

Online Sources of Information

External Class Website:

- Contact information
- Syllabus
- office hours
- exam dates

URL: www-bcf.usc.edu/~nadamchik/570

DEN Website:

- lecture videos
- lecture notes
- HW assignments
- any other documents

Your Responsibilities

- Attend lectures and discussion sessions
- Study the material from textbook
- Do HW problems
- Do as many other problems from the textbook as possible

Your grade

Exam 1	30%	Spt 29
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Exam 2	30%	Nov 3
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Exam 3	40%	Dec 1
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Prerequisites

- Discrete Math - Mathematical Induction
- Asymptotic notation
- Sorting methods
- Basic data structures: Arrays, Stacks, queues, linked lists
- Basics of graphs: Trees, cycles, DAG, adjacency list, adjacency matrix, etc.
- Graph search alg's: BFS, DFS

Corrections:

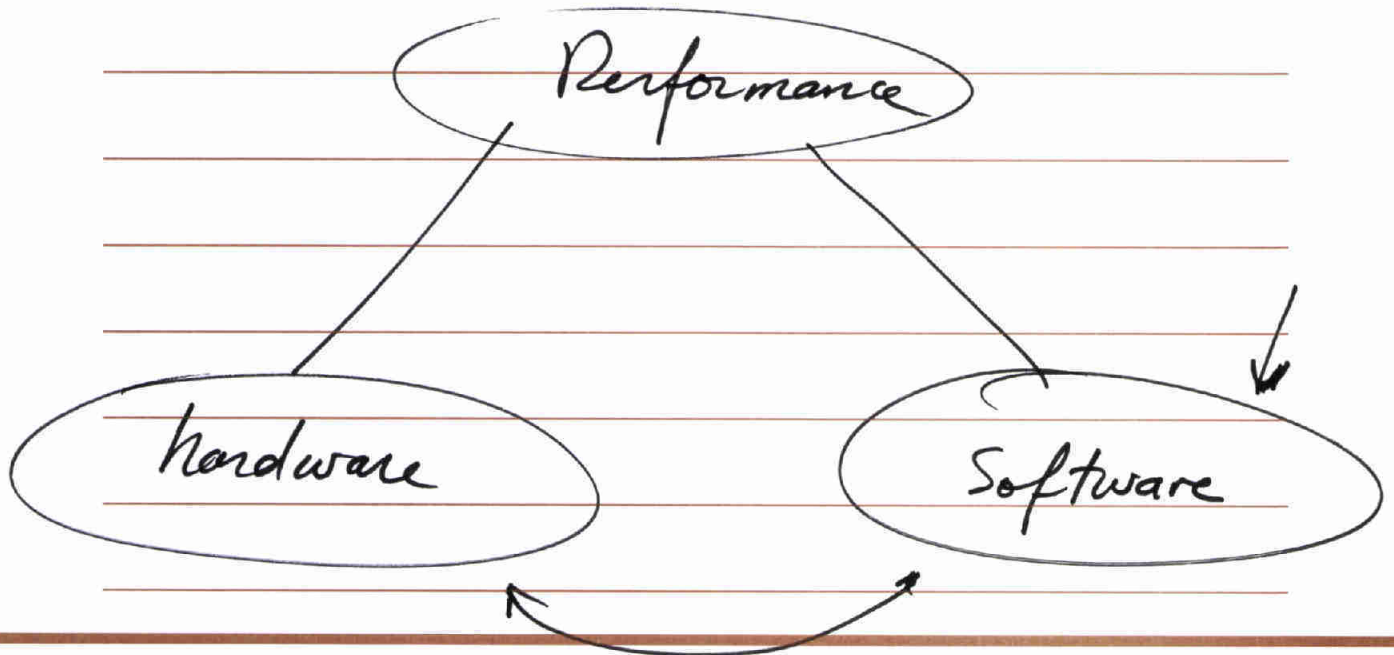
~~1- An algorithm is a set of instructions
in machine language~~

~~Khaz~~ Kharazmi 780-850

~~2- Alg. science advanced on Wall st...~~

~~3- ... Invite 6 million alg's for a listen...~~

- 1- Correctness
- 2- Performance



memory hierarchy
DMP
SMP
GPU

In studying a problem, we go through the following steps:

1- Come up with a concise problem statement

2- Present a solution

3- Prove correctness

4- Perform complexity analysis

Stable Matching

Stable Matching Example

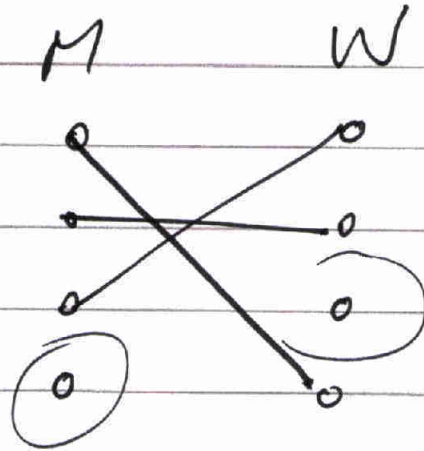
Problem: We are interested in matching n men with n women so that they could stay happily married ever after.

Step 1: Come up with a concise problem statement.

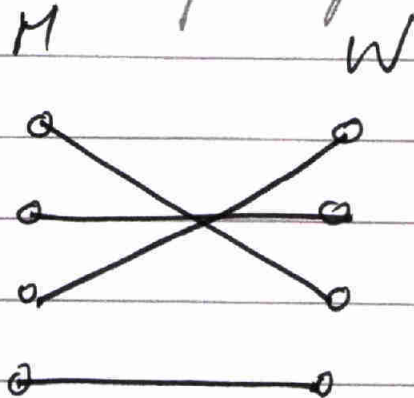
We have a set of n men $M = \{m_1, \dots, m_n\}$

We have a set of n women $W = \{w_1, \dots, w_n\}$

Def. A Matching S is a set of ordered pairs.



Def. A perfect matching S' is a matching with the property that each member of M and each member of W appear in exactly one pair in S' .



Add notion of preferences

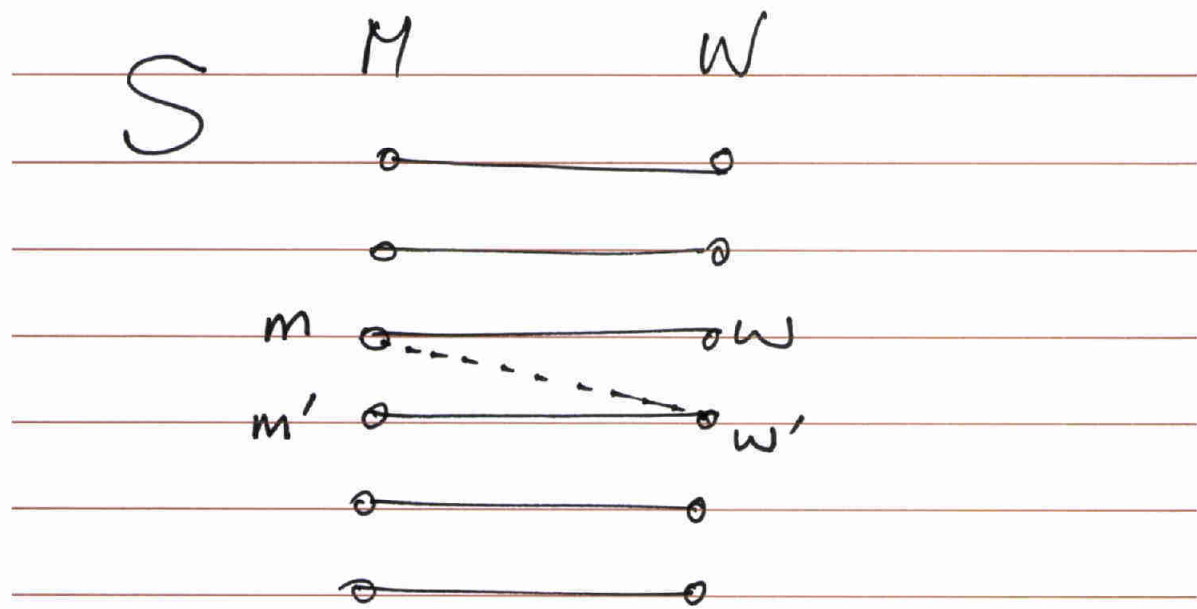
Each man $m \in M$ ranks all women

- \underline{m} prefers \underline{w} to $\underline{w'}$ if \underline{m} ranks \underline{w} higher than $\underline{w'}$.

- ordered ranking of \underline{m} is his preference list

$$P_{m_i} = \{w_{i_1}, w_{i_2}, \dots, w_{i_n}\}$$

Same for women, i.e. each woman $w \in W$ ranks all men . . .



Def. Such a pair (m, w') is
an instability wRT S .

Def. Matching S is stable if

1- It is perfect

2- There are no instabilities
WRT S

Complete Step 1: Input: Preference lists for a
set of n men & n women.

Output: Set of n marriages w/
no instabilities

Step 2 Gale-Shapley
GS

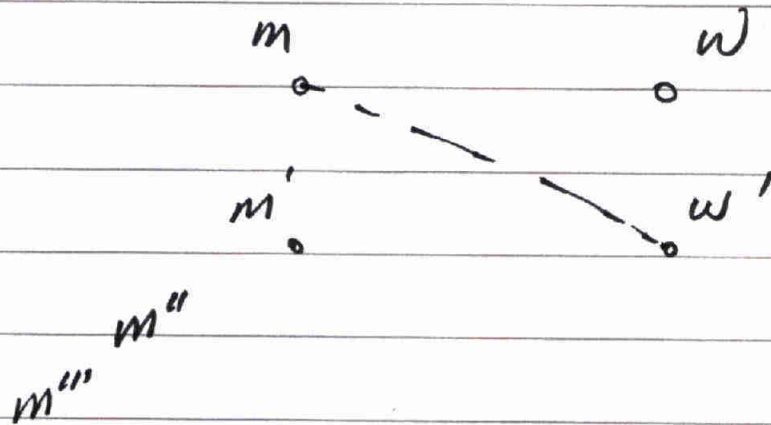
Step 3

Proof of Correctness

- ① From the woman's perspective, she starts single, and once she gets engaged she can only get into better engagement
- ② From the man's perspective, he starts single, gets engaged, and might be dropped repeatedly only to settle for a woman w/ lower ranking,
- ③ Alg. terminates after n^2 iterations
- ④ Solution is a perfect matching
- ⑤ Solution is a stable matching

Proof by Contradiction

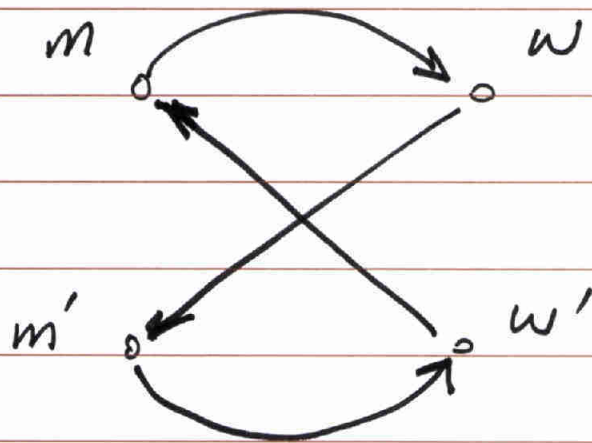
Assume an instability exists in our solution involving two pairs (m, w) , (m', w')



Q: Did m propose to w' at some point in the execution of the algorithm?

If no, Then w must be higher than w' on his list \Rightarrow Contradiction!

If yes, he must have been rejected in favor of m'' and due to ① either $m'' = m'$ or w' is better than $m'' \Rightarrow$ Contradiction!



$(m, w), (m', w')$

$(m, w'), (m', w)$

1- Identify a free man

- array
 - linked list
 - stack
 - queue
- } $O(1)$

2- Identify the highest ranked woman to whom m has not yet proposed.

Keep an array $Next[1..n]$

$Next[m]$ points to the position of the next woman that m will be proposing to

$ManPref[1..n, 1..n]$

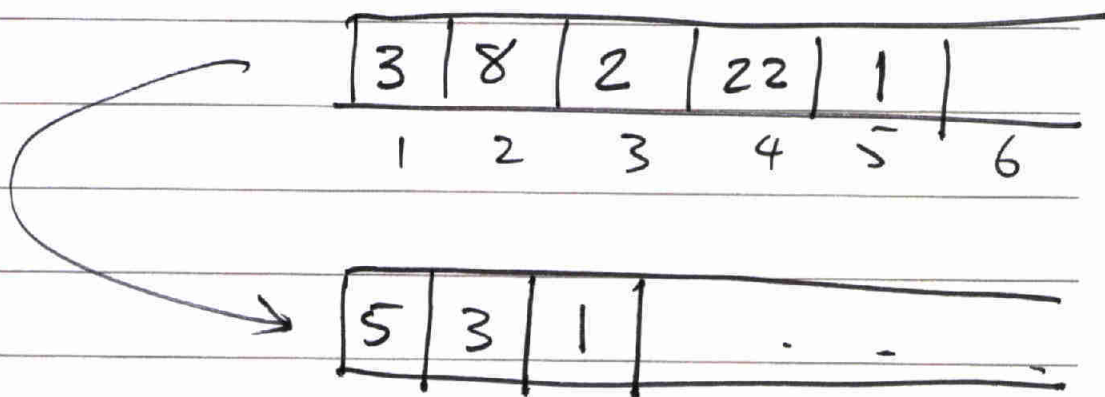
$ManPref[m, \text{Next}[m]] \quad O(1)$

3. Decide if w is engaged, and if so to whom.

Current [w] is Null if w is not engaged, and set to m if she is engaged to m

takes $O(1)$

4. Decide which man (m or m') is preferred by w.



to create Ranking lists will take $O(n^2)$

~~GS~~ GS iterations will take $O(1)$
each

we have $O(n^2)$ iterations

Preparation	+	GS
$O(n^2)$		$O(n^2)$

overall complexity = $O(n^2)$