江西理工大學信息工程学院
JIANGXI UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF INFORMATION ENGINEERING

Digital Image Processing

数字图像处理

Lecture 016:

Image restoration

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LECTURE 16:

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图像修复

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Agenda

- Image restoration
  - What is image restoration?
  - Model of Degradation/Restoration Process
  - Algebraic Approach to Image Restoration

图像修复
- 什么是图像修复?
--退化/恢复过程模型。
图像恢复的代数方法。

Image restoration

图像修复

- Image restoration is to reconstruct or recover an image that has been degraded using some prior knowledge of the degradation phenomenon.

  - 图像恢复是利用退化现象的一些先验知识来重建或恢复已经退化的图像。
- Image restoration usually involves formulating a criterion of goodness that will yield an optimal estimate of the desired result. Thus this is an objective process.

  。- 图像恢复通常涉及制定一个良性标准，该标准将产生对所需结果的最佳估计。 因此，这是一个客观的过程。

- ***Enhancement techniques basically are heuristic procedures designed to manipulate an image in order to take advantage of the psychophysical aspects of the human visual system. Thus this is a subjective process.***

  - 增强技术基本上是启发式程序，旨在操纵图像以利用人类视觉系统的心理物理方面。 因此，这是一个主观的过程

Review one point
digital image enhancement techniques

回顾一点数字图像增强技术

- There are two main methods in digital image enhancement:
  - spatial domain method
  - frequency domain method.

空间域方法
频域方法

- Spatial domain method mainly processes the pixel in the image on the basis of gray mapping transformation.

空间域方法主要是在灰度映射变换的基础上对图像中的像素进行处理

- Frequency domain method is based on convolution theorem.

频域方法是基于卷积定理的

- ***In general, frequency domain method uses frequency transform such as Fourier transform method to achieve image enhancement.***

频域方法一般是利用频率变换如傅里叶变换等方法来实现图像增强

Enhancement Vs Image restoration

增强与图像恢复

Enhancement techniques list

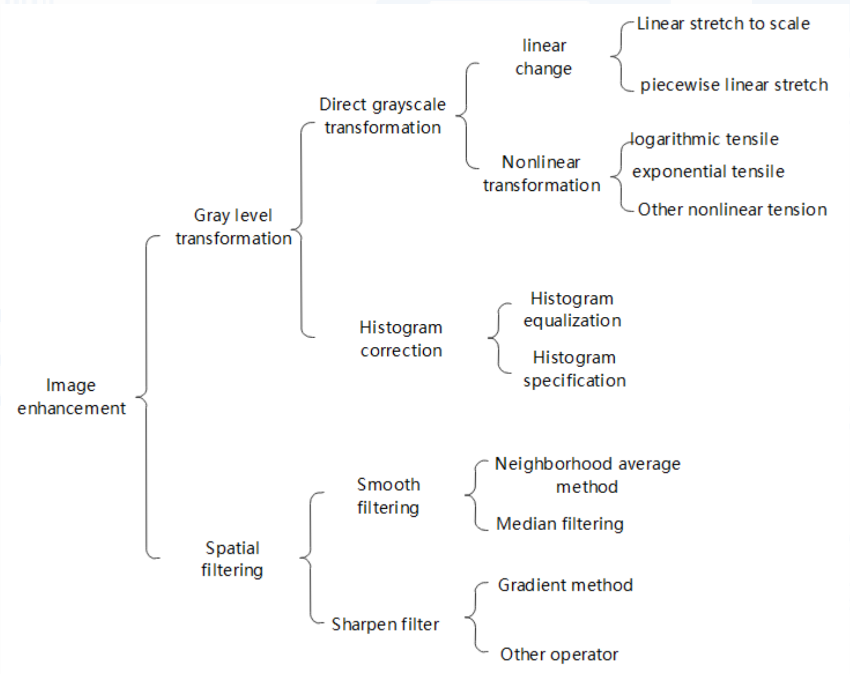

Image restoration list

- **Image Restoration** is a family of inverse problems for obtaining a high quality image from a corrupted input image.
- Corruption may occur due to the image-capture process (e.g., noise, lens blur), post-processing (e.g., JPEG compression), or photography in non-ideal conditions (e.g., haze, motion blur).

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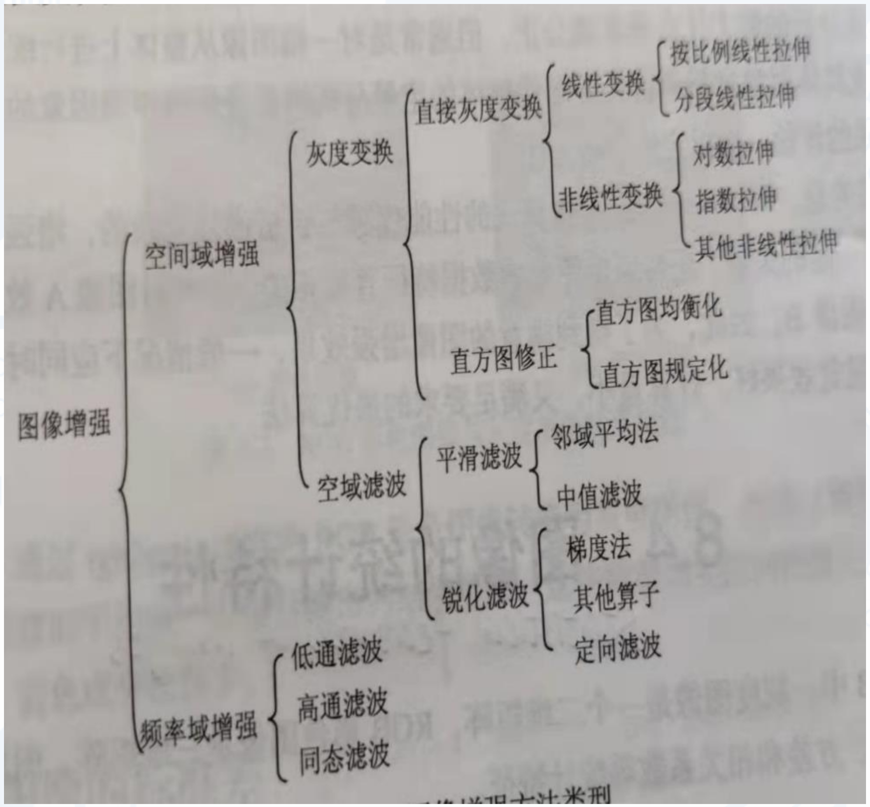
Enhancement
Vs
Image restoration

增强与图像恢复

增强技术列表



图像恢复列表

- 图像恢复是一组从损坏的输入图像中获得高质量图像的反问题。由于图像捕捉过程(如噪声、镜头模糊)、后期处理(如JPEG压缩)或非理想条件下的摄影(如雾霾、运动模糊)，可能会发生损坏。

What is Image Restoration?

什么是图像恢复?

Image restoration attempts to restore images that have been degraded

- Identify the degradation process and attempt to reverse it
- Similar to image enhancement, but more objective

图像恢复尝试恢复已降级的图像
• 识别退化过程并尝试逆转它
• 类似于图像增强，但更客观

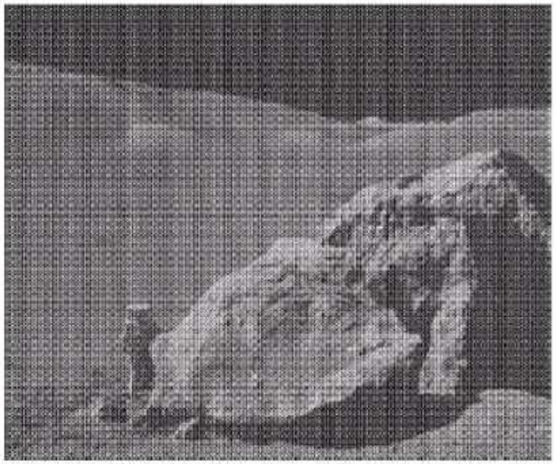


Image enhancement VS image restoration

- Image enhancement is largely a subjective process; enhancement techniques basically are heuristic procedures designed to manipulate an image in order to take advantage of the psychophysical aspects of the human visual system

- Image restoration is an objective process

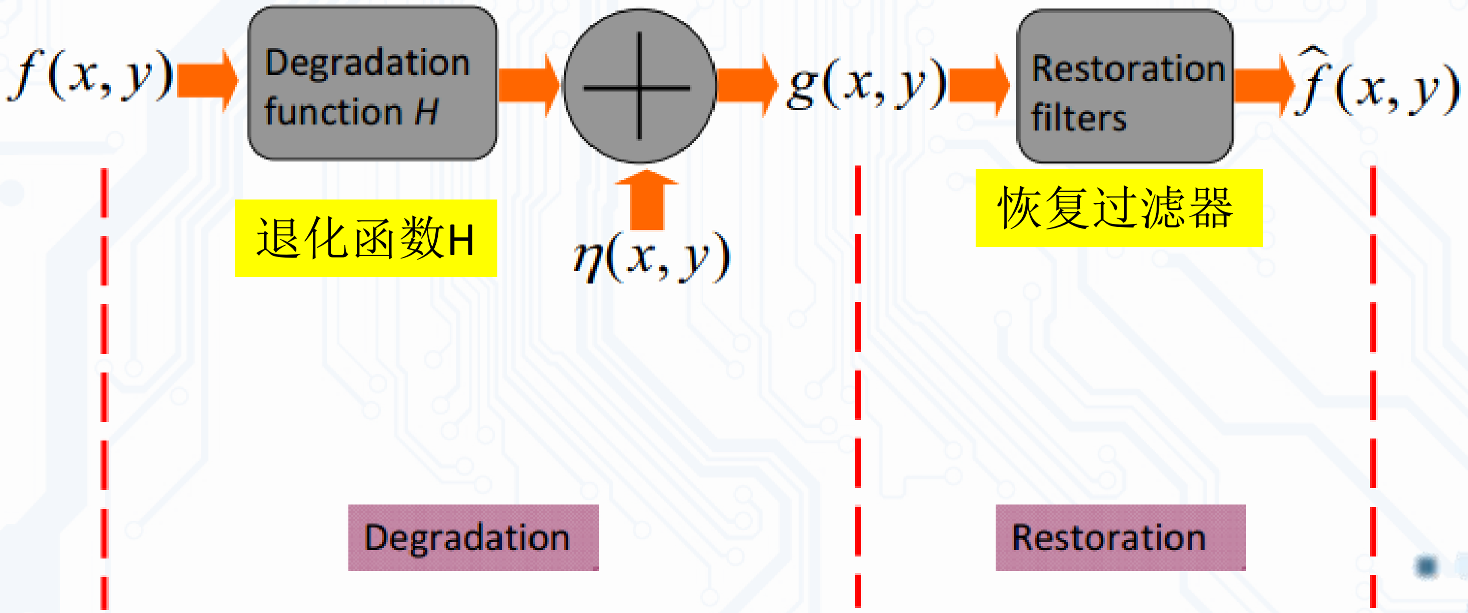
- 图像增强在很大程度上是一个主观过程； 增强技术基本上是启发式程序，旨在操纵图像以利用人类视觉系统的心理物理方面
- -图像恢复是一个客观的过程

Degradation process modeling

退化过程建模

A degradation function with an ***additive noise* term**

- 带有附加噪声项的退化函数



A Model of the Image Degradation/Restoration Process

图像退化/恢复过程的模型

退化过程建模
带有加性噪声项的退化函数

- Degradation process modeling
- A degradation function with an ***additive noise*** **term**

If *H* is a linear, position-invariant process, the degraded image is given in the spatial domain by

$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$$

如果H是一个线性的位置不变过程，则退化图像在空间域中由

where “*” denotes the convolution

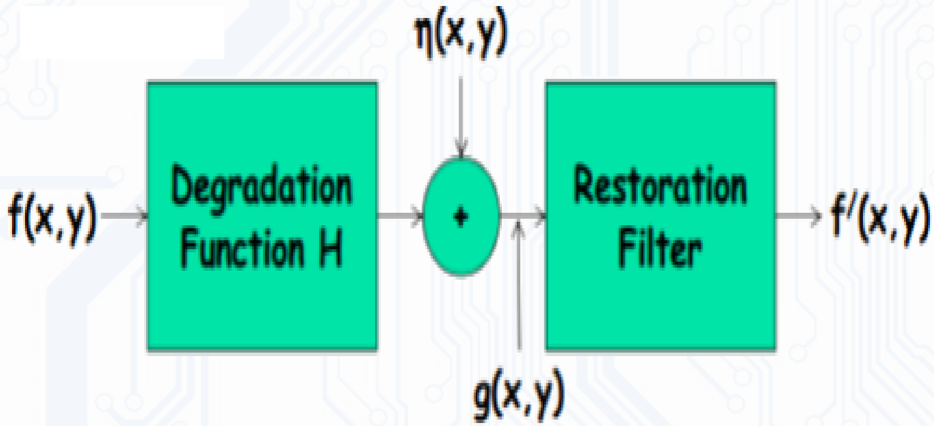
The equivalent frequency domain representation is

其中“*”表示卷积
等效频域表示为

Model of Degradation/Restoration Process

降解/恢复过程模型

- The (image) degradation process is modeled as a system, H together with an additive noise term, n(x,y) operating on an input image, f(x,y) to produce a degraded image, g(x,y) as shown below.



（图像）退化过程被建模为一个系统，H 与加性噪声项，n(x,y) 对输入图像进行操作，f(x,y) 以产生退化图像，g(x,y) 如下所示。

The degradation process is mathematically expressed as 降解过程在数学上表示为

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

Model of Degradation/Restoration Process

降解/恢复过程模型

or in matrix form as

$$g = Hf + \eta$$

where

- (i) f, g and n are MNx1 column matrices formed by stacking the rows of the MxN matrices formed from the extended (zero-padded) functions, $f_e(x,y)$, $g_e(x,y)$ and $n_e(x,y)$ of the original functions, f(x,y), g(x,y) and n(x,y), respectively,
- *(ii) H is a MNxMN* ***block circulant matrix*** *formed by stacking the circulant matrix, H which is in turn constructed from the jth row of the extended (zero-padded) function, $h_e(x,y)$ of the original impulse or unit sample or point spread function, h(x,y) of the degradation system, H.*

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Model of Degradation/Restoration Process

降解/恢复过程模型

或者以矩阵形式表示

where

$$g = Hf + \eta$$

– (i) f、g、n分别是原始函数f(x,y)、g(x,y)、n(x,y)的扩展(零填充)函数f(x,y)、g(x,y)和n(x,y)的MxN矩阵的行叠加而成的MNx1列矩阵，

– *(i) H是由扩展(零填充)函数的第j行he(x,y)，退化系统的原始脉冲或单位样本或点扩展函数H (x,y)的循环矩阵H (x,y)叠加而成的MNxMN块循环矩阵*

Model of Degradation/Restoration Process

降解/恢复过程模型

$$
H = \begin{bmatrix} H_0 & H_{M-1} & H_{M-2} & \bullet & \bullet & \bullet & H_1 \\ H_1 & H_0 & H_{M-1} & \bullet & \bullet & \bullet & H_2 \\ H_2 & H_1 & H_0 & \bullet & \bullet & \bullet & H_3 \\ \bullet & \bullet & & & & & \bullet \\ \bullet & \bullet & & & & & \bullet \\ \bullet & \bullet & & & & & \bullet \\ H_{M-1} & H_{M-2} & H_{M-3} & \bullet & \bullet & \bullet & H_0 \end{bmatrix}
$$

where

$$
H_j = \begin{bmatrix} h_e(j,0) & h_e(j,N-1) & . & . & . & h_e(j,1) \\ h_e(j,1) & h_e(j,1) & & . & . & h_e(j,2) \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ h_e(j,N-1) & h_e(j,N-2) & & . & . & h_e(j,0) \end{bmatrix}
$$

Algebraic Approach to Image Restoration

图像恢复的代数方法

- The objective of image restoration is to estimate the original image,
  - **f** from the degraded image,
  - **g** using some knowledge or assumption about **H** and ***h*.**

图像复原的目标是估计原始图像，
f 来自退化图像，
g 使用一些关于 H 和 h 的知识或假设。

- The objective of algebraic approach is to seek an estimate, **f**$^{/}$, of the original image, **f** from the degraded image, **g** such that a predefined criterion function is minimized.

代数方法的目标是从退化图像 g 中寻找原始图像 f 的估计值 f/，从而使预定义的标准函数最小化。

- There are two basic algebraic approaches:
  - unconstrained and constrained restoration.
  - The basic image degradation model

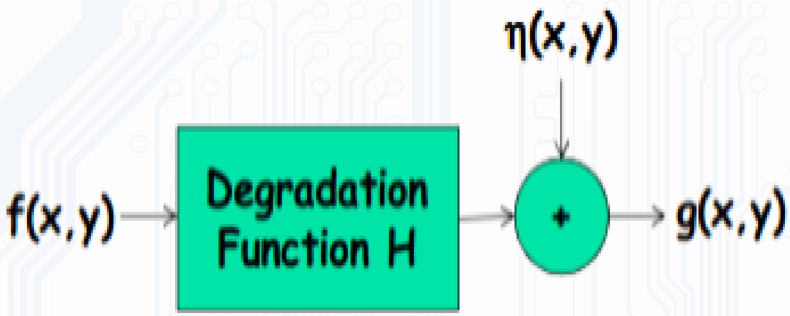
有两种基本的代数方法：
无约束和有约束的恢复。
基本的图像退化模型

Unconstrained Restoration: Inverse Filtering

无约束恢复：反向过滤

- The basic image degradation model

基本的图像退化模型



- From the basic image degradation model,

$$\eta = g - Hf \text{ - - - - - - - - } (1)$$

Unconstrained Restoration: Inverse Filtering

无约束恢复：反向过滤

In the absence of any knowledge about the noise, η, the objective of unconstrained restoration is to seek an estimate, f' of the original image, f from the degraded image, g such that Hf' approximates g and the norm of the noise term is minimized.

That is, f' is found such that

$$\|\boldsymbol{\eta}\|^2 = \|\mathbf{g} - \mathbf{Hf}'\|^2 \text{-------}(2)$$

is minimum, where

$$\|\boldsymbol{\eta}\|^2 = \eta^T\eta, \quad \text{norm of } \eta$$

$$\|\mathbf{g} - \mathbf{Hf}'\|^2 = (g - Hf')^T(g - Hf'), \quad \text{norm of } (g - Hf')$$

在没有任何关于噪声的知识的情况下，无约束恢复的目标是从退化图像中寻求原始图像的估计f'，f, g，使Hf近似于g，噪声项的范数最小化。也就是说，f'被找到时，

Unconstrained Restoration: Inverse Filtering

无约束恢复：反向过滤

- The minimization of Equ(2) is achieved by differentiating it with respect to f’and equating the result to zero.

$$\frac{\partial\left(\|\eta\|^2\right)}{\partial f'} = -2H^T\left(g - Hf'\right) = 0 \text{-------}(3)$$

方程(2)的极小化是通过对f'求导并使结果等于零来实现的

- Solving Equ(3) for f’

求f'的方程(3)

$$f' = H^{-1}g \text{-------------}(4)$$

- Equ(4) is the inverse filtering.

式(4)为反滤波。

Unconstrained Restoration: Inverse Filtering

无约束恢复：反向过滤

- The frequency domain representation of Equ(4) is

$$F'(u,v) = \frac{G(u,v)}{H(u,v)} \text{-------------} (5)$$

where F'(u,v), G(u,v) and H(u,v) are the Fourier transforms of f, g and h, respectively. 其中F'(u,v) G(u,v)和H(u,v)分别是F, G和H的傅里叶变换。

In the presence of noise, Equ(5) becomes

$$F'(u,v) = \frac{G(u,v)}{H(u,v)} + \frac{N(u,v)}{H(u,v)} \text{-------------} (6)$$

where N(u,v) is the Fourier transform of the noise, η.

其中N(u,v)是噪声N的傅里叶变换。

Unconstrained Restoration:Inverse Filtering

无约束恢复：反向过滤

The disadvantages of the inverse filtering (unconstrained restoration) are:

– (i) The complete knowledge about H and η is required and

– (ii) The restoration process is very sensitive to noise.

- That is, the restoration result is dominated by noise if H(u,v) is zero or small

反滤波(无约束恢复)的缺点是:
-(i)需要对H和n有完全的了解
- (ii)修复过程对噪音非常敏感。
. 即当H(u,v)为0或较小时，恢复结果受噪声控制

Inverse Filtering

- The idea of the inverse filtering method is to recover the original image from the blurred image by inversing blurring filter. We assume that no additional noise is present in the system.
- We need to find such a filter kernel *r(x,y)* that its convolution with the blurred image could produce a result close to the original image:
- $g * r = f,$
- $(f * h) * r = f$
- or since the convolution operation is associative and has a multiplicative identity,
- $h * r = O\tau,$
- where $O\tau(x,y)$ is the Kronecker's delta function.
- Then, applying the convolution theorem in the frequency domain we obtain:
- $H(u,w) * R(u,w) = 1.$

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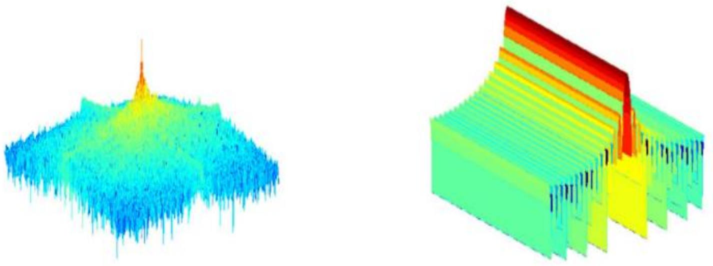
反滤波

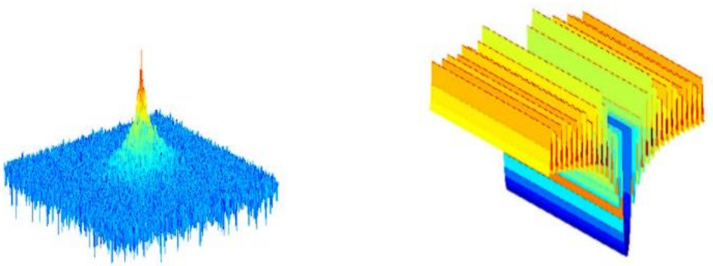
- 反滤波方法的思想是通过反模糊滤波从模糊图像中恢复出原始图像。我们假设系统中不存在额外的噪声。
- 我们需要找到这样一个滤波核r(x,y)，使其与模糊图像的卷积结果接近于原始图像:
- $g * r = f,$
- $(f * h) * r = f$
- 或者因为卷积运算是结合的并且有一个乘法恒等式，
- $h * r = O\tau,$
- 其中Or(x,y)是克罗内克函数。.
- 然后，将卷积定理应用于频域，得到:
- $H(u,w) * R(u,w) = 1.$

Inverse Filtering

- Thus, dividing the DFT of the blurred image by the DFT of the kernel, we can recover (in theory) the original image. Then the inverse filter in frequency domain is simply $R(u,w) = R_{inv}(u,w)$ in frequency domain is $1 / H(u,w)$.
- Using point-wise multiplication of inverse filter $R_{inv}(u,w)$ with Fourier transform of the blurred image $G(u,w)$, we get Fourier transform of the restored image. Now using inverse DFT we obtain the final restored image.



Picture 6: Spectrum of the original image Picture 7: Spectrum of the motion blur kernel



Picture 8: Spectrum of the degraded (blurred) image Picture 9: Spectrum of the inverse filter

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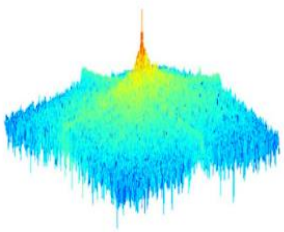
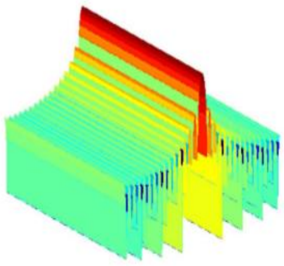
Inverse Filtering

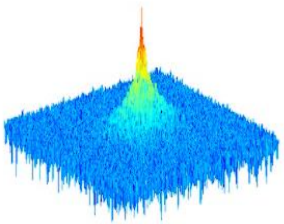
- 哦因此，将模糊图像的DFT除以核的DFT，我们可以恢复(理论上)原始图像。则频域逆滤波器为R(u，) =Rin(uw)在频域为1/H(u)。
- 将反滤波器R(u,w)与模糊图像G(u)的傅里叶变换逐点相乘，得到复原图像的傅里叶变换。现在使用DFT逆，我们得到最终的恢复图像。



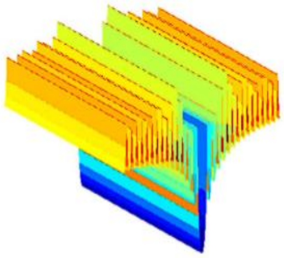
Picture 6: Spectrum of the original image



Picture 7: Spectrum of the motion blur kernel



Picture 8: Spectrum of the degraded (blurred) image



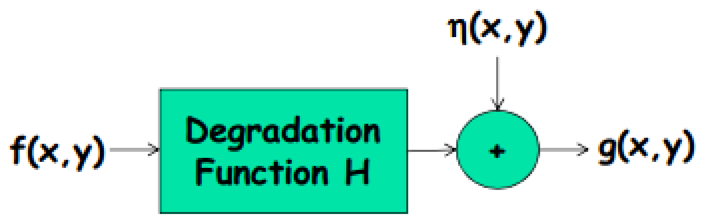
Picture 9: Spectrum of the inverse filter

Constrained Restoration

受限恢复

- The basic image degradation/restoration model

基本的图像退化/恢复模型



- From the basic image degradation/restoration model,

根据基本的图像退化/恢复模型，

$$\eta = g - Hf \text{-------} (1)$$

Constrained Restoration

受限恢复

- The objective of constrained restoration is to seek an estimate, f’ of the original image, f from the degraded image, g such that the criterion function

$$\mathbf{J(f')} = \left\|\mathbf{Qf'}\right\|^2 + \alpha\left(\left\|\mathbf{g}-\mathbf{Hf'}\right\|^2 - \left\|\boldsymbol{\eta}\right\|^2\right) ------(2)$$

- is minimum, where

限制恢复的目的是寻求一个估计。F '的原始图像，从退化图像中提取，从而得到准则函数

$\|\boldsymbol{\eta}\|^2 = \eta^T\eta$, norm of η

$\|\mathbf{g}-\mathbf{Hf'}\|^2 = (g-Hf')^T(g-Hf')$, norm of $(g-Hf')$

Q is an operator on **f**

α is Lagrange's multipler.

Constrained Restoration

受限恢复

The minimization of Equ(2) is achieved by differentiating it wrt f' and equating the result to zero.

方程(2)的极小化是通过微分它wrt f'并使结果等于零来实现的。

$$\frac{\partial[J(\mathbf{f}')]}{\partial \mathbf{f}'} = 2\mathbf{Q}^T\mathbf{Q}\mathbf{f}' - 2\alpha\mathbf{H}^T(\mathbf{g} - \mathbf{H}\mathbf{f}') = 0 ------(3)$$

- Solving for $\mathbf{f}'$,

$$\mathbf{f}' = [\mathbf{H}^T\mathbf{H} + \gamma\mathbf{Q}^T\mathbf{Q}]^{-1}\mathbf{H}^T\mathbf{g} ------(4)$$

where

$$\gamma = \frac{1}{\alpha}$$

Equ(4) yields different solutions for different choices of Q.

Constrained Restoration: LeastMean Square (Wiener) Filter

约束恢复：最小均方 (Wiener) 滤波器

- Least Mean Square or Wiener filter is obtained by defining

通过定义得到最小均方滤波器或维纳滤波器

$$\mathbf{Q}^T\mathbf{Q} = \mathbf{R}_f^{-1}\mathbf{R}_\eta \text{ -------- } (5)$$

where

$\mathbf{R}_f$ is the (auto) correlation matrix of $\mathbf{f}$.

$\mathbf{R}_\eta$ is the (auto) correlation matrix of $\boldsymbol{\eta}$.

- From Equ(4) and Equ(5)

$$\mathbf{f}' = \left[\mathbf{H}^T\mathbf{H} + \gamma\,\mathbf{R}_f^{-1}\mathbf{R}_\eta\right]^{-1}\mathbf{H}^T\mathbf{g} \text{ -------- } (6)$$

Constrained Restoration: Least Mean Square (Wiener) Filter

约束恢复：最小均方 (Wiener) 滤波器

The frequency domain representation of Equ(7) is

$$F^{/}(u,v) = \left[\frac{H^{*}(u,v)}{|H(u,v)|^{2} + \gamma\left[S_{\eta}(u,v)/S_{f}(u,v)\right]} \right] G(u,v) \text{---------} (7)$$

where $F^{/}(u,v)$, $H(u,v)$ and $G(u,v)$ are the Fourier transforms of $f^{/}$, h and g, respectively and $S_h(u,v)$ and $S_f(u,v)$ are the Power Spectral Densities (PSDs) of h and f, respectively.

- With γ=1, Equ(7) becomes the so-called Wiener filter.
- With variable γ, Equ(7) becomes the so-called parametric Wiener filter.
- With Sη(u,v)=0 (no noise), Equ(7) becomes the inverse filter.
- With H(u,v)=1 for all (u,v) (no degradation, only noise), Equ(7) becomes the smoothing (noise removal) filter.

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Constrained Restoration: Least Mean Square (Wiener) Filter

约束恢复：最小均方 (Wiener) 滤波器

The frequency domain representation of Equ(7) is

$$F'(u,v)=\left[\frac{H^{*}(u,v)}{|H(u,v)|^{2}+\gamma\left[S_{\eta}(u,v)/S_{f}(u,v)\right]}\right]G(u,v)\text{---------}(7)$$

式中F'(u,v)， H(u,v)， G(u,v)分别为F、H、G的傅里叶变换，Ss(u,v)， S(u,v)分别为H、F的功率谱密度(psd)。

- 当y=1时，式(7)成为所谓的维纳滤波器。
- 有了变量y，式(7)就变成了所谓的参数维纳滤波器。
- 当Sn(u.y)=0(无噪声)时，式(7)为反滤波器。
- 当H(uv)=1 for all (u.y)(没有退化，只有噪声)时，式(7)成为平滑(去噪)滤波器。

Wiener filtering

- The second technique widely used in image restoration is known as a **Wiener filtering**.
- This restoration method assumes that noise which is present in the system is additive white Gaussian noise and it minimizes mean square error between original and restored images.
- Wiener filtering normally requires prior knowledge of the power spectra (spectral power densities) of the noise and the original image. Spectral power density is a function that describes power distribution over the different frequencies.
- A simplified equation of the Wiener filter R_w is given below:

$$R_W(u,w) = \frac{H(u,w)^*}{|H(u,w)|^2 + \dfrac{S_n(u,w)}{S_f(u,w)}}$$

- where Sn(u,w) is the spectral power density of the noise and Sf(u,w) is the spectral power density of the image.
- Then using inverse DFT we obtain the restored image.

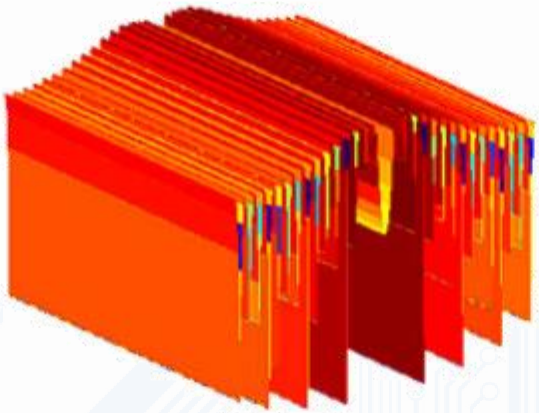
Translation is on the next page

Wiener filtering

- 第二种广泛应用于图像恢复的技术是维纳滤波
- 该方法假设系统中存在的噪声为加性高斯白噪声，使原图像和恢复图像之间的均方误差最小。
- 维纳滤波通常需要先验知识的功率谱(谱功率密度)的噪声和原始图像。
- 谱功率密度是描述不同频率上功率分布的函数。维纳滤波器R的简化方程如下:

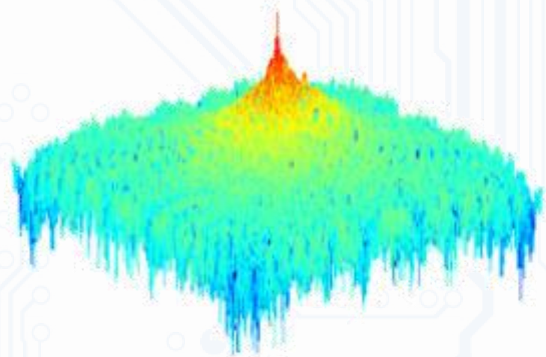
$$R_W(u,w) = \frac{H(u,w)^*}{|H(u,w)|^2 + \frac{S_n(u,w)}{S_f(u,w)}}$$

- 其中Sn(u.w)为噪声的谱功率密度，Sf(u.w)为图像的谱功率密度。
- 然后利用DFT反变换得到恢复后的图像。



Picture 10:

Spectrum of the Wiener filter



Picture 11:

Spectrum of the Wiener-filtered image

$$\frac{S_n(u,w)}{S_f(u,w)} = const = K$$

The inverse filter of a blurred image is a high pass filter. The parameter *K* of the Wiener filter is related to the low frequency aspect of the Wiener filter. The Wiener filter behaves as a band pass filter, where the high pass filter is an inverse filter and the low pass filter is specified by the parameter *K*. Note how Wiener filter becomes an inverse filter when *K* = *0*.

模糊图像的反滤波器是高通滤波器。维纳滤波器的参数K与维纳滤波器的低频方面有关。维纳滤波器表现为带通滤波器，其中高通滤波器是反滤波器，低通滤波器由参数K指定。注意，当K=0时维纳滤波器是如何变成反滤波器的。

inverse filter and Wiener filter.

逆滤波器和维纳滤波器。

The comparison of restoration results obtained by using inverse filter and Wiener filter.

对比反相滤波器和维纳滤波器的恢复结果。

Picture 12: Image restored with inverse filter

Picture 13: Image restored with Wiener filter

Constrained Least Squares Restoration

约束最小二乘法恢复

Defining

$$Q = P\text{------}(5)$$

- **where P is a Laplacian smoothing matrix, Equ(4) becomes**

$$\mathbf{f}' = \left[\mathbf{H}^T\mathbf{H} + \gamma\,\mathbf{P}^T\mathbf{P}\right]^{-1}\mathbf{H}^T\mathbf{g}\text{------}(6)$$

- **The frequency domain representation of Equ(6) is**

$$F'(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right]G(u,v)\text{---------}(7)$$

where P(u,v) is the Fourier transform of the extended version of the 2D Laplacian operator, p(x,y) given by

Constrained Least Squares Restoration

约束最小二乘法恢复

$$p(x,y)=\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

The norm of noise, η can be expressed in terms of its mean, μ_η and standard deviation, σ_η as

$$\|\boldsymbol{\eta}\|^2 = (M-1)(N-1)[\sigma_\eta^2 + \mu_\eta^2] ------(8)$$

噪声范数n可以用其平均值un和标准差Onas表示。

where M & N are dimensions of the noise matrix.

其中M和N是噪声矩阵的维数。

Constrained Least Squares Restoration

约束最小二乘法恢复

<u>Procedure or algorithm:</u> The procedure or algorithm for the constrained least squares restoration is as follows:

步骤或算法:约束最小二乘恢复的步骤或算法如下:

Step1: Initialize γ

Step2: Estimate ||η||2 using

$$\|\mathbf{\eta}\|^2 = (M-1)(N-1)[\sigma_\eta^2 + \mu_\eta^2]$$

Step3: Compute F/(u,v) and hence *f*/ using

$$F'(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right]G(u,v)$$

- **where P(u,v) is the Fourier transform of the extended version of the 2D Laplacian operator, p(x,y) given by**

其中P(u,v)是二维拉普拉斯算子的扩展版的傅里叶变换;p (x, y)给出的

Constrained Least Squares Restoration

约束最小二乘法恢复

$$p(x,y)=\begin{bmatrix}0 & -1 & 0\\ -1 & 4 & -1\\ 0 & -1 & 0\end{bmatrix}$$

Step4: Compute the residual, r and hence f(g) **using**

$$\phi(\gamma)=\|\mathbf{r}\|^2=\|\mathbf{g}-\mathbf{H}\mathbf{f}'\|^2$$

Step5: Increment γ if $\phi(\gamma)<||\eta||^2-a$

OR

Decrement γ if $\phi(\gamma)>||\eta||^2+a$

Step6: Return to Step3 and continue until the statement $\phi(\gamma)=||\eta||^2\pm a$ is true.

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Pseudo Inverse Filter

伪逆滤波器

In the inverse filtering technique, it is often practically difficult to obtain the exact inverse, (1/H) of the degradation function, H, which is stable.

Hence a stable version of the exact inverse of the degradation function, known as the pseudo inverse filter, is obtained.

在逆滤波技术中，实际上通常很难获得稳定的退化函数 H 的精确倒数 (1/H)。因此，获得了退化函数的精确逆的稳定版本，称为伪逆滤波器。

Linear & Position-Invariant (LPI) Degradation

线性和位置不变 (LPI) 退化

A degradation system, H is linear if:

$$H[a_1 f_1(x,y) + a_2 f_2(x,y)] = a_1 H[f_1(x,y)] + a_2 H[f_2(x,y)]$$

退化系统，H是位置不变的，如果:

- **A degradation system, H is position-invariant if:**

$$H[f(x,y)] = g(x,y) \Rightarrow H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

- **A degradation system, H is LPI if it is linear and position-invariant.**

一个退化系统，H是LPI，如果它是线性和位置不变的。

Geometric transformations

几何变换

- Geometric transformations often are called rubber-sheet transformations, because they may be viewed as the process of "printing" an image on a sheet of rubber and then stretching this sheet according to some predefined set of rules.

- A geometric transformation consists of two basic operations:
  - (i) A spatial transformation, which defines the "rearrangement" of pixels on the image plane
  - (ii) Gray-level interpolation, which deals with the assignment of gray levels to pixels in the spatially transformed image.

几何变换通常被称为胶板变换，因为它们可以被看作是在一张胶板上“印刷”一个图像，然后根据一些预定义的规则拉伸这张胶板的过程。
几何变换包含两个基本运算:
(i)空间变换，它定义了图像平面0上像素的“重排”
(ii)灰度插值，处理将灰度值分配到空间变换图像中的像素

Geometric transformations

几何变换

- Spatial transformation: an image with pixel coordinates (x,y) undergoes geometric distortion to produce an image g with coordinates (x',y').

空间变换:像素坐标为(x,y)的图像经过几何变形，生成坐标为(x'.y)的图像g。

- This transformation may be expressed as

$$x_1 = r(x, y)$$

$$y_1 = s(x, y)$$

其中r和s是产生几何畸变图像g(x'.y')的空间变换。

- where r and s are the spatial transformations that produced the geometrically distorted image, g(x',y').

- Example: r(x,y)=x/2 and s(x,y)=y/2. This transformation is simply shrinking the image.

例如:r(x,y)-x/2和s(x,y)=y/2。这种转换仅仅是缩小图像。

Geometric transformations

几何变换

- If r(x,y) and s(x,y) can be expressed analytically, then the original image, f(x,y) can be easily recovered from the distorted image, g(x',y') by applying the transformation in the reverse direction.

- If r(x,y) and s(x,y) cannot be expressed analytically as is the case in most practical applications, the spatial relocation of the pixels are formulated using the tiepoints which are a subset of pixels whose location in the distorted and corrected images is known precisely as shown in the following figure.

- A set of equations are derived for the spatial relocations of these tie points. The reverse transformation is achieved using them.

Translation is on the next page

Geometric transformations

几何变换

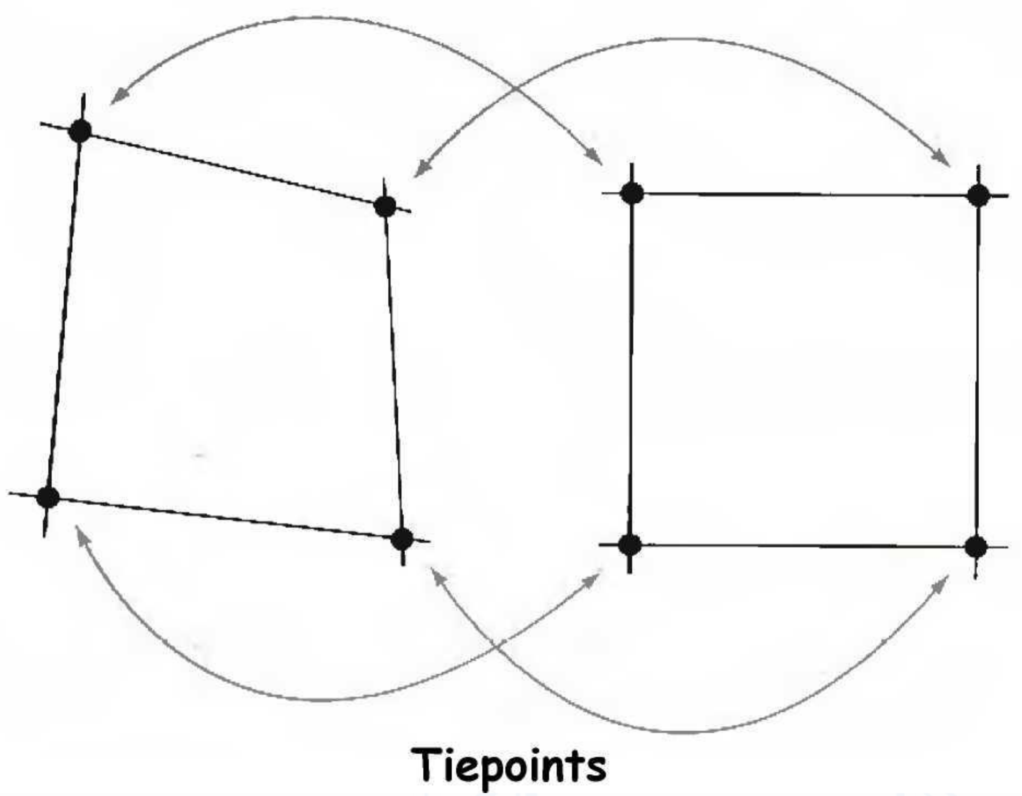
- 如果r(x,y)和s(x,y)可以解析表示，则原。通过反向变换，图像f(x,y)可以很容易地从扭曲的图像g(x.y')中恢复出来。

- 如果r(x,y)和s(x,y)不能像大多数实际应用中那样以解析的方式表达，则使用联络点来表示像素的空间重新定位，联络点是像素的子集，其在扭曲和校正图像中的位置是精确已知的，如下图所示。

- 推导了这些连接点的空间迁移方程。反向转换是使用它们实现的。

Geometric transformations

几何变换



Gray-Level Interpolation

灰度插值

- When performing the reverse geometrical transformation to obtain the original image, f(x,y) from the distorted image, g(x',y').

- Depending on the coefficients of the equations for r(x,y) and s(x,y), the coordinates, (x',y') may be integers or non-integers. For a digital image, the coordinates must be integers.

- Hence, for the non-integer coordinates, their integer equivalents may not have any gray level values. Obtaining the gray level values for such coordinates is known as the gray-level interpolation.

- **The gray-level interpolation techniques include :**
  - **(ii) cubic convolution interpolation**
  - **and (iii)bilinear interpolation.**

Translation is on the next page

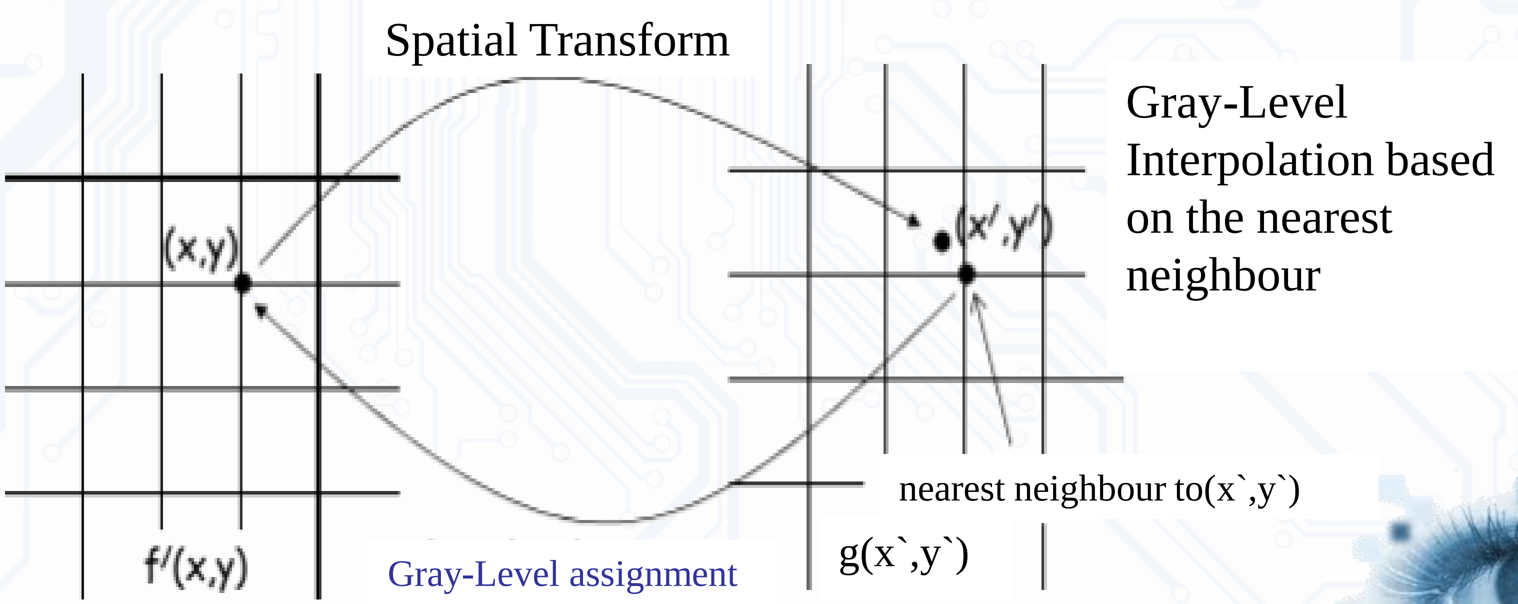
灰度插值

- 当执行逆几何变换以从失真图像中获得原始图像 f(x,y) 时， g(x',y’)。
- 根据 r(x,y) 和 s(x,y) 等式的系数， 坐标 (x',y') 可以是整数或非整数。 对于数字图像， 坐标必须是整数。
- 因此， 对于非整数坐标， 它们的整数等价物可能没有任何灰度值。 获取此类坐标的灰度值称为灰度插值。
- 灰度级插值技术包括：
  - (ii) 三次卷积插值
  - (iii) 双线性插值。

Gray-Level Interpolation

灰度插值

- **Zero-order interpolation: This is based on the nearest-neighbor approach.**
  - **1) the mapping of integer (x,y) coordinates into fractional coordinates (x',y'),**
  - **2) the selection of the closest integer coordinate neighbor to (x',y') and**
  - **3) the assignment of the gray level of this nearest neighbor to the pixel located at (x, y).**

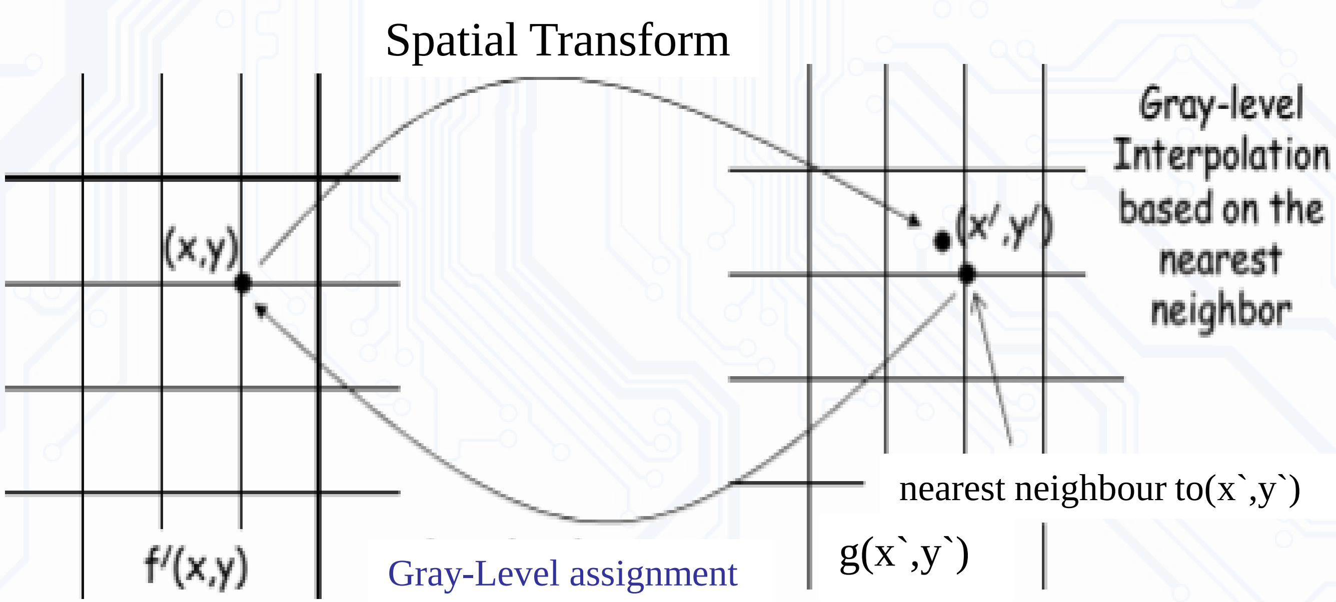


Translation is on the next page

Gray-Level Interpolation

灰度插值

- 零级插值:这是基于最近的邻居方法。
- 1)整数(x,y)坐标到分数坐标(x,y ')的映射，
  - 2)选择(x'.xy')和(x'.xy')最近的整数坐标邻居
  - 3)将最近邻的灰度值赋给位于(x, y)的像素。



Gray-Level Interpolation

灰度插值

- **Cubic convolution interpolation**: This fits a surface of the sin(z)/z type through a much larger number of neighbours (say, 16) in order to obtain a smooth estimate of the gray level at any desired point.
- **Bilinear interpolation**: This uses the gray levels of the four nearest neighbors usually is adequate. The gray-level value at the non-integral pairs of coordinates, denoted v(x',y'), can be interpolated from the values of its neighbors by using the relationship

$$v(x',y') = ax' + by' + cx'y' + d$$

- where the four coefficients are easily determined from the four equations in four unknowns that can be written using the four known neighbors of (x',y').
- When these coefficients have been determined, v(x',y') is computed and this value is assigned to the location in f(x,y) that yielded the spatial mapping into location (x',y').

Translation is on the next page

Gray-Level Interpolation

灰度插值

- **三次卷积插值:这适合一个sin(z)/z类型的曲面，通过更多的邻居(例如。16)以获得任意期望点的灰度值的平滑估计值。**

- **双线性插值:这使用了四个最近邻的灰度级通常是足够的。非整坐标对处的灰度值(记为v(x‘.y’))可以是interpo v(x. y)。V ‘)=ax +by’ +cxy'+d通过使用关系**

$$v(x',y') = ax' + by' + cx'y' + d$$

- 其中四个系数很容易由四个方程确定，四个方程由四个未知数组成，可以用(x'.x')的四个已知邻居来表示。

- 当这些系数被确定后，v(x'.x')被计算，这个值被赋给f(x,y)中的位置，从而产生到位置(xy)的空间映射。

Gray-Level Interpolation

灰度插值

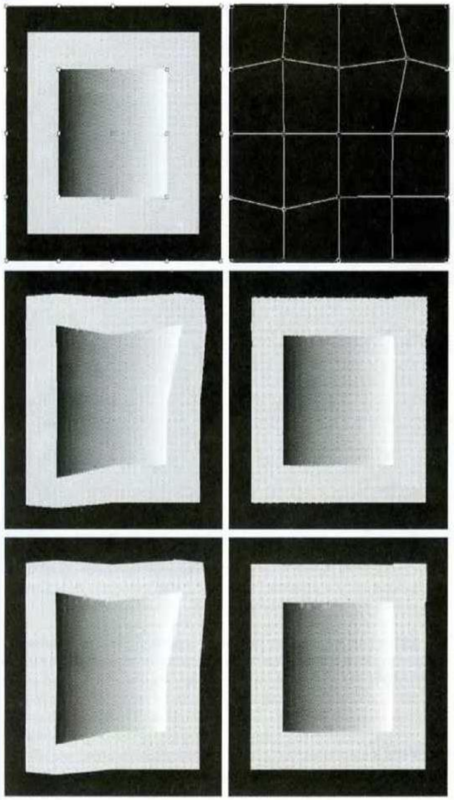


FIGURE 5.34 (a) Image showing tiepoints. (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.

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THANK YOU

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Digital Image Processing

LECTURE 16_B: Interpolation more concept

插值更多概念

Resizing Images

- Zooming :Creating new pixel locations Assigning gray-level values to these locations
- Solution: Interpolation

缩放：创建新的像素位置将灰度值分配给这些位置。
解决方案：插值

Image interpolation

- 图像插值

- Image interpolation occurs when you resize or distort your image from one pixel grid to another.
- Image resizing is necessary when you need to increase or decrease the total number of pixels, whereas remapping can occur when you are correcting for lens distortion or rotating an image.
- Zooming refers to increase the quantity of pixels, so that when you zoom an image, you will see more detail.

- 当您将图像从一个像素网格调整大小或扭曲到另一个像素网格时，就会发生图像插值。
- 当您需要增加或减少像素总数时，需要调整图像大小，而在校正镜头失真或旋转图像时可能会发生重新映射。
- 缩放是指增加像素的数量，这样当你缩放图像时，你会看到更多的细节。

Image interpolation

图像插值

- Interpolation works by using known data to estimate values at unknown points.
- Image interpolation works in two directions, and tries to achieve a best approximation of a pixel's intensity based on the values at surrounding pixels.
- **Common interpolation algorithms can be grouped into two categories: adaptive and non-adaptive.**
  - **Adaptive methods change depending on what they are interpolating, whereas non-adaptive methods treat all pixels equally.**
  - **Non-adaptive algorithms include: nearest neighbor, bilinear, bicubic, spline, sinc, lanczos and others.**
- Adaptive algorithms include many proprietary algorithms in licensed software such as: Qimage, PhotoZoom Pro and Genuine Fractals.

Translation is on the next page

Image interpolation 图像插值

- 插值的工作原理是利用已知数据估计未知点的值。
- 图像插值工作在两个方向，并试图根据周围像素的值获得像素强度的最佳近似。
- **常用的插值算法可分为自适应和非自适应两类。.**
  - **自适应方法的变化取决于它们所插值的内容，而非自适应方法对所有像素一视同仁。.**
  - **非自适应算法包括:最近邻算法、双线性算法、双三次算法、样条算法、sinc算法、lanczos算法等。.**
- 自适应算法包括许多授权软件中的专有算法。如:Qimage, PhotoZoom Pro和Genuine分形。.

Image interpolation

图像插值

- Many compact digital cameras can perform both an optical and a digital zoom.
- A camera performs an optical zoom by moving the zoom lens so that it increases the magnification of light.
- However, a digital zoom degrades quality by simply interpolating the image.
- Even though the photo with digital zoom contains the same number of pixels, the detail is clearly far less than with optical zoom.

许多小型数码相机可以执行光学变焦和数码变焦。
相机通过移动变焦镜头来执行光学变焦，从而增加光的放大倍数。
然而，数字变焦通过简单地插入图像来降低质量。
尽管数码变焦的照片包含相同数量的像素，但细节显然远不如光学变焦。

Image interpolation

图像插值

- zooming and shrinking will be introduced and for this purpose interpolation is introduced and discussed.
- Many various interpolation techniques will be briefly introduced and three of them namely, nearest neighbour, bilinear, and bicubic interpolations.

将介绍缩放和收缩，为此目的，介绍和讨论插值。
将简要介绍多种插值技术，其中三种，即最近邻插值、双线性插值和双三次插值。

3 main type of 2D Interpolations :

3 种主要类型的 2D 插值:

- Nearest neighbor interpolation
- Bilinear interpolation
- Bicubic interpolation

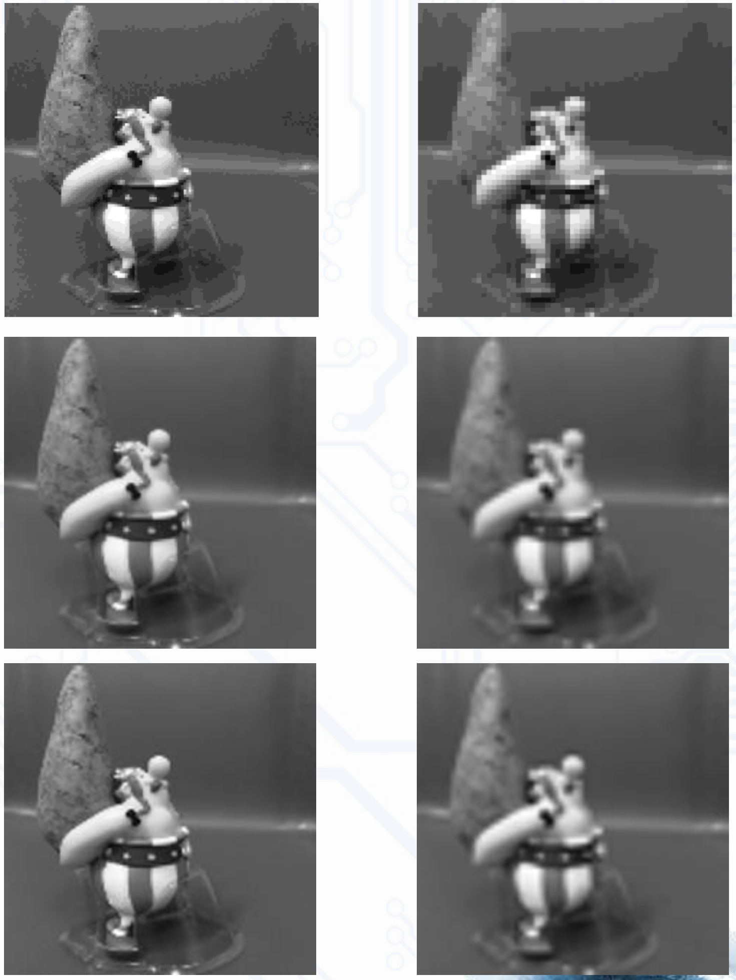
最近邻插值
双线性插值
双立方插值

We will review this points in matlab sections

- Nearest Neighbor

Bilinear

Bicubic

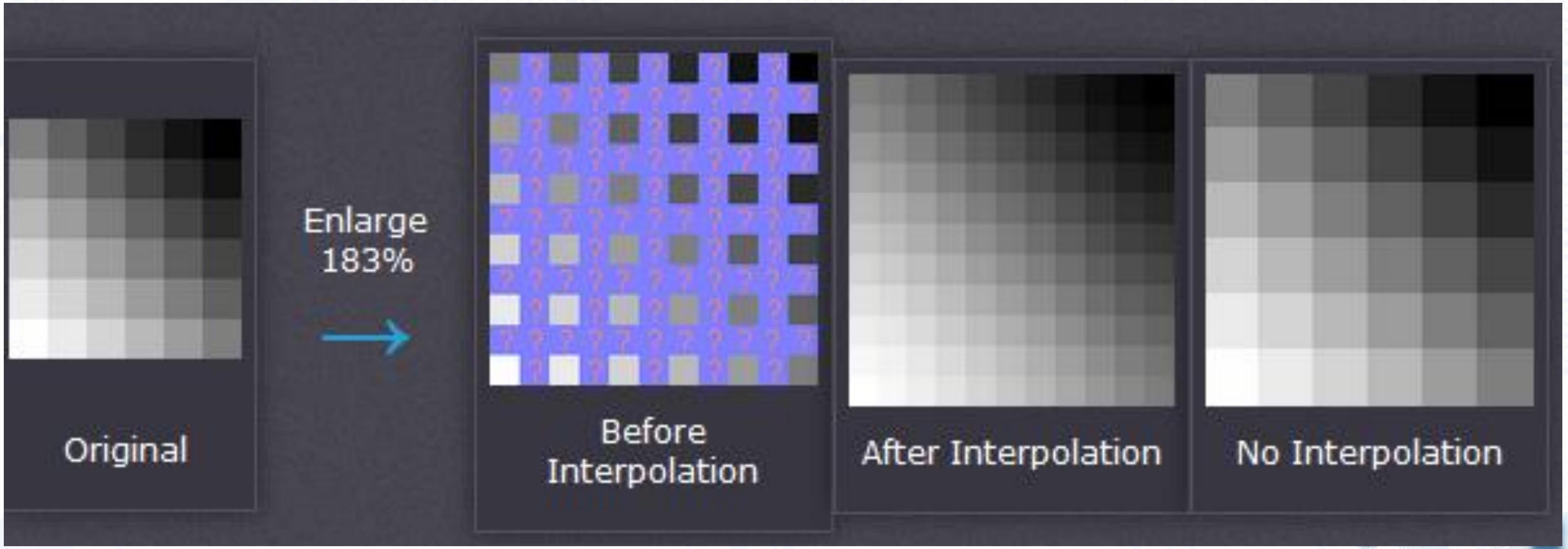


128 → 1024

64 → 1024

Image interpolation 图像插值

- Image interpolation works in two directions, and tries to achieve a best approximation of a pixel's color and intensity based on the values at surrounding pixels.
- The following example illustrates how resizing / enlargement works:



图像插值工作在两个方向，并试图根据周围像素的值获得像素的颜色和强度的最佳近似。下面的例子说明了如何调整/放大尺寸:

Image interpolation

图像插值

- Image interpolation occurs in all digital photos at some stage — whether this be in bayer demosaicing or in photo enlargement.
- It happens anytime you resize or remap (distort) your image from one pixel grid to another.
- Image resizing is necessary when you need to increase or decrease the total number of pixels, whereas remapping can occur under a wider variety of scenarios: correcting for lens distortion, changing perspective, and rotating an image.

图像插值出现在所有数码照片的某个阶段-无论是在bayer demosaicing还是在;照片放大。
它发生在任何时候，你调整大小或重新映射(扭曲)你的图像从一个像素网格到另一个。
当您需要增加或减少总像素数时，图像大小调整是必要的，而重映射可以在更广泛的场景下发生:纠正镜头失真、改变视角和旋转图像。

Image interpolation

图像插值

Even if the same image resize or remap is performed, the results can vary significantly depending on the interpolation algorithm. Itis only an approximation, therefore an image will always lose some quality each time interpolation is performed.

即使执行了相同的图像大小调整或重映射，根据插值算法的不同，结果也会有很大的不同。这只是一个近似值，因此每次进行插值时图像总是会失去一些质量。

TYPES OF INTERPOLATION ALGORITHMS

插值算法的类型

- Common interpolation algorithms can be grouped into two categories: adaptive and non-adaptive.
  – Adaptive methods change depending on what they are interpolating (sharp edges vs. smooth texture),
  – whereas non-adaptive methods treat all pixels equally.

常用的插值算法可分为自适应和非自适应两类。
-自适应方法的变化取决于它们的插值(尖锐的边缘和平滑的纹理)
-而非自适应方法对所有像素一视同仁。

Non-adaptive algorithms

非自适应算法

- **Non-adaptive algorithms** include: nearest neighbor, bilinear, bicubic, spline, sinc, lanczos and others. Depending on their complexity, these use anywhere from 0 to 256 (or more) adjacent pixels when interpolating.
- The more adjacent pixels they include, the more accurate they can become, but this comes at the expense of much longer processing time. These algorithms can be used to both distort and resize a photo.

非自适应算法包括：最近邻、双线性、双三次、样条、sinc、lanczos 等。根据它们的复杂性，它们在插值时使用 0 到 256（或更多）个相邻像素。它们包含的相邻像素越多，它们就越准确，但这是以更长的处理时间为代价的。这些算法可用于扭曲和调整照片大小。

Adaptive algorithms

自适应算法

- **Adaptive algorithms** include many proprietary algorithms in licensed software such as: Qimage, PhotoZoom Pro, Genuine Fractals and others.
- Many of these apply a different version of their algorithm (on a pixel-by-pixel basis) when they detect the presence of an edge — aiming to minimize unsightly interpolation artifacts in regions where they are most apparent.
- These algorithms are primarily designed to maximize artifact-free detail in enlarged photos, so some cannot be used to distort or rotate an image.

Translation is on the next page

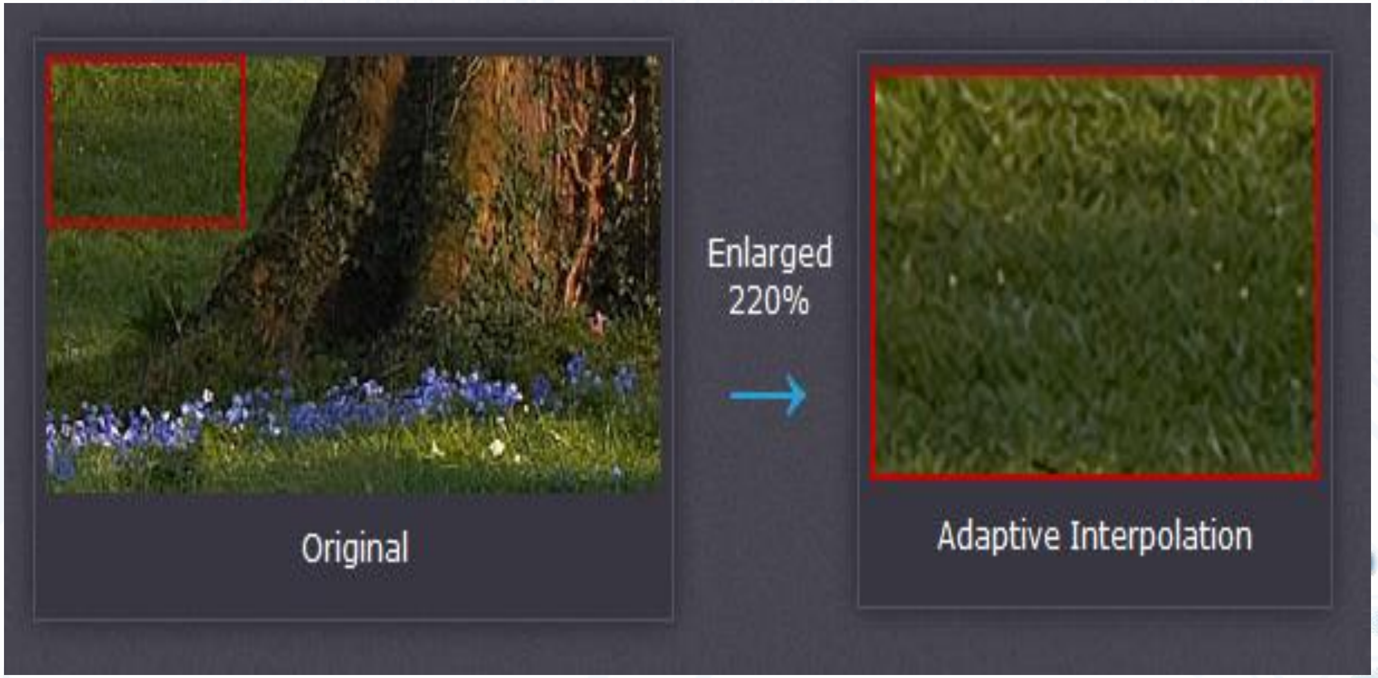
Adaptive algorithms

自适应算法

- **自适应算法包括授权软件中的许多专有算法，如:Qimage, PhotoZoom Pro, Genuine fractal等。**
- 当它们检测到ar边缘时，其中许多应用了不同版本的算法(逐像素)，目的是在最明显的区域最小化不美观的插值伪影。
- 这些算法主要是为了最大化放大照片中的无伪细节而设计的，因此有些算法不能用于扭曲或旋转图像。

Adaptive interpolators

- Adaptive interpolators may or may not produce the above artifacts, however they can also induce non-image textures or strange pixels at small-scales:



自适应插值可能会也可能不会产生上述伪影，但是它们也可以在小尺度上诱发非图像纹理或奇怪的像素:

Adaptive interpolators

自适应算法

- On the other hand, some of these "artifacts" from adaptive interpolators may also be seen as benefits. Since the eye expects to see detail down to the smallest scales in fine-textured areas such as foliage, these patterns have been argued to trick the eye from a distance (for some subject matter).

另一方面，自适应插值器的一些“人为因素”也可能被视为好处。由于眼睛期望看到精细纹理区域(如树叶)的最小尺度的细节，这些模式被认为是从远处欺骗眼睛(对于某些主题)。

NEAREST NEIGHBOR INTERPOLATION

最近邻插值

- Nearest neighbour is the most basic and requires the least processing time of all the interpolation algorithms because it only considers one pixel — the closest one to the interpolated point.
- This has the effect of simply making each pixel bigger.

最近邻是最基本的，在所有插值算法中需要最少的处理时间，因为它只考虑一个像素——最接近插值点的像素。
这具有简单地使每个像素更大的效果。

BILINEAR INTERPOLATION

双线性插值

- Bilinear interpolation considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixel.
- It then takes a weighted average of these 4 pixels to arrive at its final interpolated value.
- This results in much smoother looking images than nearest neighbor.

双线性插值考虑围绕未知像素的已知像素值的最接近的 2x2 邻域。
然后取这 4 个像素的加权平均值，以得出其最终的内插值。
这导致看起来比最近邻更平滑的图像。

BICUBIC INTERPOLATION

双三次插值

- Bicubic goes one step beyond bilinear by considering the closest 4x4 neighborhood of known pixels — for a total of 16 pixels. Since these are at various distances from the unknown pixel, closer pixels are given a higher weighting in the calculation. Bicubic produces noticeably sharper images than the previous two methods, and is perhaps the ideal combination of processing time and output quality. For this reason it is a standard in many image editing programs (including Adobe Photoshop), printer drivers and in-camera interpolation.

Bicubic通过考虑已知像素最接近的4x4邻域——总共16个像素，超越了双线性。由于这些像素与未知像素之间的距离不同，因此在计算中，距离越近的像素权重越大。与前两种方法相比，Bicubic产生的图像明显更清晰，可能是处理时间和输出质量的理想组合。由于这个原因，它是许多图像编辑程序(包括Adobe Photoshop)、打印机驱动程序和摄像头内插的标准

See once more 2D Interpolations :

再次查看 2D 插值:

- Nearest neighbor interpolation
- Bilinear interpolation
- Bicubic interpolation

最近邻插值

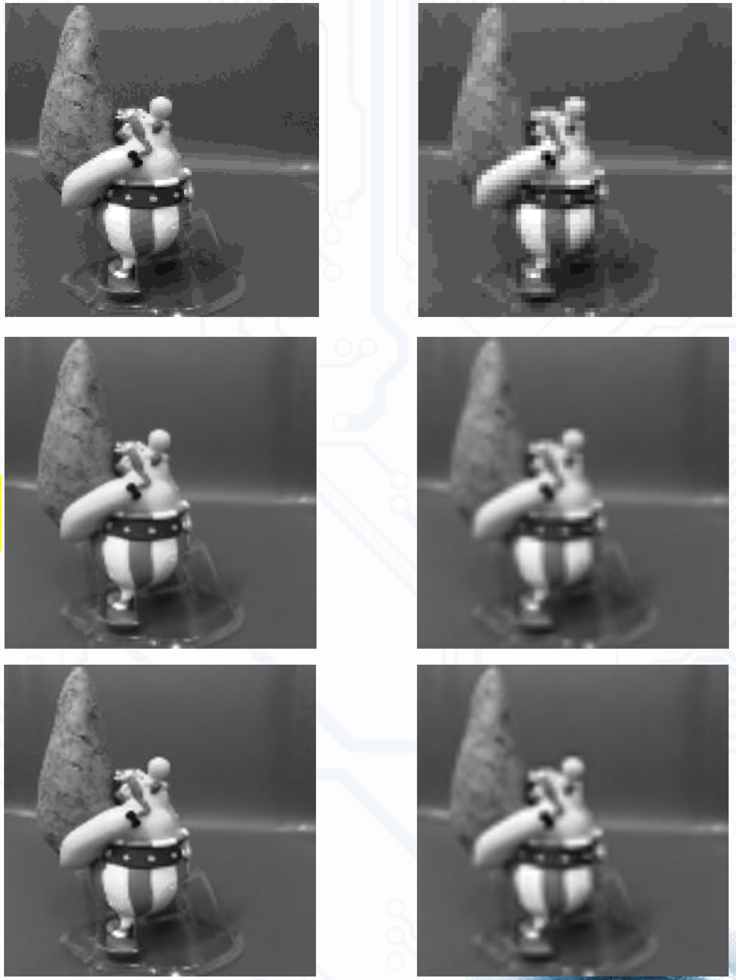
- Nearest Neighbor

双线性插值

Bilinear

双立方插值

Bicubic



128 → 1024

64 → 1024

We will review this points in matlab sections

HIGHER ORDER INTERPOLATION: SPLINE & SINC

高阶插值：SPLINE & SINC

- There are many other interpolators which take more surrounding pixels into consideration, and are thus also much more computationally intensive.
- These algorithms include spline and sinc, and retain the most image information after an interpolation.
- They are therefore extremely useful when the image requires multiple rotations / distortions in separate steps.
- However, for single-step enlargements or rotations, these higher-order algorithms provide diminishing visual improvement as processing time is increased.

Translation is on the next page

HIGHER ORDER INTERPOLATION: SPLINE & SINC

高阶插值：SPLINE & SINC

- 还有许多其他的插值器考虑更多的周围像素，因此也有更多的计算密集型。
- 这些算法包括样条算法和sinc算法，并在插值后保留大部分图像信息。
- 因此，当图像需要在不同的步骤中进行多次旋转/扭曲时，它们非常有用。
- 然而，对于单步放大或旋转，这些•随着处理时间的增加，高阶算法提供的视觉改善逐渐减少。

INTERPOLATION ARTIFACTS TO WATCH OUT FOR

需要注意的插值伪像

所有的非自适应插值都试图在三种不受欢迎的伪影之间找到一个最佳平衡:边缘光晕、模糊和混叠。

- All non-adaptive interpolators attempt to find an optimal balance between three undesirable artifacts: edge halos, blurring and aliasing.

INTERPOLATION ARTIFACTS TO WATCH OUT FOR

需要注意的插值伪像

- Even the most advanced non-adaptive interpolators always have to increase or decrease one of the above artifacts at the expense of the other two — therefore at least one will be visible
- Also note how the edge halo is similar to the artifact produced by over sharpening with an unsharp mask, and improves the appearance of sharpness by increasing acutance.

即使是最先进的非自适应插值器也必须增加或减少上述两种伪影之一，因此至少有一种伪影是可见的
还请注意边缘光晕是如何类似于伪影产生的过度锐化与非锐化罩，并改善外观锐化增加锐度。

ANTI-ALIASING

抗锯齿

- Anti-aliasing is a process which attempts to minimize the appearance of aliased or jagged diagonal edges, termed "jaggies." These give text or images a rough digital appearance:

反锯齿是一个过程，试图减少锯齿或锯齿的对角线边缘的外观，称为“锯齿”。这些给文本或图像一个粗略的数字外观:

ANTI-ALIASING

抗锯齿

- Anti-aliasing removes these jaggies and gives the appearance of smoother edges and higher resolution. It works by taking into account how much an ideal edge overlaps adjacent pixels. The aliased edge simply rounds up or down with no intermediate value, whereas the anti-aliased edge gives a value proportional to how much of the edge was within each pixel:

反锯齿消除了这些锯齿，使边缘更平滑，分辨率更高。它的工作原理是考虑理想边缘与相邻像素的重叠程度。混叠边缘只是向上或向下四舍五入，没有中间值，而抗混叠边缘给出的值与每个像素内的边缘比例成正比：

ANTI-ALIASING

抗锯齿

- A major obstacle when enlarging an image is preventing the interpolator from inducing or exacerbating aliasing.
- Many adaptive interpolators detect the presence of edges and adjust to minimize aliasing while still retaining edge sharpness.
- Since an anti-aliased edge contains information about that edge's location at higher resolutions, it is also conceivable that a powerful adaptive (edge-detecting) interpolator could at least partially reconstruct this edge when enlarging.

Translation is on the next page

ANTI-ALIASING

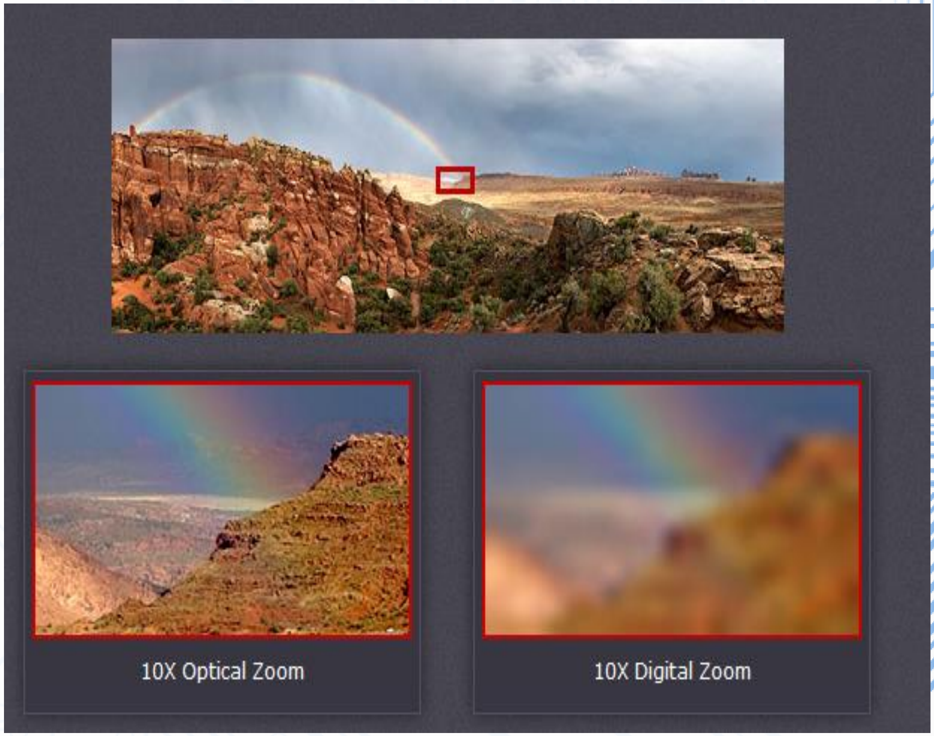
抗锯齿

- 放大图像的一个主要障碍是防止插值器诱发或加剧混叠。
- 许多自适应内插器检测到边缘的存在并调整以最小化混叠，同时仍然保持边缘锐利。
- 由于反锯齿边缘包含更高分辨率下该边缘位置的信息，因此可以想象，一个强大的自适应(边缘检测)插值器在放大时至少可以部分地重建该边缘。

NOTE ON OPTICAL vs. DIGITAL ZOOM

光学变焦与数字变焦的注意事项

- Many compact digital cameras can perform both an optical and a digital zoom. A camera performs an optical zoom by moving the zoom lens so that it increases the magnification of light before it even reaches the digital sensor. In contrast, a digital zoom degrades quality by simply interpolating the image — after it has been acquired at the sensor.



许多小型数码相机可以执行光学变焦和数码变焦。 相机通过移动变焦镜头来执行光学变焦，从而在光线到达数字传感器之前增加其放大倍数。 相比之下，数字变焦通过简单地对图像进行插值来降低质量——在图像被传感器获取之后。

Remember ……….

- Even though the photo with digital zoom contains the same number of pixels, the detail is clearly far less than with optical zoom.
- **Digital zoom should be almost entirely avoided**, unless it helps to visualize a distant object on your camera's LCD preview screen.
- If you find you are needing digital zoom too frequently, purchase a teleconverter add-on, or better yet: a lens with a longer focal length.

尽管数码变焦的照片包含相同数量的像素，但细节显然远不如光学变焦。数码变焦应该几乎完全避免，除非它有助于在相机的 LCD 预览屏幕上可视化远处的物体。如果您发现自己过于频繁地需要数码变焦，请购买增距镜附件，或者更好的是：焦距更长的镜头。

Student Task_3: DIP

- 请帮我翻译部分的朋友鼓掌
- Qǐng bāng wǒ fānyì bùfèn de péngyǒu gǔzhǎng

Solve the Question shared in mooc

解决mooc分享的问题

Send for Next lecture

发送下一个讲座

江西理工大学

Jiangxi University of Science and Technology

信息工程学院

School of information engineering

Digital Image Processing

THANK YOU

Reference

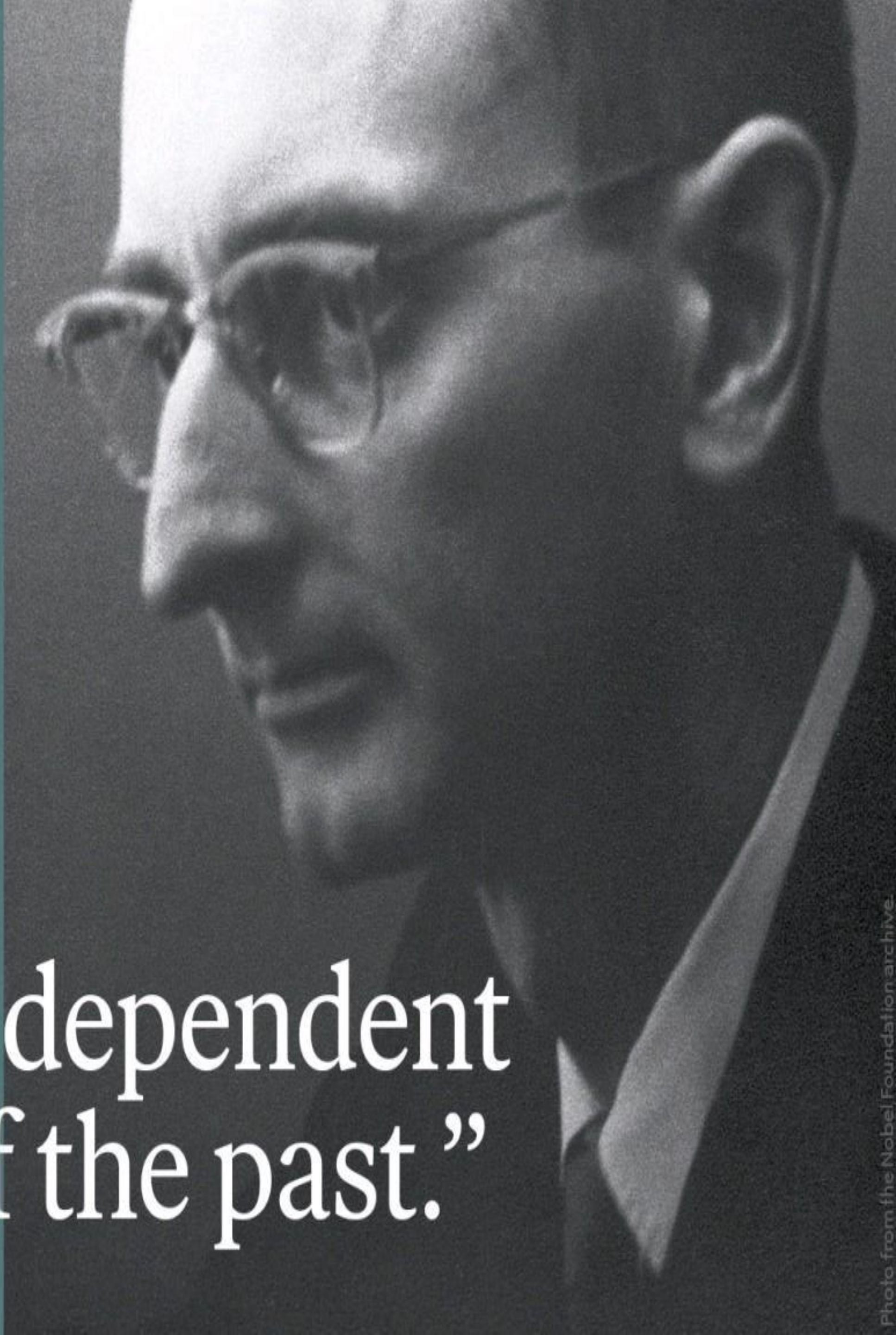
Introduction to MATLAB, *Kadin Tseng,Boston University, Scientific Computing and Visualization*

- Images taken from Gonzalez & Woods, Digital Image Processing (2002)

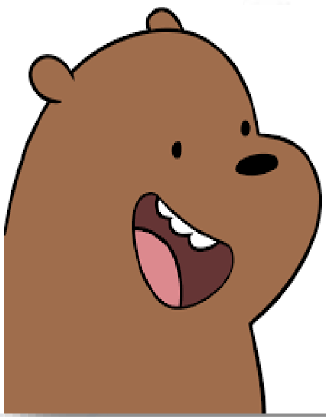
从Gonzalez & Woods拍摄的图像，数字图像处理(2002)

OWEN CHAMBERLAIN
Nobel Prize in Physics 1959

“Each new idea is dependent upon the ideas of the past.”



"BE HUMBLE. BE HUNGRY. AND ALWAYS BE THE HARDEST WORKER IN THE ROOM."



MAY-BRITT MOSER
Nobel Prize in Physiology or Medicine 2014

“I learned at an early age that work makes you happy.”