 [6] (*\mathcal{ALC} semantics*). An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a nonempty set $\Delta^{\mathcal{I}}$, called the domain of \mathcal{I} , and a function $\cdot^{\mathcal{I}}$ that maps every \mathcal{ALC} Concept to a subset of $\Delta^{\mathcal{I}}$, and every \mathcal{ALC} role to a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ such that, for all \mathcal{ALC} Concepts C, D and all role names r :

$$\begin{aligned}
 \top^{\mathcal{I}} &= \Delta^{\mathcal{I}} & \perp^{\mathcal{I}} &= \emptyset, \\
 (C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}}, \\
 (C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}}, \\
 \neg C^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}, \\
 (\exists r.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}} \text{ with } \langle x, y \rangle \in r^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\}, \\
 (\forall r.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \forall y \in \Delta^{\mathcal{I}}, \text{ if } \langle x, y \rangle \in r^{\mathcal{I}}, \text{ then } y \in C^{\mathcal{I}}\}.
 \end{aligned}$$

The examples of TBox and ABox in the previous section are in \mathcal{ALC} .

2.2.2 \mathcal{SHIQ}

S - abbreviation of \mathcal{ALC} with transitive roles.

H - Role hierarchy (role r_1 can be subrole of role r_2)

I - Inverse properties (if $a, b : r$, then $b, a : r^-$)

Q - Quantified cardinality restrictions (for example $\leq 2hasChild$)

Examples of Concepts in *SHIQ*:

Example 2.2.1 [5] $Human \sqcap \neg Female \sqcap \exists married.Doctor$
 $\sqcap (\geq 5hasChild) \sqcap \forall hasChild.Professor$.

"A man that is married to a doctor and has at least five children, all of whom are professors".

Example 2.2.2 [5] $Human \sqsubseteq \forall hasParent.Human \sqcap (\geq 2hasParent.\top)$
 $\sqcap (\leq 2hasParent.\top) \sqcap \forall hasParent^-.Human$

"If someone is a human, all their parents are human. they have exactly two parents, and everything that has them as a parent (i.e. is their child) is a human."

Example 2.2.3 [5]
 $hasParent \sqsubseteq hasAncestor$.

"hasParent is a subrole of hasAncestor (i. e. If $A hasparent.B$, then $A hasAncestor.B$)."

Example 2.2.4 $Trans(hasAncestor)$

"The role hasAncestor is transitive (i.e. if $A hasAncestor.B$ and $B hasAncestor.C$ then $A hasAncestor.C$)."

The definitions of syntax and semantics of *SHIQ* are similar to those of *ALC*.

Definition 2.2.3 [5] (*SHIQ syntax*) Let R be a set of role names, which is partitioned into a set R_+ of transitive roles and a set R_p of normal roles. The set of all *SHIQ* roles is $R \cup \{r^- | r \in R\}$, where r^- is called the inverse of the role r .

Let C be a set of concept names. The set of *SHIQ* concepts is the smallest set such that:

1. every concept $A \in C$ is a *SHIQ* concept.
2. if A and B are *SHIQ* concepts and r is a *SHIQ* role, then $A \sqcap B, A \sqcup B, \neg A, \forall r.A$, and $\exists r.A$ are *SHIQ* concepts.
3. if A is a *SHIQ* concept and r is a simple *SHIQ* role (simple role is neither transitive nor has a transitive subrole), and $n \in \mathbb{N}$, then $(\leq nr.A)$ and $(\geq nr.A)$ are *SHIQ* concepts.

TODO: nemala by tam byt spomenuta aj hiearachia roli? (v handbook to v definicii syntaxu a semantiky konceptov a roli nespominaju)

Definition 2.2.4 (*SHIQ semantics*) [5] in addition to definition ??, for all $p \in R$ and $r \in R_+$:

$$\begin{aligned} \langle x, y \rangle \in p^{\mathcal{I}} & \text{ iff } \langle y, x \rangle \in (p^-)^{\mathcal{I}}. \\ \text{if } \langle x, y \rangle \in r^{\mathcal{I}} \text{ and } \langle y, z \rangle \in r^{\mathcal{I}} & \text{ then } \langle x, z \rangle \in r^{\mathcal{I}}. \\ (\leq nr.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} | \#r^{\mathcal{I}}(x, C) \leq n\}, \\ (\geq nr.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} | \#r^{\mathcal{I}}(x, C) \geq n\}, \end{aligned}$$

where $\#M$ denotes the cardinality of the set M , and $r^{\mathcal{I}}(x, C) := \{y | \langle x, y \rangle \in r^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\}$.

OWL is based on *SHOIN*, which is very similar to *SHIQ*

2.2.3 SHOIN

2.2.4 ALCHO

\mathcal{ALC} with role hierarchy and Nominals.

Nominals are concepts with exactly one specific instance. For example, $\{\text{john}\}$ is a concept with its only instance being the individual 'john'.

Nominals can be used to express enumerations, for example [2]:

$$\text{Beatle} \equiv \{\text{john}\} \sqcup \{\text{paul}\} \sqcup \{\text{george}\} \sqcup \{\text{ringo}\}$$

Role hierarchy was already described in section 2.2.2 (example 2.2.3).

Definition 2.2.5 (*Syntax and semantics of \mathcal{ALCHO}*)

Syntax of \mathcal{ALCHO} is the syntax of \mathcal{ALC} (definition 2.2.1), with $\{a\}$ added to the set of concepts C for each individual 'a'.

Similarly, semantics of \mathcal{ALCHO} are the semantics of \mathcal{ALC} (definition 2.2.2) with the addition of $\{a\}^{\mathcal{I}} = \{a^{\mathcal{I}}\}$,

where 'a' is an individual and \mathcal{I} is the interpretation.

(TODO: nemali by sme spomenut aj hierarchiu roli?)

2.3 tableau algorithm

Definition 2.3.1 [6] (*model \mathcal{ALC}*)

A knowledge base $KB = (\mathcal{T}, \mathcal{A})$, where \mathcal{T} is TBox and \mathcal{A} is ABox.

A general concept inclusion (GCI) is of the form $C \sqsubseteq D$, where C, D are \mathcal{ALC} Concepts. \mathcal{T} is a set of GCIs. An interpretation \mathcal{I} is a model of a GCI $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.

\mathcal{I} is a model of a TBox \mathcal{T} if it is a model of every GCI in \mathcal{T} .

An assertional axiom is of the form $x : C$ or $(x, y) : r$, where C is an \mathcal{ALC} Concept, r is a role name, and x and y are individual names. \mathcal{A} is a set of assertional axioms.

An interpretation \mathcal{I} is a model of an assertional axiom $x : C$ if $x^{\mathcal{I}} \in C^{\mathcal{I}}$, and \mathcal{I} is a model of an assertional axiom $(x, y) : r$ if $x^{\mathcal{I}}, y^{\mathcal{I}} \in r^{\mathcal{I}}$.

\mathcal{I} is a model of an ABox \mathcal{A} if it is a model of every axiom in \mathcal{A} .

\mathcal{I} is a model of KB if it's a model of \mathcal{A} and \mathcal{T} .

Example 2.3.1 (*model*)

Knowledge base $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$, $\mathcal{T} = \{$

$$A \sqsubseteq B$$

$$A \sqsubseteq \exists r.C$$

$$C \sqsubseteq \forall r.D$$

$\}$

$\mathcal{A} = \{$

$$a : A$$

$$c, d : r$$

$\}$

One possible model of \mathcal{K} , \mathcal{M}_1 , would be: $\{$

$$\Delta^{\mathcal{I}} = \{ a_x, c_x, d_x \}$$

$$a^{\mathcal{I}} = a_x, c^{\mathcal{I}} = c_x, d^{\mathcal{I}} = d_x$$

$$A^{\mathcal{I}} = \{a_x\}, B^{\mathcal{I}} = \{a_x\}, C^{\mathcal{I}} = \{c_x\}, D^{\mathcal{I}} = \{d_x\}$$

$$r^{\mathcal{I}} = \{ \langle a_x, b_x \rangle, \langle a_x, c_x \rangle, \langle c_x, d_x \rangle \}$$

$\}$

But other models also exist, for example \mathcal{M}_2 and \mathcal{M}_3 . $\mathcal{M}_2 = \{$

$$\Delta^{\mathcal{I}} = \{ a_x, c_x, d_x, c_n \}$$

$$a^{\mathcal{I}} = a_x, c^{\mathcal{I}} = c_x, d^{\mathcal{I}} = d_x$$

$$\begin{aligned}
A^{\mathcal{I}} &= \{a_x\}, B^{\mathcal{I}} = \{a_x\}, C^{\mathcal{I}} = \{c_n\}, D^{\mathcal{I}} = \{\} \\
r^{\mathcal{I}} &= \{\langle a_x, b_x \rangle, \langle a_x, c_n \rangle, \langle c_x, d_x \rangle\} \\
\mathcal{M}_3 &= \{ \\
&\Delta^{\mathcal{I}} = \{i_x\} \\
&a^{\mathcal{I}} = i_x, c^{\mathcal{I}} = i_x, d^{\mathcal{I}} = i_x \\
&A^{\mathcal{I}} = B^{\mathcal{I}} = C^{\mathcal{I}} = D^{\mathcal{I}} = \{i_x\} \\
&r^{\mathcal{I}} = \{\langle i_x, i_x \rangle\} \\
&\}
\end{aligned}$$

In our algorithm, we heavily make use of tableau algorithm, which is a method of constructing a model of a knowledge base \mathcal{K} if \mathcal{K} is consistent, and stops if \mathcal{K} is inconsistent and therefore no model exists.

Tableau algorithm uses knowledge base in negation normal form (NNF), that is, every concept complement \neg applies only to a concept name [6]. Any \mathcal{ALC} concept can be transformed to an equivalent concept in NNF by using de Morgan's laws and the duality between existential and universal restrictions ($\neg\exists r.C \equiv \forall r.\neg C$). For example, the concept $\neg(\exists r.A \sqcap \forall s.B)$, where A, B are concept names, can be transformed using de Morgan's laws to $\neg(\exists r.C) \sqcup \neg(\forall s.B)$, and this can then be transformed using the existential-universal duality into $(\forall r.\neg A) \sqcup (\exists s.\neg B)$.

The idea behind the tableau algorithm for $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ is to start with the concrete situation described in \mathcal{A} and expand based on what can be inferred from \mathcal{T} and currently known ABox statements. This is done by starting with a tree where nodes are individuals, directed edges are relations between individuals, and each individual has a label consisting of concepts the individual belongs to and rules that apply to the individual. The algorithm must at some times, specifically when \sqcup rule is used, make an undeterministic

decision.

Clash is when, for an a concept C , both C and $\neg C$ are in an individuals label. if this happens, we backtrack to a previous undeterministic decision with an unexplored tree and continue from there. If there is no decision with an unexplored possibility we can backtrack to, we can declare \mathcal{K} to be inconsistent. On the other hand, if all relevant rules have been used for each individual, then \mathcal{K} is consistent and we have just found it's model.

TODO: priklad s obrazkom, formalna definicia, casova zlozitost, rozsirenje tableau pre ostatne spomenute deskriptcne logiky.

2.4 OWL (možno)

2.5 Použitia (možno, spomenúť rôzne databázy)

Chapter 3

Abduction

3.1 Definition

3.2 Uses

Chapter 4

Previous abduction solvers

4.1 theoretical approaches

,

4.2 implemented solutions

Chapter 5

Our approach

5.1 Explanation of our algorithm

```

1: function MERGE( Knowledge base K, observation O, maximum Depth
   maxD)
2:   Output: Set of minimum explanations S
3:   if KB  $\cup$  O is inconsistent then
4:     return  $\{\{\}\}$  //observation not consistent with knowledge base
5:   end if
6: end function

```

Chapter 6

Implementation

Chapter 7

Results

Chapter 8

Conclusion

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Zoznam obrázkov