江西理工大學信息工程学院
JIANGXI UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF INFORMATION ENGINEERING

Digital Image Processing

数字图像处理

Lecture 012:

Image Convolution

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LECTURE 12: Image Convolution

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Correlation and Convolution

相关和卷积 Xiāngguān hé juàn jī

- Correlation and Convolution are basic operations that we will perform to extract information from images.
- They are in some sense the simplest operations that we can perform on an image, but they are extremely useful.
- Moreover, because they are simple, they can be analyzed and understood very well, and they are also easy to implement and can be computed very efficiently.
- Theoretically, convolution are linear operations on the signal or signal modifiers, whereas correlation is a measure of similarity between two signals.

Correlation and Convolution

相关和卷积 Xiāngguān hé juàn jī

- -相关和卷积是我们将执行的从图像中提取信息的基本操作。从某种意义上说，它们是我们可以对图像执行的最简单的操作，但它们非常有用。
- - 而且，因为它们很简单，所以可以很好地分析和理解，并且它们也很容易实现并且可以非常高效地计算。
- - 理论上，卷积是对信号或信号修饰符的线性运算，而相关性是两个信号之间相似性的度量。

Image Convolution

图像卷积 Túxiàng juàn jī

- As we have discussed in the introduction to image processing and in the signal and system that image processing is more or less the study of signals and systems because an image is nothing but a two dimensional signal.
- Also we have discussed, that in image processing , we are developing a system whose input is an image and output would be an image. This is pictorially represented as.

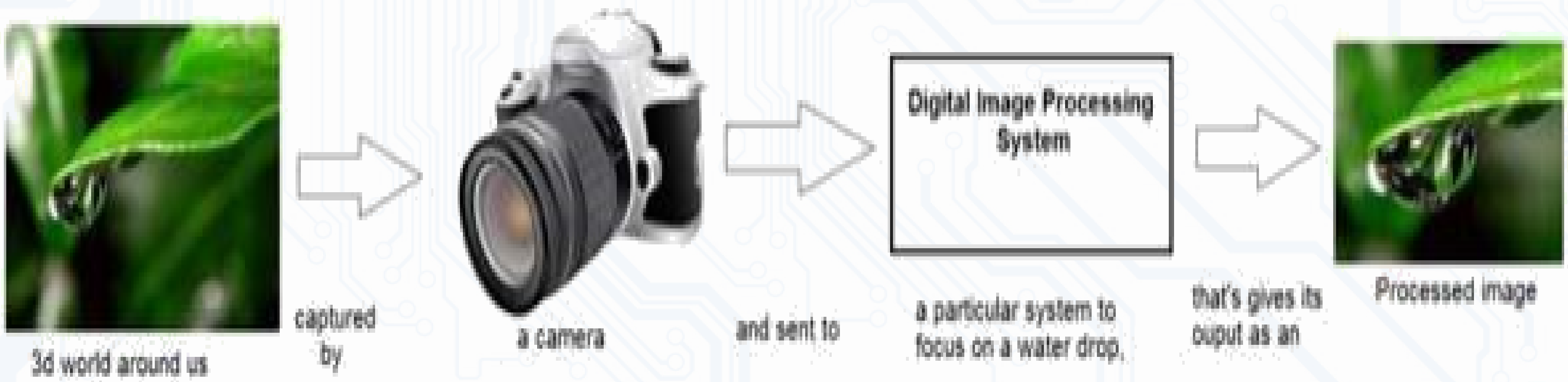


Image Convolution

图像卷积 Túxiàng juàn jī

- 正如我们在图像处理和信号与系统介绍中所讨论的那样，图像处理或多或少是对信号和系统的研究，因为图像只不过是二维信号。
- 我们还讨论过，在图像处理中，我们正在开发一个系统，其输入是图像，输出将是图像。这被形象地表示为。

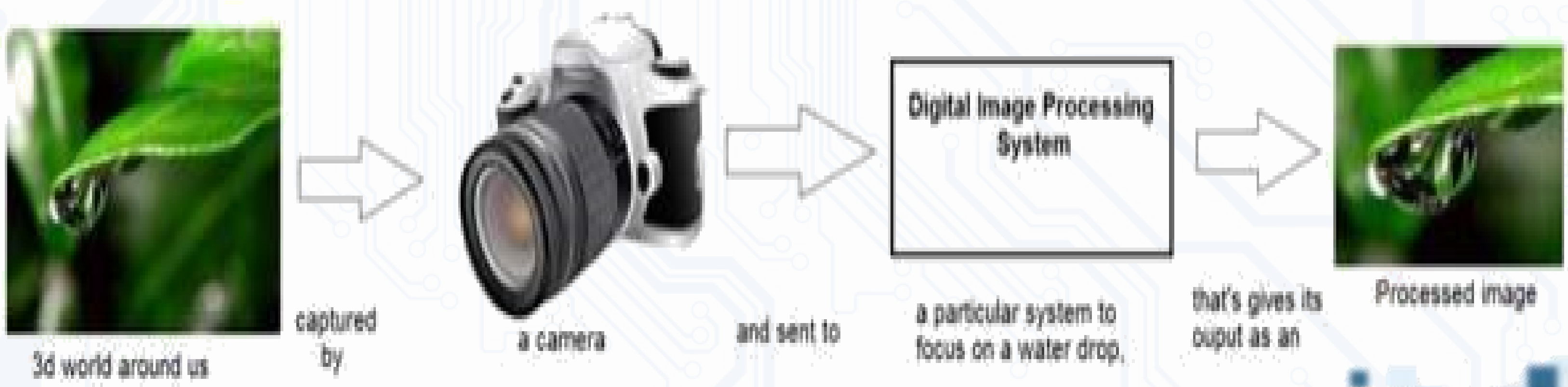


Image Convolution

图像卷积 Túxiàng juàn jī

- The box is that is shown in the above figure labeled as "Digital Image Processing system" could be thought of as a black box 数字图像处理系统
- It can be better represented as:

Image Convolution

图像卷积 Túxiàng juàn jī

- 这个盒子就是上面图中标示的“数字图像处理系统”，可以看作是一个黑盒子
- 它可以更好地表示为:

Image Convolution

图像卷积 Túxiàng juàn jī

- Till now we have discussed two important methods to manipulate images. Or in other words we can say that, our black box works in two different ways till now.
- The two different ways of manipulating images were
- **Graphs (Histograms)** 图表（直方图）

This method is known as histogram processing. We have discussed it in detail in previous for increase contrast, image enhancement, brightness e.t.c

这种方法被称为直方图处理。 我们已经在前面详细讨论了它的增强对比度，图像增强，亮度等

Image Convolution

图像卷积 Túxiàng juàn jī

- 到目前为止，我们已经讨论了两种重要的处理图像的方法。 换句话说，到目前为止，我们的黑盒有两种工作方式。
- 两种不同的处理图像的方法
- 图表(柱状图)

这种方法被称为直方图处理。 我们已经在前面详细讨论了它的增强对比度，图像增强，亮度等

Transformation functions

变换函数 Biànhuàn hánshù

- This method is known as transformations, in which we discussed different type of transformations and some gray level transformations

这种方法称为变换，其中我们讨论了不同类型的变换和一些灰度变换

Another way of dealing images

处理图像的另一种方式 Chǔlǐ túxiàng de lìng yī zhǒng fāngshì

- another method of dealing with images. This other method is known as convolution.
- Usually the black box(system) used for image processing is an LTI system or linear time invariant system.
- By linear we mean that such a system where output is always linear , neither log nor exponent or any other.
- And by time invariant we means that a system which remains same during time.

It can be mathematically represented as two ways

g(x,y) = h(x,y) * f(x,y) It can be explained as the “mask convolved with an image”.

Or

g(x,y) = f(x,y) * h(x,y) It can be explained as “image convolved with mask”.

There are two ways to represent this because the convolution operator(*) is commutative.

The h(x,y) is the mask or filter.

Another way of dealing images

处理图像的另一种方式 Chǔlǐ túxiàng de lìng yī zhǒng fāngshì

- 另一种处理图像的方法。 另一种方法被称为卷积。
- 通常用于图像处理的黑盒子(系统)是LTI系统或线性时不变系统。
- 线性是指输出总是线性的系统，既不是对数也不是指数。
- 我们说的时不变是指一个在时间上保持不变的系统。

它可以用两种方法在数学上表示

g(x,y) = h(x,y) * f(x,y) 它可以解释为“蒙版与图像的卷积”.

或者

g(x,y) = f(x,y) * h(x,y) 它可以解释为“图像与掩模的卷积”。

有两种表示方法，因为卷积运算符(*)是可交换的。

h(x,y)是掩码或过滤器。

1D Continuous Convolution - Definition

一维连续卷积 - 定义

- Convolution is defined as follows:

卷积定义如下： Juàn jī dìngyì rúxià:

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(a)g(x-a)da$$

卷积是可交换的 Juàn jī shì kě jiāohuàn de

- Convolution is commutative

$$f(x) * g(x) = g(x) * f(x)$$

2D Convolution

二维卷积

- Definition 定义 Dìngyì

$$f(x,y) * g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)g(x-a,y-b)\,da\,db$$

- 2D convolution theorem 二维卷积定理 Èr wéi juàn jī dìnglǐ

$$f(x,y) * g(x,y) <---> F(u,v)\ G(u,v)$$

$$f(x,y)\ g(x,y) <---> F(u,v) * G(u,v)$$

Convolution Theorem

卷积定理 Juàn jī dìnglǐ

- Convolution in the spatial domain is equivalent to multiplication in the frequency domain.

$$f(x) * g(x) <\!\!-\!\!> F(u)G(u)$$

$$f(x) \longleftrightarrow F(u)$$
$$g(x) \longleftrightarrow G(u)$$

Multiplication in the spatial domain is equivalent to convolution in the frequency domain.

$$f(x)g(x) <\!\!-\!\!> F(u) * G(u)$$

空间域中的卷积相当于频域中的乘法。
Kōngjiān yù zhōng de juàn jī xiāngdāng yú pín yù zhōng de chéngfǎ.

空间域中的乘法相当于频域中的卷积。
Kōngjiān yù zhōng de chéngfǎ xiāngdāng yú pín yù zhōng de juàn jī.

Why Convolution?

为什么是卷积? Wèishéme shì juàn jī?

- Convolution can achieve something, that the previous two methods of manipulating images can’t achieve.
- Those include the blurring, sharpening, edge detection, noise reduction e.t.c.

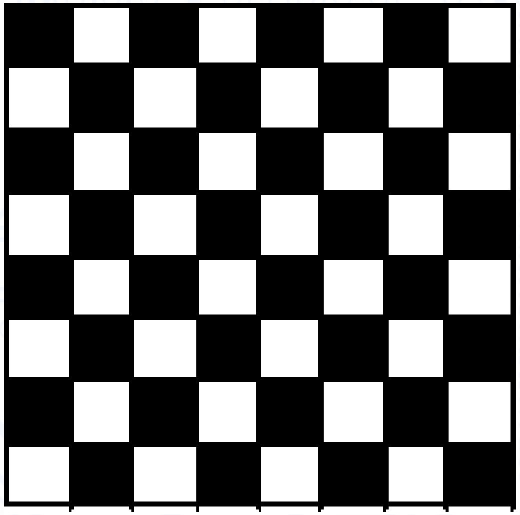
卷积可以实现前两种处理图像的方法无法实现的功能。
这些包括模糊，锐化，边缘检测，降噪等。

Spatial frequencies

空间频率 Kōngjiān pínlǜ

- Convolution filtering is used to modify the spatial frequency characteristics of an image.
  卷积滤波用于修改图像的空间频率特性。

- zero spatial frequency 零空间频率
- low spatial frequency 低空间频率
- high spatial frequency 高空间频率



What is convolution?

什么是卷积? Shénme shì juàn jī?

- Convolution is a general purpose filter effect for images. 卷积是一种通用的图像滤波效果。
  - Is a matrix applied to an image and a mathematical operation comprised of integers

    一个矩阵应用于一个图像和一个由整数组成的数学运算
  - It works by determining the value of a central pixel by adding the weighted values of all its neighbors together

    它的工作原理是通过将所有相邻像素的加权值相加来确定中心像素的值
  - The output is a new modified filtered image

    输出的是一个新的修改过的滤波图像

Convolution

什么是卷积？ Shénme shì juàn jī?

- Convolution is a simple mathematical operation which is fundamental to many common image processing operators.
- Convolution provides a way of `multiplying together' two arrays of numbers, generally of different sizes, but of the same dimensionality, to produce a third array of numbers of the same dimensionality.

Convolution

什么是卷积？ Shénme shì juàn jī?

- 卷积是一种简单的数学运算，是许多常见图像处理操作的基础。
- 卷积提供了一种将两个大小不同但维度相同的数字数组"相乘"的方法，从而生成第三个维度相同的数字数组。

Convolution

什么是卷积？ Shénme shì juàn jī?

This can be used in image processing to implement operators whose output pixel values are simple linear combinations of certain input pixel values.

这可以用于图像处理，以实现其输出像素值是某些输入像素值的简单线性组合的操作符。

Convolution

什么是卷积？ Shénme shì juàn jī?

- in an image processing context, one of the input arrays is normally just a graylevel image.
- The second array is usually much smaller, and is also two-dimensional (although it may be just a single pixel thick), and is known as the kernel.

在图像处理上下文中，一个输入数组通常只是一个灰度图像。

第二个数组通常要小得多，也是二维的(尽管它可能只有一个像素厚)，被称为内核。

Convolution

什么是卷积？ Shénme shì juàn jī?

- Figure 1 shows an example image and kernel that we will use to illustrate convolution.

图1显示了一个示例图像和内核，我们将使用它来说明卷积。

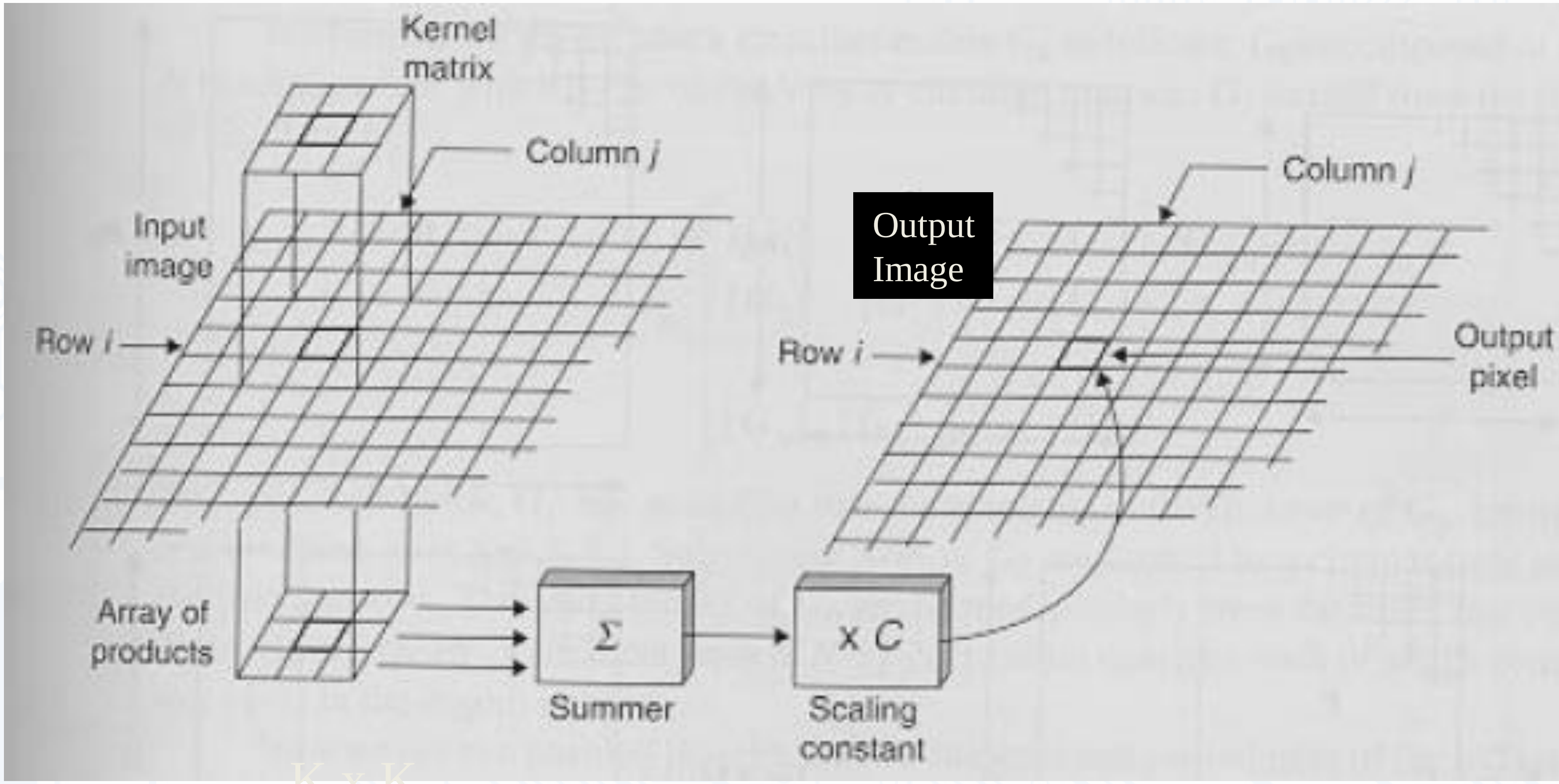
I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	I_{16}	I_{17}	I_{18}	I_{19}
I_{21}	I_{22}	I_{23}	I_{24}	I_{25}	I_{26}	I_{27}	I_{28}	I_{29}
I_{31}	I_{32}	I_{33}	I_{34}	I_{35}	I_{36}	I_{37}	I_{38}	I_{39}
I_{41}	I_{42}	I_{43}	I_{44}	I_{45}	I_{46}	I_{47}	I_{48}	I_{49}
I_{51}	I_{52}	I_{53}	I_{54}	I_{55}	I_{56}	I_{57}	I_{58}	I_{59}
I_{61}	I_{62}	I_{63}	I_{64}	I_{65}	I_{66}	I_{67}	I_{68}	I_{69}

K_{11}	K_{12}	K_{13}
K_{21}	K_{22}	K_{23}

图1示例小图像(左)和核(右)来说明卷积。每个方格内的标签用于标识每个方格。

Figure 1 An example small image (left) and kernel (right) to illustrate convolution. The labels within each grid square are used to identify each square.

Correlation - Review



$$g(x,y)=w(x,y)\bullet f(x,y)=\sum_{s=-K/2}^{K/2}\ \sum_{t=-K/2}^{K/2} w(s,t)\,f(x+s,y+t)$$

Convolution

什么是卷积?   Shénme shì juàn jī?

- The convolution is performed by sliding the kernel over the image, generally starting at the top left corner, so as to move the kernel through all the positions where the kernel fits entirely within the boundaries of the image. (Note that implementations differ in what they do at the edges of images, as explained below.)
- Each kernel position corresponds to a single output pixel, the value of which is calculated by multiplying together the kernel value and the underlying image pixel value for each of the cells in the kernel, and then adding all these numbers together.
- **So, in our example, the value of the bottom right pixel in the output image will be given by:**

$$O_{57} = I_{57}K_{11} + I_{58}K_{12} + I_{59}K_{13} + I_{67}K_{21} + I_{68}K_{22} + I_{69}K_{23}$$

Convolution

什么是卷积？ Shénme shì juàn jī?

- 卷积是通过在图像上滑动核来执行的，通常从左上角开始，以便在核完全适合于图像边界的所有位置移动核。(注意在图像边缘的实现是不同的，如下所述。)
- 每个核位置对应一个输出像素，其值是通过将核值与核中每个细胞的底层图像像素值相乘，然后将所有这些数字相加计算出来的。
- 因此，在我们的例子中，输出图像中右下角像素的值是:

$$O_{57} = I_{57}K_{11} + I_{58}K_{12} + I_{59}K_{13} + I_{67}K_{21} + I_{68}K_{22} + I_{69}K_{23}$$

Convolution

什么是卷积？ Shénme shì juàn jī?

If the image has M rows and N columns, and the kernel has m rows and n columns, then the size of the output image will have M - m + 1 rows, and N - n + 1 columns.

如果图像有M行N列，核有M行N列，那么输出图像的大小将有M - M + 1行，N - N + 1列。

Mathematically we can write the convolution as:
数学上，我们可以把卷积写成:

$$O(i,j) = \sum_{k=1}^{m} \sum_{l=1}^{n} I(i+k-1, j+l-1) K(k,l)$$

where i runs from 1 to M - m + 1 and j runs from 1 to N - n + 1.

其中i从1到M - M + 1 j从1到N - N + 1。

Convolution

- Note that many implementations of convolution produce a larger output image than this because they relax the constraint that the kernel can only be moved to positions where it fits entirely within the image.
- Instead, these implementations typically slide the kernel to all positions where just the top left corner of the kernel is within the image.
- Therefore the kernel `overlaps' the image on the bottom and right edges.
- One advantage of this approach is that the output image is the same size as the input image.

注意，许多卷积实现产生的输出图像比这个更大，因为它们放宽了内核只能移动到它完全适合图像的位置的限制。

相反，这些实现通常会将内核滑动到图像中仅包含内核左上角的所有位置。

因此，内核在图像的底部和右侧边缘“重叠”。

这种方法的一个优点是输出图像与输入图像的大小相同。

Convolution

- Unfortunately, in order to calculate the output pixel values for the bottom and right edges of the image, it is necessary to *invent* input pixel values for places where the kernel extends off the end of the image.
- Typically pixel values of zero are chosen for regions outside the true image, but this can often distort the output image at these places.
- Therefore in general if you are using a convolution implementation that does this, it is better to clip the image to remove these spurious regions. Removing *n* - 1 pixels from the right hand side and *m* - 1 pixels from the bottom will fix things.

然而，为了计算图像的底部和右侧边缘的输出像素值，必须为内核延伸到图像末端的地方创造输入像素值。
通常在真实图像之外的区域选择像素值为零，但这通常会使这些地方的输出图像失真。
因此，一般来说，如果你使用卷积实现来做到这一点，最好是剪辑图像以去除这些虚假区域。从右边移除n - 1个像素，从底部移除m - 1个像素，就可以修复问题。

Convolution

- Convolution can be used to implement many different operators, particularly spatial filters and feature detectors.
- Examples include
  – Gaussian smoothing
  – and the Sobel edge detector.

卷积可用于实现许多不同的算子，特别是空间滤波器和特征检测器。
例子包括
—高斯平滑
—和索贝尔边缘检测器。

Gaussian smoothing

高斯平滑 Gāosī pínghuá

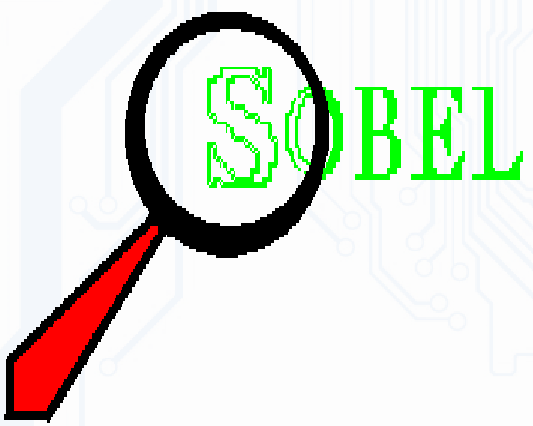
- The Gaussian smoothing operator is a 2-D convolution operator that is used to `blur' images and remove detail and noise.
- In this sense it is similar to the mean filter, but it uses a different kernel that represents the shape of a Gaussian (`bell-shaped') hump.
- This kernel has some special properties which are detailed below.

高斯平滑算子是一种二维卷积算子，用于“模糊”图像并去除细节和噪声。从这个意义上说，它类似于均值滤波器，但它使用不同的内核来表示高斯（“钟形”）驼峰的形状。 这个内核有一些特殊的属性，下面详细介绍。

Sobel edge detector

索贝尔边缘检测器 Suǒ bèi'ěr biānyuán jiǎncè qì

- The Sobel operator performs a 2-D spatial gradient measurement on an image and so emphasizes regions of high spatial frequency that correspond to edges.
- Typically it is used to find the approximate absolute gradient magnitude at each point in an input grayscale image.



Sobel 算子对图像执行二维空间梯度测量，因此强调与边缘对应的高空间频率区域。通常，它用于查找输入灰度图像中每个点的近似绝对梯度幅度。

Reference

参考（cān kǎo ）

- https://homepages.inf.ed.ac.uk/rbf/HIPR2/gsmooth.htm

Student Task_: DIP

- 请帮我翻译部分的朋友鼓掌
- Qǐng bāng wǒ fānyì bùfèn de péngyǒu gǔzhǎng

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THANK YOU

MAY-BRITT MOSER
Nobel Prize in Physiology or Medicine 2014

"I learned at an early age that work makes you happy."

Student Task_3: DIP

- 请帮我翻译部分的朋友鼓掌
- Qǐng bāng wǒ fānyì bùfèn de péngyǒu gǔzhǎng

Solve the Question shared in mooc

解决mooc分享的问题

Send for Next lecture

发送下一个讲座

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THANK YOU

"BE HUMBLE. BE HUNGRY. AND ALWAYS BE THE HARDEST WORKER IN THE ROOM."