



江西理工大学 信息工程学院
JIANGXI UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF INFORMATION ENGINEERING



Prof Associate ,
School of information engineering Jiangxi university of
science and technology, China

EMAIL: ajm@jxust.edu.cn

Digital Image Processing

数字图像处理



Lecture 011: Image enhancement Fourier Transform

Dr Ata Jahangir Moshayedi

图像增强
傅里叶变换

Autumn _2021



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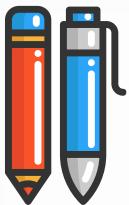
Digital Image Processing

LECTURE 011:

Transformation and Image

Fourier Transform

图像增强
傅里叶变换



Agenda

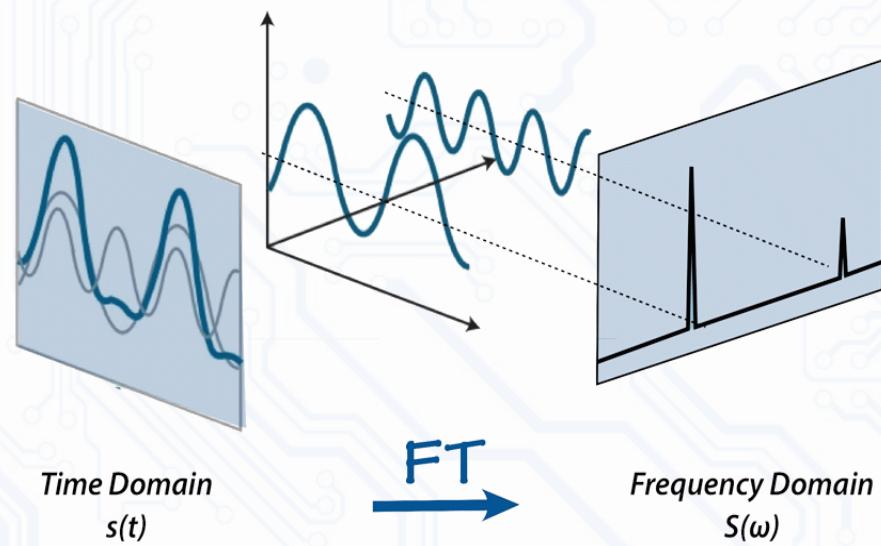


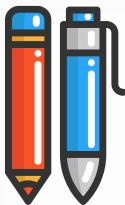
Transformation and Image

1. Fourier Transform
2. Discrete Cosine Transformation (DCT)

变换与形象

1. 傅立叶变换
2. 离散余弦变换 (DCT)





Review some word.....

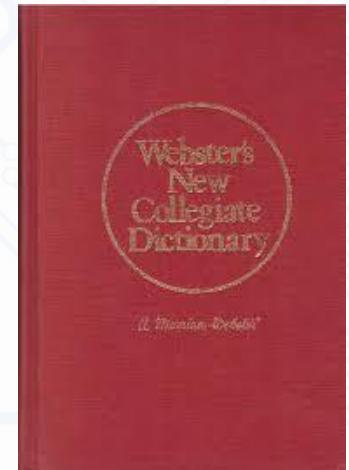


Enhance: 提高:

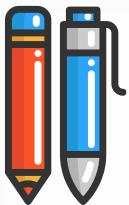
To make greater< as in value, desirability, or attractiveness>

Frequency: 频率:

The number of times that a periodic Function repeats the same sequence of value during a unit variation of the independent variable



Webster's New Collegiate Dictionary



Review some word.....



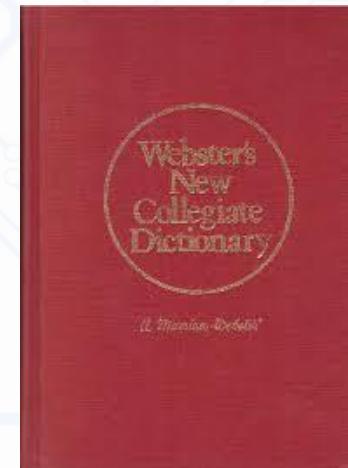
提高:

为了取得更大的<价值、可取性或吸引力>

频率:

在独立变量的单位变体中，周期函数重复相同值序列的次数

Webster's New Collegiate Dictionary



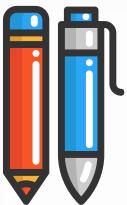
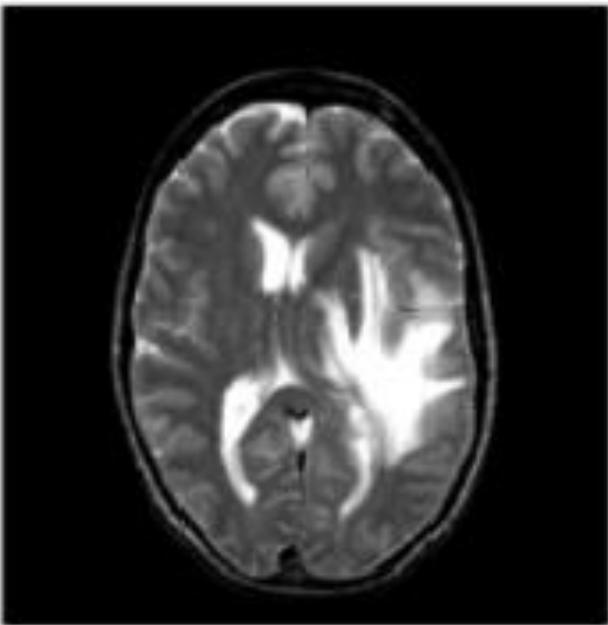
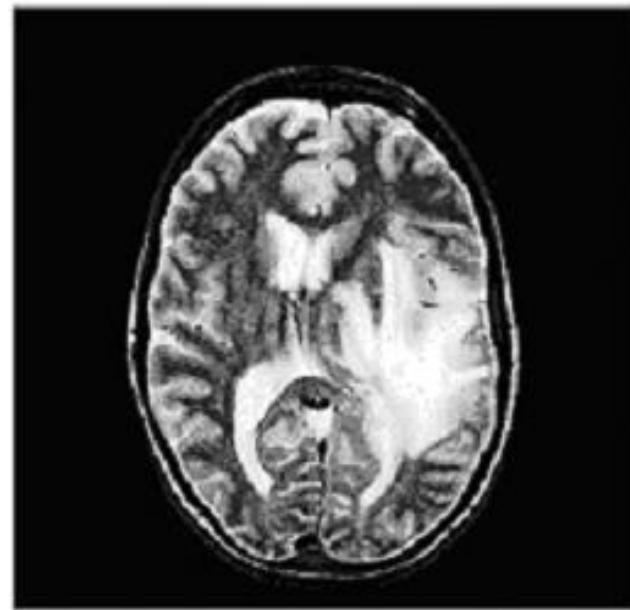


Image Enhancement

图像增强 Túxiàng zēngqiáng



Original image



Enhanced image

Enhancement: to process an image for more suitable output for a specific application.

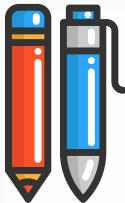
增强：

处理图像以获得更适合特定应用程序的输出。



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Background (Fourier Series)

定期



- ✓ Any function that **periodically** repeats itself can be expressed as the sum of **sines** and **cosines** of different **frequencies** each multiplied by a different **coefficient**

正弦

余弦

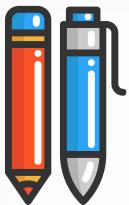
- ✓ This sum is known as **Fourier Series**

复杂的 Fùzá de

- ✓ It does not matter how **complicated** the function is; as long as it is **periodic** and meet some mild conditions it can be represented by such as a sum

定期的 Dìngqí de

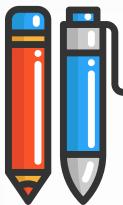
- ✓ It was a revolutionary discovery



背景（傅立叶级数）



- ✓ 任何周期性重复的函数都可以表示为不同频率的正弦和线圈的总和，每个因子乘以不同的系数
- ✓ 此总和称为傅立叶级数
- ✓ 功能有多复杂并不重要：只要它是周期性的，并满足一些温和的条件，它可以代表，如总和
- ✓ 这是一个革命性的发现



Fourier Transform: a review



- Basic ideas: 基本思路: Jīběn sīlù:

- A periodic function can be represented by the sum of sines/cosines functions of different frequencies, multiplied by a different coefficient.
- Non-periodic functions can also be represented as the integral of sines/cosines multiplied by weighing function.

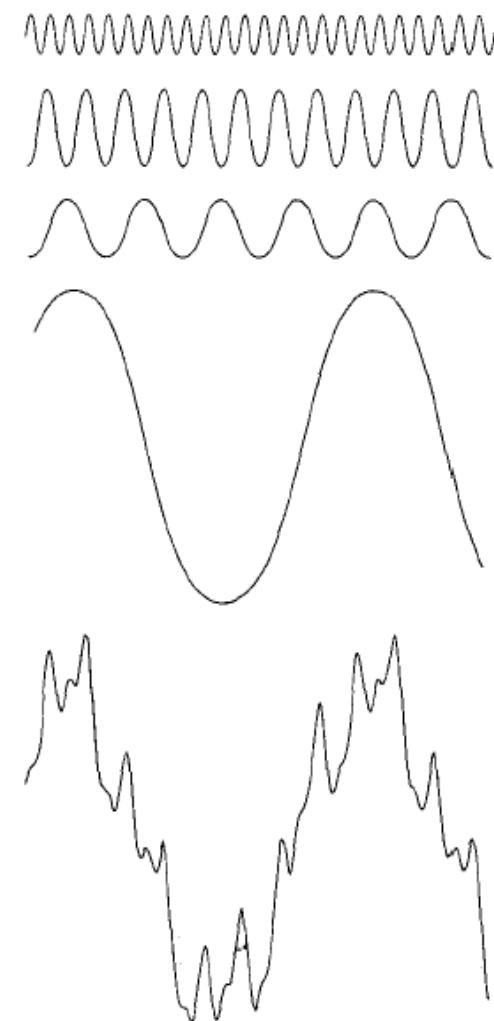
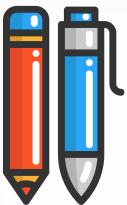


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.



傅立叶转换：回顾



- 基本思想：
- 周期函数可以由不同频率的正弦/阴性函数之和表示，乘以不同的系数。
- - 非周期性函数也可以表示为正弦/阴性乘以称重函数的组成部分。
 -

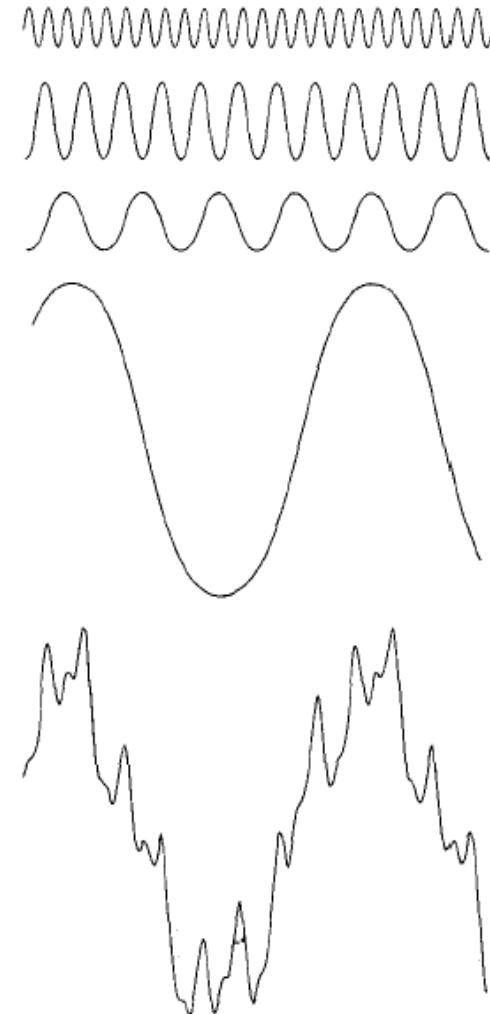
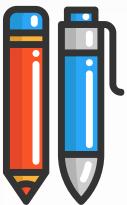


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

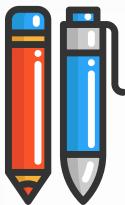


Joseph Fourier

(1768-1830)

Fourier was obsessed with the physics of heat and developed the Fourier transform theory to model heat-flow problems.





Background (Fourier Transform)



周期性但有限 Zhōuqí xìng dàn yǒuxiàn

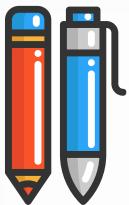
- Even functions that are not **periodic but Finite** can be expressed as the integrals of sines and cosines multiplied by a weighing function.
- This is known as Fourier Transform.

傅立叶级数 Fùlìyè jí shù

转变 Zhuǎnbiàn

- A function expressed in either a **Fourier Series** or **transform** can be reconstructed completely via an inverse process with no loss of information.
- This is one of the important characteristics of these representations because they allow us to work in the **Fourier Domain** and then return to the **original domain** of the function

傅立叶域 Fùlìyè yù

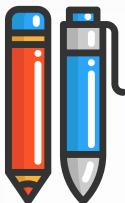


背景（傅立叶转换）



- 即使是不是周期性的，但有限性的功能也可以表示为正弦和余弦的积分乘以称重函数。
- 这被称为傅立叶转换。
-
- 傅立叶表示的函数或转换可以通过反向过程完全重建，而不会丢失信息。
-
-
- 这是这些表示的重要特征之一，因为它们允许我们在傅立叶域中工作，然后返回到函数的原始域
-



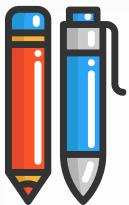


Fourier Transform



- ‘Fourier Transform’ *transforms* one function into another domain , which is called the *frequency domain representation* of the original function
- The original function is often a function in the *Time domain*
- In image Processing the original function is in the *Spatial Domain*
- The term *Fourier transform* can refer to either the Frequency domain representation of a function or to the process/formula that "*transforms*" one function into the other.

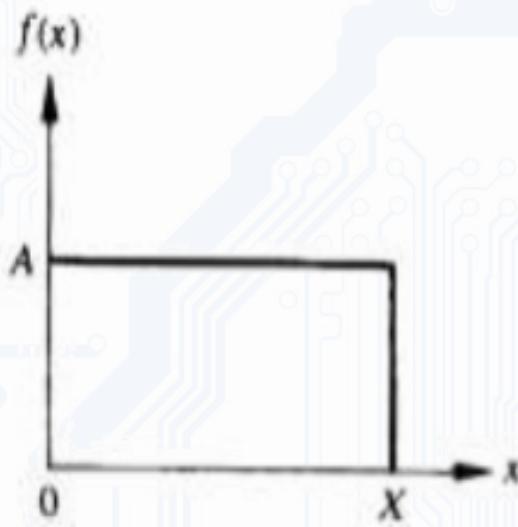
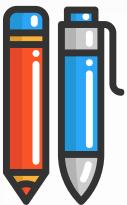
空间域 Kōngjiān yù



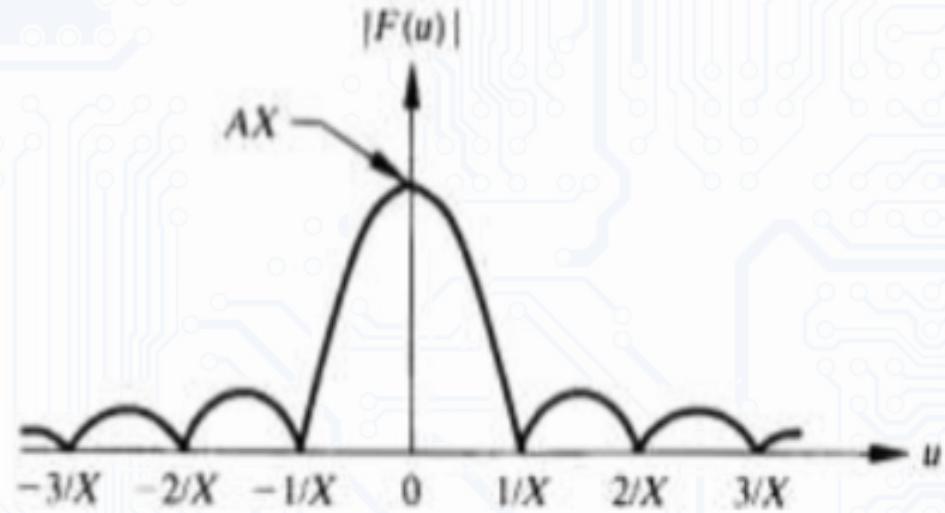
傅立叶转换



- “傅立叶转换”将一个函数转换为另一个域，称为原始函数的频率域表示
- 原始功能通常是时间域中的函数
- 在图像处理中，原始功能位于空间域中
- “傅立叶转换”一词可以指函数的频率域表示，也可以指将一个函数“转换”到另一个函数的过程/公式。

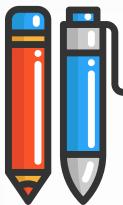


Simple function



Fourier Transform

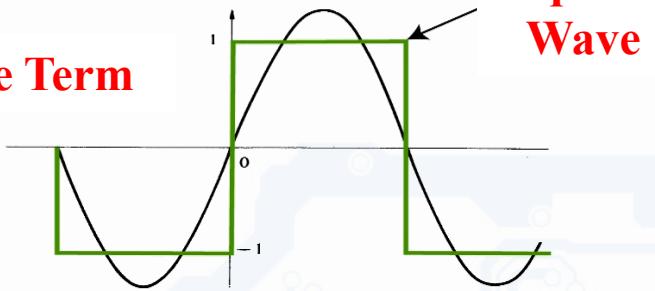




Fourier transform basis functions

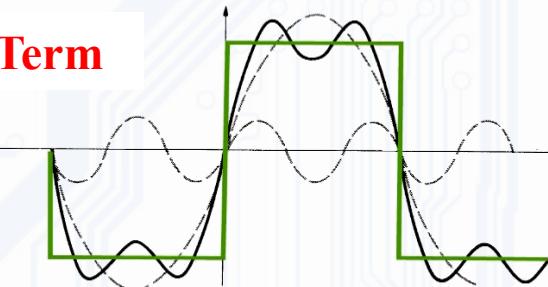


One Term



Square
Wave

Two Term

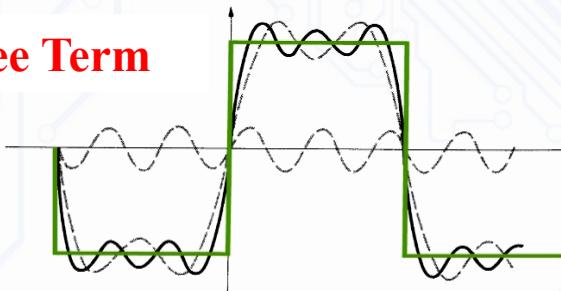


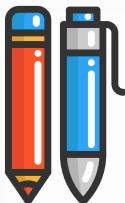
Approximating a square wave
as the sum of sine waves.

将方波近似为正弦波的总和。

Jiāng fāng bō jìnsì wèi zhèngxián bō de zǒnghé.

Three Term



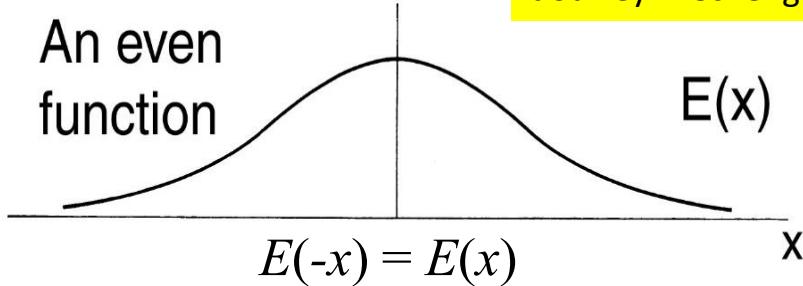


Any function can be written as the sum of an even and an odd function

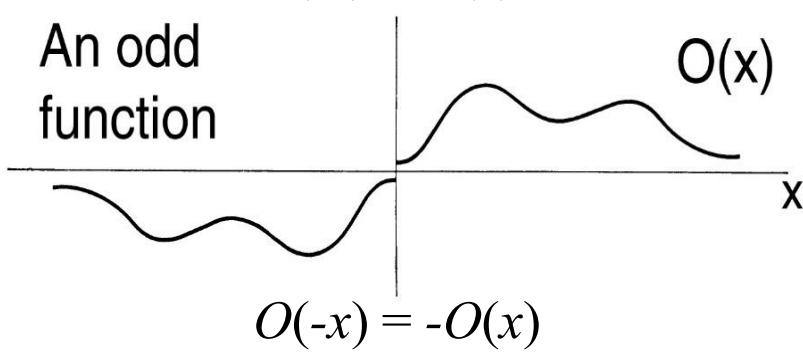


任何函数都可以写成偶函数和奇函数的和 Rènhé hánshù
dōu kěyǐ xiěchéng ǒu hánshù hé qí hán shǔ de hé

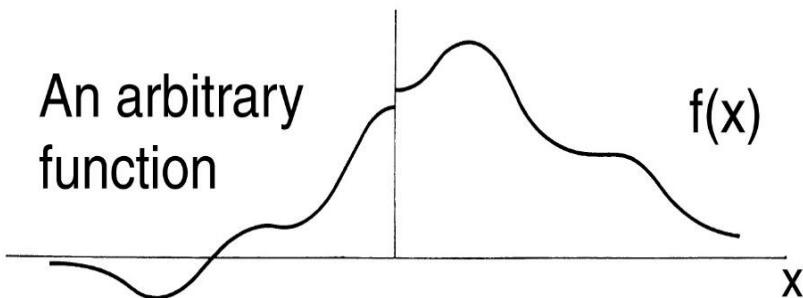
An even
function



An odd
function



An arbitrary
function



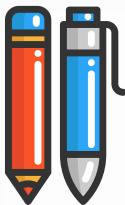
$$E(x) \equiv [f(x) + f(-x)] / 2$$

$$O(x) \equiv [f(x) - f(-x)] / 2$$



$$f(x) = E(x) + O(x)$$





Fourier Cosine Series

傅立叶余弦级数 Fùlìyè yúxián jí shù



Because $\cos(mt)$ is an even function, we can write an even function, $f(t)$, as:

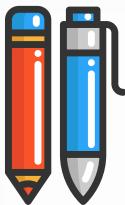
$$f(t) = \frac{1}{\pi} \sum_{m=0}^{\infty} F_m \cos(mt)$$

where series F_m is computed as

$$F_m = \int_{-\pi}^{\pi} f(t) \cos(mt) dt$$

Here we suppose $f(t)$ is over the interval $(-\pi, \pi)$.





Fourier Sine Series

傅立叶正弦系列 Fùlìyè zhèngxián xìliè



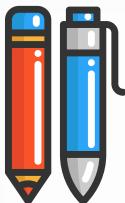
Because $\sin(mt)$ is an odd function, we can write any odd function, $f(t)$, as:

$$f(t) = \frac{1}{\pi} \sum_{m=0}^{\infty} F'_m \sin(mt)$$

where the series F'_m is computed as

$$F'_m = \int_{-\pi}^{\pi} f(t) \sin(mt) dt$$





Fourier Series

傅立叶级数 Fùlìyè jí shù



So if $f(t)$ is a general function, neither even nor odd, it can be written:

$$f(t) = \frac{1}{\pi} \sum_{m=0}^{\infty} F_m \cos(mt) + \frac{1}{\pi} \sum_{m=0}^{\infty} F'_m \sin(mt)$$

Even component

偶数组件 Ōushù zǔjiàn

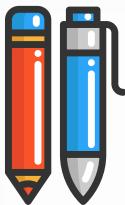
Odd component

奇数分量 Jīshù fènliàng

where the Fourier series is

$$F_m = \int f(t) \cos(mt) dt$$

$$F'_m = \int f(t) \sin(mt) dt$$



The Fourier Transform



Let $F(m)$ incorporates both cosine and sine series coefficients, with the sine series distinguished by making it the imaginary component:

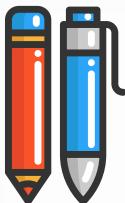
$$F(m) = F_m - jF_m' = \int f(t) \cos(mt) dt - j \cdot \int f(t) \sin(mt) dt$$

Let's now allow $f(t)$ range from $-\infty$ to ∞ , we rewrite:

$$\mathcal{F}\{f(t)\} = F(u) = \int_{-\infty}^{\infty} f(t) \exp(-j2\pi ut) dt$$

$F(u)$ is called the **Fourier Transform** of $f(t)$. We say that $f(t)$ lives in the “**time domain**,” and **$F(u)$** lives in the “**frequency domain**.” **u** is called the **frequency variable**.

$F(u)$ 被称为 $f(t)$ 的傅立叶变换。我们说 $f(t)$ 存在于“时域”中，而 $F(u)$ 存在于“频域”中。 u 称为频率变量。



The Inverse Fourier Transform

One Dimensional Fourier Transform and its Inverse



逆傅里叶变换 Nì fù lǐ yè biān huàn

The Fourier transform $F(u)$ of a single variable, continuous function $f(x)$ is

We go from $f(t)$ to $F(u)$ by

$$\mathfrak{F}\{f(t)\} = F(u) = \int_{-\infty}^{\infty} f(t) \exp(-j2\pi ut) dt$$

where $j = \sqrt{-1}$.

*Fourier
Transform*

Given $F(u)$, $f(t)$ can be obtained by the inverse Fourier transform

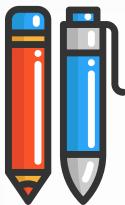
$$\mathfrak{F}^{-1}\{F(u)\} = f(t) = \int_{-\infty}^{\infty} F(u) \exp(j2\pi ut) du$$

*Inverse
Fourier
Transform*

傅立叶转变
Fùlìyèzhuǎnbìan

逆傅立叶转变
Nìfùlìyèzhuǎnbìan





Applications of Fourier Transforms

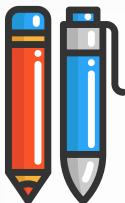
傅里叶变换的应用 Fù lǐ yè biàn huàn de yìng yòng



- 1-D Fourier transforms are used in Signal Processing
- 2-D Fourier transforms are used in Image Processing
- 3-D Fourier transforms are used in Computer Vision
- Applications of Fourier transforms in Image processing: –
 - *Image enhancement,*
 - *Image restoration,*
 - *Image encoding / decoding,*
 - *Image description*

傅里叶变换在图像处理中的应用：

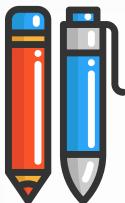
- 图像增强,
- 图像修复,
- 图像编码/解码,
- 图片说明



Fourier Transform



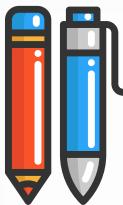
- A useful analogy is to compare the Fourier transform to a glass prism.
- The prism is a physical device that separates light into various color components, each depending on its wavelength (or frequency) content.
- The Fourier transform may be viewed as a “mathematical prism” that separates a function into various components, also based on frequency content.
- When we consider light, we talk about its spectral or frequency content.
- Similarly, the Fourier transform lets us characterize a function by its frequency content.
- This is a powerful concept that lies at the heart of linear filtering.



傅立叶转换



- 一个有用的类比是把傅里叶变换比作玻璃棱镜，
- 棱镜是一种物理装置，它将光分成不同的颜色分量，每个分量取决于其波长(或频率)含量。
- 傅里叶变换可以看作是一种数学棱镜，它将函数分成不同的分量，也是基于频率内容的。
- 当我们同意时，我们谈论它的光谱 or 频率的内容。
- 很小程度上，傅里叶变换使我们可以用函数的频率内容来刻画它。
- 这是一个强大的概念，是线性滤波的核心。



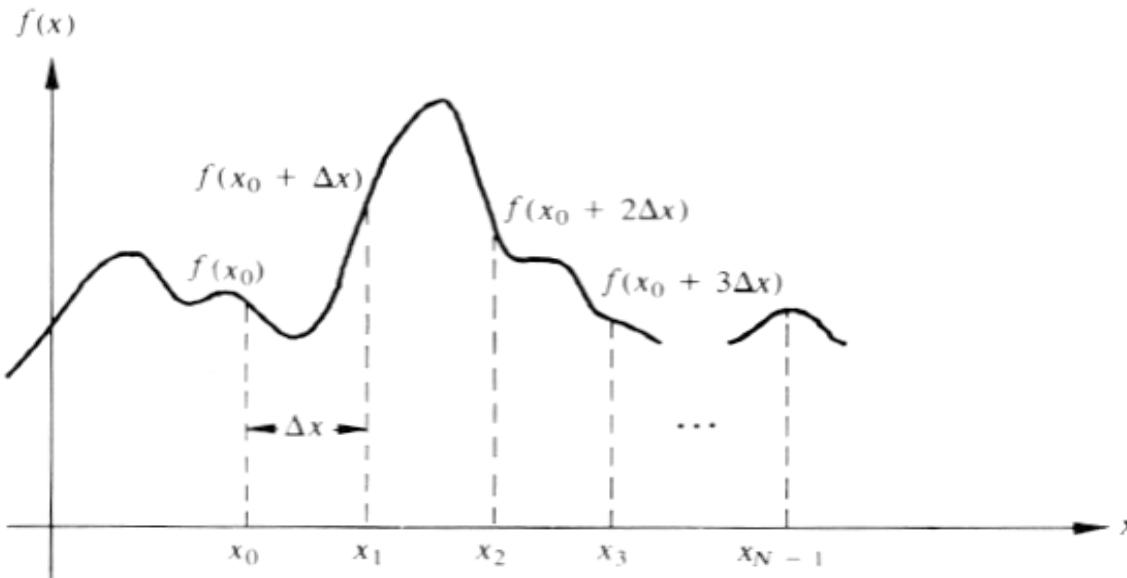
Discrete Fourier Transform (DFT)

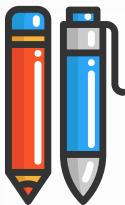


离散傅立叶变换 (DFT) Lísàn fùlìyè biànhuàn (DFT)

- A continuous function $f(x)$ is discretized as:

$$\{f(x_0), f(x_0 + \Delta x), f(x_0 + 2\Delta x), \dots, f(x_0 + (M-1)\Delta x)\}$$





Discrete Fourier Transform (DFT)



离散傅立叶变换 (DFT) Lísàn fùlìyè biànhuàn (DFT)

Let x denote the discrete values ($x=0,1,2,\dots,M-1$), i.e.

$$f(x) = f(x_0 + x\Delta x)$$

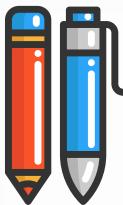
then

$$\{f(x_0), f(x_0 + \Delta x), f(x_0 + 2\Delta x), \dots, f(x_0 + (M-1)\Delta x)\}$$



$$\{f(0), f(1), f(2), \dots, f(M-1)\}$$





Discrete Fourier Transform (DFT)

离散傅立叶变换 (DFT) Lísàn fùlìyè biànhuàn (DFT)



- The discrete Fourier transform pair that applies to sampled functions is given by:

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) \exp(-j2\pi ux/M)$$

$$u=0, 1, 2, \dots, M-1$$

and

$$f(x) = \sum_{u=0}^{M-1} F(u) \exp(j2\pi ux/M)$$

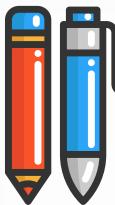
$$x=0, 1, 2, \dots, M-1$$

Since $e^{j\theta} = \cos\theta + j\sin\theta$, Euler's formula

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) [\cos(2\pi ux/M) - j \sin(2\pi ux/M)].$$

$$F(u) = \begin{matrix} R(u) & jI(u) \\ \text{Real} & \text{Imaginary} \end{matrix}$$





2-D Fourier Transform

Two Dimensional Fourier Transform and its Inverse

二维傅立叶变换
二维傅里叶变换及其逆

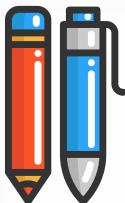


- The Fourier transform $F(u,v)$ of a two variable, continuous function $f(x,y)$ is

$$\mathcal{F}\{f(x,y)\} = F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \exp(-j2\pi(ux+vy)) dx dy$$

- Given $F(u,v)$ we can obtain $f(x,y)$ by means of the Inverse Fourier Transform

$$\mathcal{F}^{-1}\{F(u,v)\} = f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) \exp(j2\pi(ux+vy)) du dv$$



2-D DFT

二维傅立叶变换
二维傅里叶变换及其逆



2D DFT pair $f(x,y) \longleftrightarrow F(u,v)$

$$F(u, v) = R(u, v) + jI(u, v)$$

real part

imaginary part

(Fourier)
Magnitude spectrum

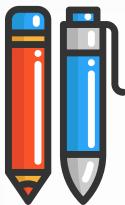
$$|F(u, v)| = \sqrt{R^2(u, v) + I^2(u, v)}$$

(Fourier)
Phase spectrum

$$\phi(u, v) = \angle F(u, v) = \tan^{-1} \left[\frac{I(u, v)}{R(u, v)} \right]$$

Power spectrum

$$P(u, v) = |F(u, v)|^2$$



Polar Coordinate Representation of FT

FT 的极坐标表示 FT de jí zuòbiāo biǎoshì



- The Fourier transform of a real function is generally **complex** and we use polar coordinates:

$$F(u, v) = R(u, v) + j \cdot I(u, v)$$

极坐标 Jí zuòbiāo



Polar coordinate

$$F(u, v) = |F(u, v)| \exp(j\phi(u, v))$$

Magnitude: $|F(u, v)| = [R^2(u, v) + I^2(u, v)]^{1/2}$

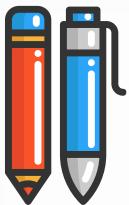
震级: Zhènjí:

Phase:

$$\phi(u, v) = \tan^{-1} \left[\frac{I(u, v)}{R(u, v)} \right]$$

阶段: Jiēduàn:





2-D Discrete Fourier Transform

二维离散傅立叶变换 Èr wéi lísàn fùlìyè biànhuàn



- In 2-D case, the DFT pair is: for MxN sample

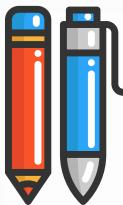
$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \exp(-j2\pi(ux/M + vy/N))$$

$u=0, 1, 2, \dots, M-1$ and $v=0, 1, 2, \dots, N-1$

and:

$$f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \exp(j2\pi(ux/M + vy/N))$$

$x=0, 1, 2, \dots, M-1$ and $y=0, 1, 2, \dots, N-1$



Properties of Discrete cosine transformation

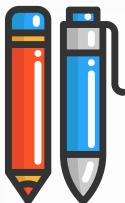
离散余弦变换的性质 Lísàn yúxián biànhuàn dì xìngzhì



Properties of Discrete cosine transformation are as following:

- Real and Orthogonal: $C=C^*$ $\rightarrow C^{-1}=C^T$
- Not! Real part of DFT
- Fast Transform
- Excellent Energy Compaction (Highly Correlated Data)





Fast Fourier Transform (FFT):

快速傅立叶变换 (FFT): Kuàisù fùlìyè biān huàn (FFT):

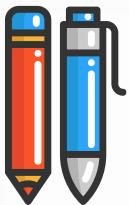


- Due to the property of separability of 2D DFT, the FFT algorithm developed for 1D DFT is applied without any modification for 2D DFT twice successively along each dimension.

由于 2D DFT 的可分离性，为 1D DFT 开发的 FFT 算法在每个维度上连续两次对 2D DFT 进行任何修改。

$$F(u, v) = \sum_{m=0}^{N-1} \left[\left(\frac{1}{N} \right) \sum_{n=0}^{N-1} f(m, n) e^{-j \frac{2\pi nv}{N}} \right] e^{-j \frac{2\pi mu}{N}}$$

$$F(u, v) = \sum_{m=0}^{N-1} F(m, v) e^{-j \frac{2\pi mu}{N}}$$

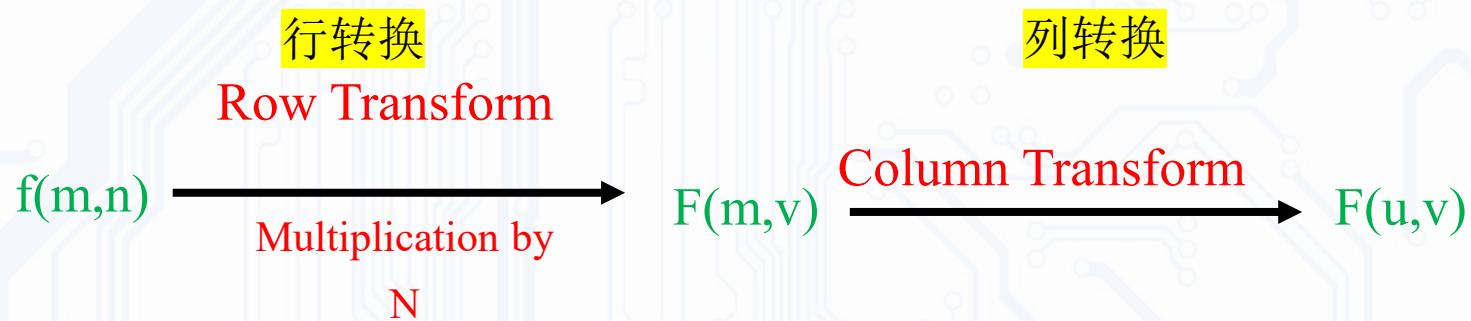


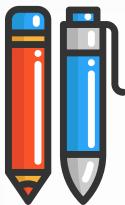
Fast Fourier Transform (FFT):

快速傅立叶变换 (FFT): Kuàisù fùlìyè biàn huàn (FFT):



Fast Fourier Transform (FFT):





Fourier Transform: shift



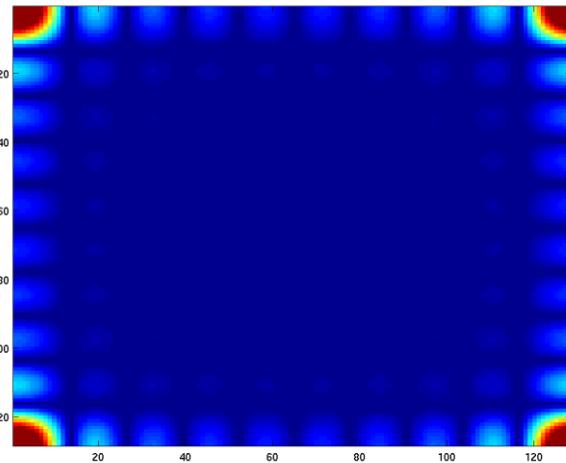
Shifting the Origin to the Center

将原点移到中心 Jiāng yuándiǎn yí dào zhōngxīn

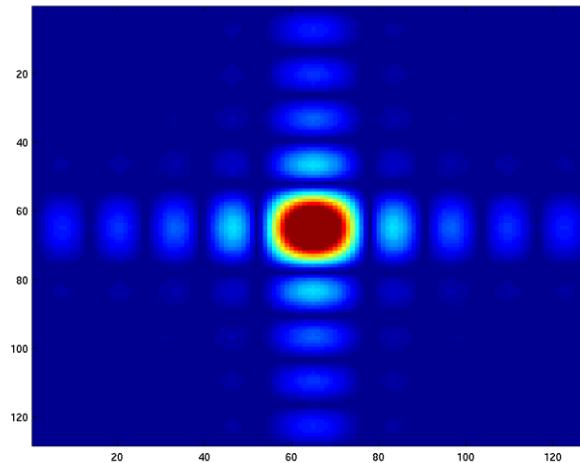
- It is common to multiply input image by $(-1)^{x+y}$ prior to computing the FT.
- This shift the center of the FT to $(M/2, N/2)$.

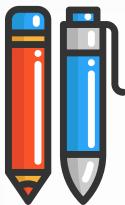
$$\Im\{f(x, y)\} = F(u, v)$$

$$\Im\{f(x, y)(-1)^{x+y}\} = F(u - M/2, v - N/2)$$



Shift
→





Shifting the Origin to the Center

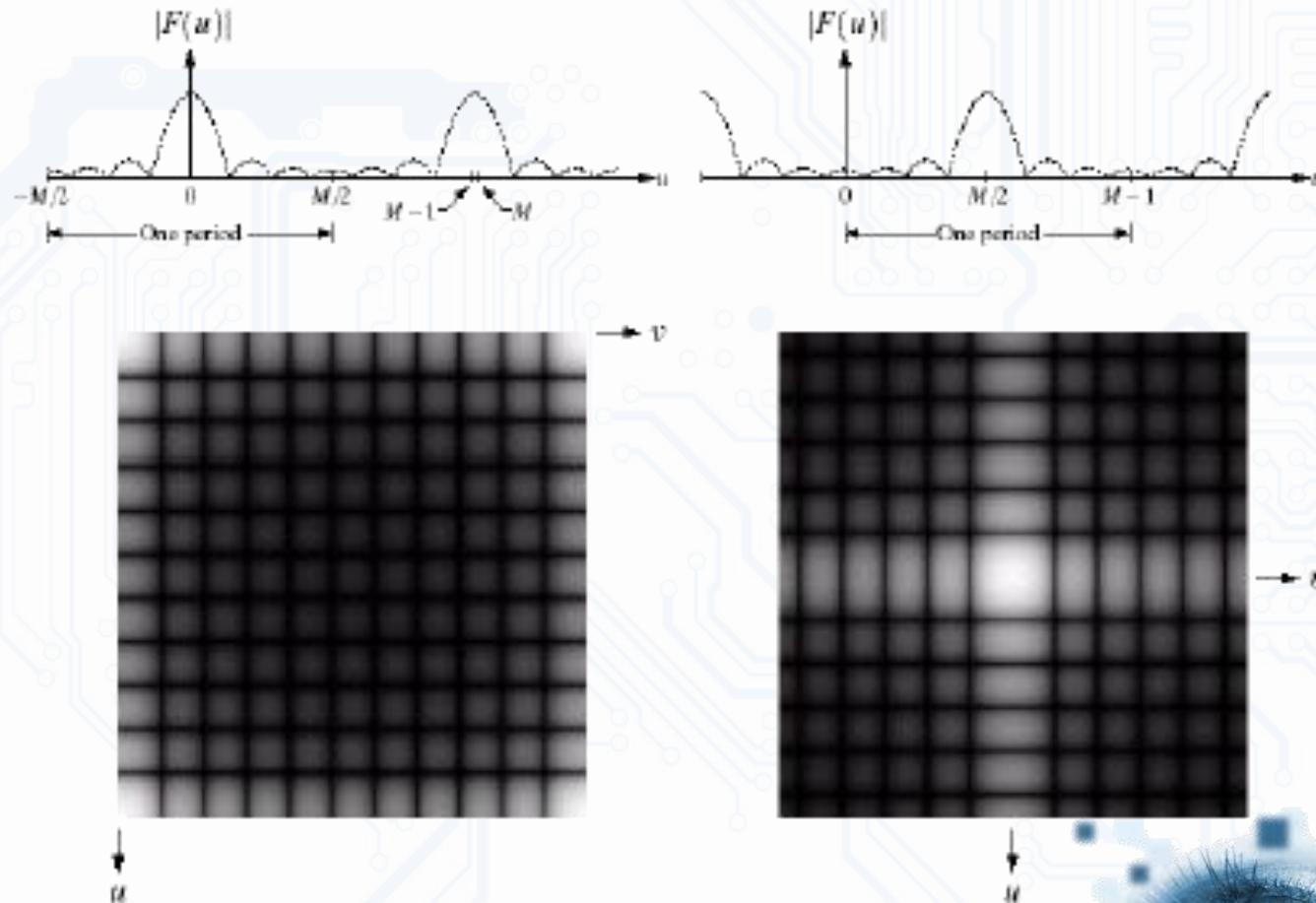
将原点移到中心 Jiāng yuándiǎn yí dào zhōngxīn

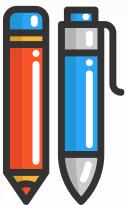


a
b
c
d

FIGURE 4.34

- (a) Fourier spectrum showing back-to-back half periods in the interval $[0, M - 1]$.
- (b) Shifted spectrum showing a full period in the same interval.
- (c) Fourier spectrum of an image, showing the same back-to-back properties as (a), but in two dimensions.
- (d) Centered Fourier spectrum.





Fourier Transform

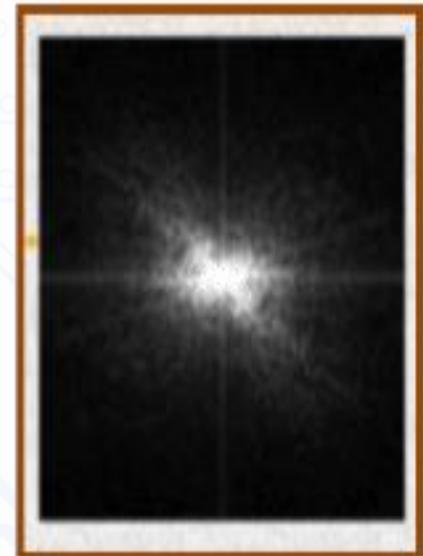


Properties of Fourier transformation are as follows:

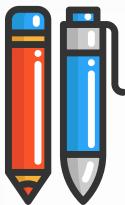
- Symmetric Unitary
- Periodic Extension
- Sampled Fourier
- Fast
- Conjugate Symmetry
- Circular Convolution

傅里叶变换的性质如下：

- 对称幺正；
- 定期延期；
- 采样傅立叶；
- 快速地；
- 共轭对称；
- 循环卷积；



Fast Fourier transformation of the image

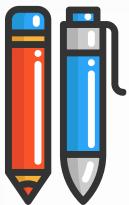


Properties of Fourier Transform



傅里叶变换的性质如下：

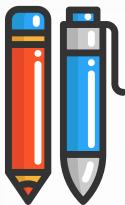
- ✓ As we move away from the origin in $F(u,v)$ the lower frequencies corresponding to slow gray level changes
- ✓ Higher frequencies correspond to the fast changes in gray levels (smaller details such edges of objects and noise)
- ✓ The direction of amplitude change in spatial domain and the amplitude change in the frequency domain are orthogonal



傅立叶转换的性质



- ✓ 当我们离开 $F(u, v)$ 的源时，与缓慢的灰色水平变化相对应的较低频
- ✓ 较高的频率与灰度水平的快速变化相对应（较小的细节，如物体边缘和噪声）
- ✓ 空间域振幅变化的方向和频率域的振幅变化是正交的



Properties of Fourier Transform

傅里叶变换的性质如下：



The Fourier Transform pair has the following translation property

Average value 平均值

$F(0,0)$ gives the average intensity value of an image

$$F(0,0) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y). \quad F(0,0) \text{给出了一张图像的平均强度值}$$

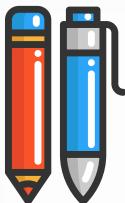
Translation 转换

$$f(x, y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$$

Using this property $f(x, y)(-1)^{(x+y)} \Leftrightarrow F(u - M/2, v - N/2)$

Similar to the above case

$$f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$$



Properties of Fourier Transform

傅里叶变换的性质如下：



Distributive property: 分配属性

$$\Im[f_1(x, y) + f_2(x, y)] = \Im[f_1(x, y)] + \Im[f_2(x, y)],$$
$$\Im[f_1(x, y) \cdot f_2(x, y)] \neq \Im[f_1(x, y)] \Im[f_2(x, y)],$$

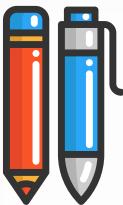
Fourier transform is distributive over addition but not over multiplication

傅立叶转换可分配成加法而不是乘法

Linearity and scaling 线性和缩放

Linearity: $af_1(x, y) + bf_2(x, y) \Leftrightarrow aF_1(u, v) + bF_2(u, v)$

Scaling: $af(x, y) \Leftrightarrow aF(u, v)$



Properties of Fourier Transform

傅里叶变换的性质如下：



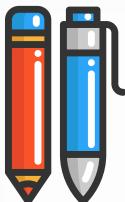
Rotation 旋转

The rotation property states that,

$$f(r, \theta + \theta_0) \Leftrightarrow F(w, \phi + \theta_0).$$

If the image is rotated in spatial domain by a fixed angle, the Fourier transform is also rotated at the same angle.

如果图像在空间域上通过一个像素角度被旋转，
那么傅立叶转换也以同样的角度旋转



Properties of Fourier Transform

傅里叶变换的性质如下：



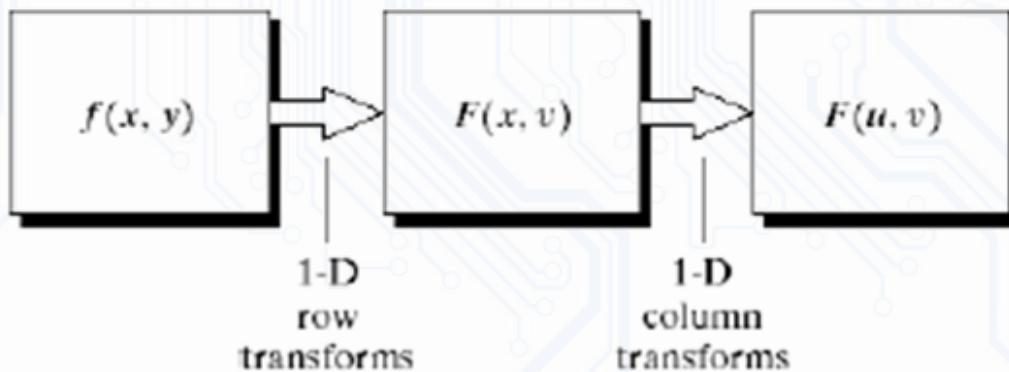
Separability 可分离性

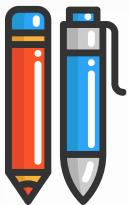
二级傅立叶转换可以看成是由一系列的一级傅立叶转换所构成。简称DFT（复杂的指数是可分离的）

The 2D Fourier transform can be performed as a series of 1D DFT (complex exponential is separable)

$$F(u, v) = \frac{1}{M} \sum_{x=0}^{M-1} e^{-j2\pi ux/M} \frac{1}{N} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi vy/N} = \frac{1}{M} \sum_{x=0}^{M-1} F(x, v) e^{-j2\pi ux/M}$$

where $F(x, v) = \frac{1}{N} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi vy/N}$.





Properties of Fourier Transform



傅里叶变换的性质如下：

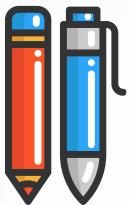
Derivatives and Laplacian 衍生物和拉普拉西安

Considering 1-D, it can be shown that

$$\Im \left[\frac{d^n f(x)}{dx^n} \right] = (ju)^n F(u),$$

Implementing the above equation for 2-D Laplacian

$$\begin{aligned} \Im \left[\frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2} \right] &= (ju)^2 F(u, v) + (jv)^2 F(u, v) \\ &= -(u^2 + v^2) F(u, v). \end{aligned}$$



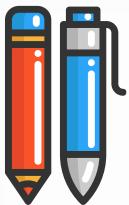
Properties of Fourier Transform



傅里叶变换的性质如下：

- ✓ The lower frequencies corresponds to slow gray level changes

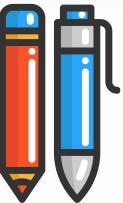
- ✓ Higher frequencies correspond to the fast changes in gray levels (smaller details such edges of objects and noise)



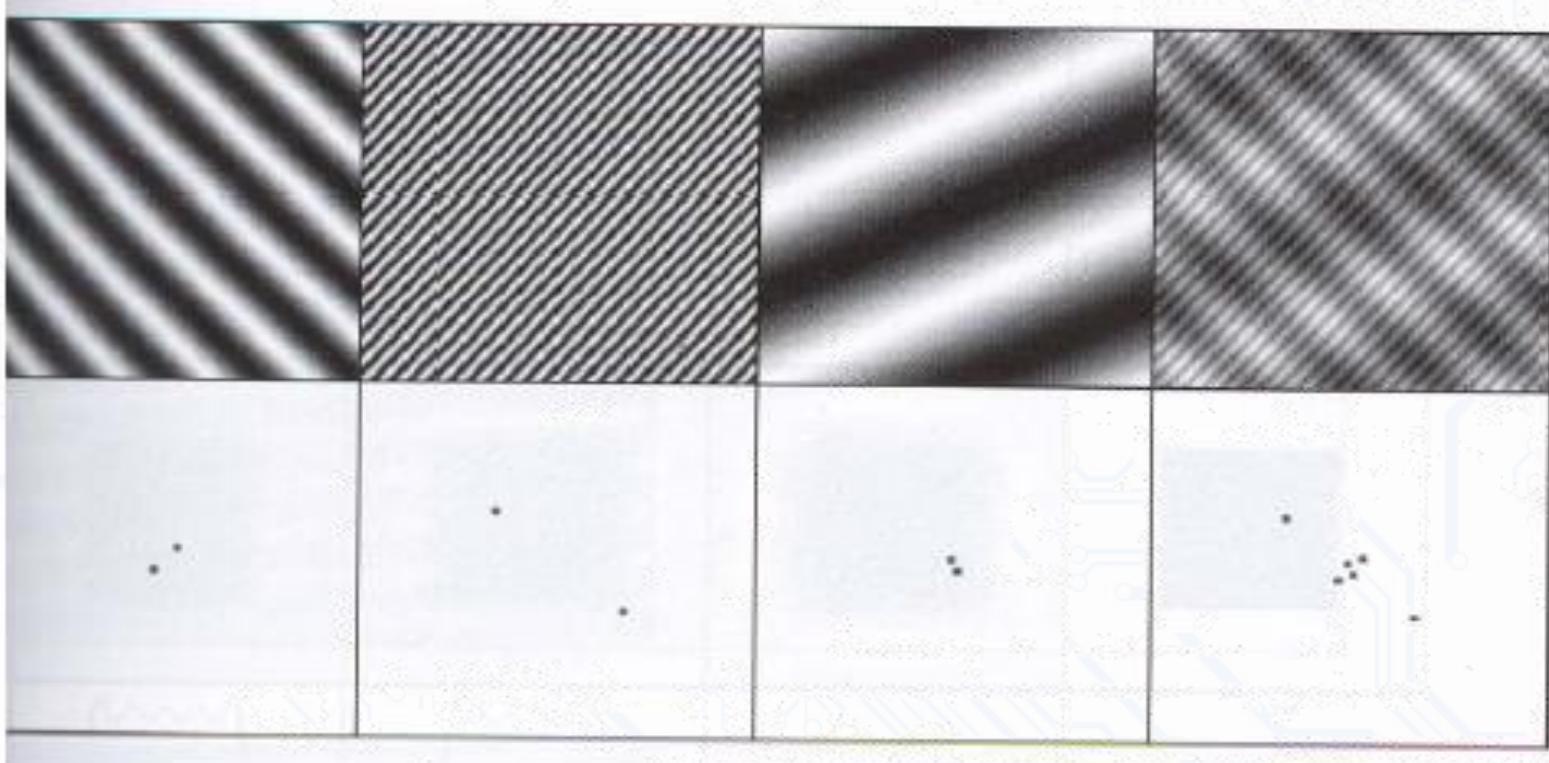
傅立叶转换的性质



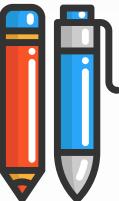
- ✓ 较低的频率对应于缓慢的灰色水平变化
- ✓ 较高的频率与灰度水平的快速变化相对应
(较小的细节, 如物体边缘和噪声)



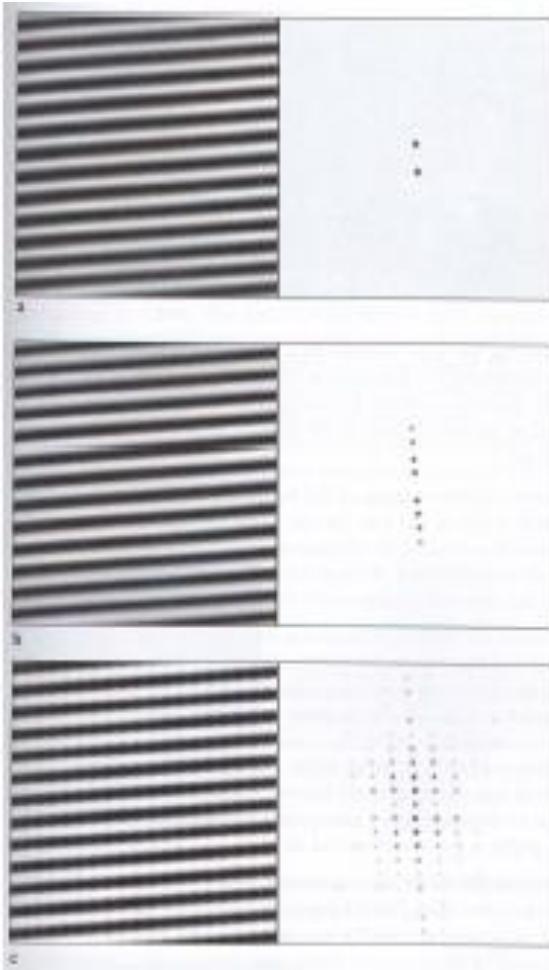
DFT Examples



Three sinusoidal patterns and their sum with their Fourier Transforms



DFT Examples



Sinusoidal lines and its DFT

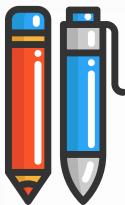
正弦线条和它的DFT

Sinusoidal lines with non-sinusoidal brightness profile and its DFT

有着非正弦亮度配置文件的正弦线条和它的DFT

Display defects (lines on smaller angles are shown in steps) complicates the DFT

展示缺陷（小角度的线条被分步呈现）使DFT变得复杂

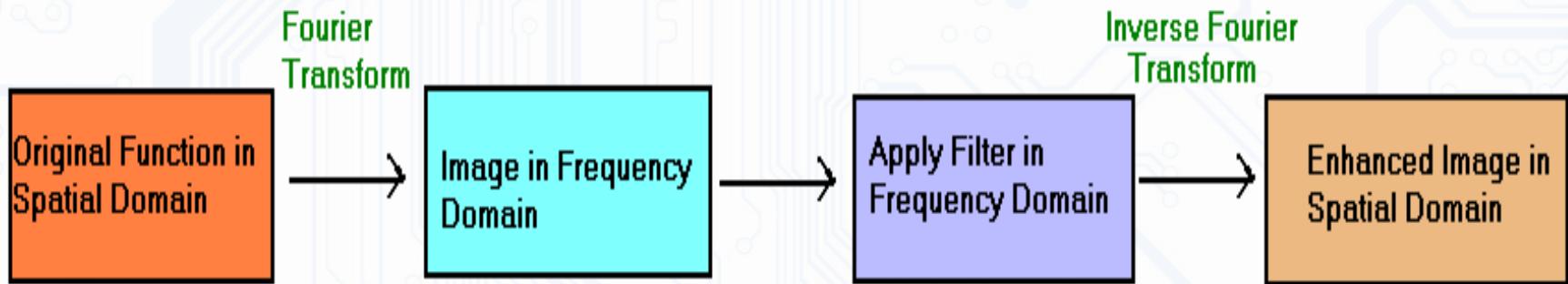


Our Interest in Fourier Transform

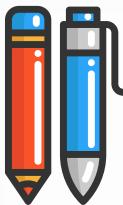
我们对傅立叶变换的兴趣



- We will be dealing only with functions (images) of finite duration so we will be interested only in Fourier Transform



我们将只处理有限持续时间的函数（图像），所以我们只对傅立叶变换感兴趣
Wǒmen jiāng zhǐ chǔlǐ yǒuxiànr chíxù shíjiān de hánshù (túxiàng), suǒyǐ wǒmen zhǐ duì fùlìyè biàn huàn gǎn xìngqù



Frequency Domain Filtering



频域滤波 Pín yù lùbō

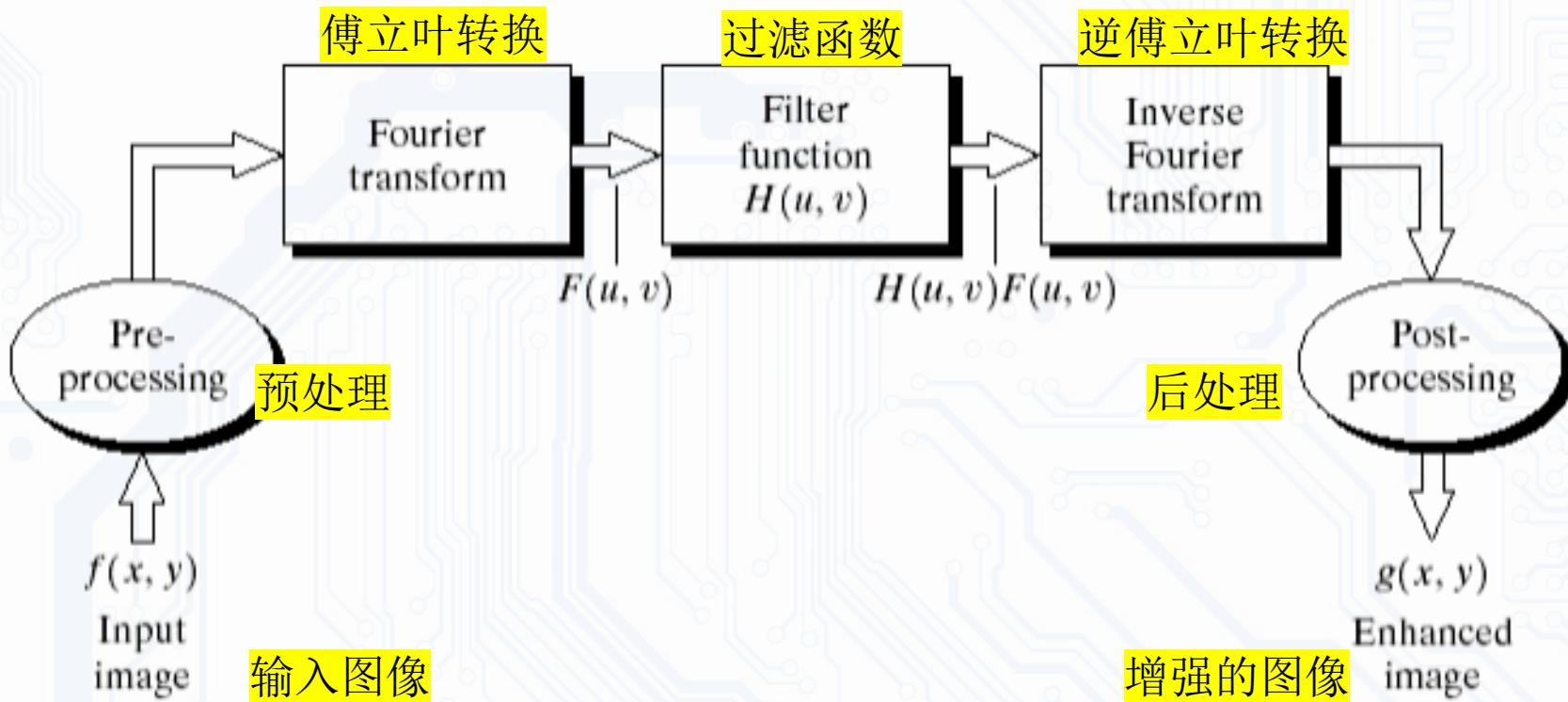
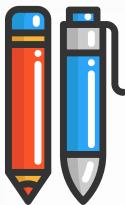


FIGURE 4.5 Basic steps for filtering in the frequency domain.

频率滤波的基本步骤

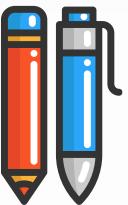


Frequency Domain Filtering

频域滤波 Pín yù lǜbō



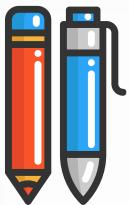
- Edges and sharp transitions (e.g., noise) in an image contribute significantly to high-frequency content of FT.
- Low frequency contents in the FT are responsible to the general appearance of the image over smooth areas.
- Blurring (smoothing) is achieved by attenuating range of high frequency components of FT.



频率滤波



- 图像中的边缘和急剧转换（例如噪音）对 频率滤波 的高频内容有显著贡献。
- 在频率滤波的低频内容代表图像在光滑区域的总体外观。
- 模糊（平滑）是通过降低频率滤波高频组件的范围来实现的。



Symmetry of FT

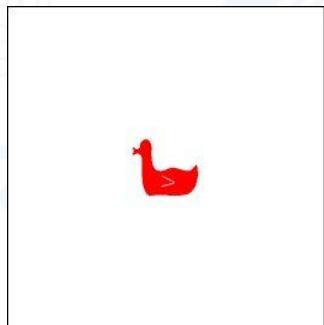
FT的对称性 FT de duìchèn xìng



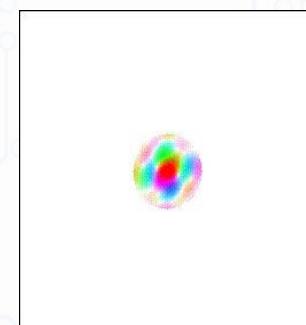
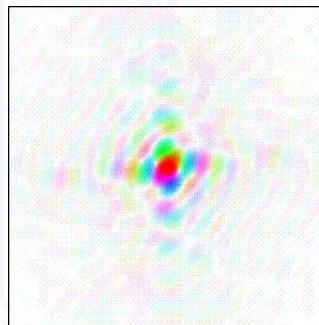
- For real image $f(x,y)$, FT is conjugate symmetric:

$$F(u,v) = F^*(-u,-v) \quad |F(u,v)| = |F(-u,-v)|$$

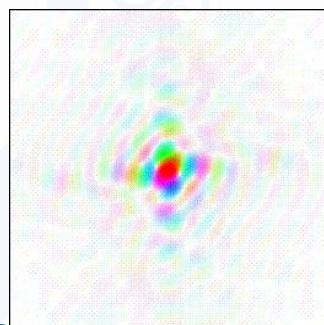
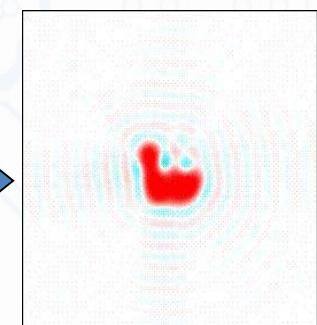
- The magnitude of FT is symmetric:



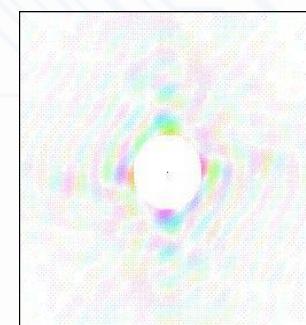
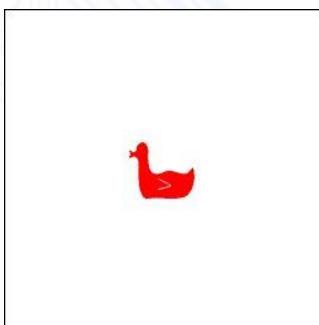
FT



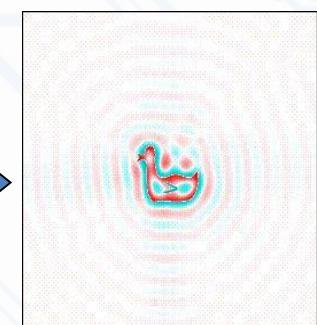
IFT

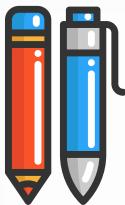


IFT



IFT





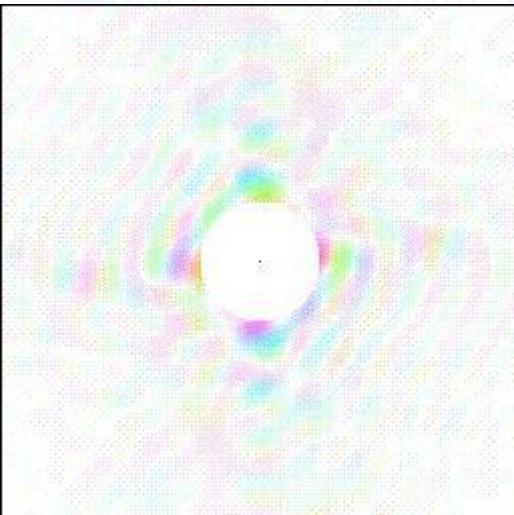
Symmetry of FT

FT的对称性 FT de duìchèn xìng



The central part of FT, i.e. the low frequency components are responsible for the general gray-level appearance of an image.

FT的中心部分，即低频组件，代表图像的一般灰度水平外观



The high frequency components of FT are responsible for the detail information of an image.

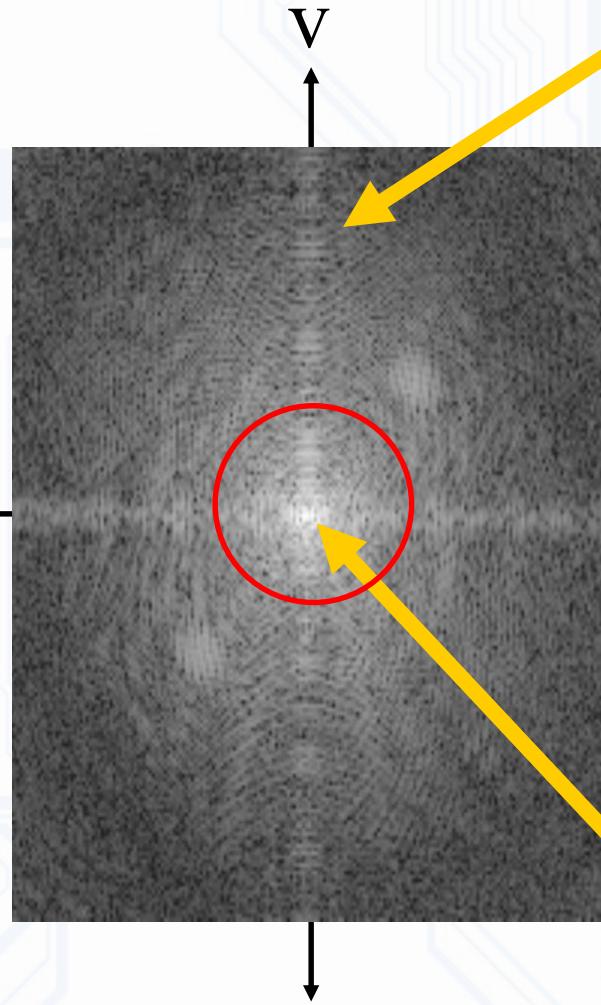
FT的高频组件代表图像的详细信息



Image



Frequency Domain (log magnitude)



频域 (对数幅度)
Pín yù (duì shù fúdù)

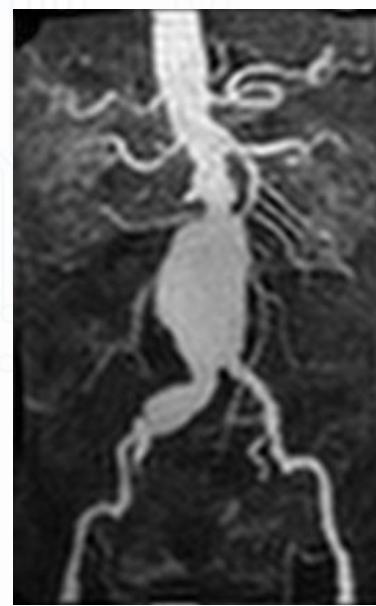
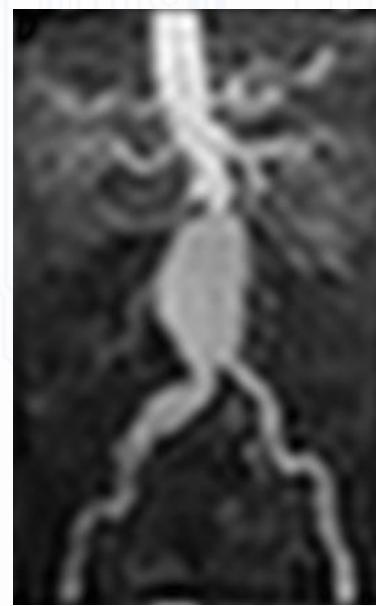
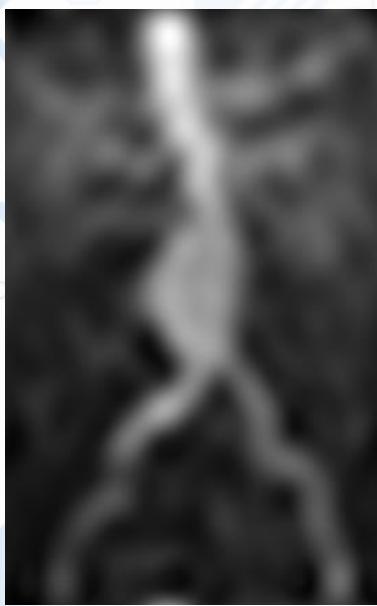
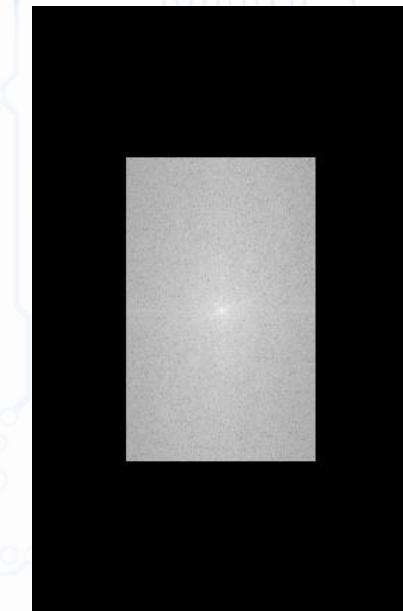
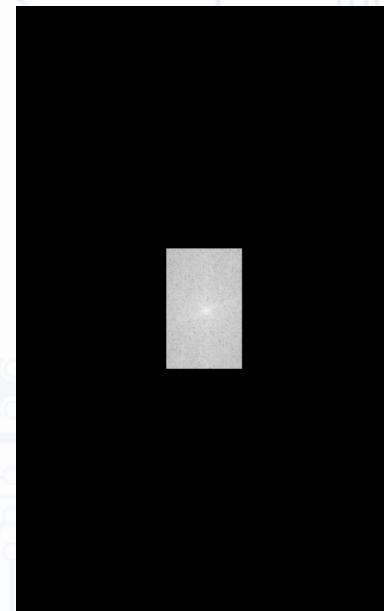
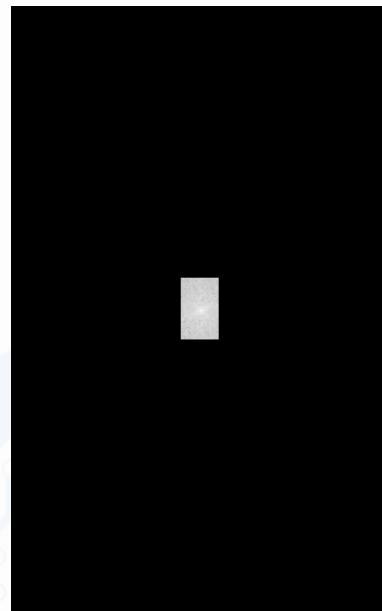
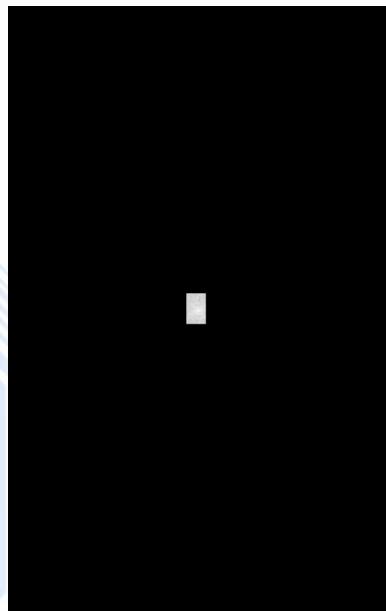
Detail

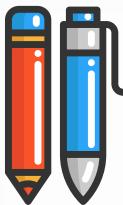
细节 xìjié

→ u

总体外观
Zǒngtǐ wàiguān

General
appearance





Transformation and Image

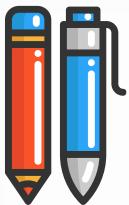
变换与形象 Biàn huàn yǔ xíngxiàng



- Transformation is a function. A function that maps one set to another set after performing some operations.
- **IMAGE TRANSFORMATION**
 - A function or operator that takes an image as its input and produces an image as its output.
 - Depending on the transform chosen, the input and output images may appear entirely different and have different interpretations.
 - Fourier transforms
 - principal component analysis (also called Karhunen-Loeve analysis)
 - various spatial filters

are examples of frequently used image transformation procedures.





转换和图像

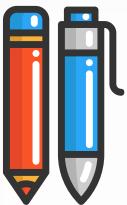


- 转换是一种功能。在执行某些操作后将一个设置映射到另一组的函数。

• 图像转换

- 以图像为输入并生成图像作为输出的功能或操作员。
- 根据选择的转换，输入和输出图像可能看起来完全不同，有不同的解释。
- 傅立叶转换
 - 主要组件分析（也称为卡胡宁-洛夫分析）
 - 各种空间过滤器是常用的图像转换程序示例。





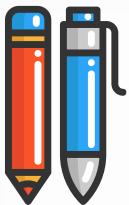
Fourier Transform

变换与形象 ...Biànhuàn yǔ xíngxiàng



- Fourier transform is mainly used for image processing. In the Fourier transform, the intensity of the image is transformed into frequency variation and then to the frequency domain.
- It is used for slow varying intensity images such as the background of a passport size photo can be represented as low-frequency components and the edges can be represented as high-frequency components. Low-frequency components can be removed using filters of FT domain.
- When an image is filtered in the FT domain, it contains only the edges of the image. And if we do inverse FT domain to spatial domain then also an image contains only edges. Fourier transform is the simplest technique in which edges of the image can be fined.



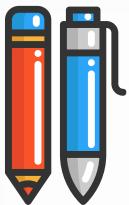


傅立叶转换



- 傅立叶变换主要用于图像处理。在 Fourier 转换中，图像的强度转换为频率变化，然后转换为频率域。
- 它用于慢速变化强度图像，如护照大小照片的背景可以表示为低频组件，边缘可以表示为高频组件。低频组件可以使用 FT 域的过滤器删除。
- 当图像在 FT 域中过滤时，它只包含图像的边缘。如果我们将 FT 域与空间域进行反向，则图像也只包含边缘。傅立叶转换是最简单的技术，其中图像的边缘可以罚款。
-



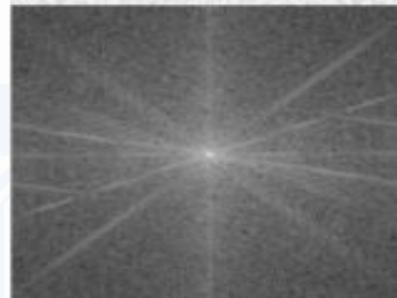


Fourier Transform

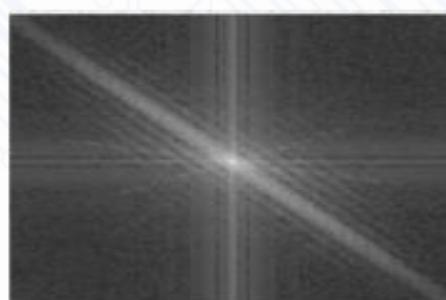
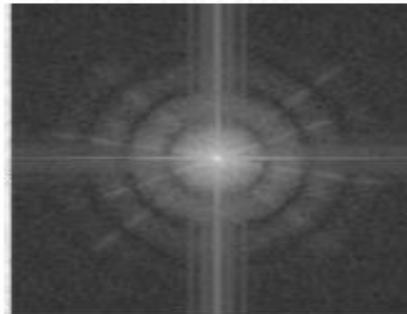
变换与形象 ...Biān huàn yǔ xíngxiàng

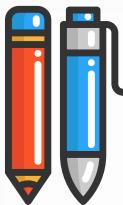


Example of Blurred image and its Fourier transformation



模糊图像的示例及其傅立叶转型
Móhú túxiàng de shìlì jí qí fùlìyèzhuǎnxíng





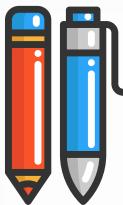
Discrete Cosine Transformation (DCT)

离散余弦变换 (DCT) Lísàn yúxián biànhuàn (DCT)



- In Discrete Cosine Transformation, coefficients carry information about the pixels of the image. Also, much information is contained using very few coefficients, and the remaining coefficient contains minimal information.
 - These coefficients can be removed without losing information. By doing this, the file size is reduced in the DCT domain.
 - DCT is used for lossy compression.
- 在离散余弦变换中，系数携带关于图像的像素的信息。
• 此外，使用很多系数包含了很多信息，并且剩余系数包含最小信息。
• 可以在不丢失信息的情况下移除这些系数。
• 通过这样做，DCT域中的文件大小会减少。
• DCT用于有损压缩。





Convolution Theorem

卷积定理 Juàn jī dìnglǐ



$$G(u,v) = F(u,v) \cdot H(u,v)$$



$$g(x,y) = h(x,y) * f(x,y)$$

Multiplication in Frequency Domain

频域中的乘法 Pín yù zhōng de chéngfǎ



Convolution in Time Domain

时域的卷积 Shí yù de juàn jī

- $f(x,y)$ is the input image
- $g(x,y)$ is the filtered
- $h(x,y)$: impulse response

Filtering in Frequency Domain with $H(u,v)$ is equivalent to filtering in Spatial Domain with $f(x,y)$.

在具有 $H(u, v)$ 的频域中过滤相当于使用 $f(x, y)$ 的空间域中过滤。

Zài jù yǒu $H(u,v)$ de pín yù zhōng guòlù xiāngdāng yú shíyòng $f(x,y)$ de kōngjiān yù zhōng guòlù.



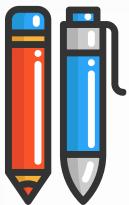


IMAGE TRANSFORMS APPLICATION

图像转换应用程序

Túxiàng zhuǎnhuàn yìngyòng chéngxù



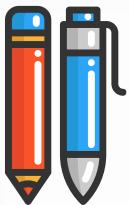
Applications of image transforms are as follows:

- Fourier transform is used for Edge Detection.
- Discrete Cosine Transform is used for image compression.

图像变换的应用如下：

- 更改的变换用于边缘检测。
- Discrete余弦变换用于图像压缩。

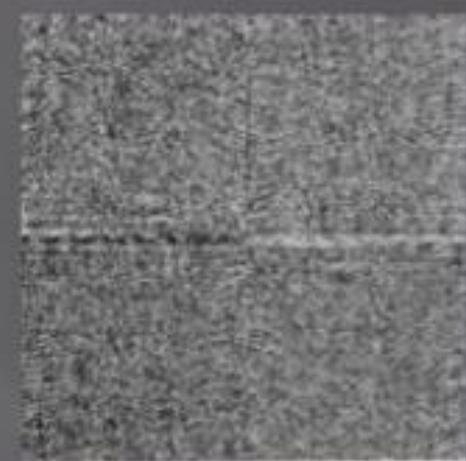




At the end

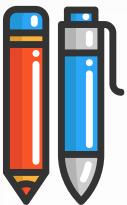


magnitude



phase



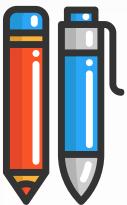


Notice



@dip学生：亲爱的学生，关于你的时期：你应该在matlab中使用代码来在图像处理中找到一个主题，并通过演示文稿中的ppt，但我应该在此之前批准主题。所有学生都被邀请确定您的主题并在10月30日之前将其发送给我。请注意，这次不会扩大。该计划是这样，学生将在可能的课程中讨论他们的主题和显示代码，学生可以用中文说话。学生演讲时间将于11月开始，每个学生都有10分钟的会话时间



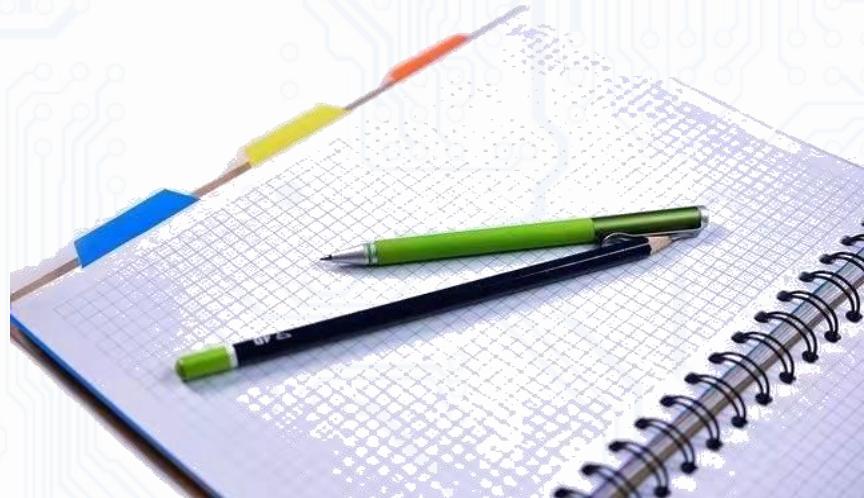


Student Task : DIP



- Select and send your Project topic from the link and list and run it in matlab make the report about project

Send after 2 week

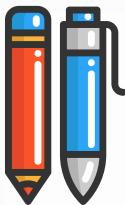


- You have time to send your task before 9 am (beijing time) of lecture
- Send the file in PPT(power point format) to this email :

drajm@yahoo.com

- Your file should have this format of name
<Task number><student name><Student ID>.ppt





Student Task_3: DIP



Write about Fourier Transform and its Application in Image Processing+ppt



- 请帮我翻译部分的朋友鼓掌
- Qǐng bāng wǒ fānyì bùfèn de péngyǒu gǔzhǎng

Solve the Question shared in mooc

解决MOOC 分享的问题

Send for Next lecture

发送下一个讲座



江西理工大学信息工程学院
JIANGXI UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF INFORMATION ENGINEERING



江西理工大学

Jiangxi University of Science and Technology

信息工程学院

School of information engineering

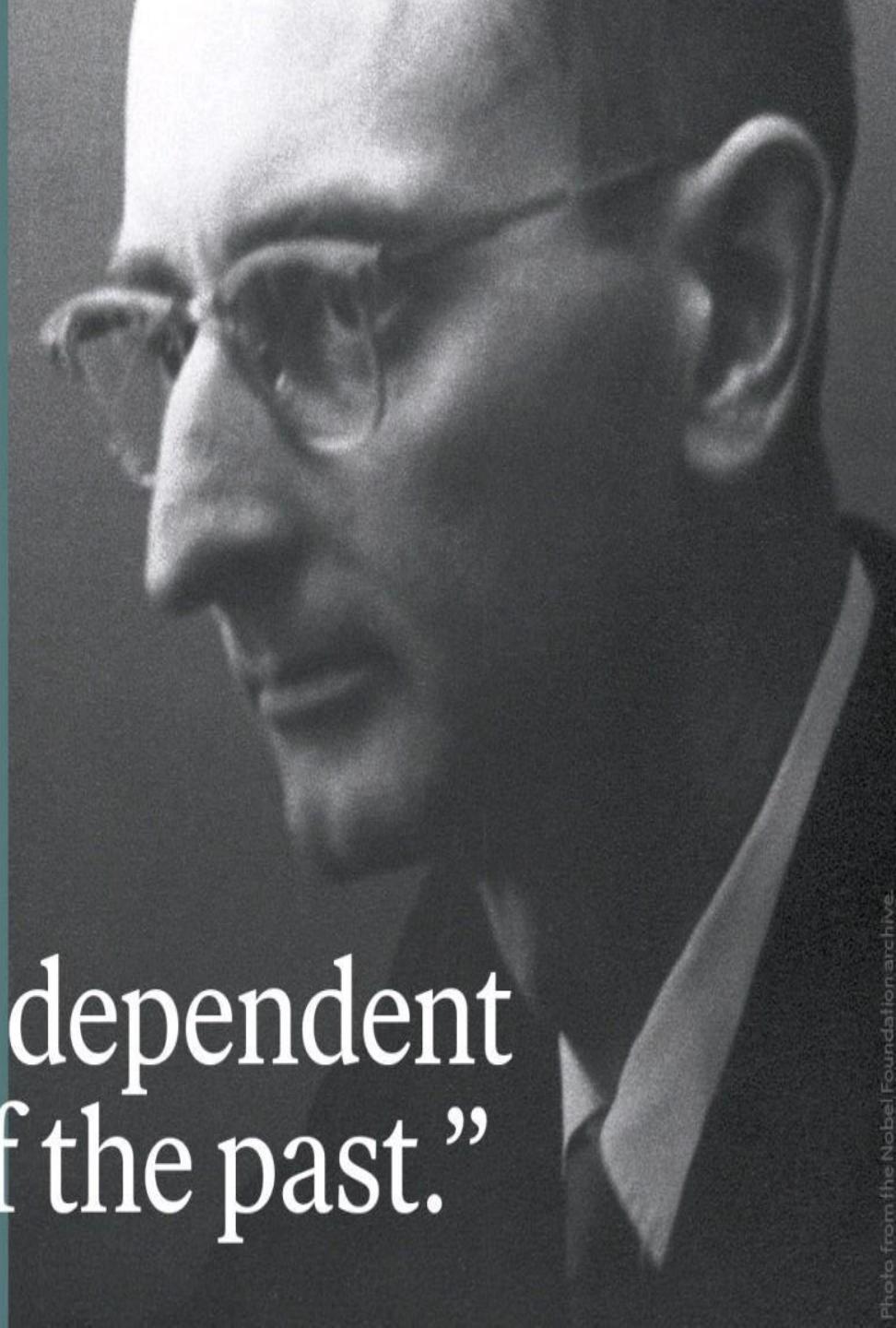
Digital Image Processing

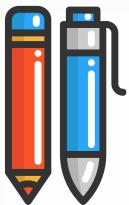
THANK YOU



OWEN CHAMBERLAIN
Nobel Prize in Physics 1959

“Each new idea is dependent upon the ideas of the past.”



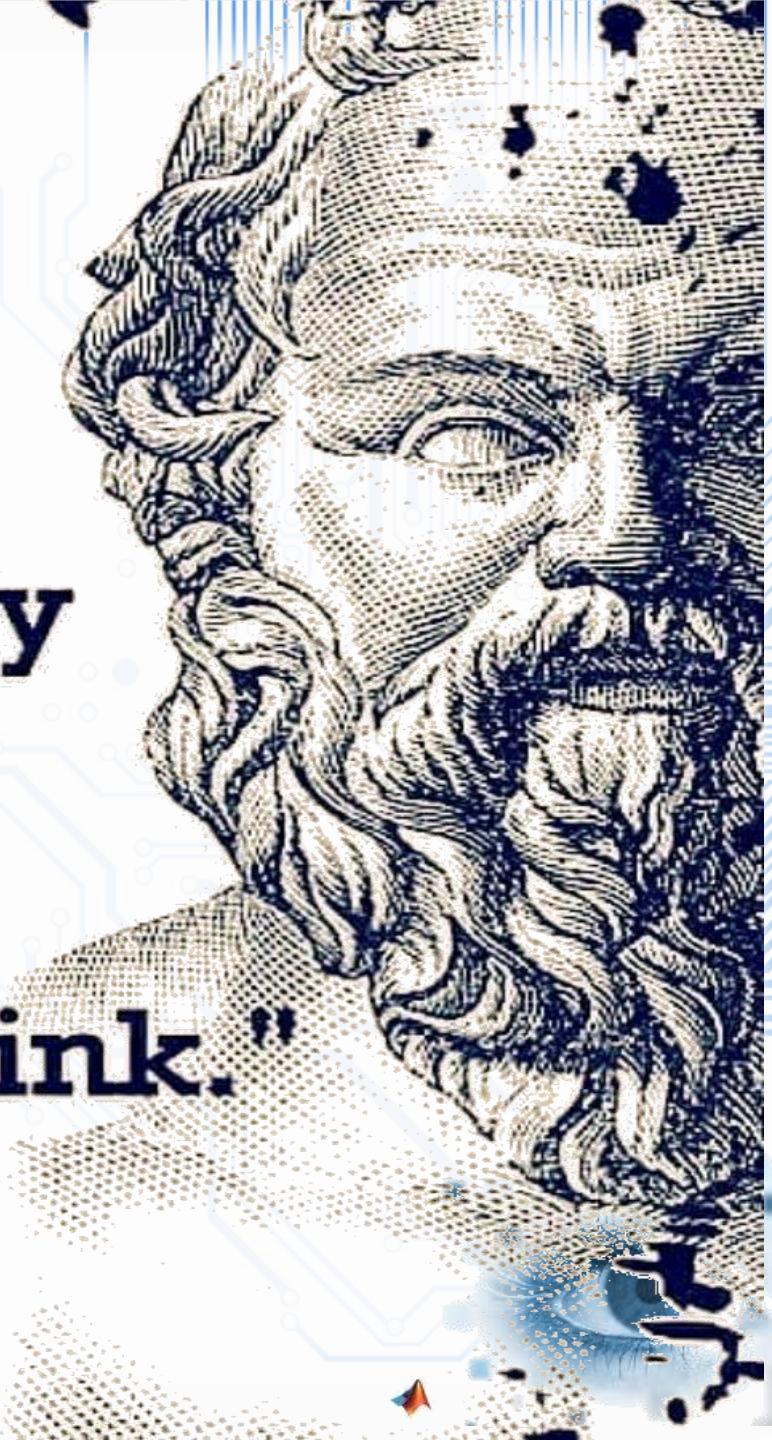


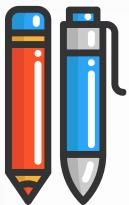
**“BE HUMBLE. BE HUNGRY.
AND ALWAYS BE THE
HARDEST WORKER
IN THE ROOM.”**



**"I cannot
teach anybody
anything,
I can only
make them think."**

~Socrates





Reference



Introduction to MATLAB, *Kadin Tseng, Boston University,
Scientific Computing and Visualization*

- Images taken from Gonzalez & Woods, Digital Image Processing (2002)

