江西理工大學信息工程学院
JIANGXI UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF INFORMATION ENGINEERING

Digital Image Processing

数字图像处理

Lecture 013:

Image enhancement_ Filter

Filtering using Fourier Transforms

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Digital Image Processing

LECTURE 13: Image enhancement section B

Spatial Filtering using Fourier Transforms

利用傅里叶变换进行空间滤波

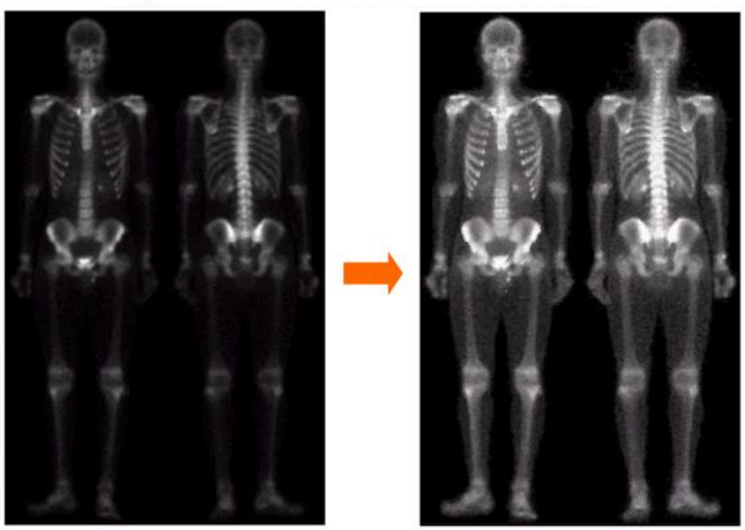
Agenda

- **Low pass filters(smoothing filters)**
  - Ideal low pass filters
  - Butterworth low pass filters
  - Gaussian low pass filters
- **High pass filters(sharpening filters)**
  - Ideal high pass filters
  - Butterworth high pass filters
  - Gaussian high pass filters
  - Laplacian in frequency domain
  - Unsharp masking high boost filtering and High frequency emphasis filters
- **Homomorphic filters**

低通滤波器（平滑滤波器）
　　理想的低通滤波器'；
　　巴特沃斯低通滤波器；
　　高斯低通滤波器；
高通滤波器（锐化滤波器）；
　　理想的高通滤波器；
　　巴特沃斯高通滤波器；
　　高斯高通滤波器；
　　频域中的拉普拉斯算子；
　　非锐化掩蔽高增强滤波和高频加重滤波器；
同态滤波器；

Review:
About Image Enhancement—Some Examples

original image result of image enhancement



original image result of image enhancement

About Image Enhancement

- Spatial domain VS frequency domain
- Spatial domain techniques:
  directly manipulate image pixels
- Frequency domain techniques

✓ Fourier transform or wavelet transform of an image

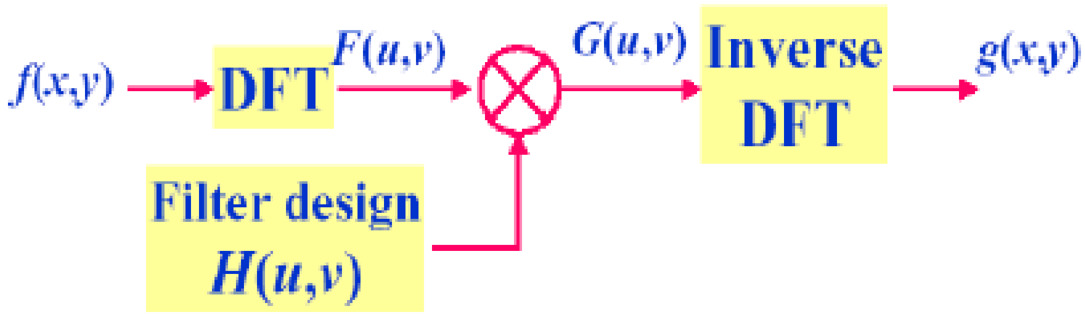
对图像增强
•空间域VS频率域
•空间领域技术:
直接操作图像像素
•频域技术
图像的傅里叶变换
或小波变换

REVIEW

Filtering using Fourier Transforms

使用傅立叶变换进行过滤 Shǐyòng fùlìyè biànhuàn jìnxíng guòlǜ

Basic steps for filtering in the frequency domain



H(u,v)起着重要的作用——决定增强后的图像看起来如何
低通H(u,v)导致模糊效果;高通H(u,v)产生锐化效果
高频:图像的边缘和清晰细节。
低频:图像的缓慢变化特征(如整体对比度和平均强度)。

- *H*(*u*,*v*) plays an important role⇨ determine how the enhanced image looks like
- A lowpass *H*(*u*,*v*) results in a *blurring* effect; a highpass *H*(*u*,*v*) results in a *sharpening* effect

模糊

锐化

- 高频 High frequency: edges and sharp details in an image
- 低频 Low frequency: slowly varying characteristics of an image (e.g. overall contrast and average intensity)

Examples of Filters

滤波器的例子

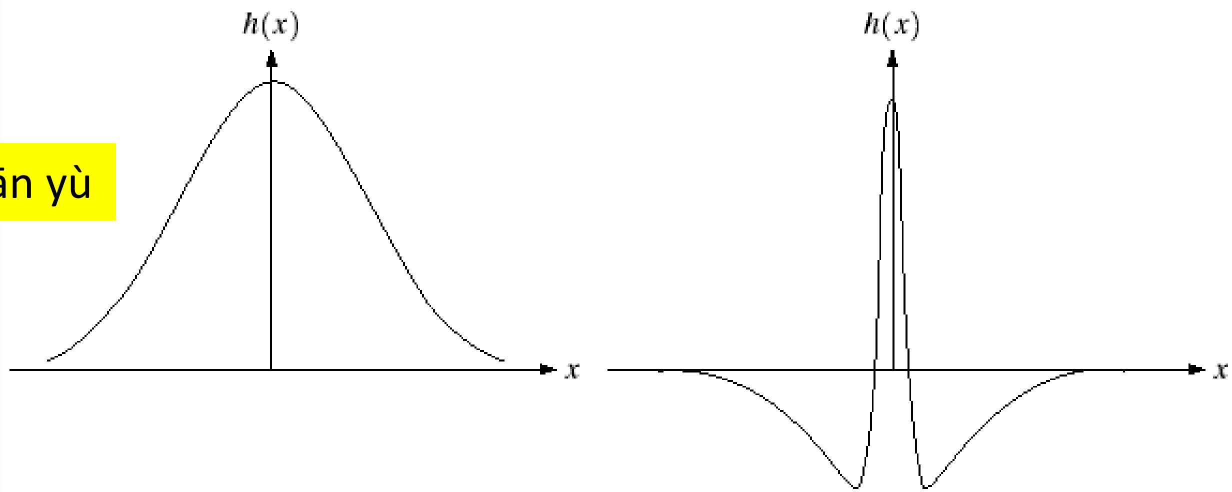
Frequency domain

频域 Pín yù

Gaussian lowpass filter   Gaussian highpass filter

Spatial domain

空间域 Kōngjiān yù



Smoothing Spatial Filters

平滑空间滤波器 Pínghuá kōngjiān lǜbō qì

- Smoothing filters ***are*** used for blurring and noise reduction.
- blurring is used in pre-processing steps, such as
  - -removal of small details from an image prior to object extraction
  - -bridging of small gaps in lines or curves
- noise reduction can be accomplished by blurring with a linear filter and also by a nonlinear filter.

模糊被用于预处理步骤，例如
-在提取目标之前去除图像中的小细节
-在直线或曲线上的小缝隙间架起桥梁
降噪可以用线性滤波器和非线性滤波器进行模糊处理。

Smoothing Spatial Filters

平滑空间滤波器 Pínghuá kōngjiān lǜbō qì

Image filtering can be grouped in two depending on the effects:

Low pass filters (Smoothing)

Spatial Averaging or Lowpass Filter

Mean Filters

Median Filters

High pass filters (Edge Detection, Sharpening

图像过滤可以根据效果分为两种:
- 低通滤波器（平滑）
- 空间平均或低通滤波器-均值滤波器
- 中值过滤器

高通滤波器（边缘检测、锐化

Low pass (Spatial) Filter

低通（空间）滤波器 Dī tōng (kōngjiān) lǜbō qì

- **Low pass spatial filter removes the sharp gray-level transitions while retaining the relatively smooth areas, hence producing blurring effect.**
- In a low pass spatial filter, the gray level of the pixel at the center of the spatial mask is replaced by the weighted average of the pixels under the spatial mask i.e., by the weighted average of its neighbourhood.
- Hence, lowpass spatial filtering is also called the ***neighbourhood averaging.*** Thus, for a mask of size mxn for lowpass filtering, the response of the mask or spatial filter is given by

$$R = \frac{1}{mn}\sum_{i=1}^{mn} z_i$$

- 低通空间滤波器去除锐利的灰度过渡，同时保留相对平滑的区域，从而产生模糊效果。
- 在低通空间滤波器中，空间掩码中心像素的灰度级被空间掩码下像素的加权平均值代替，即由其邻域的加权平均值代替。
- 因此，低通空间滤波也称为邻域平均。
- 因此，对于用于低通滤波的大小为 mxn 的掩膜，掩膜或空间滤波器的响应由下式给出

Low pass filters(Smoothing)

低通滤波器（平滑） Dī tōng lǜbō qì (pínghuá)

- A low-pass filter attenuates high frequencies and retains low frequencies unchanged.
- Low pass filtering (smoothing), is employed to remove high spatial frequency noise from a digital image.

低通滤波器衰减高频而保持低频不变

- The low-pass filters usually employ moving window operator which affects one pixel of the image at a time, changing its value by some function of a local region (window) of pixels.

低通滤波器通常采用移动窗口算子，每次影响图像的一个像素点，通过像素点的局部区域(窗口)函数改变其值

- The operator moves over the image to affect all the pixels in the image.
- The result in the spatial domain is equivalent to that of a smoothing filter; as the blocked high frequencies correspond to sharp intensity changes, *i.e.* to the fine-scale details and noise in the spatial domain image.

在空间域的结果等价于平滑滤波器的结果; 因为被遮挡的高频对应的是剧烈的强度变化，即空间域图像中的细尺度细节和噪声

Lowpass (Spatial) Filter

Examples

$$R = \frac{1}{mn}\sum_{i=1}^{mn} z_i$$

$\frac{1}{9}\times$

1	1	1
1	1	1
1	1	1

$3\times 3\ mask$

$\frac{1}{25}\times$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

$5\times 5\ mask$

Lowpass (spatial) Filter -Blurring

Original image

原图 Yuán tú

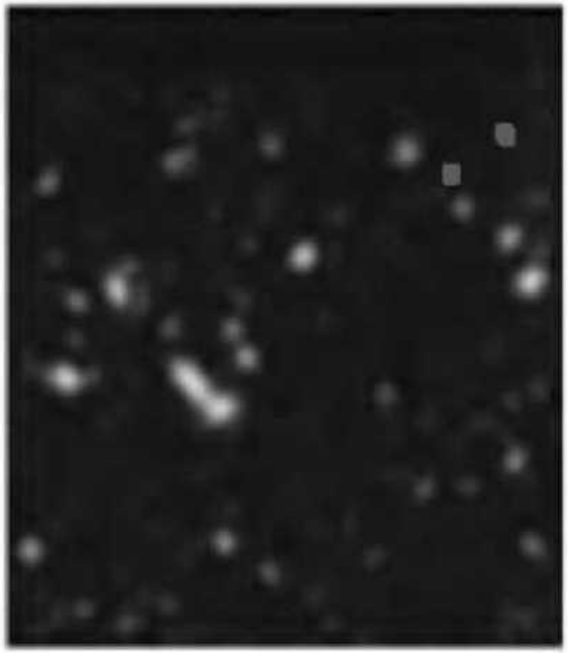
Result of lowpass (spatial) filtering - blurring

低通（空间）过滤的结果 - 模糊
Dī tōng (kōngjiān) guòlǜ de jiéguǒ - móhú

Lowpass (spatial) Filter

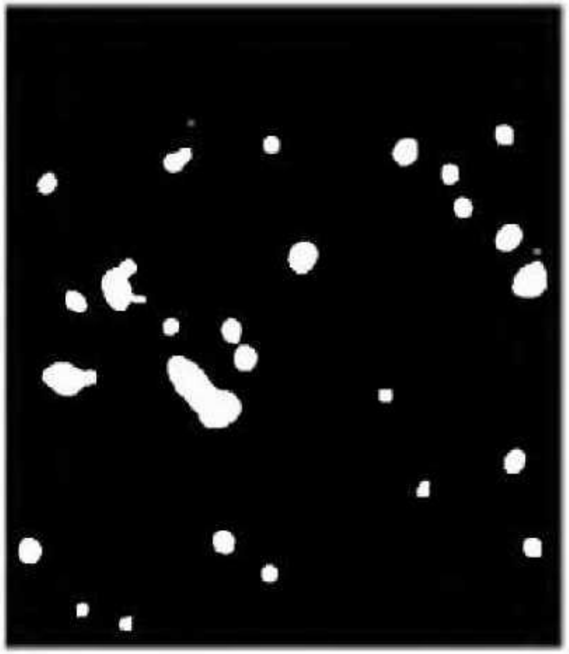
1. Image from Hubble Space Telescope

图片来自哈勃太空望远镜



2.Result of a 15x15 averaging spatial mask

一个15x15平均空间掩模的结果



3.Result of thresholding

阈值的结果

Enhancement by Mask Processing or Spatial Filtering

通过遮罩处理或空间过滤进行增强
Tōngguò zhē zhào chǔlǐ huò kōngjiān guòlǜ jìnxíng zēngqiáng

A spatial mask is a nxn (often n being odd) matrix with matrix elements being called as the mask ***coefficients*** or ***weights***.

一个空间掩码是一个nxn(通常n为奇数)矩阵，其矩阵元素被称为掩码系数或权重

w1	w2	w3
w4	w5	w 6
w7	w8	w9

- These masks are called the ***spatial filters.*** 空间过滤器

Enhancement by Mask Processing or Spatial Filtering

通过遮罩处理或空间过滤进行增强
Tōngguò zhē zhào chǔlǐ huò kōngjiān guòlǜ jìnxíng zēngqiáng

- These masks are called the ***spatial filters.*** The gray level of the pixel at the center of the spatial mask is replaced by the weighted sum, R given by

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_9 z_9 = \sum_{i=1}^{9} w_i z_i$$

- where $z_{i\#}$ i=1,2,...,9 is the gray level of the pixel under the mask weight, w_i. The value, R is called the response of the spatial mask.

- The response, R is a linear relation for the linear spatial filters.
- The response, R is a non-linear relation for the non-linear spatial filters

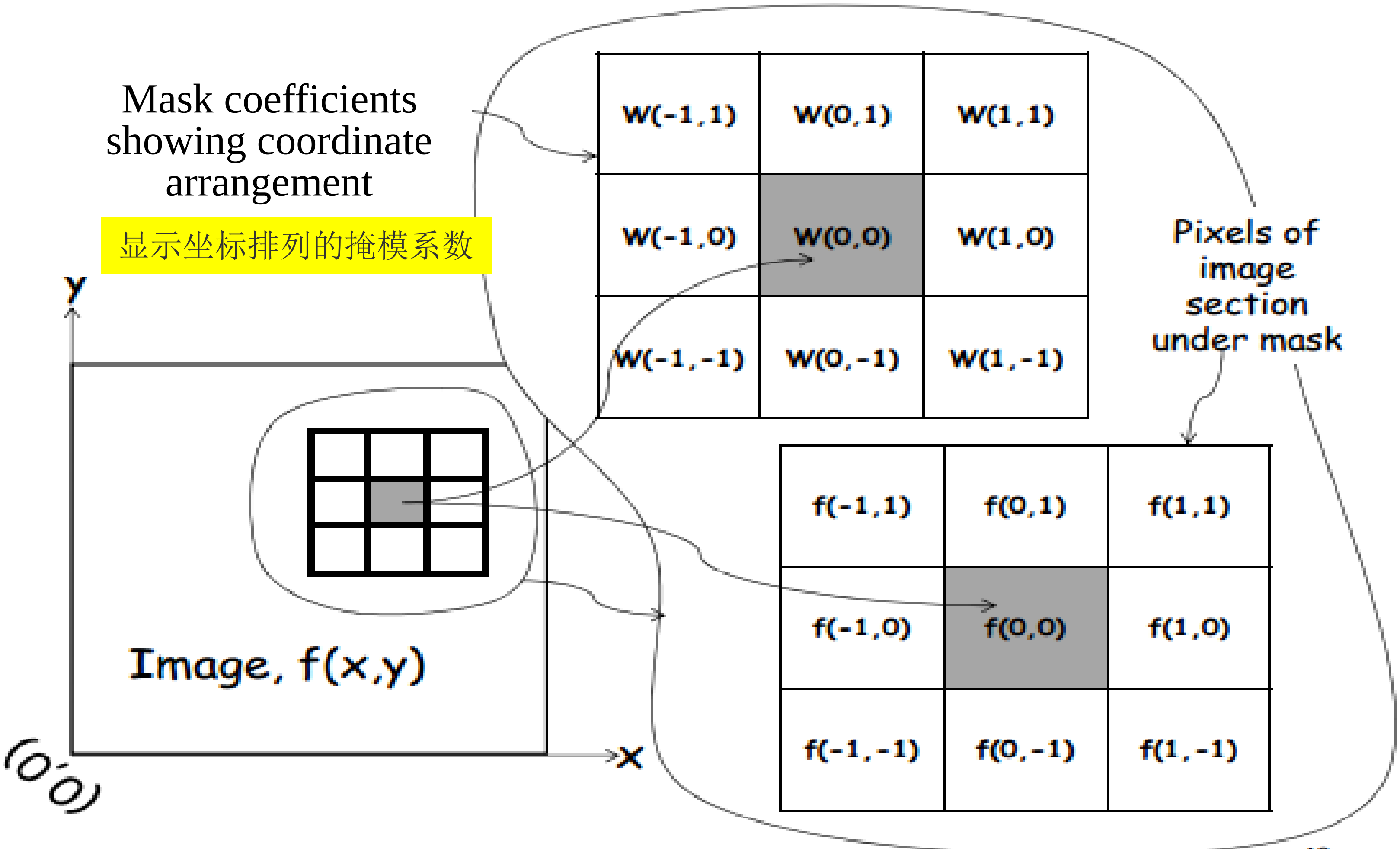
Enhancement by Mask Processing or Spatial Filtering

通过遮罩处理或空间过滤进行增强
Tōngguò zhē zhào chǔlǐ huò kōngjiān guòlǜ jìnxíng zēngqiáng

- 这些遮罩被称为空间过滤器。 将空间掩模中心像素的灰度级替换为加权和R

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_9 z_9 = \sum_{i=1}^{9} w_i z_i$$

- 在zi# = 1,2,... ， 9为掩码权重wi下像素的灰度级。 值R称为空间掩模响应。
- 响应R是线性空间滤波器的线性关系。
- 对于非线性空间滤波器，响应R是一个非线性关系


Mask coefficients showing coordinate arrangement
显示坐标排列的掩模系数
W(-1,1)
W(0,1)
W(1,1)
W(-1,0)
W(0,0)
W(1,0)
W(-1,-1)
W(0,-1)
W(1,-1)
Pixels of image section under mask
f(-1,1)
f(0,1)
f(1,1)
f(-1,0)
f(0,0)
f(1,0)
f(-1,-1)
f(0,-1)
f(1,-1)
y
x
Image, f(x,y)
(0,0)

Mean Filters

均值滤波器 Jūnzhí lǜbō qì

Arithmetic mean filter

$$\hat{f}(x,y)=\frac{1}{mn}\sum_{(s,t)\in S_{xy}} g(s,t)$$

A mean filter simply smoothes local variations in an image.
Noise is reduced as a result of blurring.

平均滤波器只是平滑图像的局部变化。
由于模糊，噪音会减少。

Geometric mean filter

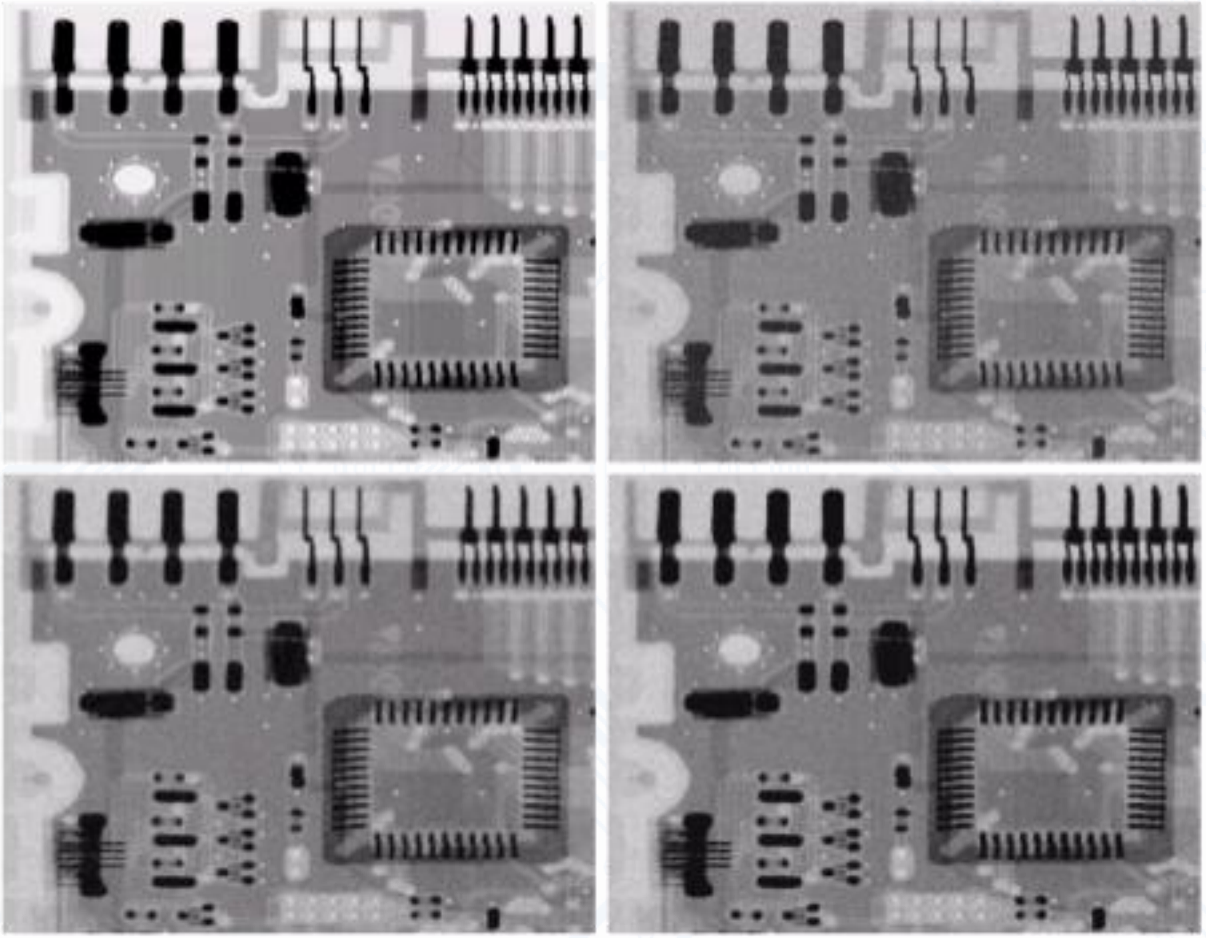
$$\hat{f}(x,y)=\left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$$

- A geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process.

几何平均滤波器的平滑效果与算术平均滤波器相当，但它往往在此过程中损失较少的图像细节

Mean Filters

均值滤波器 Jūnzhí lǜbō qì



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Mean Filters

均值滤波器 Jūnzhí lǜbō qì

-

Harmonic Mean Filter

$$\hat{f}(x,y)=\frac{mn}{\sum_{(s,t)\in S_{xy}} g(s,t)}$$

It does well also with other types of noise like Gaussian noise.

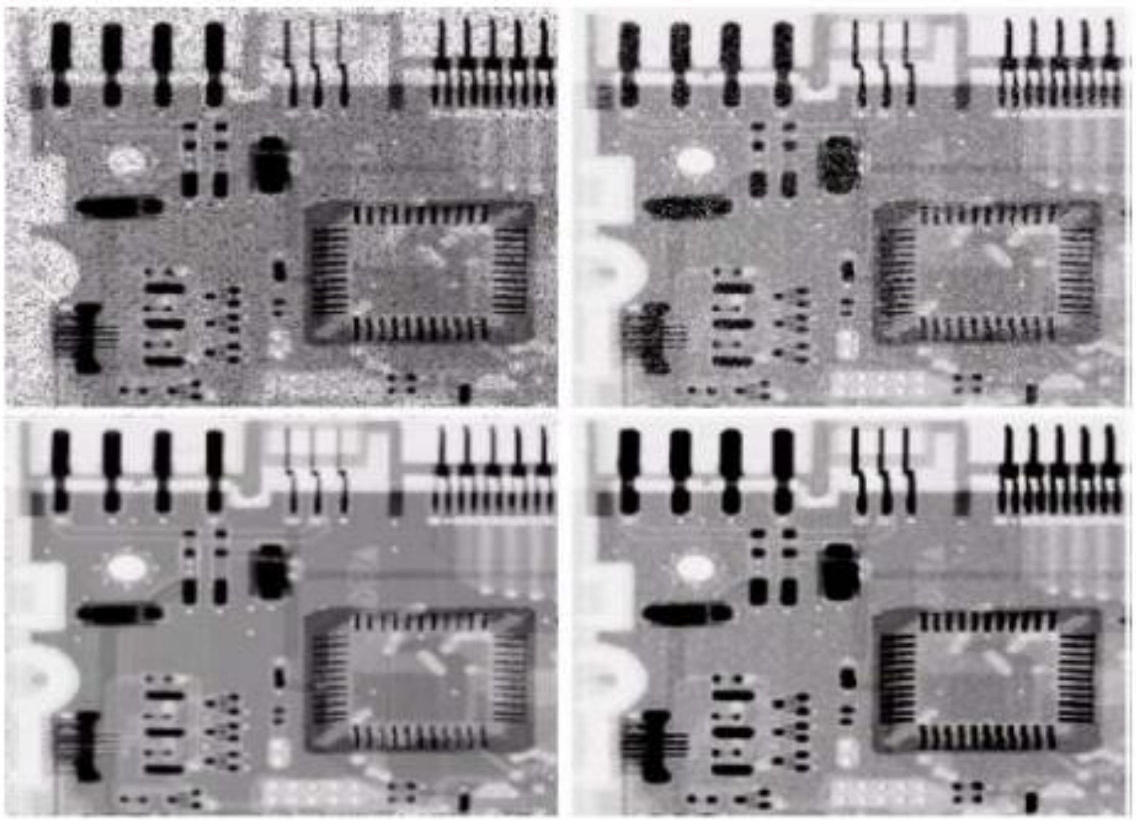
Contraharmonic Mean Filter

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q = order
+Q remove pepper noise
-Q remove salt noise
Q = 0 (arithmetic mean)
Q= -1 (Harmonic mean)

Mean Filters

均值滤波器 Jūnzhí lùbō qì

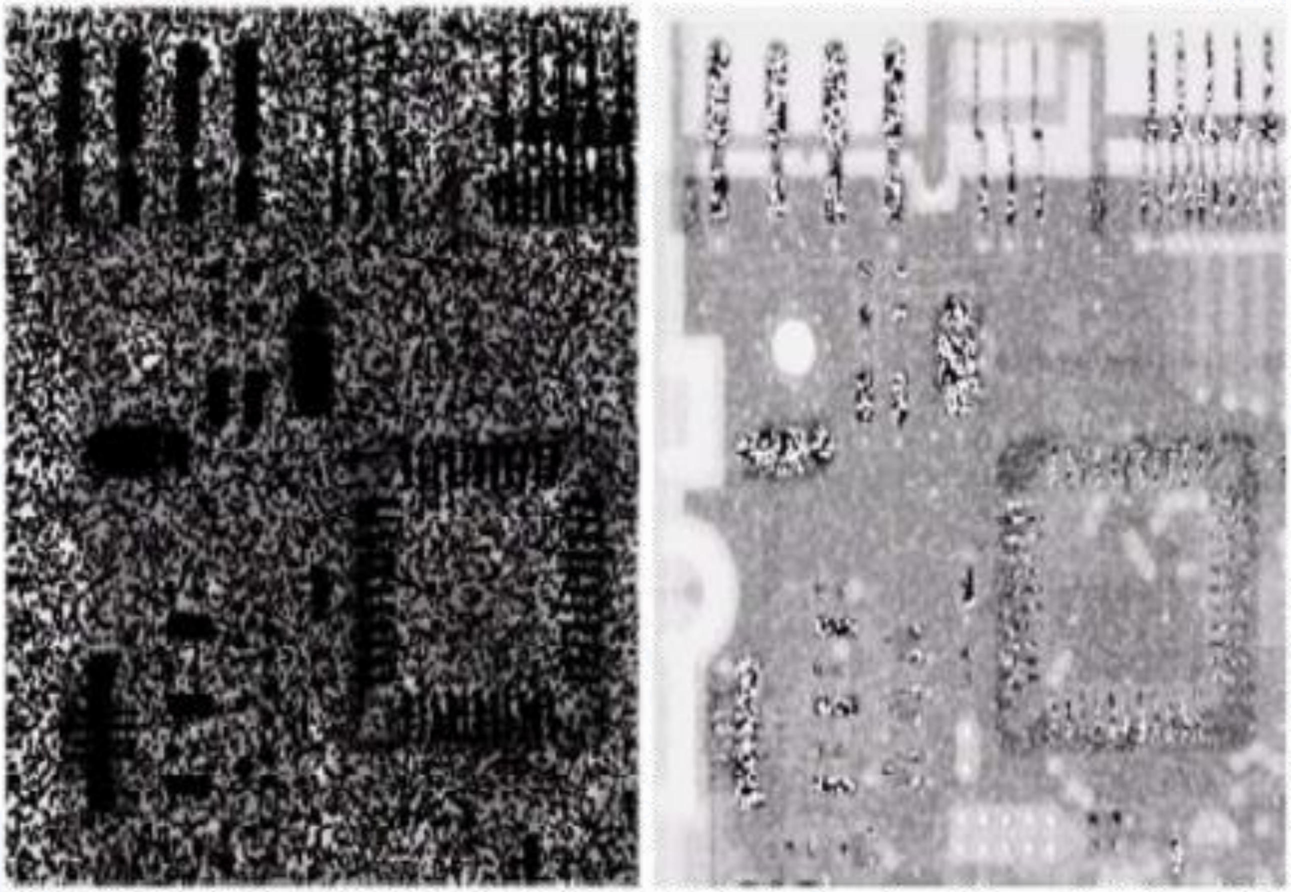


a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Mean Filters

均值滤波器 Jūnzhí lǜbō qì



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Image Averaging

图像平均 Túxiàng píngjūn

- Consider a noisy image g(x,y) formed by the addition of noise h(x,y) to an original image f(x,y)

$$g(x,y) = f(x,y) + \eta(x,y)$$

If noise has zero mean and be uncorrelated then it can be shown that if

$\bar{g}(x,y)$ = image formed by averaging K different noisy images

对k幅不同噪声图像进行平均得到的图像

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

Image Averaging

图像平均 Túxiàng píngjūn

- 考虑噪声h(x,y)加到原始图像f(x,y)上形成的噪声图像g(x,y)

$$g(x,y) = f(x,y) + \eta(x,y)$$

如果噪声均值为零且不相关，则可以证明

$\bar{g}(x,y)$ = image formed by averaging K different noisy images

对k幅不同噪声图像进行平均得到的图像

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

Image Averaging

图像平均 Túxiàng píngjūn

- Then

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$

$\sigma^2_{\bar{g}(x,y)}, \sigma^2_{\eta(x,y)}$ = variances of $\bar{g}$ and η

- if K increase, it indicates that the variability (noise) of the pixel at each location (x,y) decreases. 如果K增大，则表示每个位置(x,y)像素点的变异性(噪声)减小

  - Then

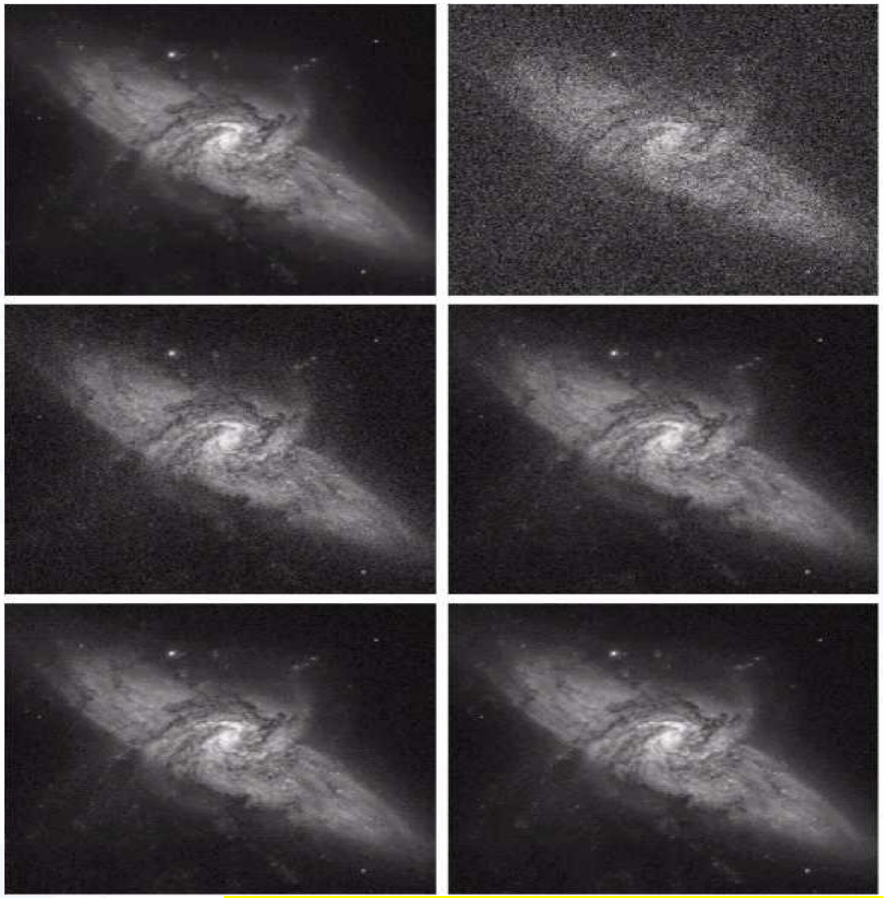
$$E\{\bar{g}(x,y)\} = f(x,y)$$

$E\{\bar{g}(x,y)\}$ = expected value of $\bar{g}$ (output after averaging)

= original image f(x,y)

Image Averaging

图像平均 Túxiàng píngjūn



a	b
c	d
e	f

a) original image

b) image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels.

c) . -f). results of averaging K = 8, 16, 64 and 128 noisy images

1)原始图像
b)被平均值为零、标准差为64个灰度级的加性高斯噪声污染的图像。
c)到 f)。 分别对K = 8,16,64和128的噪声图像进行平均

Median (Spatial) Filter

中值（空间）过滤器 Zhōng zhí (kōngjiān) guòlǜ qì

- ***A lowpass filter, if used for noise reduction, blurs edges and other sharp details.*** 低通滤波器，用于降噪，模糊边缘和其他尖锐的细节
- **An alternate approach for noise reduction without blurring effect is the use of median spatial filters.**

  另一种不产生模糊效果的降噪方法是使用中值空间滤波器
- In a median filter, the gray level of the pixel at the center of the spatial mask is replaced by the median of its neighbourhood i.e., by the median of the gray levels of the pixels under the spatial mask.
- Median spatial filters are very effective when the noise pattern consists of strong, spike-like components.
- Median filters are non-linear spatial filters. Median filters are the best-known in the category of the **order-statistics filters**. 次序统计滤波器

Median (Spatial) Filter

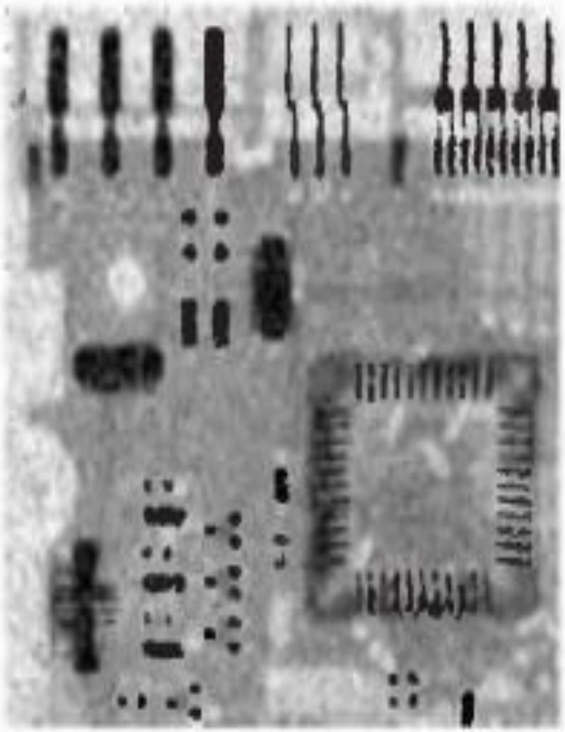
中值（空间）过滤器 Zhōng zhí (kōngjiān) guòlǜ qì

- 低通滤波器，用于降噪，模糊边缘和其他尖锐的细节
- .
- 另一种不产生模糊效果的降噪方法是使用中值空间滤波器

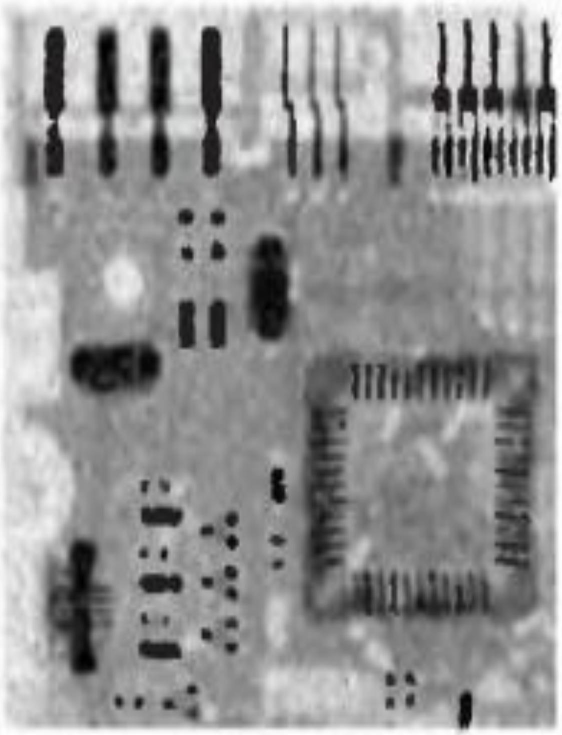
- 在中值滤波器中，空间掩模中心像素的灰度级被其邻域的中值替换，即被空间掩模下像素的灰度级的中值替换。
- 中值空间滤波器是非常有效的，当噪声模式组成强，尖状成分
- 中值滤波器是非线性空间滤波器。中值过滤器是顺序统计过滤器中最著名的

Median (Spatial) Filter

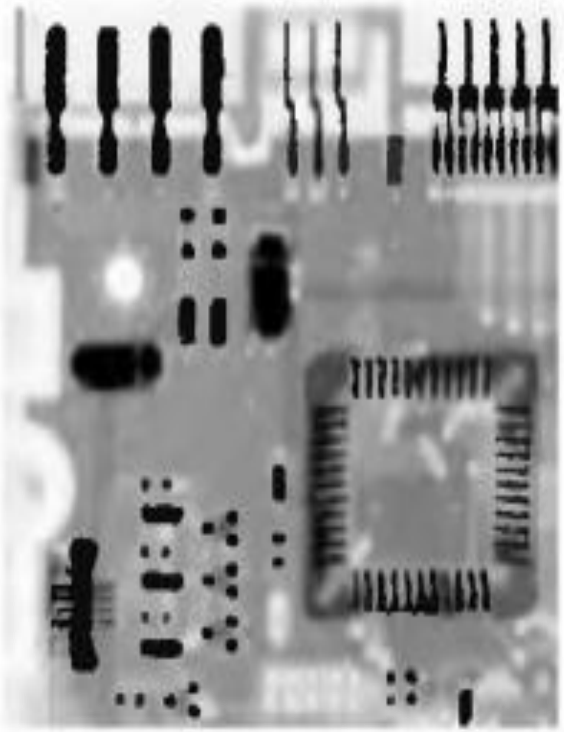
中值（空间）过滤器 Zhōng zhí (kōngjiān) guòlǜ qì



1. X-ray image of circuit board corrupted by speckle or salt & pepper noise



2. Result of a 3x3 averaging spatial mask



3. Result of a 3x3 median filter

Directional Smoothing

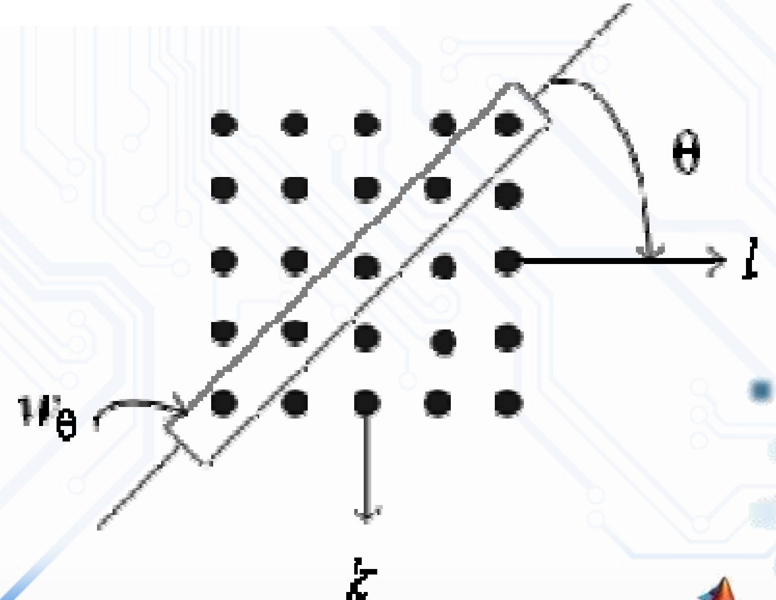
方向平滑 Fāngxiàng pínghuá

- Smoothing often results in blurring of edges.
- To protect edges from being blurred, directional averaging filters are used.
- Spatial averages are calculated in several directions as

$$v(m,n,\theta)=\frac{1}{N_\theta}\sum\sum_{(k,l\in w_\theta)} y(m-k,n-l)$$

- The direction, *6* is chosen such that minimum. Then it is set that

$$v(m,n)=v(m,n,\theta)$$



Directional Smoothing

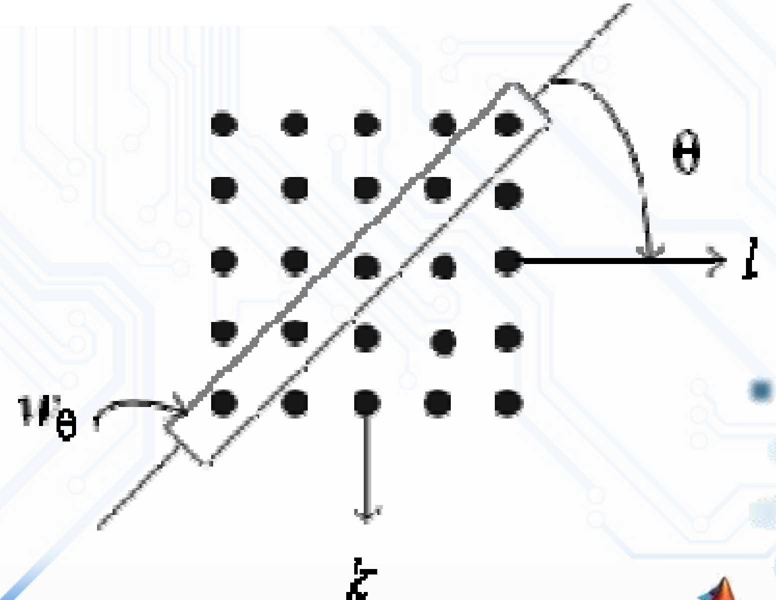
方向平滑 Fāngxiàng pínghuá

- 平滑经常导致边缘模糊。
- 为了保护边缘不被模糊，使用方向平均滤波器。
- 空间平均是按如下几个方向计算的

$$v(m,n,\theta)=\frac{1}{N_\theta}\sum\sum_{(k,l\in w_\theta)} y(m-k,n-l)$$

- 方向，6被选为最小值。 然后就设置好了

$$v(m,n)=v(m,n,\theta)$$



Notch Filters

陷波滤波器 Xiàn bō lǜbō qì

- Notch filter

A constant with a hole (notch) in the center

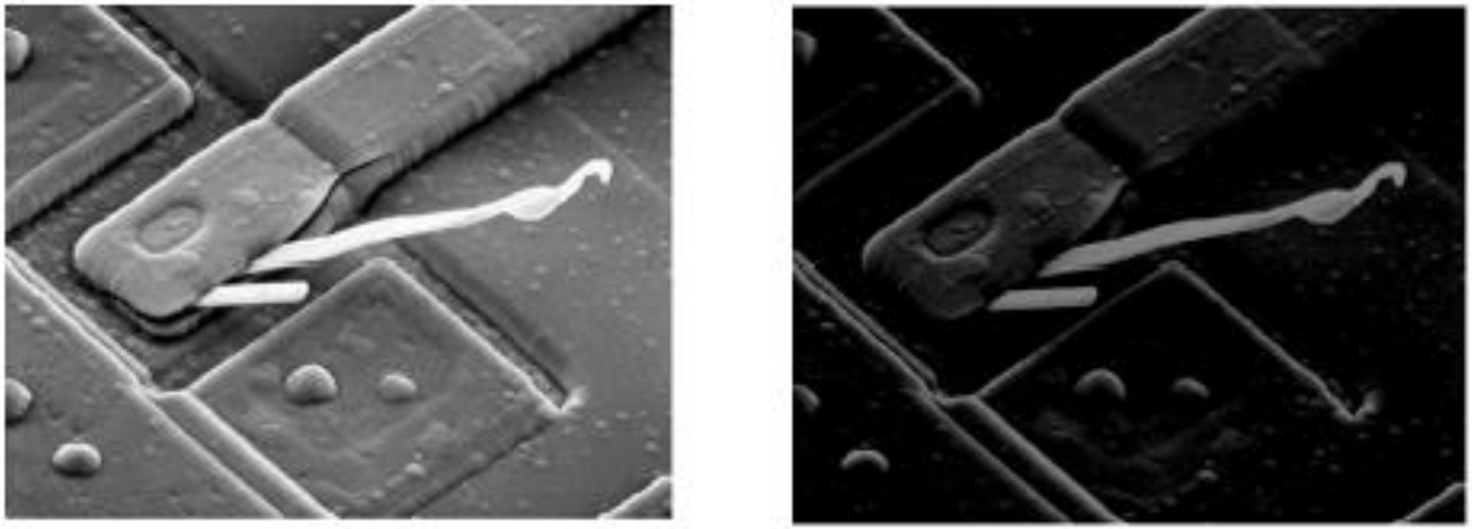
$$H(u,v) = \begin{cases} 0 & \text{if } (u,v) = (M/2, N/2) \\ 1 & \text{Otherwise} \end{cases}$$

This filter takes out the average gray level of the image. The resulting image has 0 value of $F(0,0)$ i.e. average value.

(To display it, a common practice is to display the most negative value as 0) And scaling rest of values.

这个过滤器去掉图像的平均灰度。得到的图像F(0,0)的值为0，即平均值。(为了显示它，通常的做法是将最负的值显示为0)和剩余的值进行缩放。然后缩放剩下的值

Example (Notch Function)



Notch function that sets F(0,0) to zero is applied to the original image (left)

将F(0.0)设为0的Notch函数应用于原始图像(左)

Ideal low-pass filter (ILPF)

理想低通滤波器 (ILPF) Lǐxiǎng dī tōng lǜbō qì (ILPF)

$$H(u,v) = \begin{cases} 1 & D(u,v) \le D_0 \\ 0 & D(u,v) > D_0 \end{cases}$$

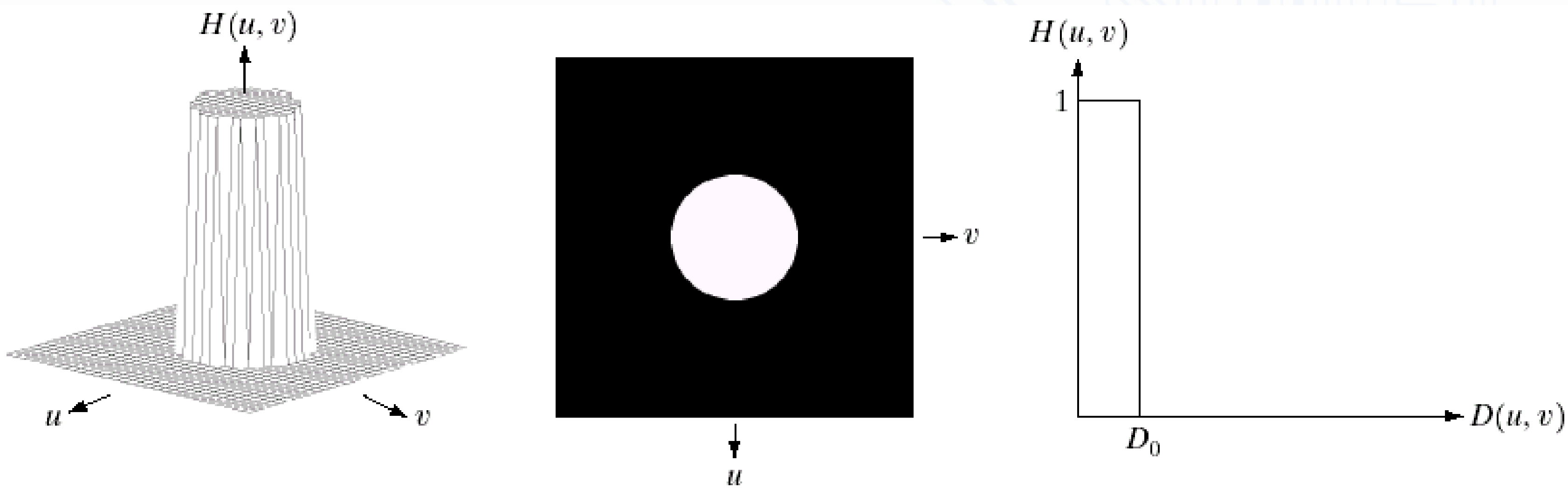
$$D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$$

(M/2,N/2): center in frequency domain

D_0 is called the *cutoff* frequency.

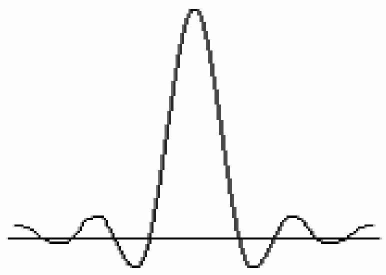
Shape of Ideal low-pass filter (ILPF)

理想低通滤波器 (ILPF) 的形状 Lǐxiǎng dī tōng lǜbō qì (ILPF) de xíngzhuàng



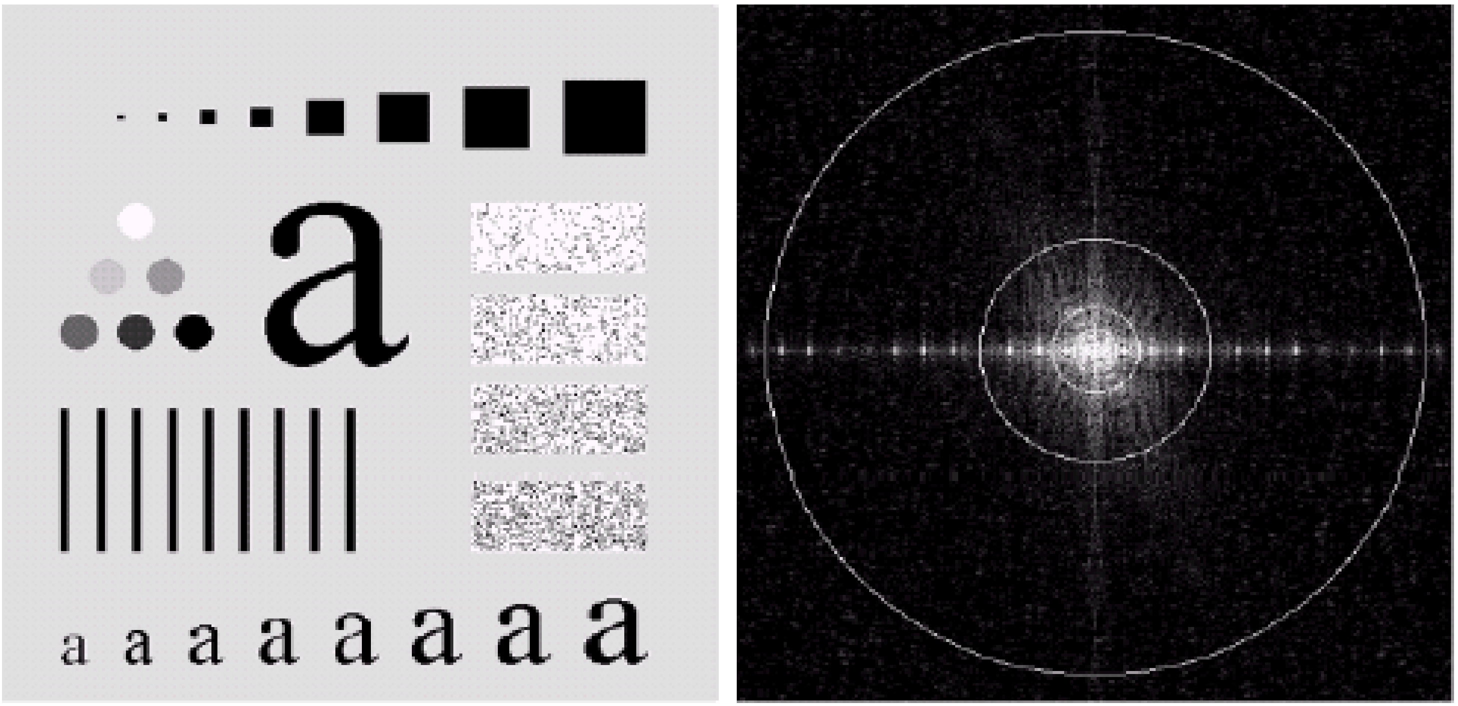
Frequency domain

Spatial domain



Ideal Low Pass Filter

理想低通滤波器 Lǐxiǎng dī tōng lǜbō qì



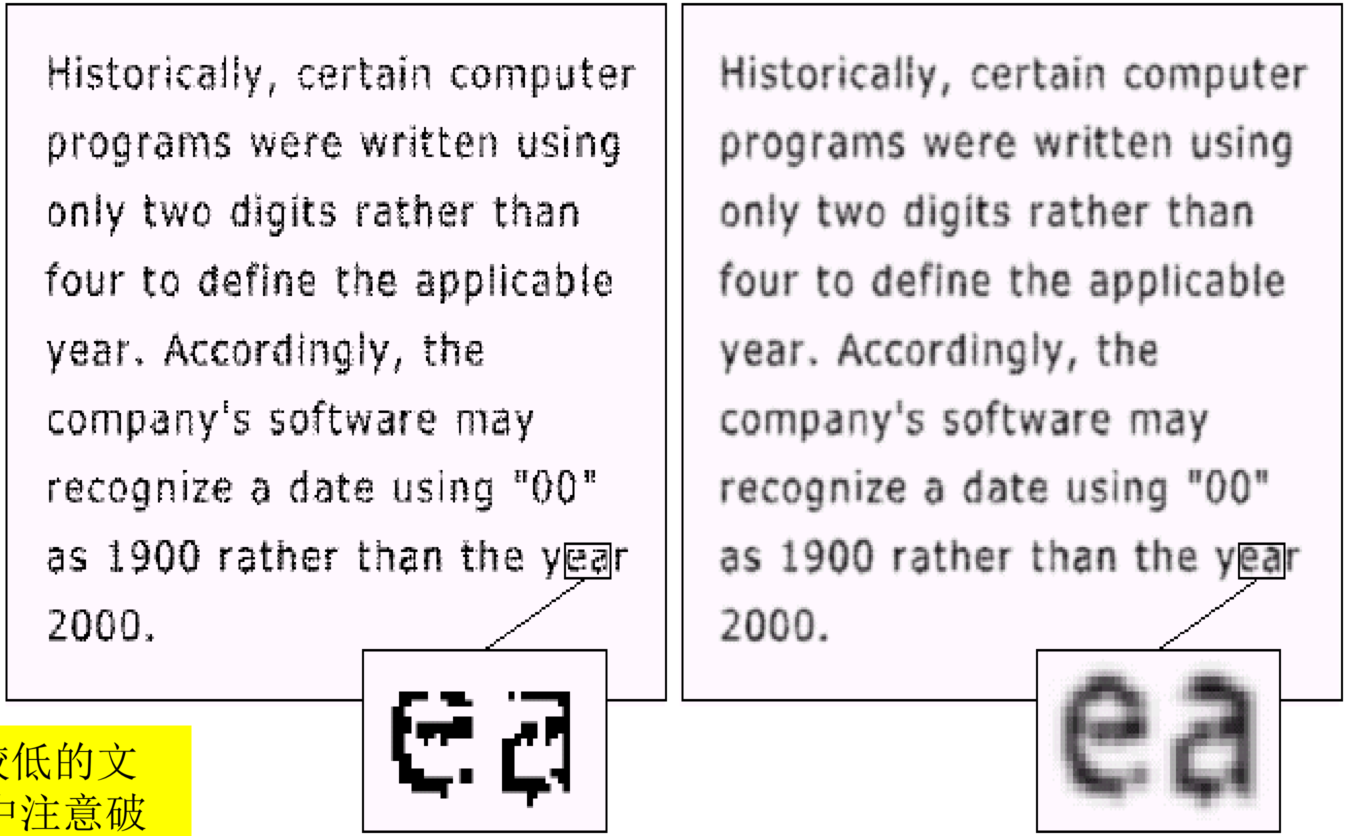
a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

图4.11 (a) 500 × 500像素的图像和(b)其傅里叶光谱。重叠圆的半径值为5。15. 30 80和230。附92.0,94.6,96.4。分别为图像功率的98.0和99.5%。

Examples of Lowpass Filtering

低通滤波示例 Dī tōng lǜbō shìlì



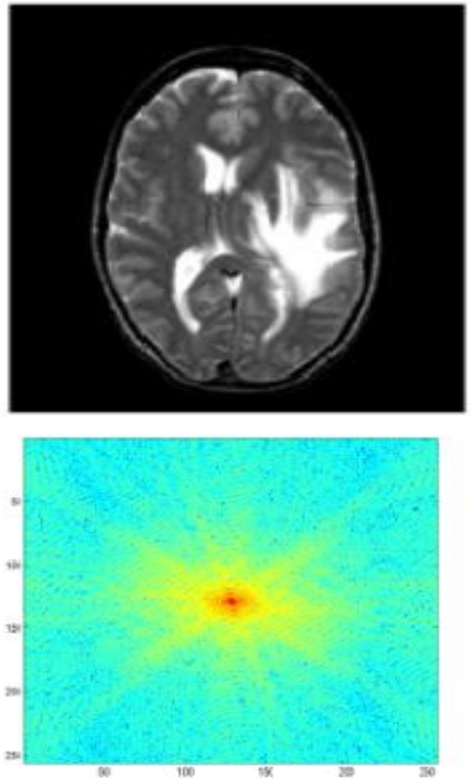
a b

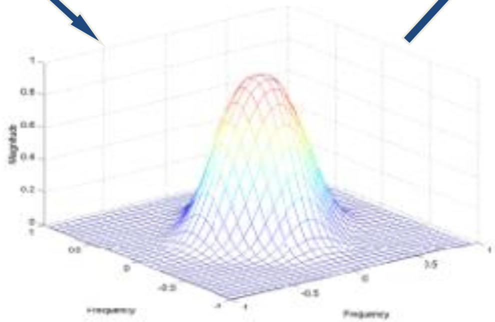
FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

图4.19 (a)分辨率较低的文本样本(放大视图中注意破碎的字符)(b)使用GLPF过滤的结果(合并了破碎的字符段)

Examples of Lowpass Filtering

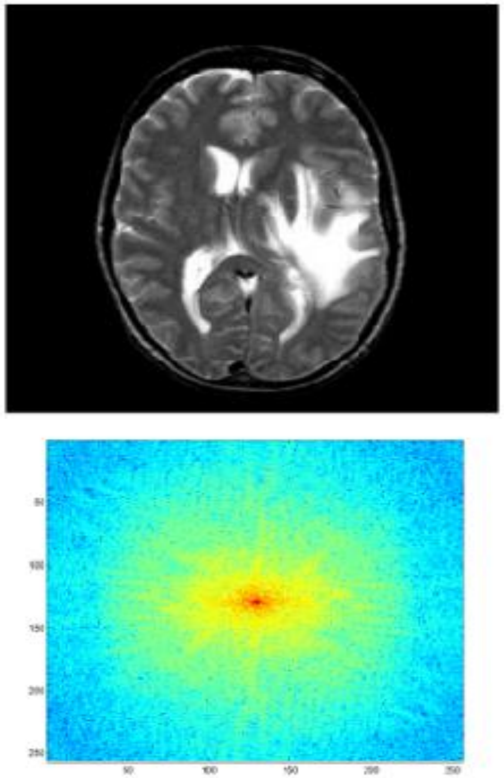
低通滤波示例 Dī tōng lǜbō shìlì





Low-pass filter H(u,v)

低通滤波器 H(u,v)

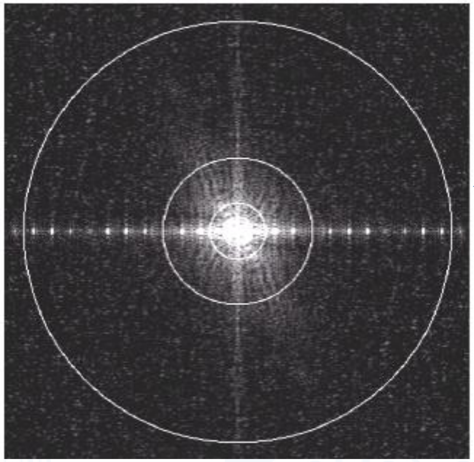


Original image and its FT

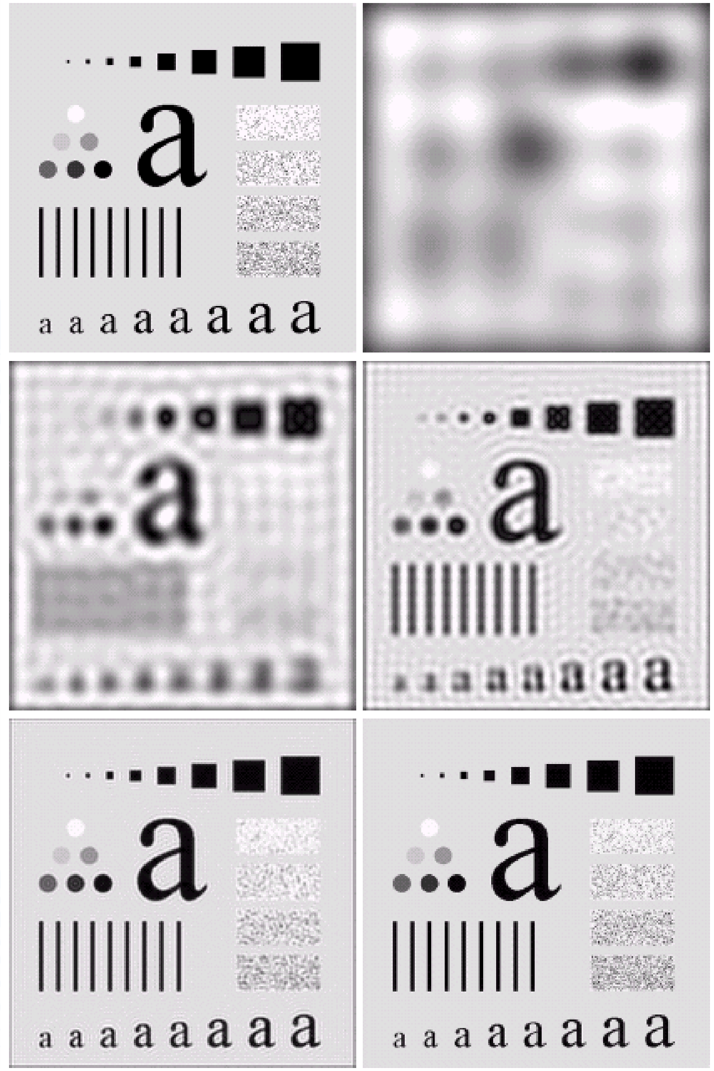
原始图像及其 FT

Filtered image and its FT

过滤后的图像及其 FT



FT



ringing and blurring

Ideal in frequency domain means non-ideal in spatial domain, vice versa.

频域理想意味着空间域不理想，反之亦然。

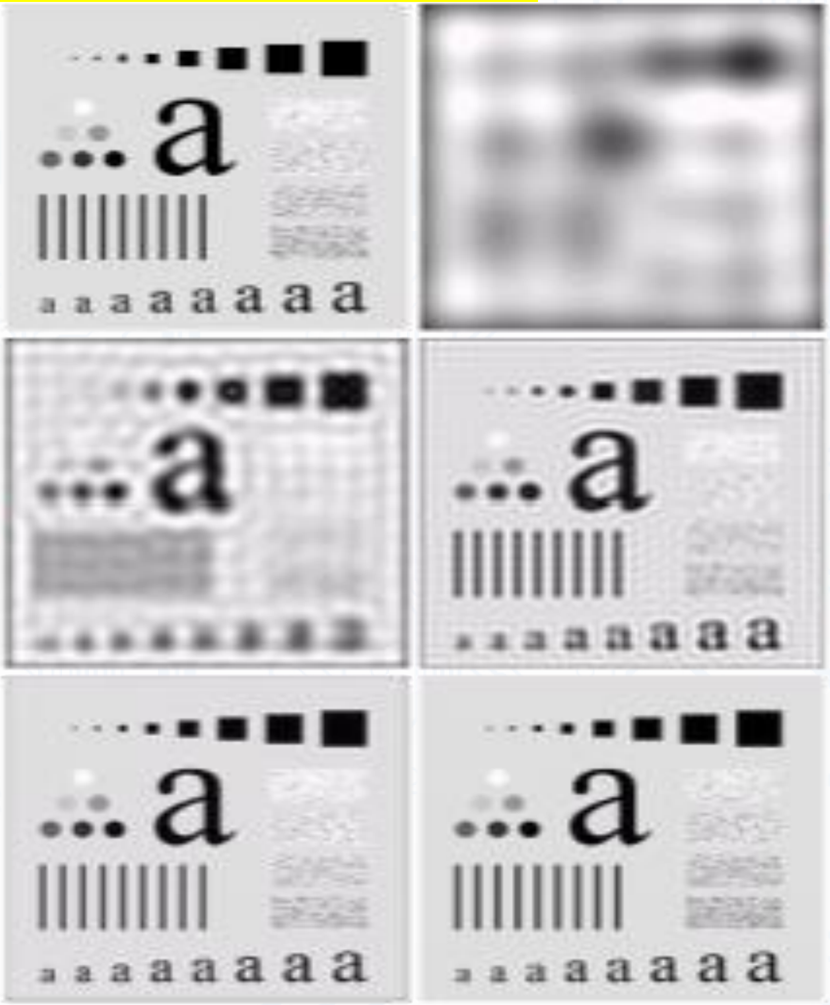
a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

图4.12 (a)原始图像。 (b)-(f)将截止频率设置为半径值5、15、30、80、230的理想低通滤波结果，如图4.11(b)所示。 这些滤波器的功率分别为8、5.4、3.6、2和0.5%

Ideal Low Pass Filter (example)

理想低通滤波器(示例)



original

5 pixels
92.0%
Useless, severe blurring (8%→ detail)

15 pixels
94.6%
Ringing effect due to ideal filter behavior

30 pixels
96.4%
Ringing effect due to ideal filter behavior

80 pixels
98.9%

230 pixels
99.5%

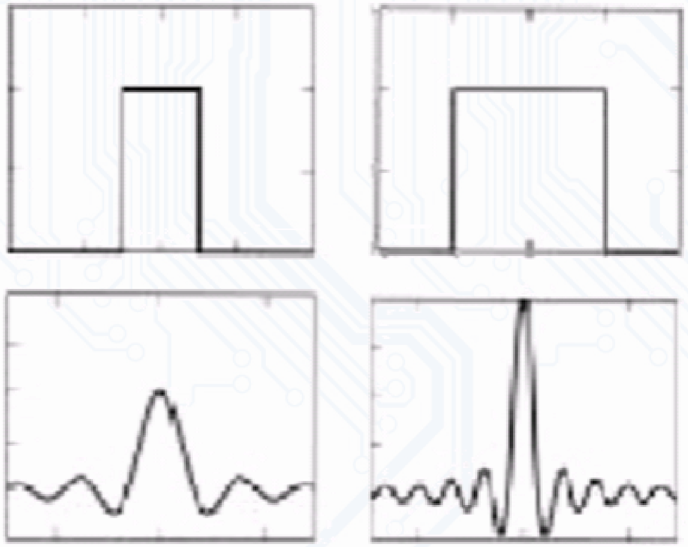
Why Ringing Effect

为什么会有振铃效应

According to convolution theorem

$$F(u,v)H(u,v) \Leftrightarrow f(x,y) * h(x,y).$$

where $H(u,v) \Leftrightarrow h(x,y).$



$H(u,v)$

$h(x,y)$

Butterworth Lowpass Filters (BLPF)

巴特沃斯低通滤波器 (BLPF) Bātè wò sī dī tōng lǜbō qì (BLPF)

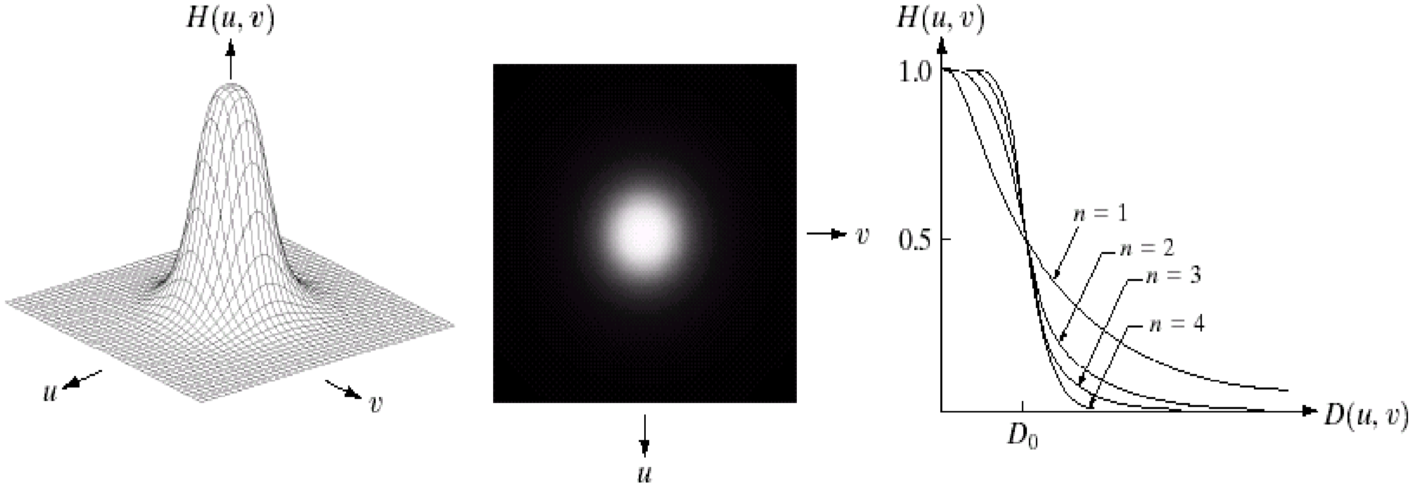
Smooth transfer function,

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

no sharp discontinuity,

no clear cutoff frequency.

光滑的传递函数，
没有锋利的不连续，
没有清晰的截止频率。



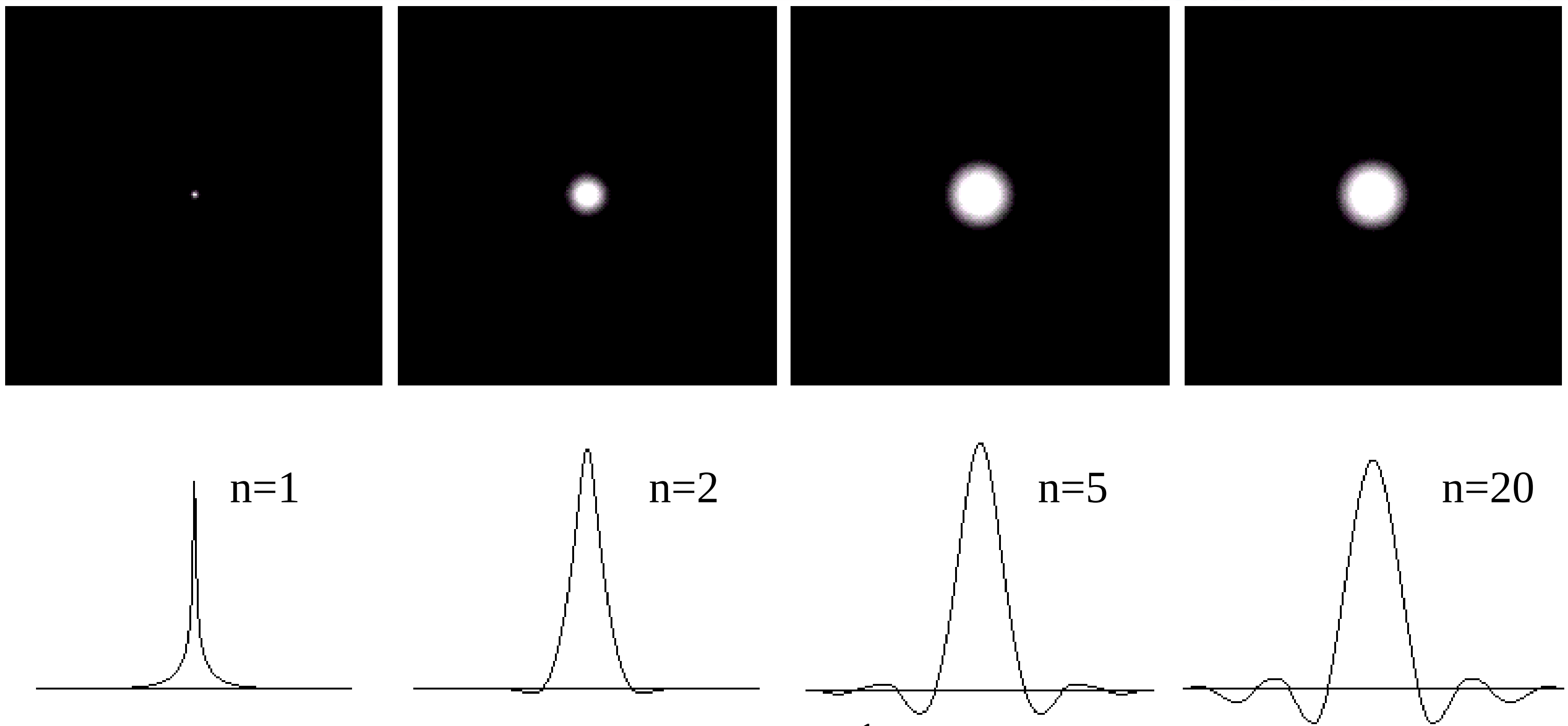
a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

图4.14 (a) 巴特沃斯低通滤波器传递函数的透视图。(b)滤镜显示为图像。(c)过滤1至4阶的径向截面

Butterworth Lowpass Filters (BLPF)

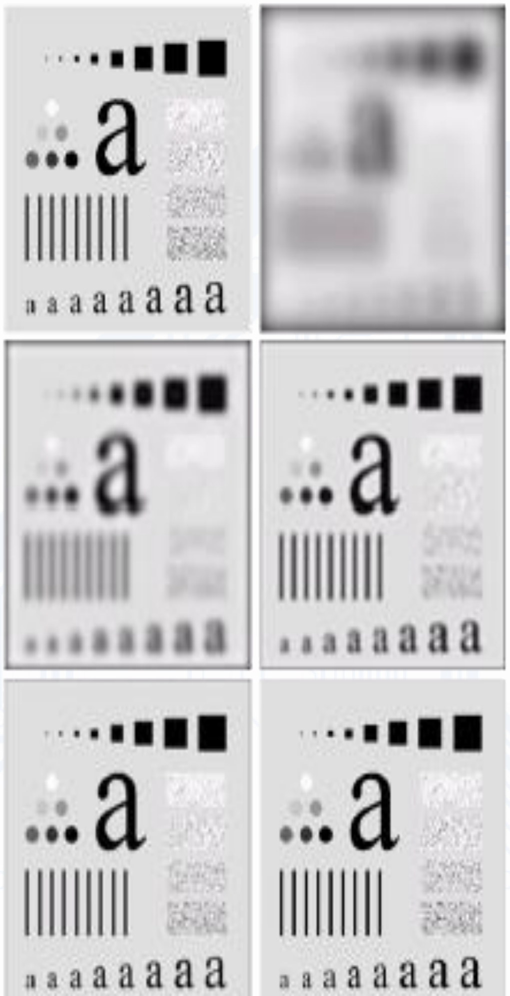
巴特沃斯低通滤波器 (BLPF) Bātè wò sī dī tōng lùbō qì (BLPF)



$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

Butterworth Lowpass Filters (BLPF)

巴特沃斯低通滤波器 (BLPF) Bātè wò sī dī tōng lǜbō qì (BLPF)



D_0: 5 pixels

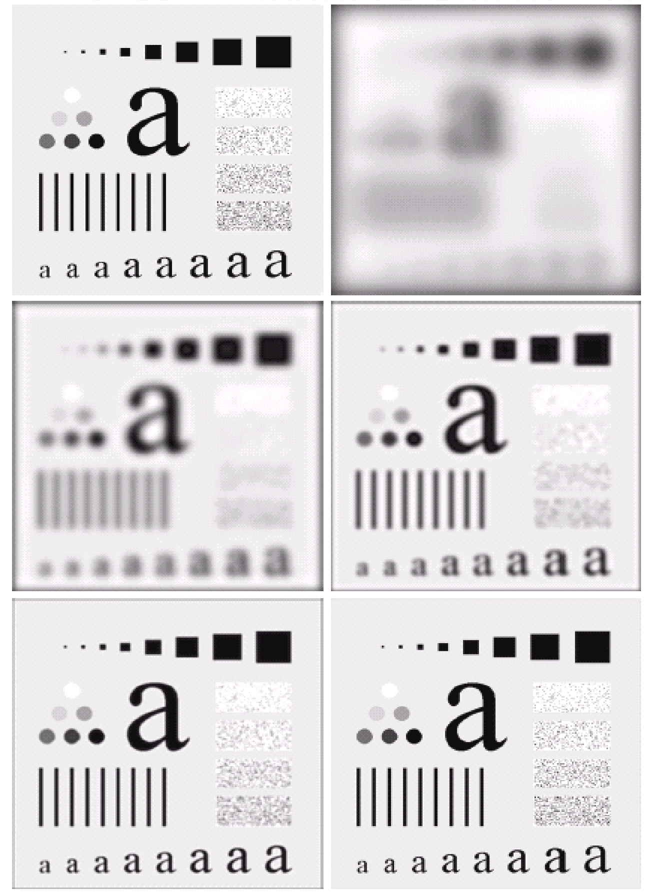
D_0: 15

D_0: 30

D_0: 80

D_0: 230

Butterworth LPF (n=2)



a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

No serious ringing artifacts

Gaussian Lowpass Filters (GLPF)

高斯低通滤波器 (LPF) Gāosī dī tōng lǜbō qì (LPF)

- Smooth transfer function, smooth impulse response, no ringing

传递函数平稳，脉冲响应平稳，无振铃现象

$$H(u,v) = e^{-\frac{D^2(u,v)}{2D^2_0}}$$

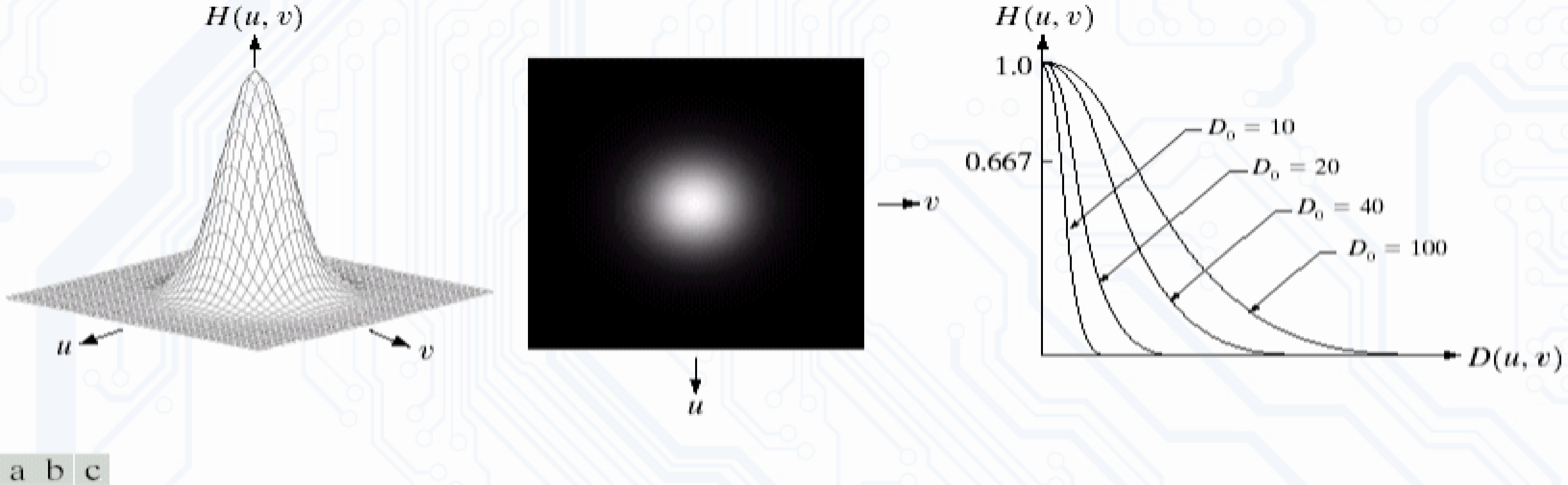


FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

图4.17 (a) GLPF传递函数的透视图。(b)滤镜显示为图像。(c)对不同的D值进行径向截面滤波。

Gaussian Low Pass Filters

高斯低通滤波器 (LPF) Gāosī dī tōng lǜbō qì (LPF)

$$H(u,v) = e^{-\frac{D^2(u,v)}{2\sigma^2}} \xrightarrow{\sigma = D_0} H(u,v) = e^{-\frac{D^2(u,v)}{2D_0^2}}$$

$$D^2(u,v) = \left(u - M/2\right)^2 + \left(v - N/2\right)^2$$

$$H(D(u,v) = 0) = 1$$

$$H(D(u,v) = D_0) = e^{-0.5} = 0.607$$

Inverse Fourier transform of GLPF is also Gaussian

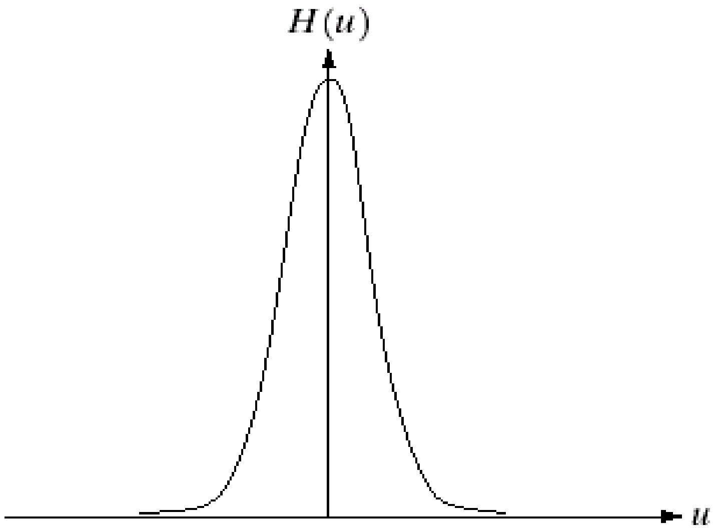
GLPF的傅里叶反变换也是高斯分布

Gaussian Low Pass Filters

高斯低通滤波器 (LPF) Gāosī dī tōng lǜbō qì (LPF)

Frequency domain

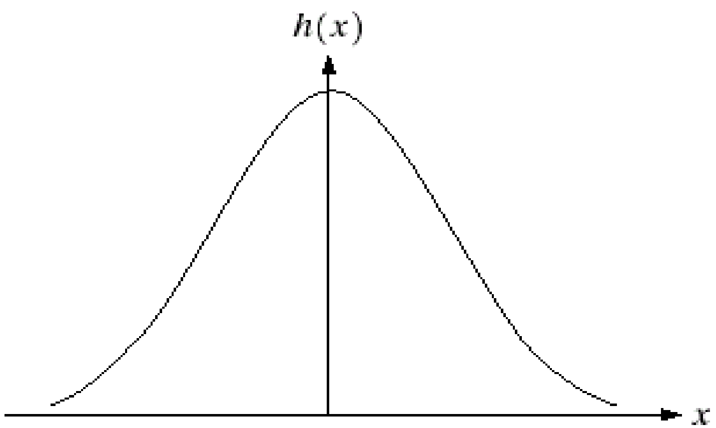
频域 Pín yù



Gaussian lowpass filter

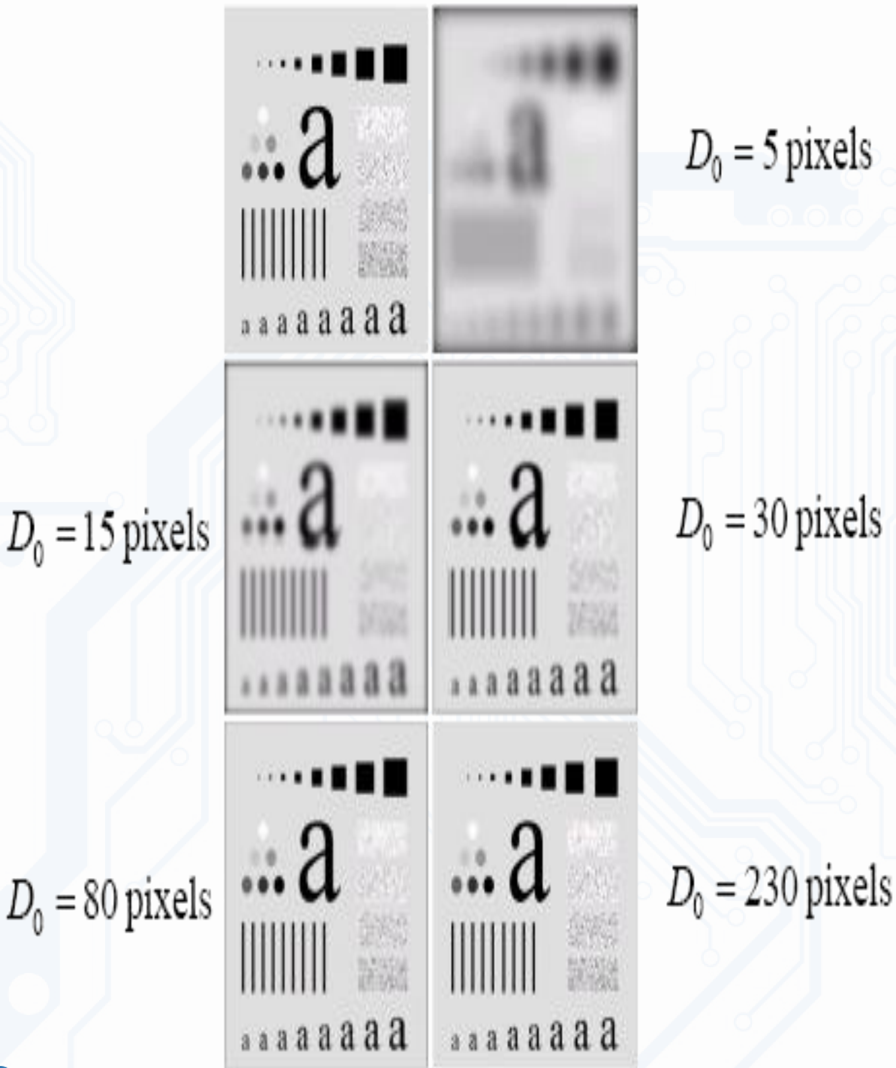
Spatial domain

空间域 Kōngjiān yù



Gaussian Low Pass Filters (example)

高斯低通滤波器 (LPF) Gāosī dī tōng lǜbō qì (LPF)



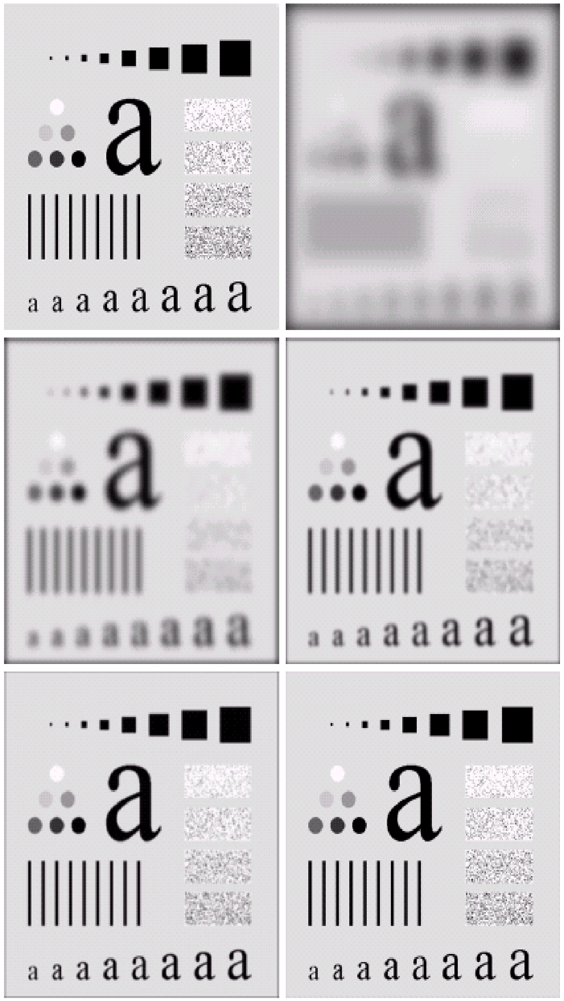
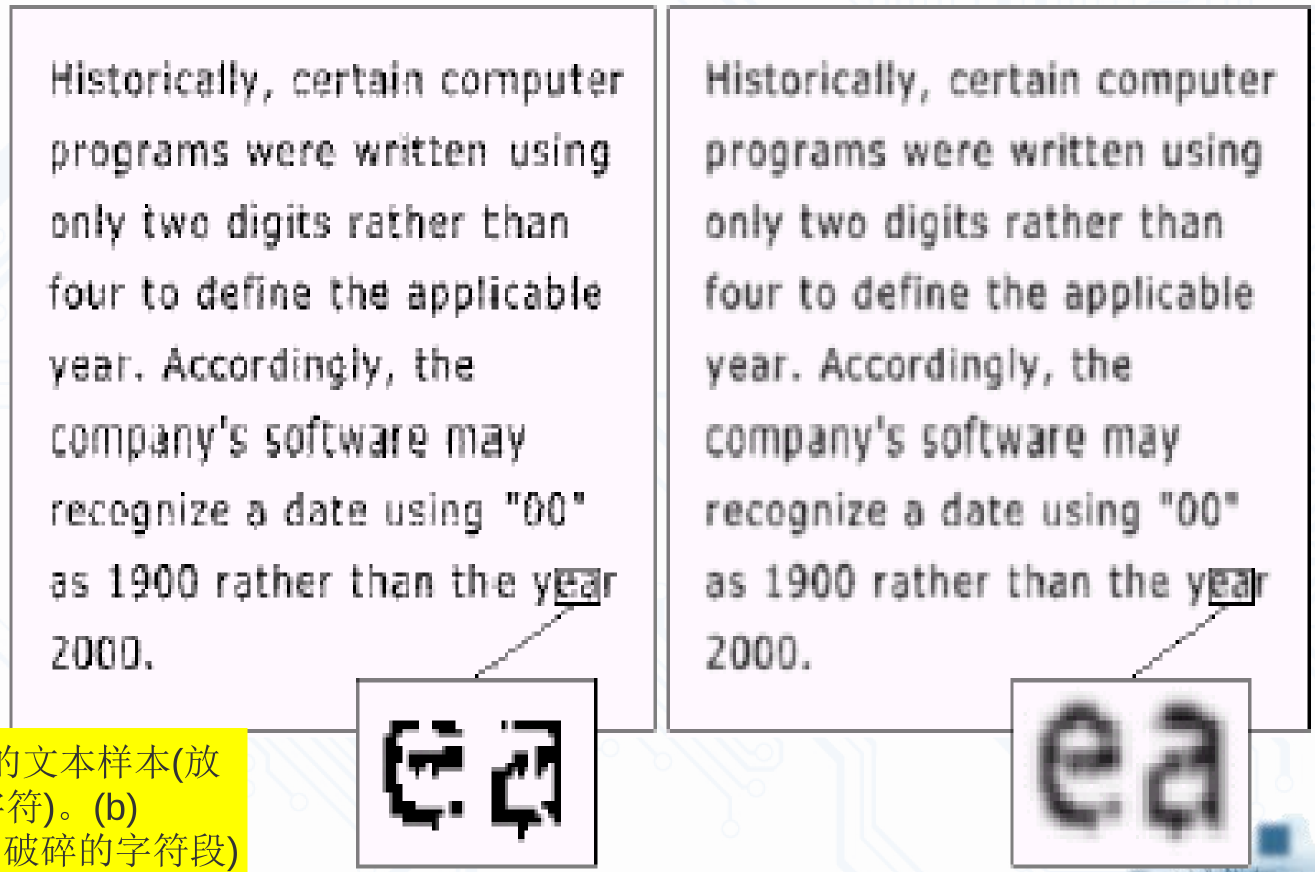


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

No ringing artifacts

Gaussian Low Pass Filters (example)

高斯低通滤波器 (LPF) Gāosī dī tōng lǜbō qì (LPF)



a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

图4.19 (a)分辨率较低的文本样本(放大视图中注意破碎的字符)。(b) GLPF过滤结果(合并了破碎的字符段)

High pass filters
(Edge Detection, Sharpening)

高通滤波器（边缘检测、锐化）

- A high-pass filter can be used to make an image appear sharper. ***These filters emphasize fine details in the image - the opposite of the low-pass filter.*** 高通滤波器可用来使图像显得更清晰。这些滤镜强调图像中的细节——与低通滤镜相反
- High-pass filtering works in the same way as low-pass filtering; it just uses a different convolution kernel.
- A high pass filter, on the other hand, yields ***edge enhancement or edge detection in the spatial domain***, because edges contain many high frequencies. 另一方面，高通滤波器在空间域产生边缘增强或边缘检测，因为边缘包含许多高频。
- Areas of rather constant gray level consist of mainly low frequencies and are therefore suppressed.

High pass filters
(Edge Detection, Sharpening)

高通滤波器（边缘检测、锐化）

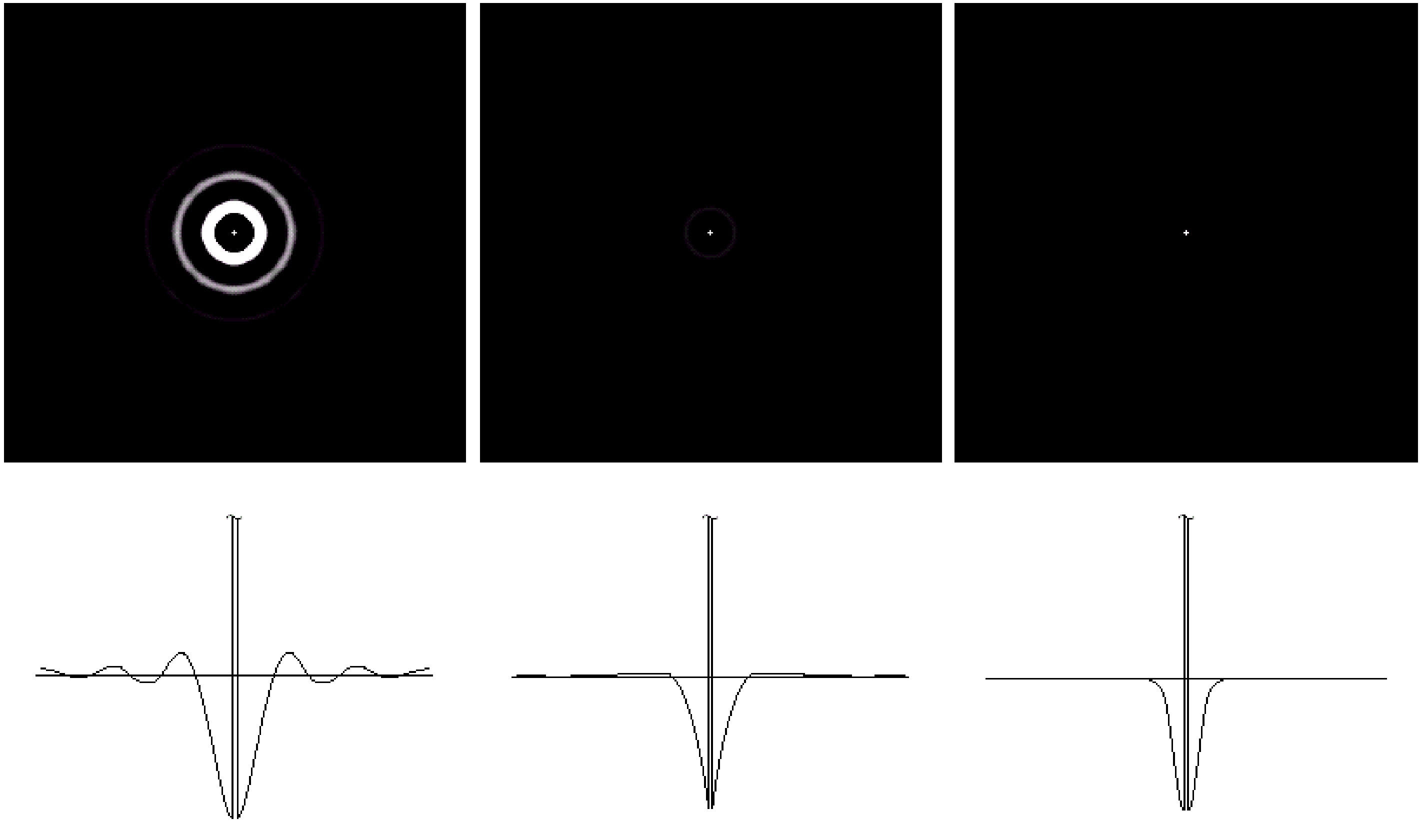
- 高通滤波器可用来使图像显得更清晰。这些滤镜强调图像中的细节——与低通滤镜相反

- 高通滤波的工作原理与低通滤波相同;它只是用了不同的卷积核。.
- 另一方面，高通滤波器在空间域产生边缘增强或边缘检测，因为边缘包含许多高频。

- 灰度相当恒定的区域主要由低频率组成，因此受到抑制。.

High-pass Filters

高通滤波器



a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

图4.23典型(a)理想的空间表示。巴特沃斯(h)。(c)高斯频域高通滤波器。以及相应的灰度轮廓

Ideal High-pass Filtering

ringing artifacts

理想的高通滤波 Lǐxiǎng de gāotōng lùbō

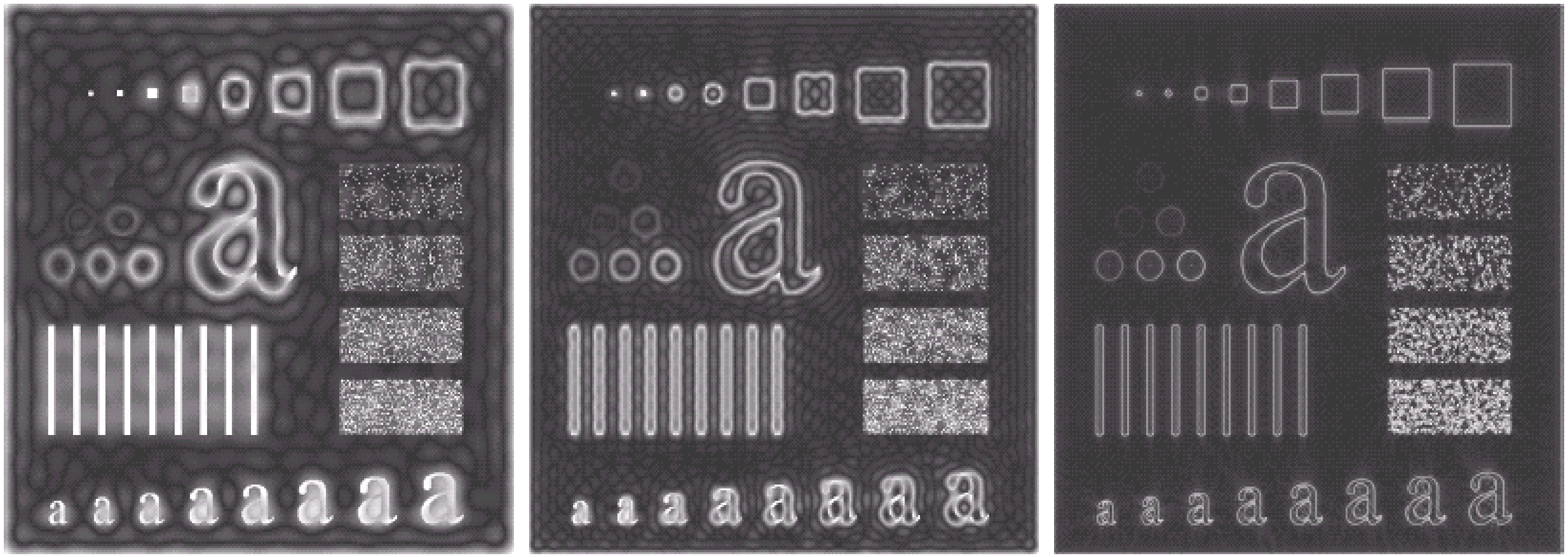


FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

图4.24 D0 = 15、30和80.时，对图4.11(a)图像进行理想高通滤波的结果在(a)和(b)中，铃声的问题是相当明显的。

Sharpening High-pass Filters

锐化高通滤波器 Ruì huà gāotōng lǜbō qì

- $H_{hp}(u,v)=1-H_{lp}(u,v)$
- Ideal:

  理想的:

$$H(u,v)=\begin{cases}1 & D(u,v)>D_0\\ 0 & D(u,v)\le D_0\end{cases}$$

- Butterworth:

  巴特沃思:

$$|H(u,v)|^2=\frac{1}{1+\left[\dfrac{D_0}{D(u,v)}\right]^{2n}}$$

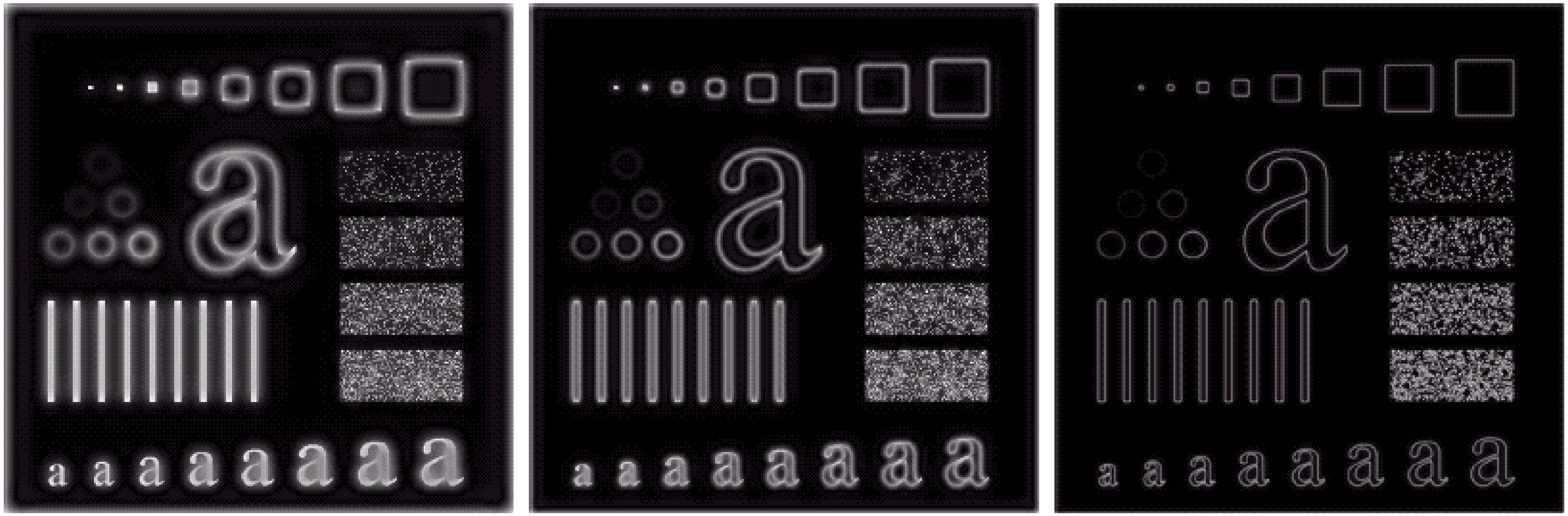
- Gaussian:

  高斯:

$$H(u,v)=1-e^{-D^2(u,v)/2D_0^2}$$

Butterworth High-pass Filtering

巴特沃斯高通滤波 Bātè wò sī gāotōng lǜbō



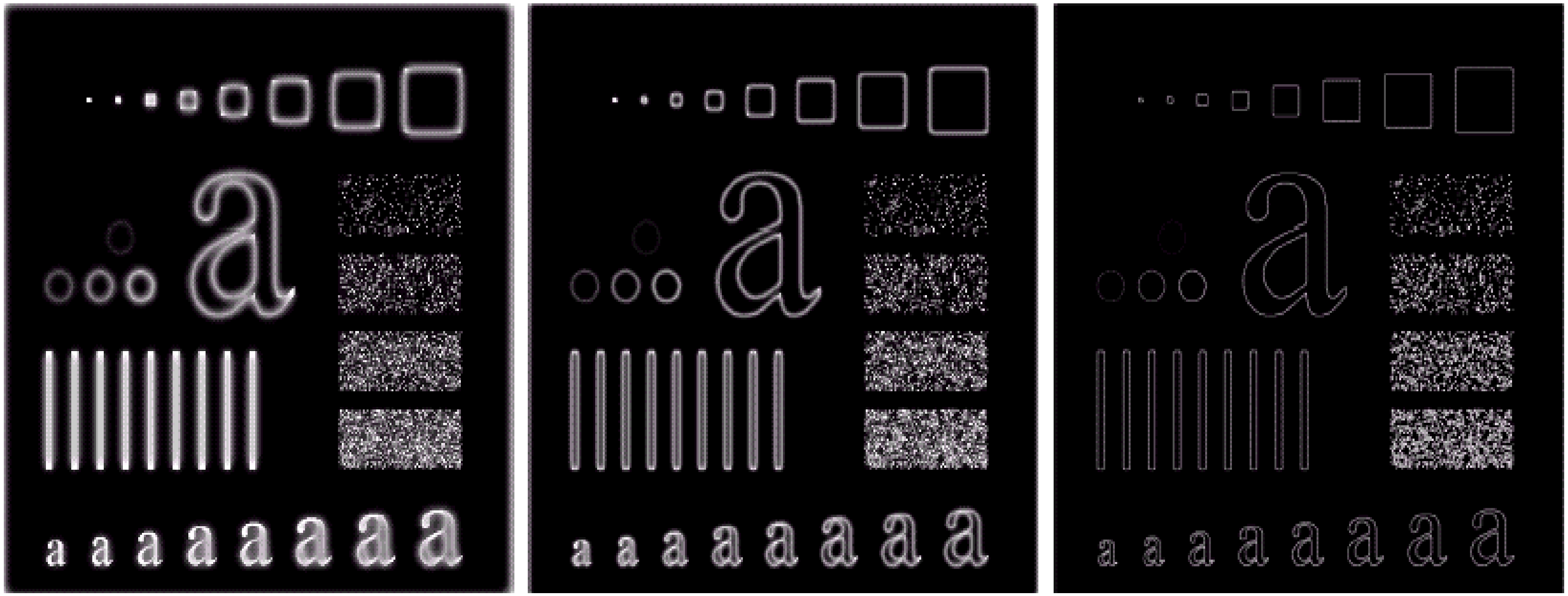
a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

图4.25使用阶2、D0= 15、30和80的BHPF对图4.11(a)中的图像进行高通滤波的结果分别。这些结果比ILPF得到的结果平滑得多。

Gaussian High-pass Filtering

高斯高通滤波 Gāosī gāotōng lǜbō



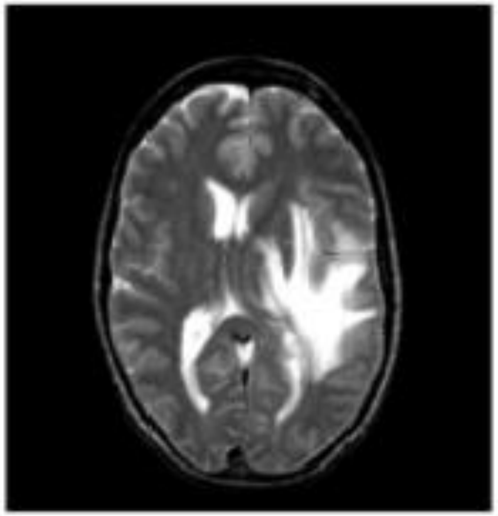
a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

图4.26使用阶2、D0=15、30和80的GHPF对图4.11(a)图像进行高通滤波的结果与图4.24和4.25比较。

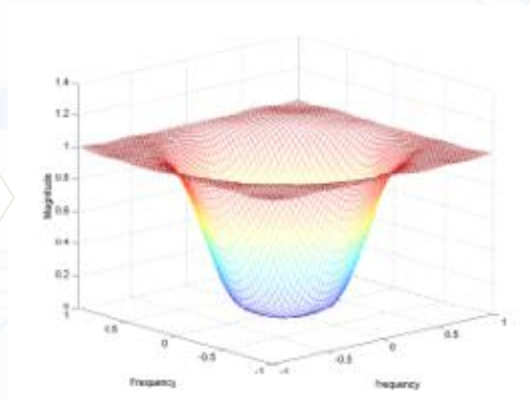
Gaussian High-pass Filtering

高斯高通滤波 Gāosī gāotōng lǜbō



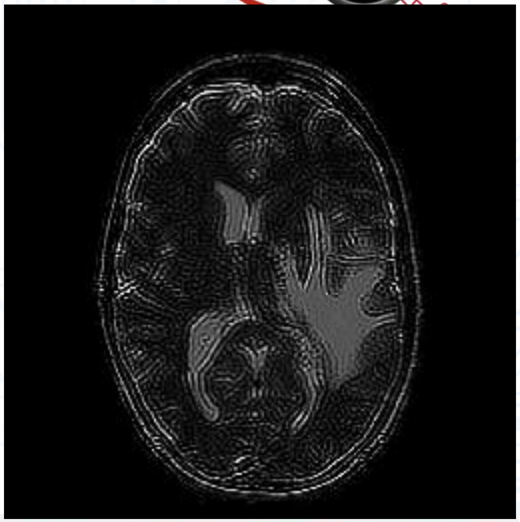
Original image

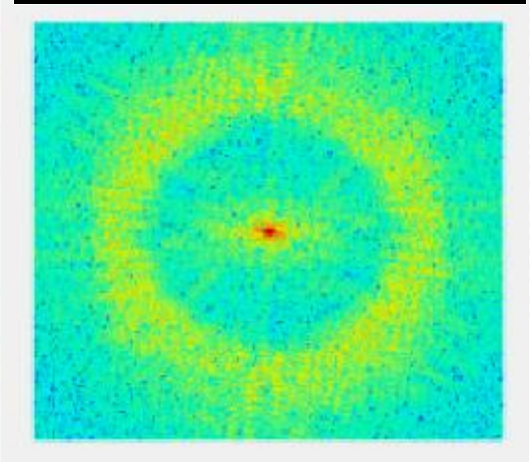
原图 Yuán tú



Gaussian filter H(u,v)

高斯滤波器 H(u,v)



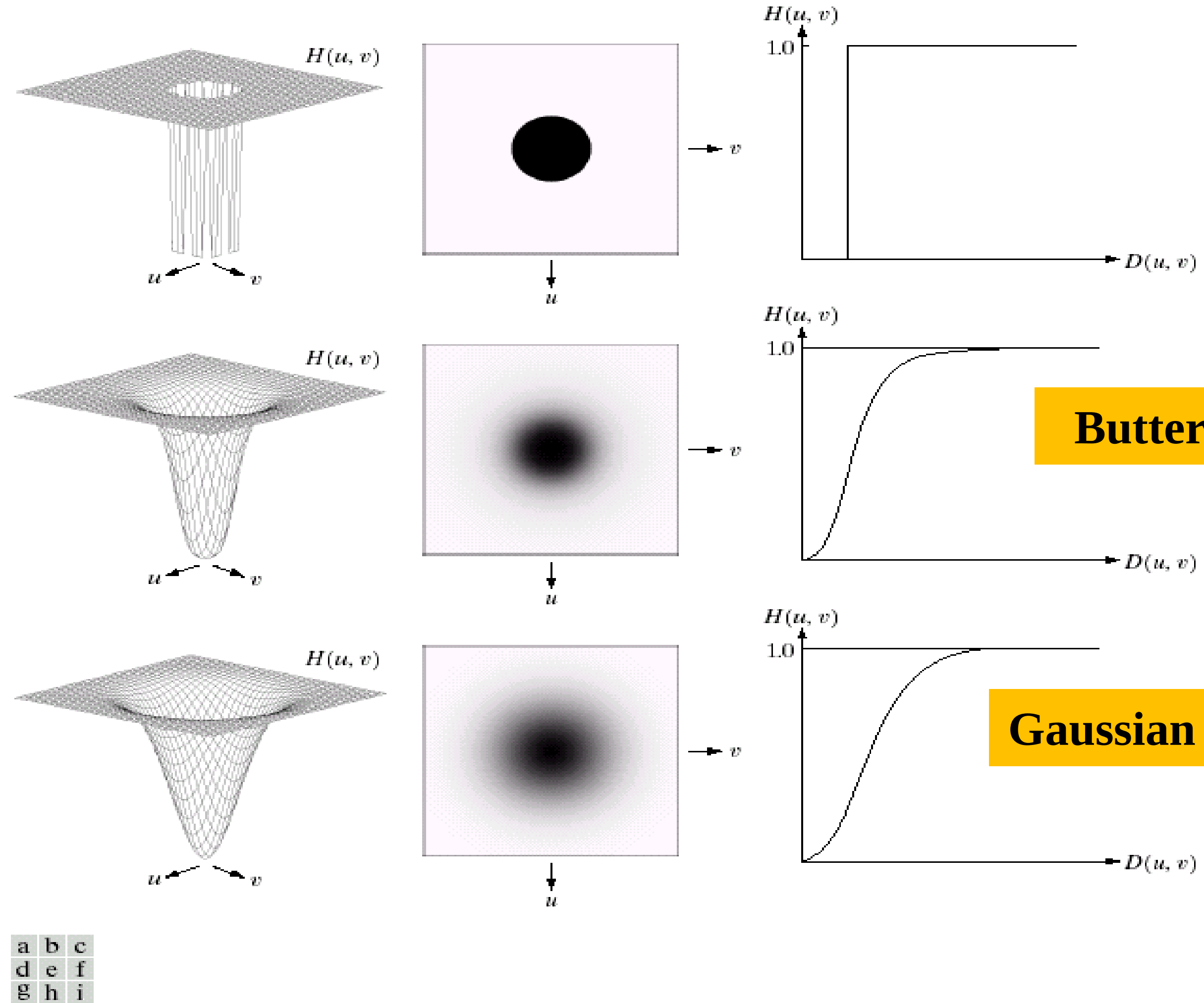


Filtered image and its FT

过滤后的图像及其 FT

Gaussian high pass filter Filters

高斯高通滤波器 Filters Gāosī gāotōng lǜbō qì Filters



Butter worth

黄油价值

Gaussian high pass

高斯高通

FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

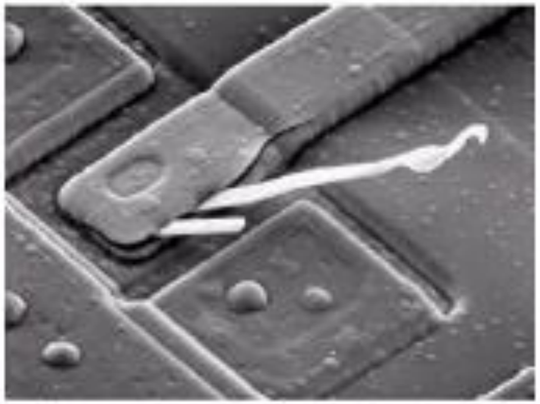
图4.22顶部一行:一个典型的理想高通滤波器的透视图、图像表示和截面。中行和底部行:对于典型的巴特沃斯和高斯高通滤波器相同的序列。

Example of Gaussian LPF and HPF

高斯 LPF 和 HPF 示例

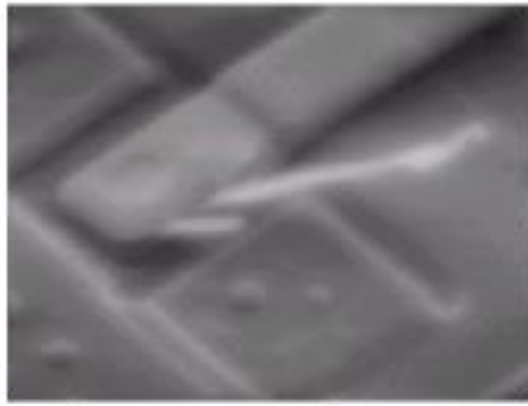
原图 Yuán tú

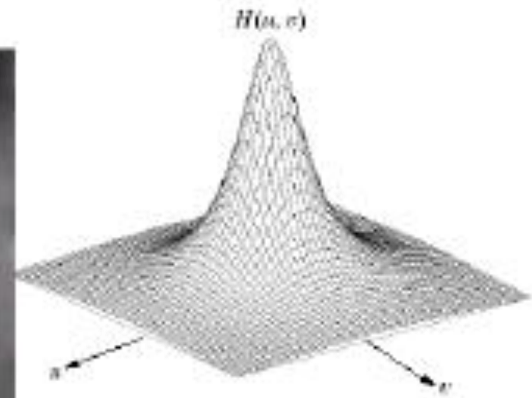
Original Image



应用LPF Yìngyòng LPF

LPF applied



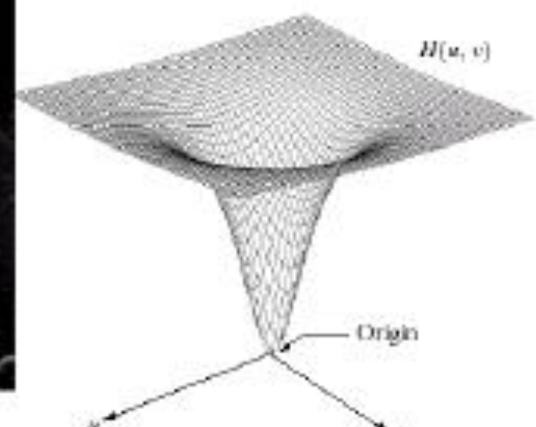


DFT applied

应用DFT

HPF applied

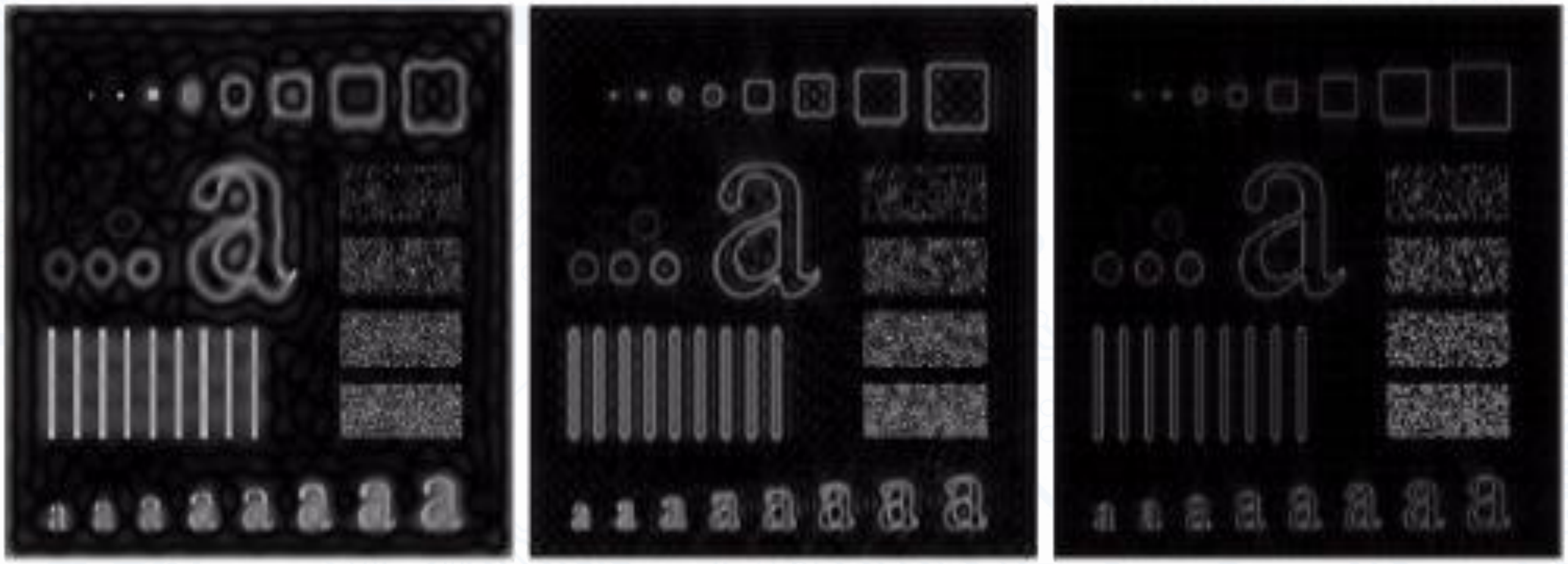
应用HPF



Results Of high Pass Filters (Examples)

高通滤波器的结果（示例）

高通滤波器D0=15	高通滤波器D0=30	高通滤波器D0=80
high Pass Filters D0=15	**high Pass Filters D0=30**	**high Pass Filters D0=80**



a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

图4.24 D0 = 15、30、80时，对图4.11(a)图像进行理想高通滤波的结果振铃的问题在(a)和(b)中相当明显。

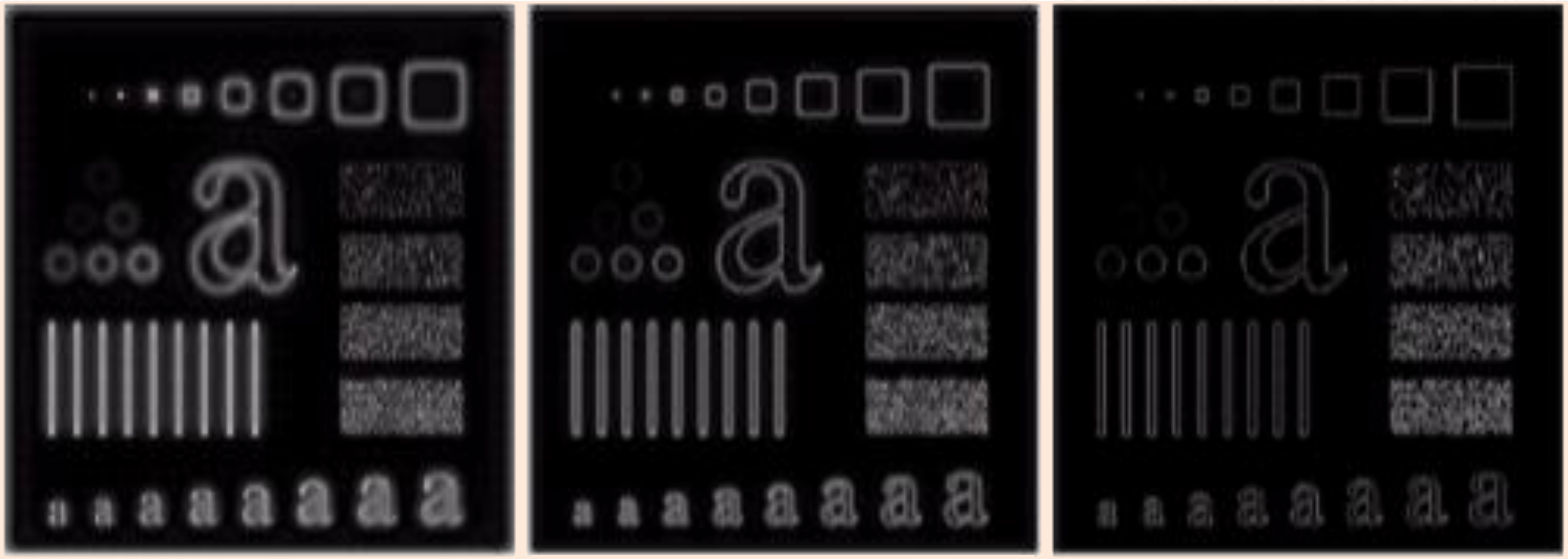
Results Of Butterworth high Pass Filters

(Examples)

巴特沃斯高通滤波的结果（示例）

Butterworth high Pass Filters D0=15

Butterworth high Pass Filters D0=80



Butterworth high Pass Filters D0=30

a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

图4.25使用D0=15、30和80的二阶BHPF对图4.11(a)中的图像进行高通滤波的结果这些结果比ILPE得到的结果平滑得多

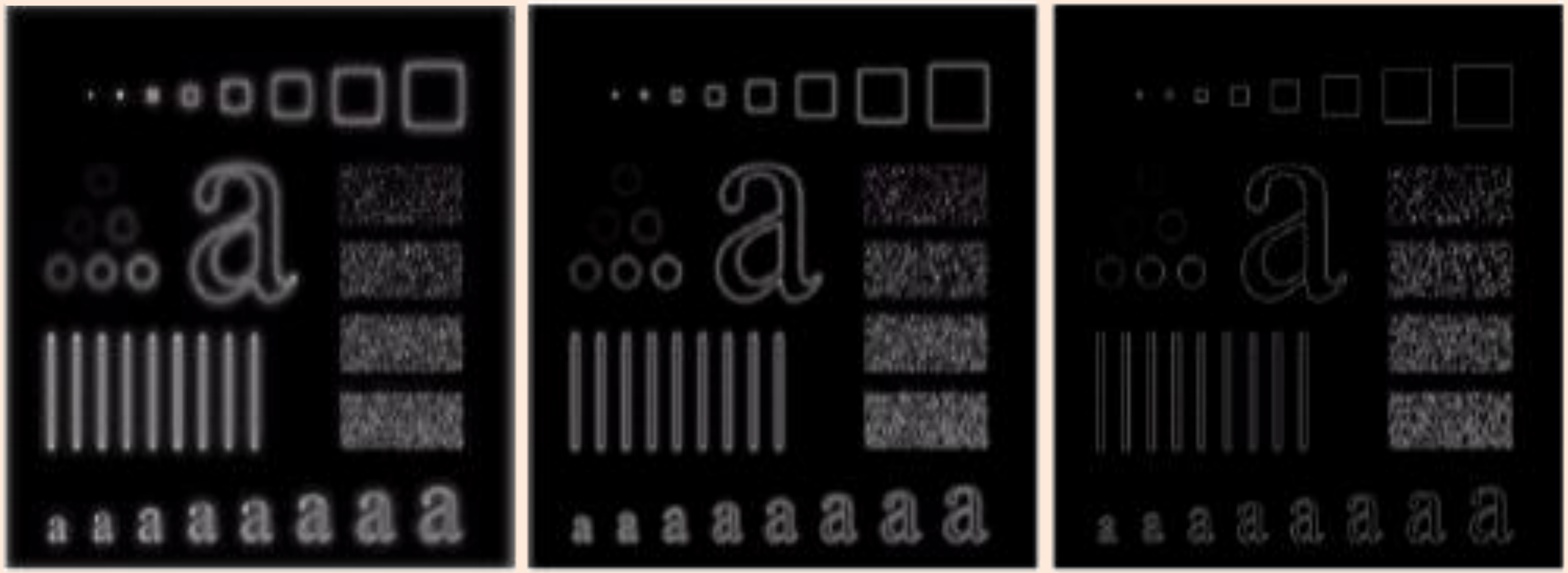
Results Of Gaussian high Pass Filters

(Examples)

高斯高通滤波器的结果（示例）

Gaussian high Pass Filters D0=15

Gaussian high Pass Filters D0=80



Gaussian high Pass Filters D0=30

a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

图4.26使用GHPF wiln D0 = 15,30和80对图4.11(a)图像进行高通滤波的结果。与图4.24和4.25比较

Band Pass Filtering

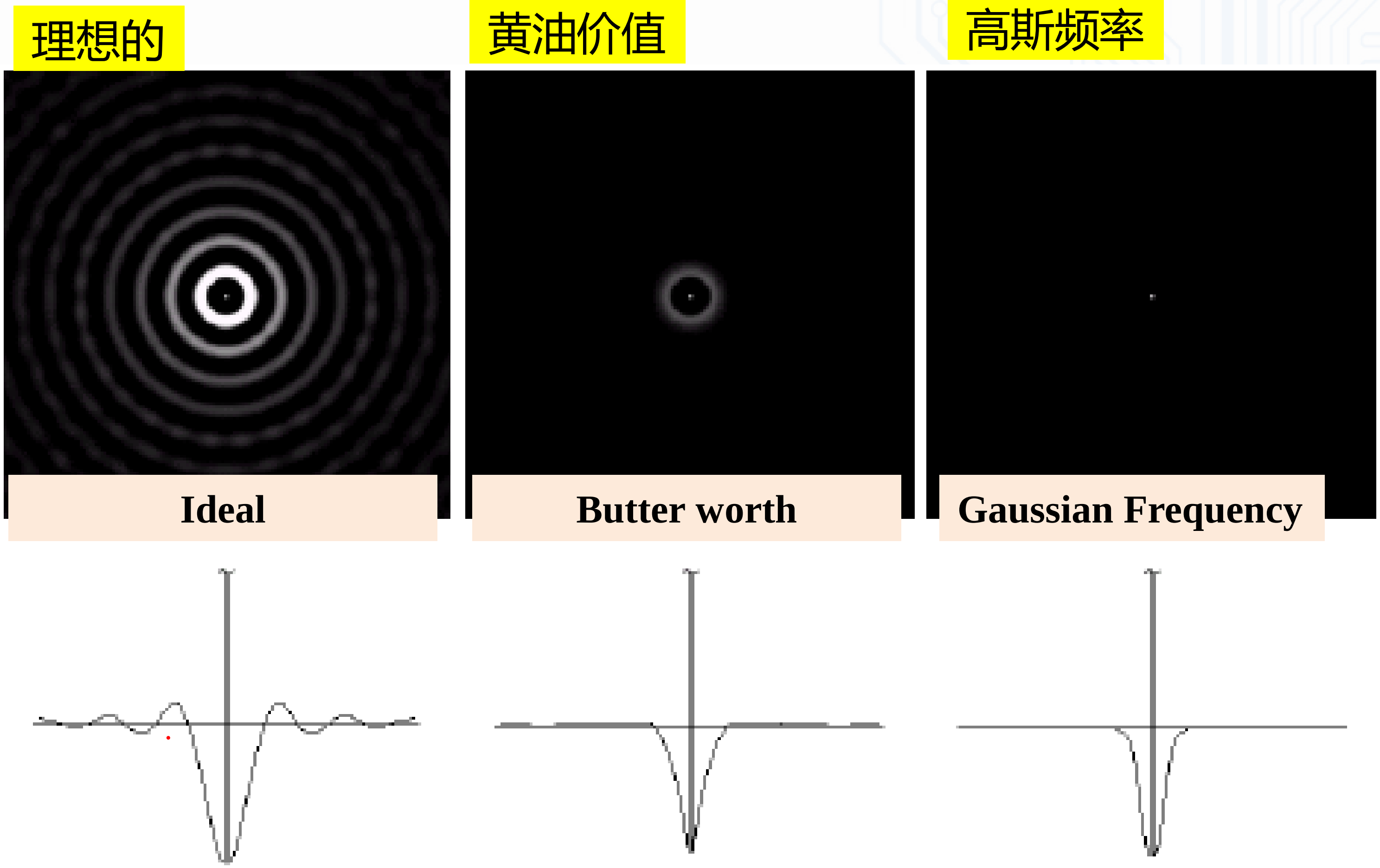
带通滤波 Dài tōng lǜbō

- A bandpass attenuates very low and very high frequencies, but retains a middle range band of frequencies.
- Bandpass filtering can be used to enhance edges (suppressing low frequencies) while reducing the noise at the same time (attenuating high frequencies).
- **Bandpass filters are a combination of both lowpass and highpass** filters. They attenuate all frequencies smaller than a frequency Do and higher than a frequency D1 , while the frequencies between the two cut-offs remain in the resulting output image.

- 带通衰减非常低和非常高的频率，但保留了中频带。
- 带通滤波可用于增强边缘（抑制低频），同时降低噪声（衰减高频）。
- 带通滤波器是低通和高通滤波器的组合。
- 它们衰减所有小于频率 Do 且大于频率 D1 的频率，而两个截止频率之间的频率保留在结果输出图像中。

Spatial Representations

空间表示 Kōngjiān biǎoshì



a b c

图4.23典型(a)理想的空间表征。(b) 巴特沃斯，和(e)高斯频域高通滤波器和相应的灰度轮廓

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

Laplacian in Frequency Domain

频域中的拉普拉斯算子 Pín yù zhōng de lā pǔ lā sī suàn zi

It was shown in the properties that

$$\pounds[\frac{\sigma^2 f(x,y)}{\sigma x^2}+\frac{\sigma^2 f(x,y)}{\sigma y^2}] = -(u^2+v^2)F(u,v)$$

The above equation shows that Laplacian in spatial domain is equal to applying the Fourier domain filter:

由上式可知空间域拉普拉斯等于应用傅里叶域滤波器:

$$H(u,v) = -(u^2+v^2).$$

which is centered and given by:

$$H(u,v) = -[(u-M/2)^2+(v-N/2)^2].$$

therefore

$$\nabla^2 f(x,y) \Leftrightarrow -[(u-M/2)^2+(v-N/2)^2]F(u,v).$$

Laplacian in Frequency Domain

频域中的拉普拉斯算子 Pín yù zhōng de lā pǔ lā sī suàn zi

$$£[\frac{σ^2 f(x,y)}{σx^2} + \frac{σ^2 f(x,y)}{σy^2}] = -(u^2+v^2)F(u,v)$$

$$H_1(u,v) = -(u^2+v^2)$$

Frequency domain

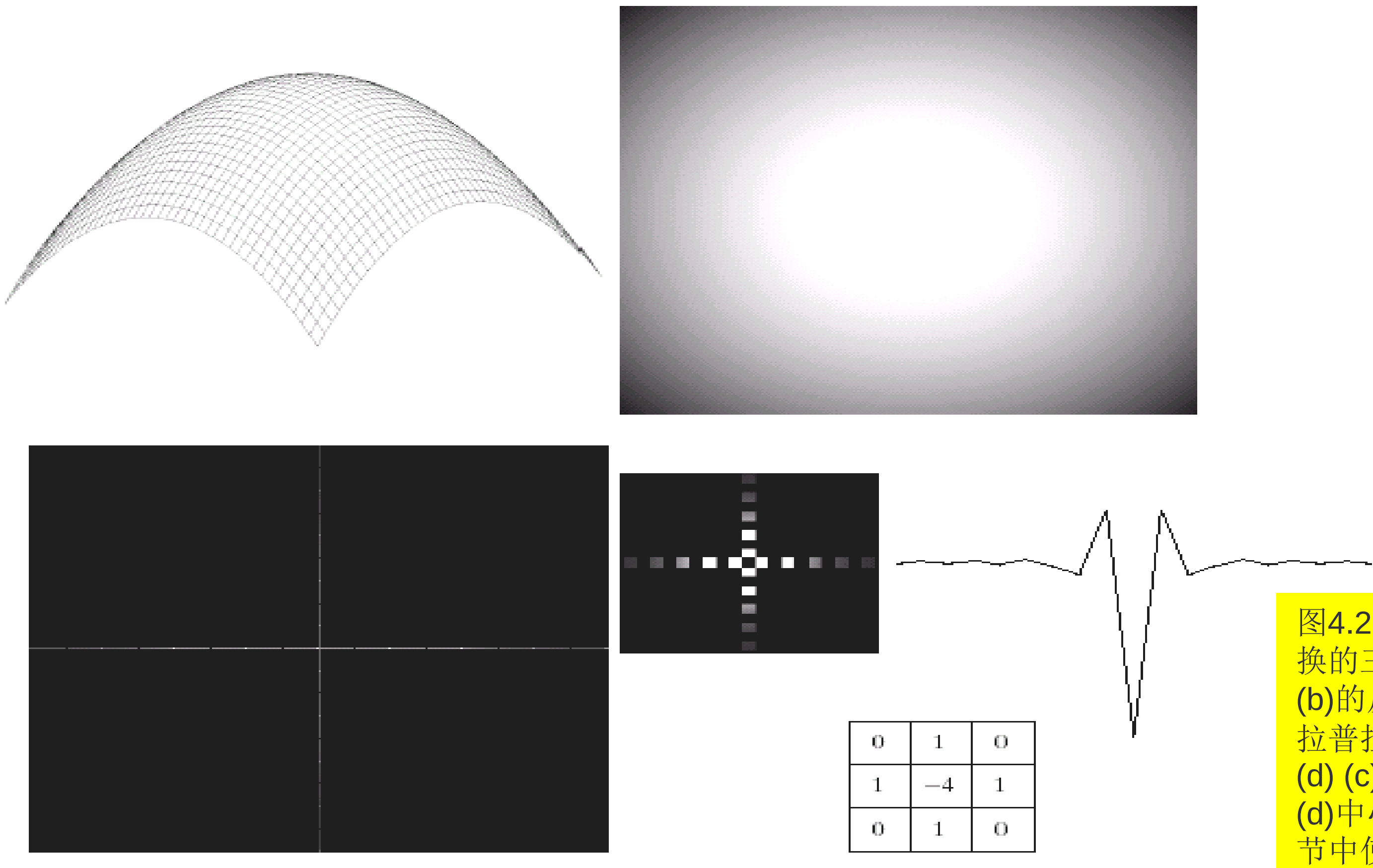
Spatial domain

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Laplacian operator

Laplacian in Frequency Domain

频域中的拉普拉斯算子 Pín yù zhōng de lā pǔ lā sī suàn zi



0	1	0
1	−4	1
0	1	0

图4.27 (a)频域拉普拉斯变换的三维图。(b) (a).(c)从(b)的反DFT得到的空间域拉普拉斯图的图像表示。(d) (c)原点的放大截面。(e) (d)中心灰度轮廓。(f) 3.7节中使用的拉普拉斯掩模。

a b
c d e
f

FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

Unsharp Masking, High Boost Filtering

非锐化掩蔽、高增强滤波

Unsharp Masking in Spatial domain is given by

$$f_{hp}(x,y) = f(x,y) - f_{lp}(x,y)$$

High boost filtering is given by

$$f_{hp}(x,y) = Af(x,y) - f_{lp}(x,y), \qquad A \geq 1$$

$$\Rightarrow f_{hp}(x,y) = (A-1)f(x,y) + f(x,y) - f_{lp}(x,y),$$

In frequency domain the unsharp masking is implemented as

$$H_{hp}(u,v) = 1 - H_{lp}(u,v).$$

在频域，非锐化掩蔽被实现为

And high boost filtering is implemented as

实现了高升压滤波

$$H_{hp}(u,v) = (A-1) + H_{hp}(u,v).$$

Example of Modified High Pass Filtering

修改后的高通滤波示例 Xiūgǎi hòu de gāotōng lǜbō shìlì

a b
c d

FIGURE 4.29 Same as Fig. 3.43, but using frequency domain filtering. (a) Input image. (b) Laplacian of (a). (c) Image obtained using Eq. (4.4-17) with $A = 2$. (d) Same as (c), but with $A = 2.7$. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

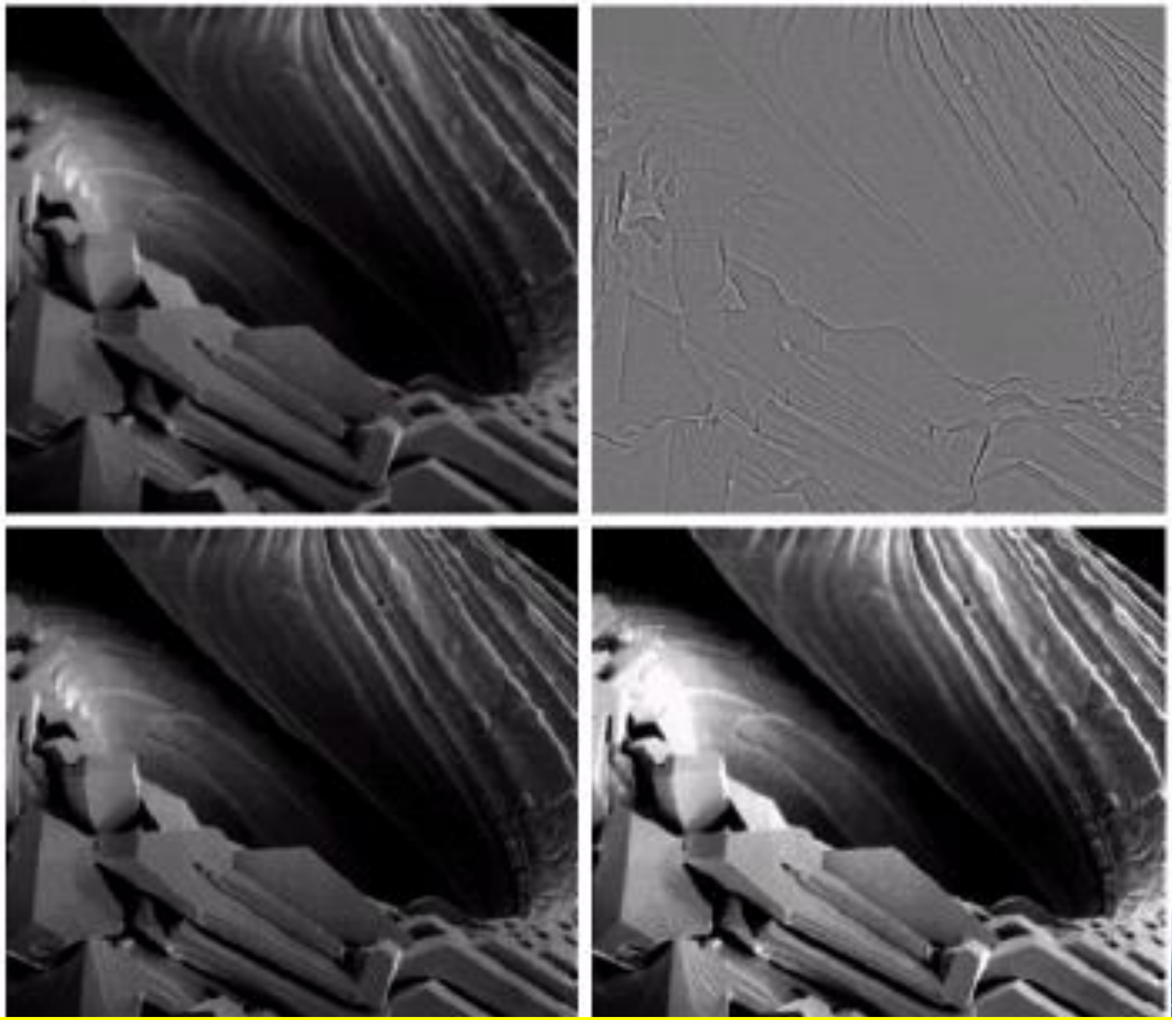


图4.29与图3.43相同，但采用频域滤波。(a)输入图像(b) (a)的拉普拉斯算子。(c) E.q (4.4-17)， A=2，(d)与(c)相同，但A= 2.7。(原图片由Michael Shaffer先生提供。俄勒冈大学地质科学系)

Subtract Laplacian from the Original Image to Enhance It

从原始图像中减去拉普拉斯算子以增强它

增强图像 enhanced image

原图 Original image

拉普拉斯算子 Laplacian output

空间域 Kōngjiān yù

Spatial domain

$$g(x,y) = f(x,y) - \tilde{N}^2 f(x,y)$$

频域 Pín yù

Frequency domain

$$G(u,v) = F(u,v) + (u^2 + v^2)F(u,v)$$

new operator

新运营商
Xīn yùnyíng shāng

$$H_2(u,v) = 1 + (u^2 + v^2) = 1 - H_1(u,v)$$

拉普拉斯算子 Laplacian

Laplacian filtering example

拉普拉斯滤波示例

a b
c d

FIGURE 4.28
(a) Image of the North Pole of the moon.
(b) Laplacian filtered image.
(c) Laplacian image scaled.
(d) Image enhanced by using Eq. (4.4-12).
(Original image courtesy of NASA.)

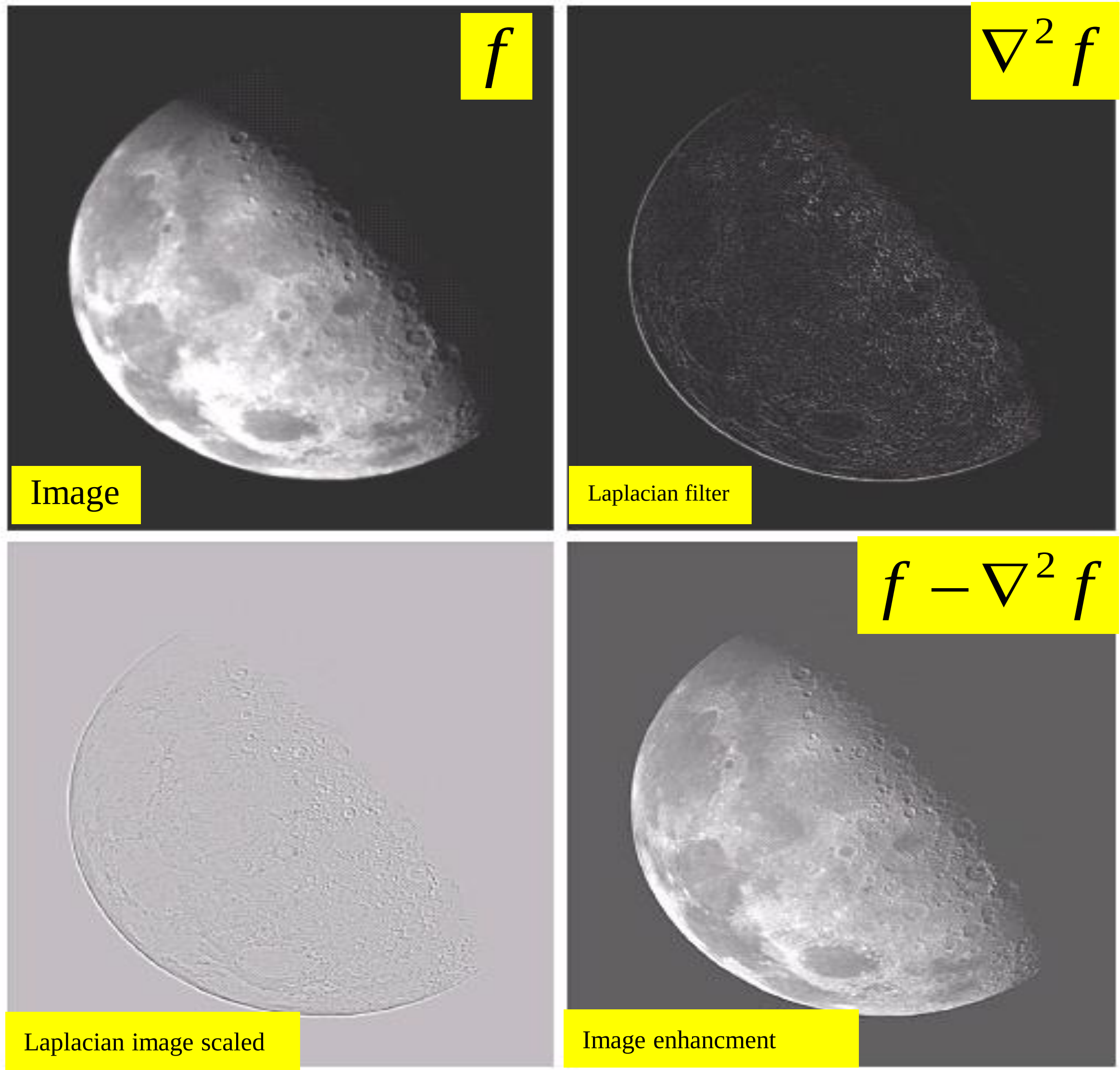


图4.28 (a)月球北极图像。(b)拉普拉斯滤波图像。(c)拉普拉斯图像缩放。(d) Eq.(4.4-12)增强图像。(原图像由NASA提供)

Unsharp Masking, High-boost Filtering

非锐化掩蔽、高升压滤波 Fēi ruì huà yǎnbì, gāo shēng yā lǜbō

- **Unsharp masking:**

锐化遮罩： Ruì huà zhē zhào:

$$f_{hp}(x,y)=f(x,y)-f_{lp}(x,y)$$

$$H_{hp}(u,v)=1-H_{lp}(u,v)$$

- **High-boost filtering:**

高升压过滤： Gāo shēng yā guòlǜ:

One more parameter to adjust the enhancement

还有一个参数用于调整增强

$$f_{hb}(x,y)=Af(x,y)-f_{lp}(x,y)$$

$$f_{hb}(x,y)=(A-1)f(x,y)+f_{hp}(x,y)$$

$$H_{hb}(u,v)=(A-1)+H_{hp}(u,v)$$

Laplacian filtering example

拉普拉斯滤波示例

a b
c d

FIGURE 4.29 Same as Fig. 3.43, but using frequency domain filtering. (a) Input image. (b) Laplacian of (a). (c) Image obtained using Eq. (4.4-17) with $A = 2$. (d) Same as (c), but with $A = 2.7$. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

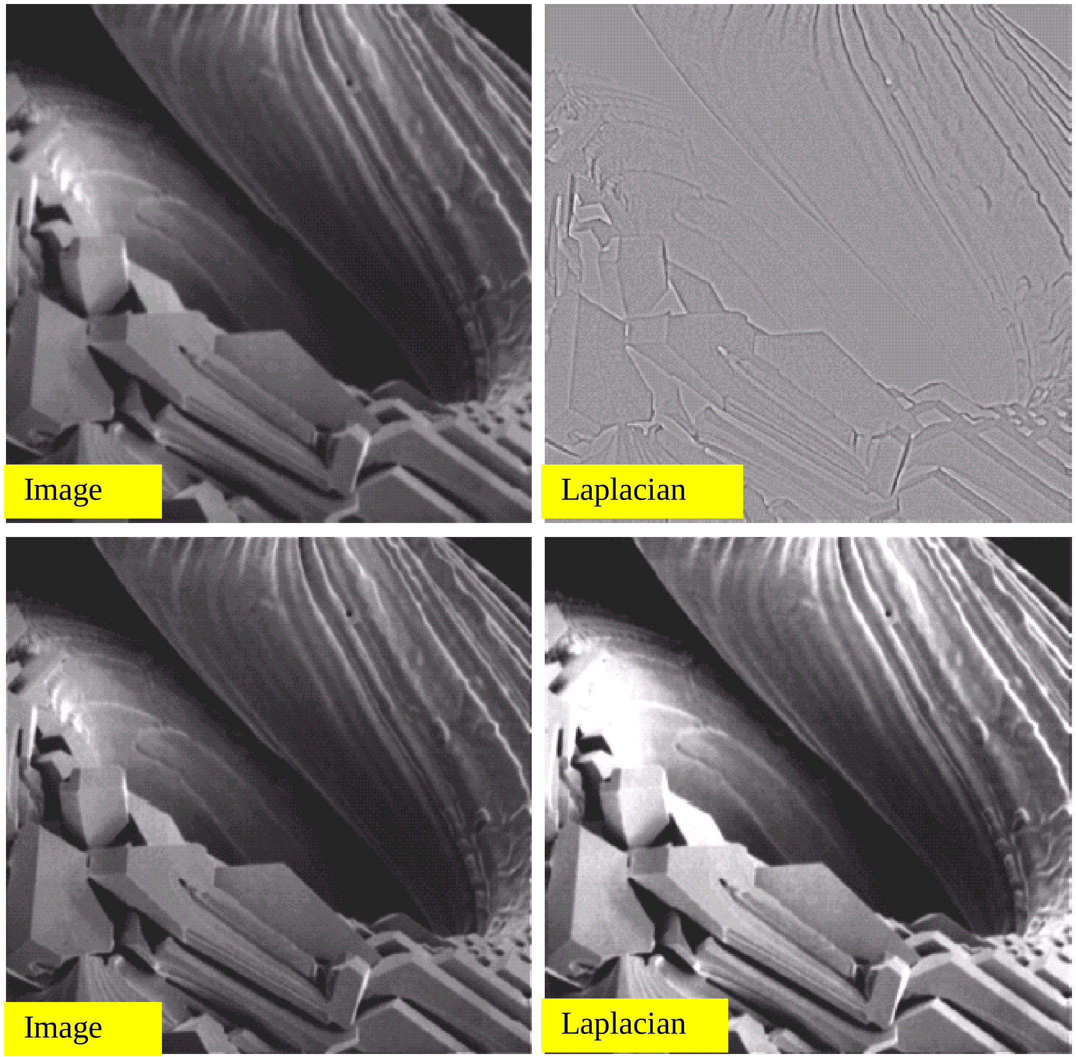


图4.29与图3.43相同。而是使用频域滤波。(a)输入图像(b) (a)的拉普拉斯算子(c) Eq.(4.4-17)得到的图像(d)与(c)相同，但a =2.7。(原始图片由俄勒冈大学地质科学系Michael Shaffer先生提供,尤金。)

Homomorphic filtering

同态过滤 Tóng tài guòlǜ

亮度范围压缩

对比增强

- Goal: simultaneous **Brightness range compression** and **Contrast enhancement**
- An image, f(x,y) is expressed in terms of its illumination and reflectance components as

照明和反射组件为

$$f(x,y)=i(x,y)r(x,y)$$

illumination and reflectance components model

照明和反射组件模型

illumination

reflectance

亮度 反射系数

- It is impossible to operate separately on the frequency components i(x,y) and r(x,y) since Fourier transform of product of two functions is not separable.
  - Taking logarithm of both sides of Equ(1), we have

$$\Im\left[f(x,y)\right] \neq \Im\left[i(x,y)\right]\Im\left[r(x,y)\right]$$

  - Then taking Fourier transform, we have

$$z(x,y)=\ln\left[f(x,y)\right]=\ln\left[i(x,y)\right]+\ln\left[r(x,y)\right]$$

and $$\Im\left[z(x,y)\right]=\Im\left[\ln\left[f(x,y)\right]\right]=\Im\left[\ln\left[i(x,y)\right]\right]+\Im\left[\ln\left[r(x,y)\right]\right]$$

Homomorphic filtering

亮度范围压缩　同态过滤 Tóng tài guòlǜ　对比增强

- 目标:同时压缩亮度范围和增强对比度
- 图像f(x,y)用它的光照和反射分量表示为

$$f(x,y)=i(x,y)r(x,y)$$

照明和反射组件模型

亮度　反射系数

- 对频率分量i(x,y)和r(x,y)分别进行操作是不可能的，因为两个函数乘积的傅里叶变换是不可分离的。.
  - 对等式(1)两边取对数，得到

$$\Im[f(x,y)] \neq \Im[i(x,y)]\Im[r(x,y)]$$

  - 然后进行傅里叶变换

$$z(x,y) = \ln[f(x,y)] = \ln[i(x,y)] + \ln[r(x,y)]$$

and $\Im[z(x,y)] = \Im[\ln[f(x,y)]] = \Im[\ln[i(x,y)]] + \Im[\ln[r(x,y)]]$

Homomorphic filtering

同态过滤 Tóng tài guòlǜ

Which can be written as

$$Z(u,v) = F_i(u,v) + F_r(u,v).$$

$F_i(u,v) \Rightarrow$ Fourier transform of $\ln[i(x,y)]$ and

$F_r(u,v) \Rightarrow$ Fourier transform of $\ln[r(x,y)]$.

Applying a filter $H(u,v)$ on $Z(u,v)$

$$S(u,v) = Z(u,v)H(u,v) = F_i(u,v)H(u,v) + F_r(u,v)H(u,v)$$

The spatial domain result can be obtained as

$$s(x,y) = \Im^{-1}[Z(u,v)] = \Im^{-1}[F_i(u,v)H(u,v)] + \Im^{-1}[F_r(u,v)H(u,v)]$$

$$\boxed{s(x,y) = i'(x,y) + r'(x,y).}$$

where $i_0(x,y)$ and $r\text{-}_0(x,y)$ are the illumination and reflectance components of the output image.

其中i0(x,y)和r-0(x,y)为输出图像的照明和反射分量。

where $i'(x,y) = \Im^{-1}[F_i(u,v)H(u,v)]$, $\quad r'(x,y) = \Im^{-1}[F_r(u,v)H(u,v)]$

This approach is termed as ***homomorphic filtering.***

Homomorphic Filtering and

An image formation model

- In some images, the quality of the image has reduced because of non-uniform illumination.
- Homomorphic filtering can be used to perform illumination correction.

$$f(x,y)=i(x,y)\cdot r(x,y)$$

- The above equation cannot be used directly in order to operate separately on the frequency components of illumination and reflectance.
- We can view an image f(x,y) as a product of two components:

$$f(x,y)=i(x,y)\cdot r(x,y)$$
$$0<i(x,y)<\infty$$
$$0<r(x,y)<1$$

  – i(x,y): illumination. It is determined by the illumination source. 照明。 它是由照明光源决定的
  – r(x,y): reflectance (or transmissivity). It is determined by the characteristics of imaged objects.

反射(或透射率)。 它是由成像物体的特性决定的

Homomorphic Filtering and

An image formation model

- 在某些图像中，由于光照不均，图像质量降低。
- 同态滤波可用来进行光照校正。

$$f(x,y)=i(x,y)\cdot r(x,y)$$

- 上式不能直接使用，要分别对光照和反射率的频率分量进行运算。.
- 我们可以把图像f(x.y)看作是两个分量的乘积:

$$f(x,y)=i(x,y)\cdot r(x,y)$$

$$0<i(x,y)<\infty$$

$$0<r(x,y)<1$$

– i(x,y):照明。 它是由照明光源决定的 .
– r(x,y):反射(或透射率)。 它是由成像物体的特性决定的

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Homomorphic filtering

同态过滤 Tóng tài guòlǜ

$i(x,y)$: characterized by slow spatial variations(Low frequency components)

$r(x,y)$: tend to vary abruptly, particularly at the junctions of dissimilar objects(High- frequency components)

趋向于突然变化，特别是在不同物体的连接处(高频成分)

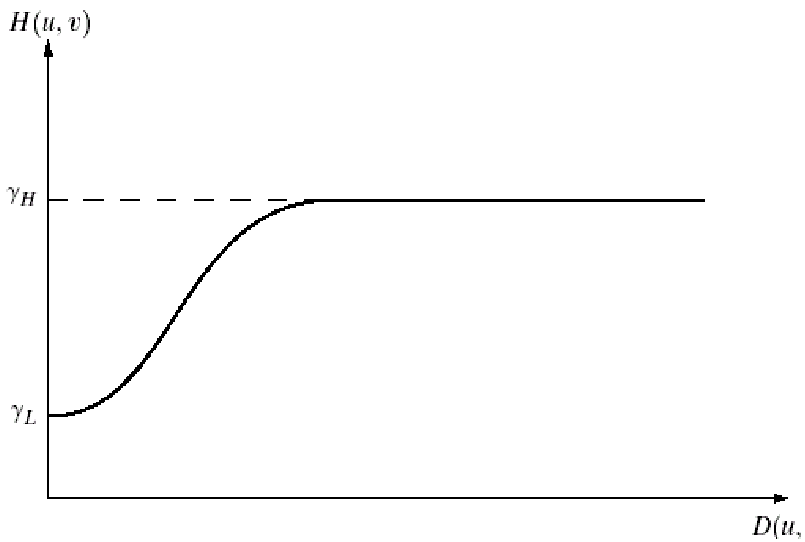

FIGURE 4.32 Cross section of a circularly symmetric filter function. $D(u, v)$ is the distance from the origin of the centered transform.

图4.32圆对称滤波器函数的截面。D(u,v)是到圆心变换原点的距离。

A filter example which suppresses low frequency components (illumination) and enhances high frequency component (reflectance)

趋向于突然变化，特别是在不同物体的连接处(高频成分)

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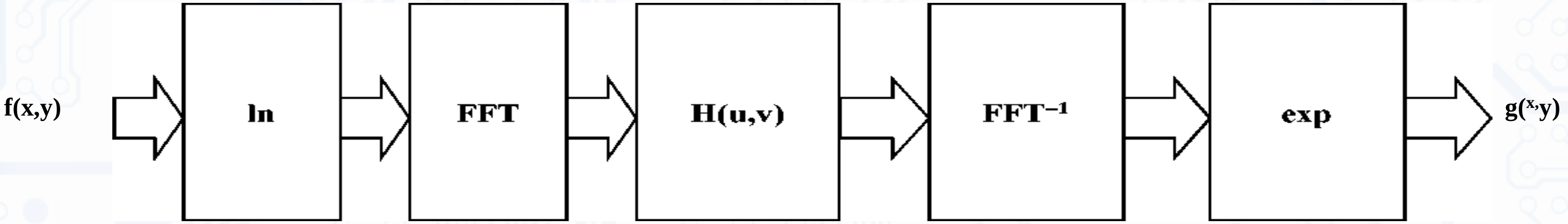
Homomorphic filtering

同态过滤 Tóng tài guòlǜ

- The homomorphic filtering is used to achieve contrast stretching and dynamic range compression simultaneously since the contrast of an image depends on the reflectance component and the dynamic range, on the illumination component.



Homomorphic Filtering

同态滤波用于同时实现对比度拉伸和动态范围压缩，因为图像的对比度取决于反射分量和动态范围，取决于照明分量。

Homomorphic filtering

同态过滤 Tóng tài guòlǜ

The enhanced image is given by

$$g(x,y) = \exp[s(x,y)] = \exp[i'(x,y)] + \exp[r'(x,y)]$$

$$\boxed{g(x,y) = i_0(x,y)\,r_0(x,y).}$$

where $i_0(x,y) = \exp[i'(x,y)]$ and $r_0(x,y) = \exp[r'(x,y)]$.

The Homomorphic filtering steps are given as

Homomorphic filtering

同态过滤 Tóng tài guòlǜ

ln : $z(x,y)=\ln f(x,y)=\ln i(x,y)+\ln r(x,y)$

DFT : $Z(u,v)=F_i(u,v)+F_r(u,v)$

H(u,v) : $S(u,v)=H(u,v)Z(u,v)$

$(\mathrm{DFT})^{-1}$: $s(x,y)=i'(x,y)+r'(x,y)$

exp : $g(x,y)=e^{s(x,y)}=i_0(x,y)r_0(x,y)$

图4.31图像增强的同态滤波方法

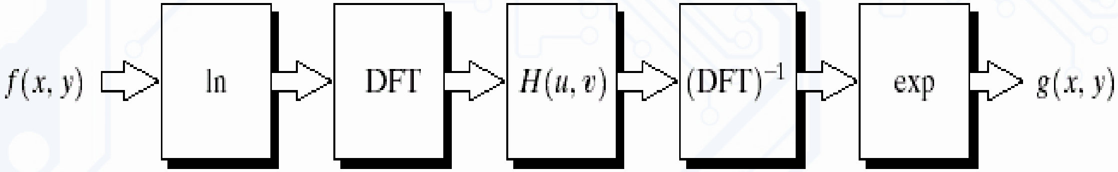


FIGURE 4.31 Homomorphic filtering approach for image enhancement.

Homomorphic filtering

同态过滤 Tóng tài guòlǜ

- By separating the illumination and reflectance components, homomorphic filter can then operate on them separately.
- Illumination component of an image generally has slow variations, while the reflectance component vary abruptly.
- By removing the low frequencies (highpass filtering) the effects of illumination can be removed .

通过分离照明和反射分量，同态滤波器可以分别对它们进行操作。图像的照明分量通常变化缓慢，而反射分量变化突然。通过去除低频（高通滤波），可以去除照明的影响。

Homomorphic Filtering: Example 1

同态过滤 Tóng tài guòlǜ

a b

FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)

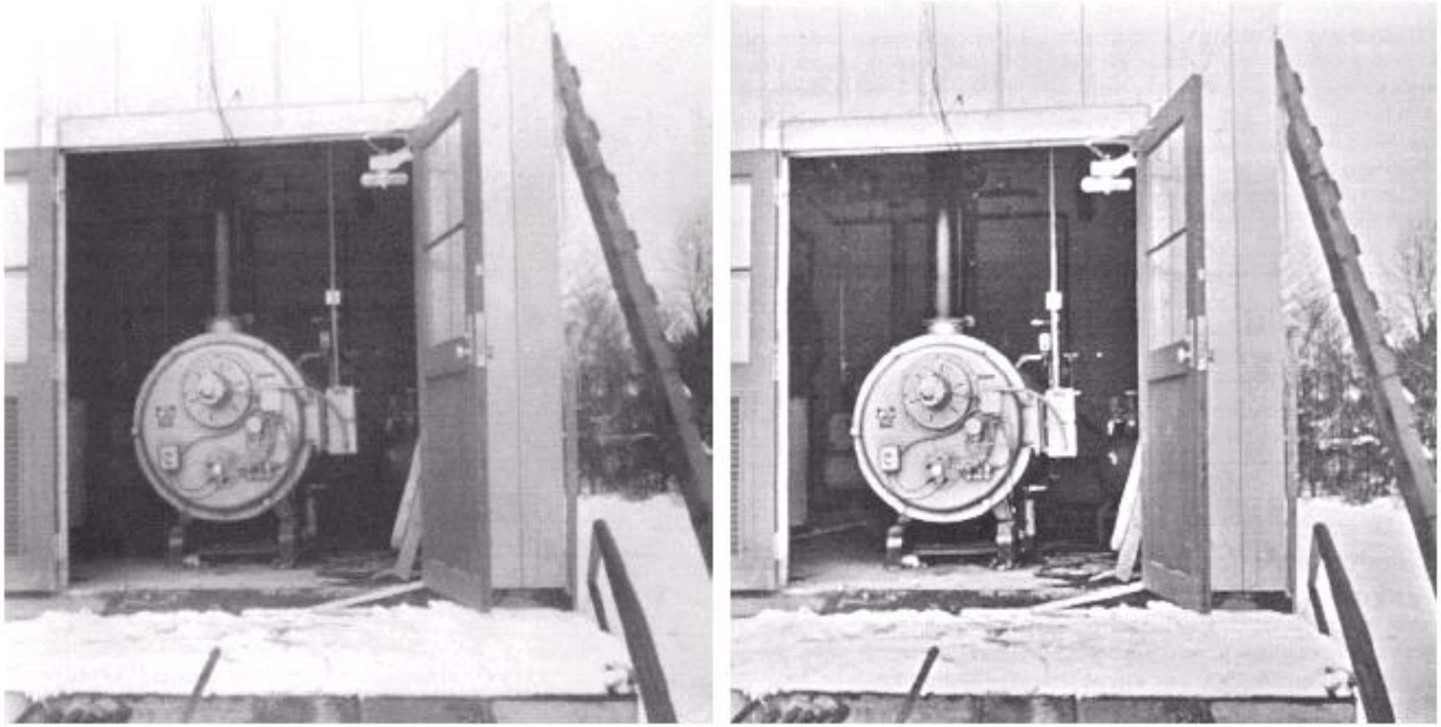
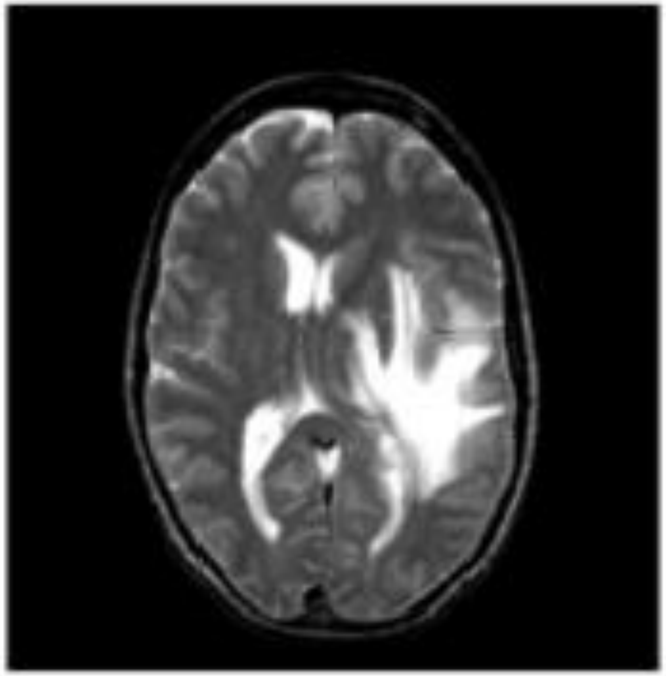
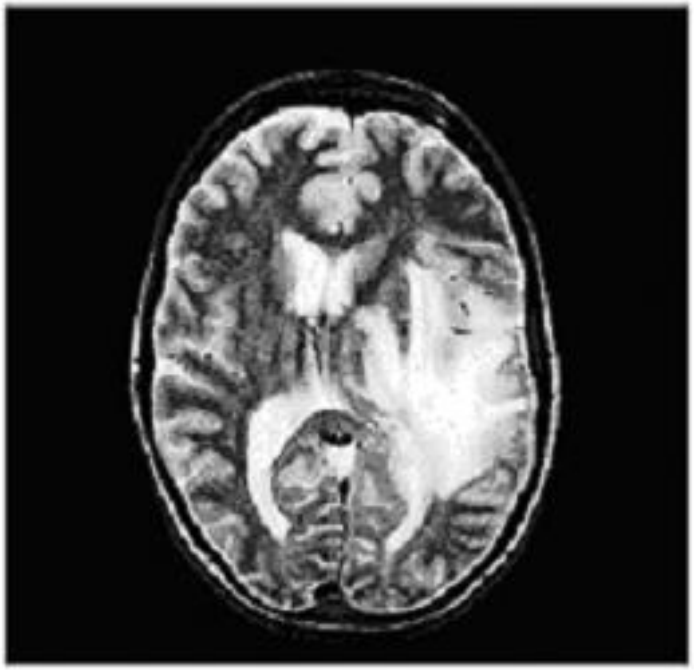


图4.33 (a)原始图像。 (b)同态滤波处理后的图像(注意遮挡内部的细节)。

Homomorphic Filtering: Example 2



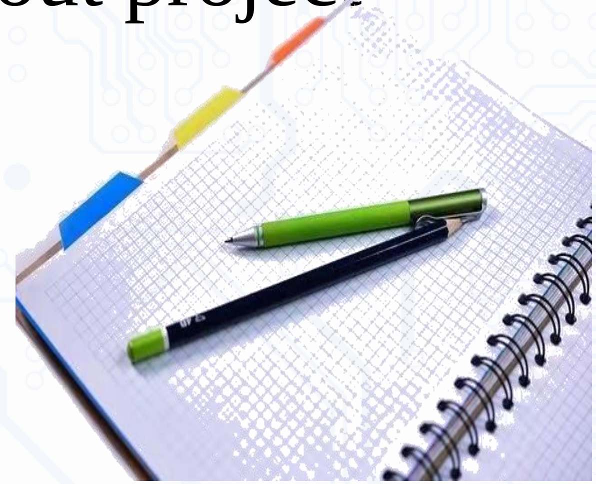
Original image



Filtered image

Student Task : DIP

- Select Project from the link and list and run it in matlab make the report about project



@dip学生：亲爱的学生，关于你的时期：你应该在matlab中使用代码来在图像处理中找到一个主题，并通过演示文稿中的ppt，但我应该在此之前批准主题。所有学生都被邀请确定您的主题并在10月30日之前将其发送给我。请注意，这次不会扩大。该计划是这样，学生将在可能的课程中讨论他们的主题和显示代码，学生可以用中文说话。学生演讲时间将于11月开始，每个学生都有10分钟的会话时间

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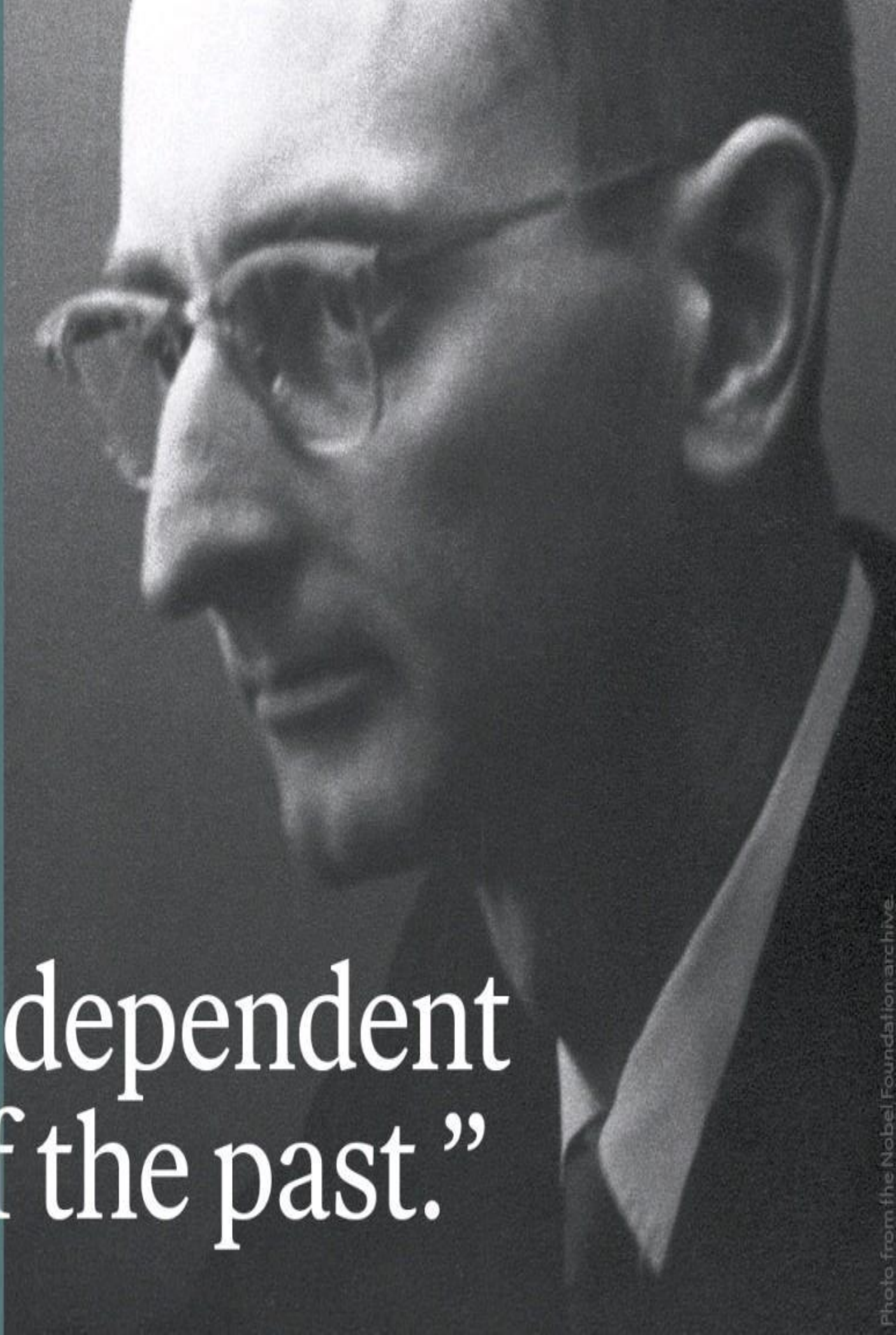
Digital Image Processing

- 请帮我翻译部分的朋友鼓掌
- Qǐng bāng wǒ fānyì bùfèn de péngyǒu gǔzhǎng

THANK YOU

OWEN CHAMBERLAIN
Nobel Prize in Physics 1959

“Each new idea is dependent upon the ideas of the past.”



MAY-BRITT MOSER
Nobel Prize in Physiology or Medicine 2014

"I learned at an early age that work makes you happy."

"BE HUMBLE. BE HUNGRY. AND ALWAYS BE THE HARDEST WORKER IN THE ROOM."