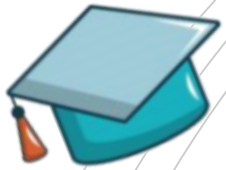

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Digital Image Processing
数字图像处理

Fourier transform and its application in image processing

Jiangxi University of Science and Technology
School of information engineering

Applications of Fourier transforms in Image processing

Fourier transform

- The Fourier transform is the transformation of a function in the space domain and in the frequency domain, the transformation from the space domain to the frequency domain is the Fourier transform, and the transformation from the frequency domain to the space domain is the inverse of the Fourier transform.
- The Fourier transform is the summation of the time domain signal into the sinusoids or cosines of different frequencies.
- The continuous case requires the original signal to satisfy the absolute integrability condition within a period.
- In the discrete case, the Fourier transform must exist. The Fourier transform can be thought of as a mathematical prism, breaking the function into its components based on frequency.
- The Fourier transform allows us to analyze a function by its frequency component. The properties of Fourier transform are linearity, symmetry, time shift, frequency shift and convolution theorem.

Applications of Fourier transforms in Image processing

Applications of Fourier Transforms

- **Fourier transform plays a very important role in image processing, it is widely used in**
  - Image Enhancement
  - Image Denoising,
  - Image Segmentation
  - Edge Detection,
  - Image Feature Extraction (Shape, Texture),
  - Image Restoration
  - Image Encoding/Decoding
  - Image Description
  - Image Compression

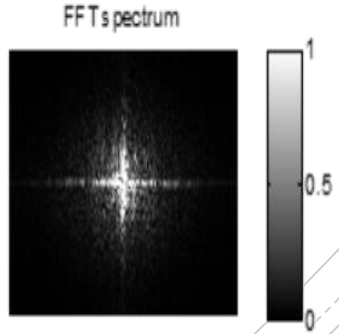


Image Sharpening

- The primary mode of image sharpening is to enhance the edge portion of the image, in fact, the intense portion of the gradation change in the image, so the image edge can be enhanced by enhanced the high frequency signal in the image, thereby achieving image sharpening

Applications of Fourier transforms in Image processing

Image Enhancement

- The image enhancement needs to enhance the edge of the image by increasing the high frequency component of the burst, so as to obtain a better display effect.
- Common image enhancement methods include contrast stretching, histogram equalization and so on. Contrast pull-up is based on pixel transformation in spatial domain while image sharpening is processed in frequency domain.
- As the image using the spatial domain methods can not obtain good image enhancement effect, consider using the method of Fourier transform, the image to Fourier transform can be converted into frequency domain system, and then processed (usually design a low-pass or high-pass filter), to convert the image in the spatial domain, will obtain good effect.
- The high frequency signal in the enhanced image can reach the purpose of image enhancement.

Applications of Fourier transforms in Image processing

Image denoising

- Image denoising is to suppress the noise part of the image. Most of the noise is high frequency component. Therefore, if the noise is in the high frequency domain, the image needs to be processed by a low pass filter, and then the Fourier transform is used.
- We can obtain the frequency domain to process the image as needed, such as when the noise in the image is required, we can design a low-pass filter, remove high frequency noise in the image.

Applications of Fourier transforms in Image processing

Image feature extraction

The shape features of images can be extracted by Fourier descriptors, the texture features of images can be calculated by Fourier coefficients, and then the features can have translation and rotation invariance by Fourier transform of extracted eigenvalues.

Applications of Fourier transforms in Image processing

Image Restoration

- The image is two-dimensional discrete, and both continuous and discrete can be transformed by Fourier, so the two-dimensional signal is obtained by one-dimensional Fourier transform in both x direction and Y direction, and its frequency forms a grid matrix.
- All images of frequencies that is that it is a picture of the different of the amplitude and phase, amplitude and phase at this time is also a grid matrix), after the Fourier transform of the image that you could get the image of the amplitude and phase diagram, and wants to space recovery from frequency domain to time domain space, It is necessary to have the amplitude diagram and phase diagram of the image at the same time, the lack of one is incomplete restoration.
- image restoration is the process of information filling in the incomplete area of the image. The mean value filter and sequential statistical filter can be used to restore the image.

Applications of Fourier transforms in Image processing

Application In Image Encoding/Decoding

- The image signal can be expressed as a series of sinusoidal signal superposition. In the field of image, it is the image brightness variation as the sinusoidal variable. Sinusoidal mode can be encoded by three components in a single Fourier: frequency, amplitude and phase. These three values can describe all the information in a sinusoidal image. A Fourier transform code is a series of sinusoidal codes, and the Fourier transform codes all the frequencies in the image simultaneously.
- Image compression is to transform and combine the image data according to certain rules and represent the image with less data. There is a large correlation between adjacent pixels in the image, which is spatial redundancy. There is a large correlation between adjacent frames of sequential images, which is time redundancy. Image compression is to reduce these redundancies. The data can be compressed by Fourier coefficients

Applications of Fourier transforms in Image processing

Image Description

- The Fourier transform of an image is the decomposition of the image function into a superposition of sines and cosines of different frequencies. In terms of physical effects, the Fourier transform transforms the image from the spatial domain to the frequency domain, and its inverse transform transforms the image from the frequency domain to the spatial domain.
- In other words, the physical meaning of Fourier transform is to transform the gray distribution function of the image into the frequency distribution function of the image.

Applications of Fourier transforms in Image processing

Image Compression

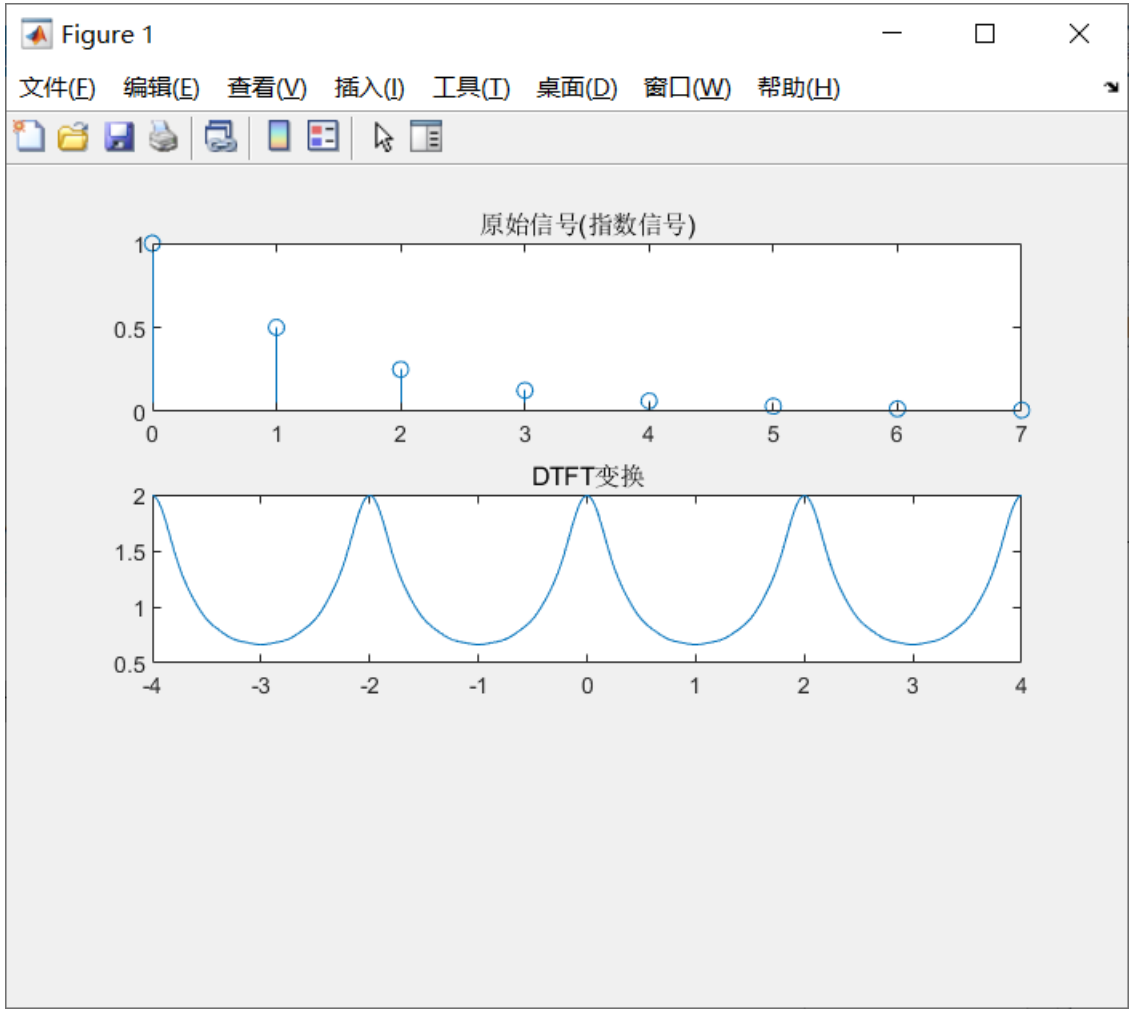
- The data can be compressed directly through the Fourier coefficient; the commonly used discrete cosine transformation is the real transformation of Fourier transform;

DTFT(Discrete Time Fourier Transform)

Code

```
clc
clear all
N=8;
n=[0:1:N-1]
xn=0.5.^n;
w=[-800:1:800]*4*pi/800;
X=xn*exp(-j*(n'*w));
subplot(311)
stem(n,xn);
title('原始信号(指数信号)');
subplot(312);
plot(w/pi,abs(X));
title('DTFT变换')
```

Result

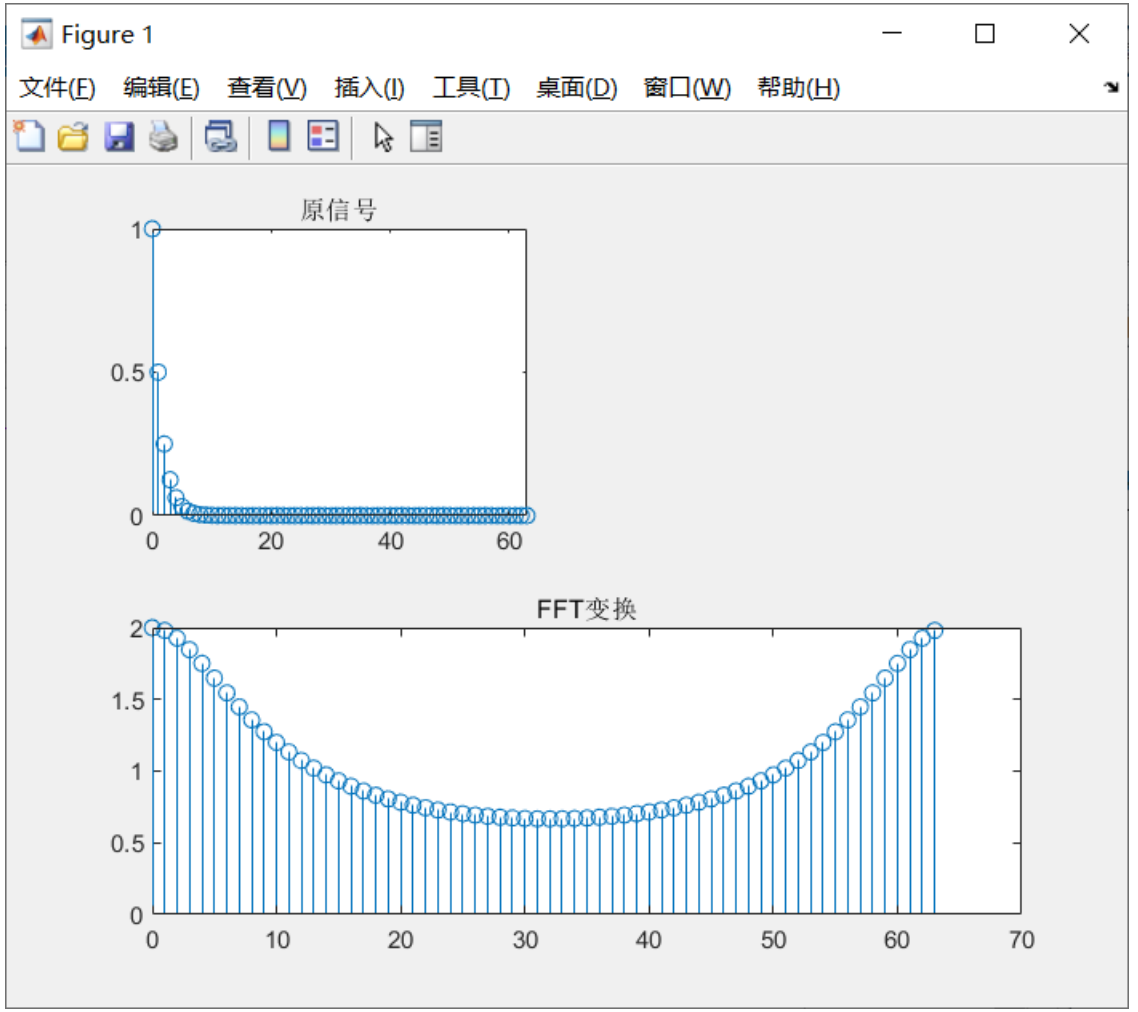


FFT（Fast Fourier Transform）

Code

```
clc
clear all
N=64;
n=[0:1:N-1]
xn=0.5.^n;
Xk=fft(xn,N);
subplot(221);
stem(n,xn);
title('原信号');
subplot(212);
stem(n,abs(Xk));
title('FFT变换')
```

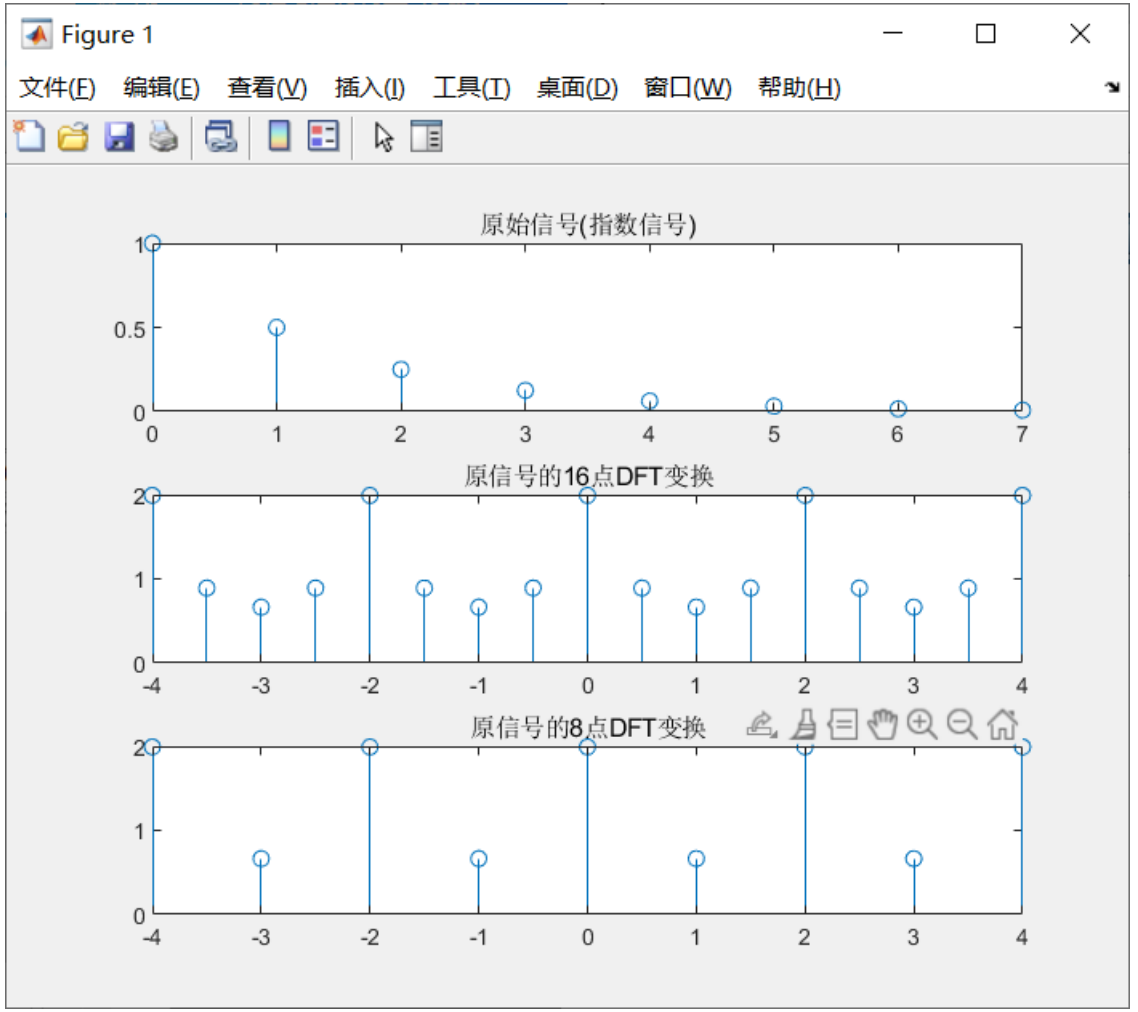
Result



DFT(Discrete Fourier Transform)

- Code

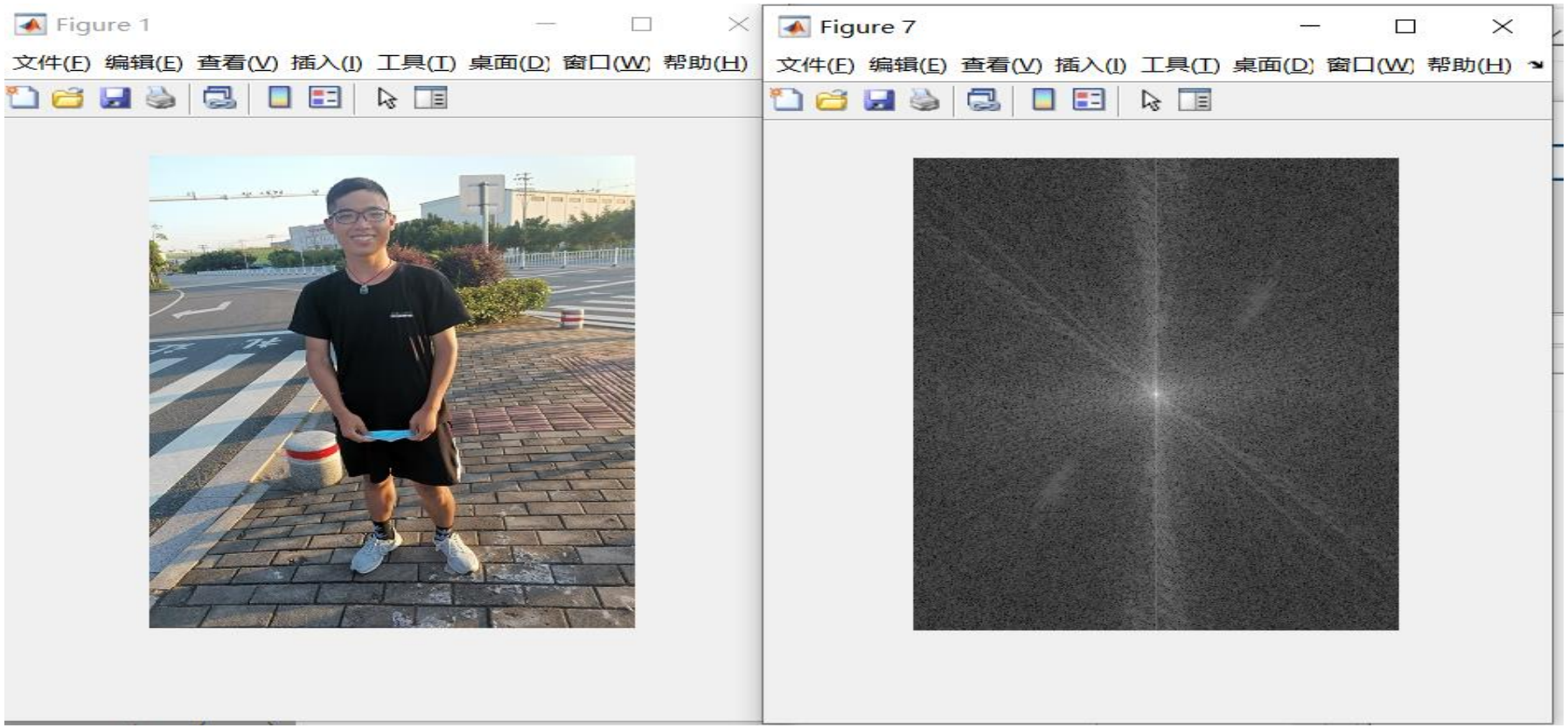
```
w=[-8:1:8]*4*pi/8;
X=xn*exp(-j*(n'*w));
subplot(311)
stem(n,xn);
w1=[-4:1:4]*4*pi/4;
X1=xn*exp(-j*(n'*w1));
title('原始信号(指数信号)');
stem(w/pi,abs(X));
title('原信号的16点DFT变换')
subplot(313)
stem(w1/pi,abs(X1));
title('原信号的8点DFT变换')
```

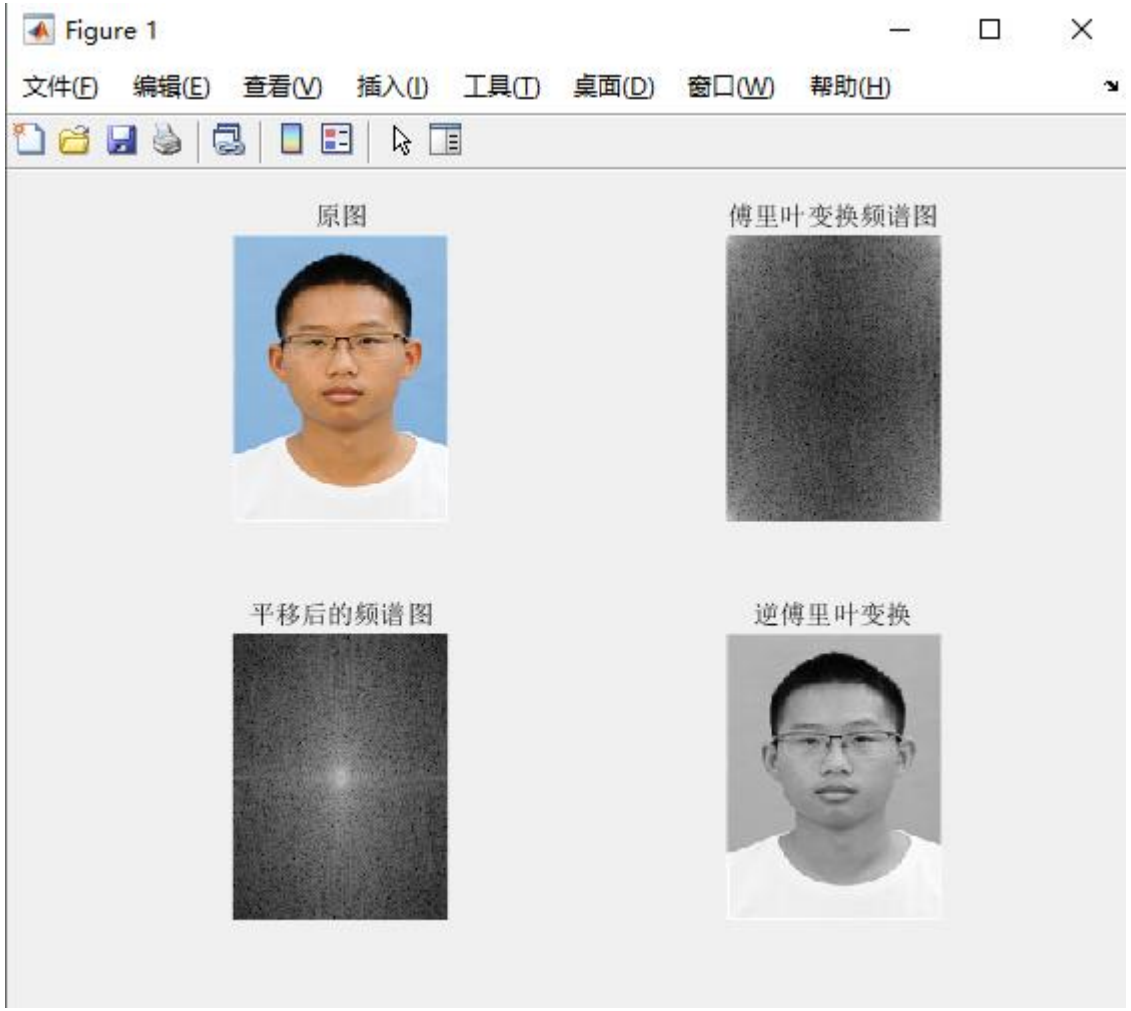


two-dimensional Fast Fourier Transform

- Code

```
clc
clear all
close all
   I=imread('1520192702.jpg');
   imshow(I);
   I=rgb2gray(I);
   I=im2double(I);
   [x,y] = size(I);
   Ax = ones(x,y);
   anss = ones(x,y);
   com = 0+1i;
 for m=1:y
      Ax(:,m) = fft(I(:,m));
   end
   for k=1:x
      anss(k,:) = fft(Ax(k,:));
   end
   F=fftshift(anss);
   F= abs(F);
   F=log(F+1);
   figure(7);
   imshow(F,[]);
```






```
clear clc
img=imread('3.jpg');
subplot(2,2,1);
imshow(img);
title('原图');
f=rgb2gray(img);
F=fft2(f);
F1=log(abs(F)+1);
subplot(2,2,2);
imshow(F1,[]);
title('傅里叶变换频谱图');
Fs=fftshift(F);
S=log(abs(Fs)+1);
subplot(2,2,3);
imshow(S,[]);
title('平移后的频谱图');
fr=real(ifft2(ifftshift(Fs)));
ret=im2uint8(mat2gray(fr));
subplot(2,2,4);
imshow(ret),title('逆傅里叶变换');
```