Jiangxi University of Science and Technology

DIGITAL DESIGN

Lecture 11:
The Karnaugh Map_1

Reminder

SIMULATION

TASKS

Attendance

Karnaugh Maps (K-Maps)

- A visual way to simplify logic expressions
- It gives the most simplified form of the expression
- Simplification of logic expression using Boolean algebra is awkward because:
  - it lacks specific rules to predict the most suitable next step in the simplification process
  - it is difficult to determine whether the simplest form has been achieved.

K_map

- A Karnaugh map is a graphical method used to obtained the most simplified form of an expression in a standard form (Sum-of-Products or Product-of-Sums).

- The simplest form of an expression is the one that has the minimum number of terms with the least number of literals (variables) in each term.
- By simplifying an expression to the one that uses the minimum number of terms, we ensure that the function will be implemented with the minimum number of gates.
- By simplifying an expression to the one that uses the least number of literals for each terms, we ensure that the function will be implemented with gates that have the minimum number of inputs.

K_MAP

- A Karnaugh map provides a pictorial method of grouping together expressions with common factors and therefore eliminating unwanted variables.
- The Karnaugh map can also be described as a special arrangement of a truth table.
- The diagram illustrates the correspondence between the Karnaugh map and the truth table for the general case of a two variable problem.

A	B	F
0	0	a
0	1	b
1	0	c
1	1	d

Truth Table.

B \ A	0	1
0	a	b
1	c	d

F.

- Using a K-map, expressions with two to four variables are easily minimized.
- Expressions with five to six variables are more difficult but achievable, and expressions with seven or more variables are extremely difficult (if not impossible) to minimize using a K-map.

K_MAP

- The values inside the squares are copied from the output column of the truth table, therefore there is one square in the map for every row in the truth table.
- Around the edge of the Karnaugh map are the values of the two input variable. A is along the top and B is down the left hand side.
- The diagram explains this:

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table.

B \ A	0	
0	0	1
1	1	1

F.

The values around the edge of the map can be thought of as coordinates. So as an example, the square on the top right hand corner of the map in the above diagram has coordinates A=1 and B=0.

This square corresponds to the row in the truth table where A=1 and B=0 and F=1.

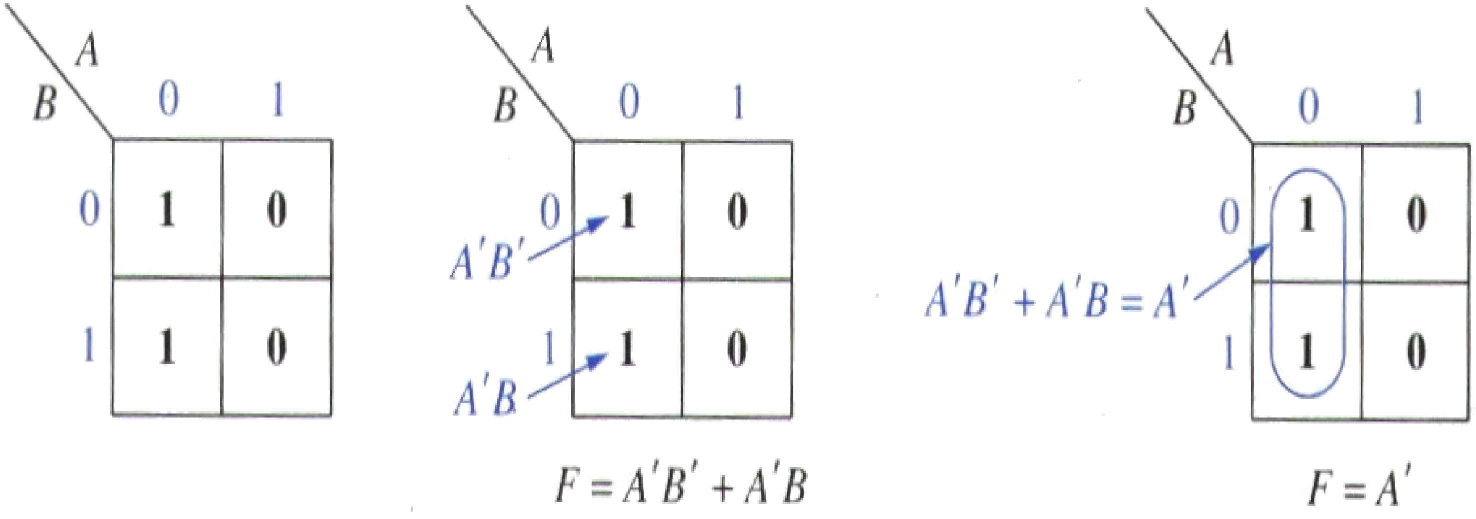
***Note that the value in the F column represents a particular function to which the Karnaugh map corresponds*.**

K_MAP

- Place 1s and 0s from the truth table in the K-map.
- Each square of 1s = minterms.
- Minterms in adjacent squares can be combined since they differ in only one variable. Use XY’ + XY = X.

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

(a)



(b) (c) (d)

Re-arranging the truth table

- A two-variable function has four possible minterms. We can re-arrange these minterms into a Karnaugh map.

x	y	minterm
0	0	x'y'
0	1	x'y
1	0	xy'
1	1	xy

x \ y	0	1
0	x'y'	x'y
1	xy'	xy

- Now we can easily see which minterms contain common literals.
  - Minterms on the left and right sides contain y' and y respectively.
  - Minterms in the top and bottom rows contain x' and x respectively.

x \ y	0	1
0	x'y'	x'y
1	xy'	xy

	y'	y
x'	x'y'	x'y
x	xy'	xy

Karnaugh map simplifications

- Imagine a two-variable sum of minterms: $x'y' + x'y$

- Both of these minterms appear in the top row of a Karnaugh map, which means that they both contain the literal x'.

		y
	$x'y'$	$x'y$
x	xy'	xy

- What happens if you simplify this expression using Boolean algebra?

$$
\begin{aligned}
x'y' + x'y &= x'(y' + y) && [\text{ Distributive }] \\
&= x' \bullet 1 && [\ y + y' = 1\ ] \\
&= x' && [\ x \bullet 1 = x\ ]
\end{aligned}
$$

More two-variable examples

- Another example expression is $x'y + xy$.
  - Both minterms appear in the right side, where y is uncomplemented.
  - Thus, we can reduce $x'y + xy$ to just y.

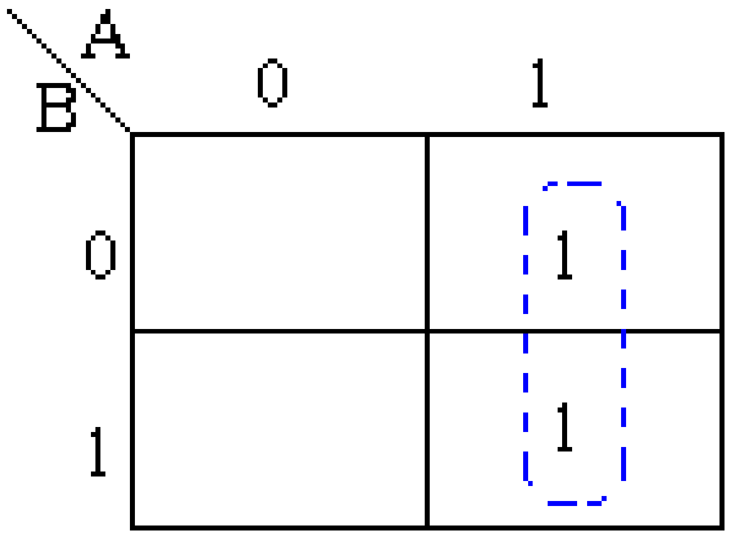
		Y
	$x'y'$	$\boxed{x'y}$
X	xy'	$\boxed{xy}$

- How about $x'y' + x'y + xy$?
  - We have $x'y' + x'y$ in the top row, corresponding to x'.
  - There's also $x'y + xy$ in the right side, corresponding to y.
  - This whole expression can be reduced to $x' + y$.

		Y
	$x'y'$	$x'y$
X	xy'	xy

K_MAP: Example 1:

- Consider the following map. The function plotted is: $Z = f(A,B) = A\bar{B} + AB$



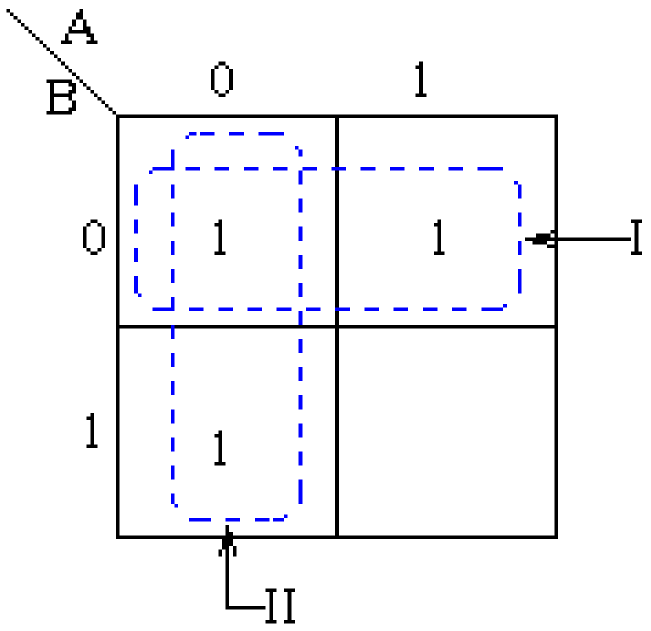
•Note that values of the input variables form the rows and columns. That is the logic values of the variables A and B (with one denoting true form and zero denoting false form) form the head of the rows and columns respectively.
•Bear in mind that the above map is a one dimensional type which can be used to simplify an expression in two variables.
•There is a two-dimensional map that can be used for up to four variables, and a three-dimensional map for up to six variables.
Using algebraic simplification,

Using algebraic simplification,
$Z = A\bar{B} + AB$
$Z = A(\bar{B} + B)$
$Z = A$

Consider the expression $Z = f(A,B) = \bar{A}\bar{B} + A\bar{B} + \bar{A}B$ plotted on the Karnaugh map:

Pairs of 1's are *grouped* as shown above, and the simplified answer is obtained by using the following steps:
Note that two groups can be formed for the example given above, bearing in mind that the largest rectangular clusters that can be made consist of two 1s. Notice that a 1 can belong to more than one group.

The first group labelled I, consists of two 1s which correspond to A = 0, B = 0 and A = 1, B = 0. Put in another way, all squares in this example that correspond to the area of the map where B = 0 contains 1s, independent of the value of A. So when B = 0 the output is 1. The expression of the output will contain the term (B bar)



$$Z = \bar{A} + \bar{B}$$

For group labelled II corresponds to the area of the map where A = 0. The group can therefore be defined as A bar .

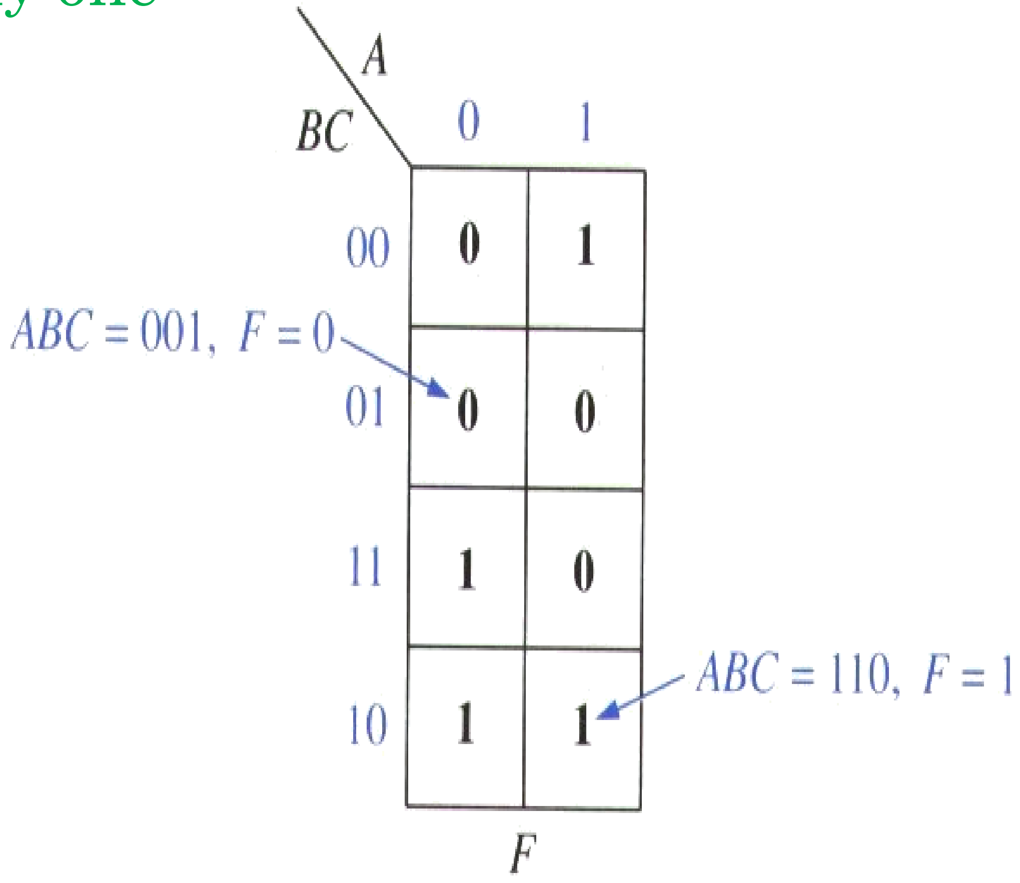
This implies that when A = 0 the output is 1. The output is therefore 1 whenever B = 0 and A = 0

3-Variable K-map

- Note BC is listed in the order of 00, 01, 11, 10. (Gray code)
- Minterms in adjacent squares that differ in only one variable can be combined using XY' + XY = X.

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

(a)



(b)

A three-variable Karnaugh map

- For a three-variable expression with inputs x, y, z, the arrangement of minterms is more tricky:

		YZ 00	01	11	10
X	0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
	1	$xy'z'$	$xy'z$	xyz	xyz'

		YZ 00	01	11	10
X	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

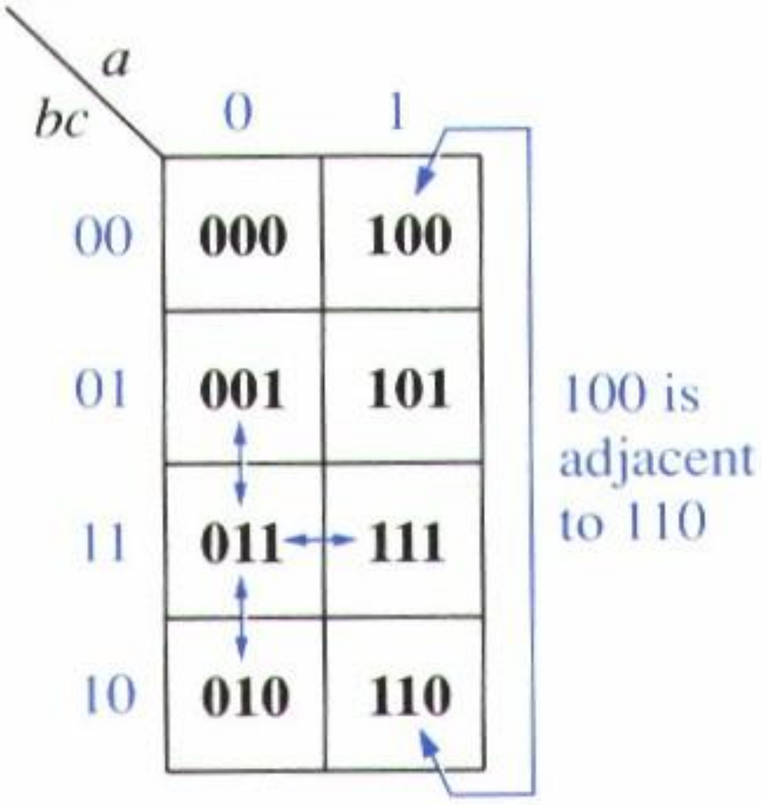
- Another way to label the K-map (use whichever you like):

			Y	Y
	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
X	$xy'z'$	$xy'z$	xyz	xyz'
		Z	Z	

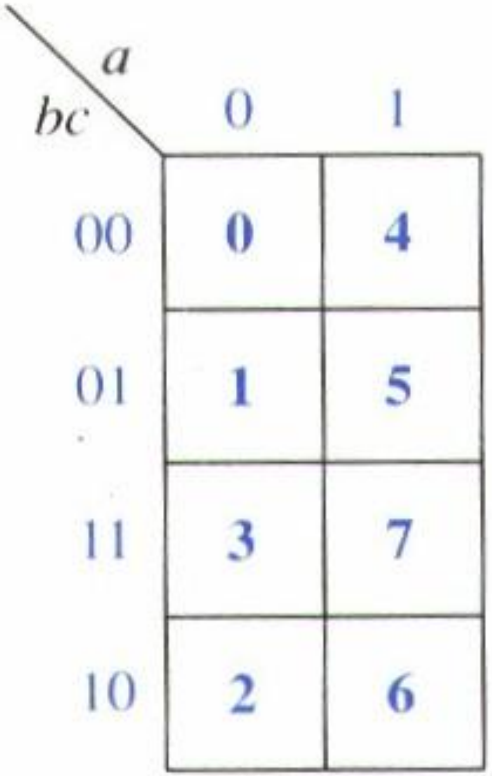
			Y	Y
	m_0	m_1	m_3	m_2
X	m_4	m_5	m_7	m_6
		Z	Z	

Location of Minterms

Adjacent terms in 3-variable K map.



(a) Binary notation



(b) Decimal notation

K Map Example

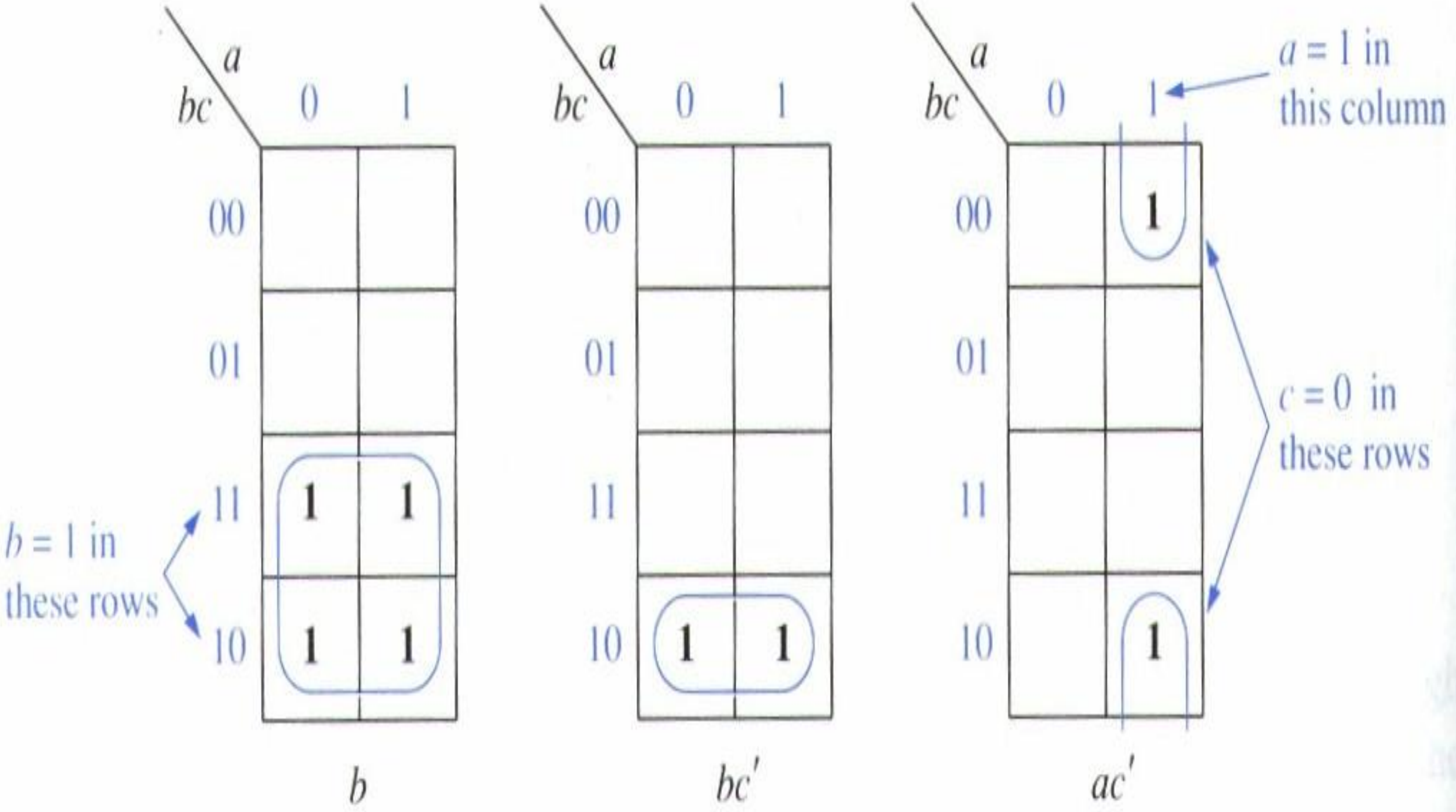
K-map of $F(a,b,c) = \sum m(1,3,5) = \prod M(0,2,4,6,7)$

bc \ a	0	1
00	0 (0)	0 (4)
01	1 (1)	1 (5)
11	1 (3)	0 (7)
10	0 (2)	0 (6)

Karnaugh Map of
$F(a, b, c) = \sum m(1, 3, 5) = \prod M(0, 2, 4, 6, 7)$

Place Product Terms on K Map

- Example:Place b, bc’ and ac’ in the 3-variable K map.



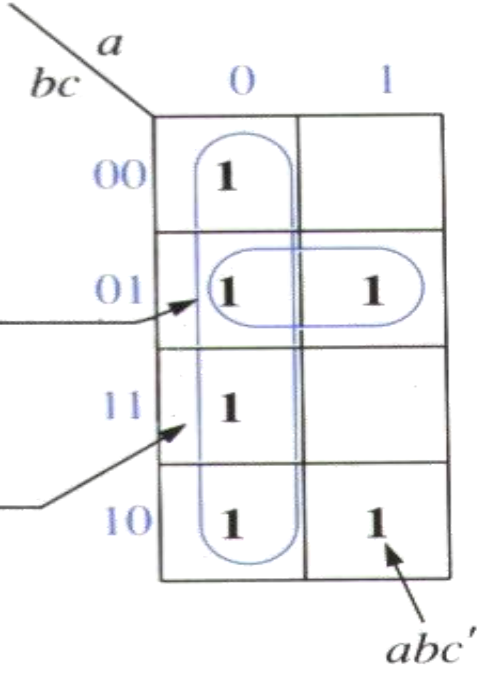
More Example

- **Exercise. Plot f(a, b, c) = abc' + b'c + a' into the K-map.**

$$f(a, b, c) = abc' + b'c + a'$$

we would plot the map as follows:

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1's in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1's in all the squares of the $a = 0$ column of the map. (*Note:* since there already is a 1 in the $abc = 001$ square, we do not have to place a second 1 there because $x + x = x$.)



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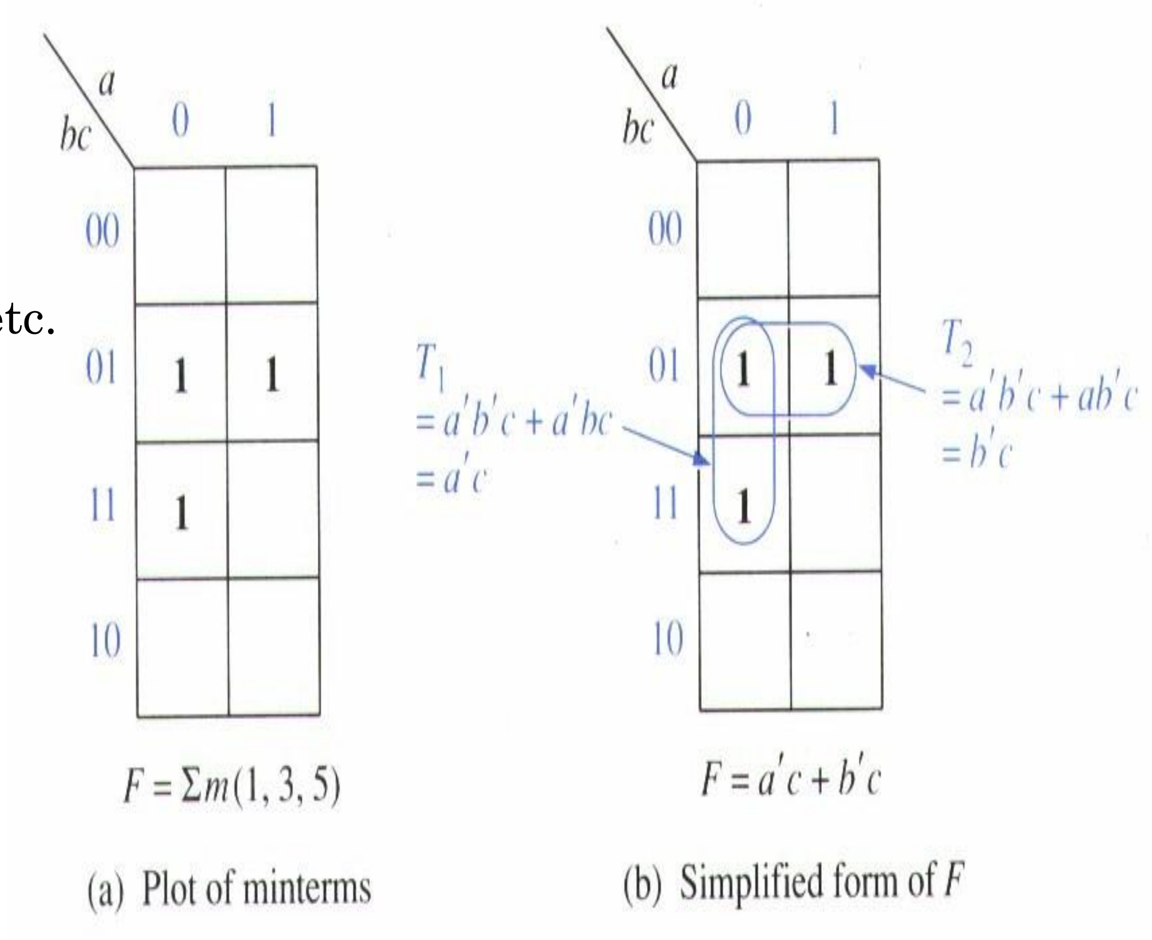
Simplication Example

- Exercise. Simplify:

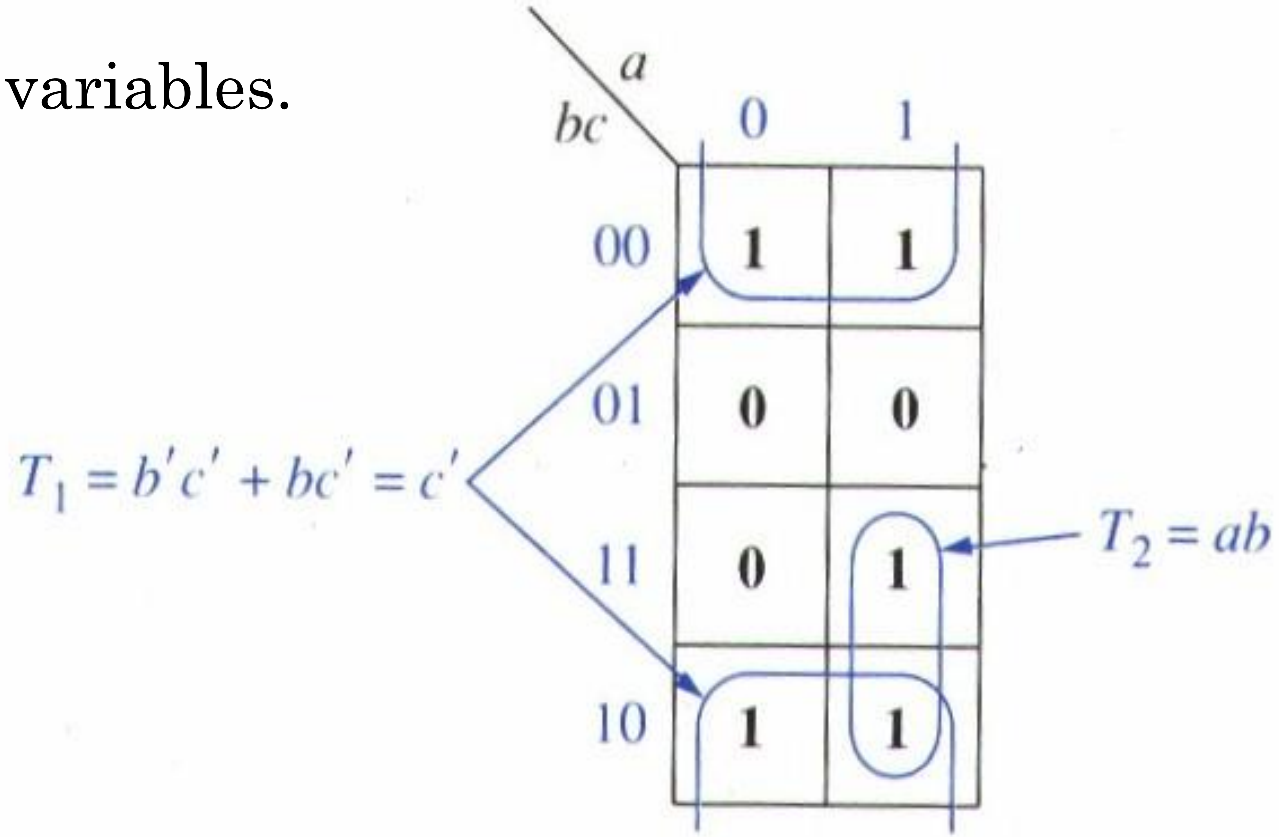
$F(a,b,c) = \sum m(1,3,5)$

  - Procedure: place minterms into map.
  - Select adjacent 1's in group of two 1's or four 1's etc.
  - Kick off x and x'.



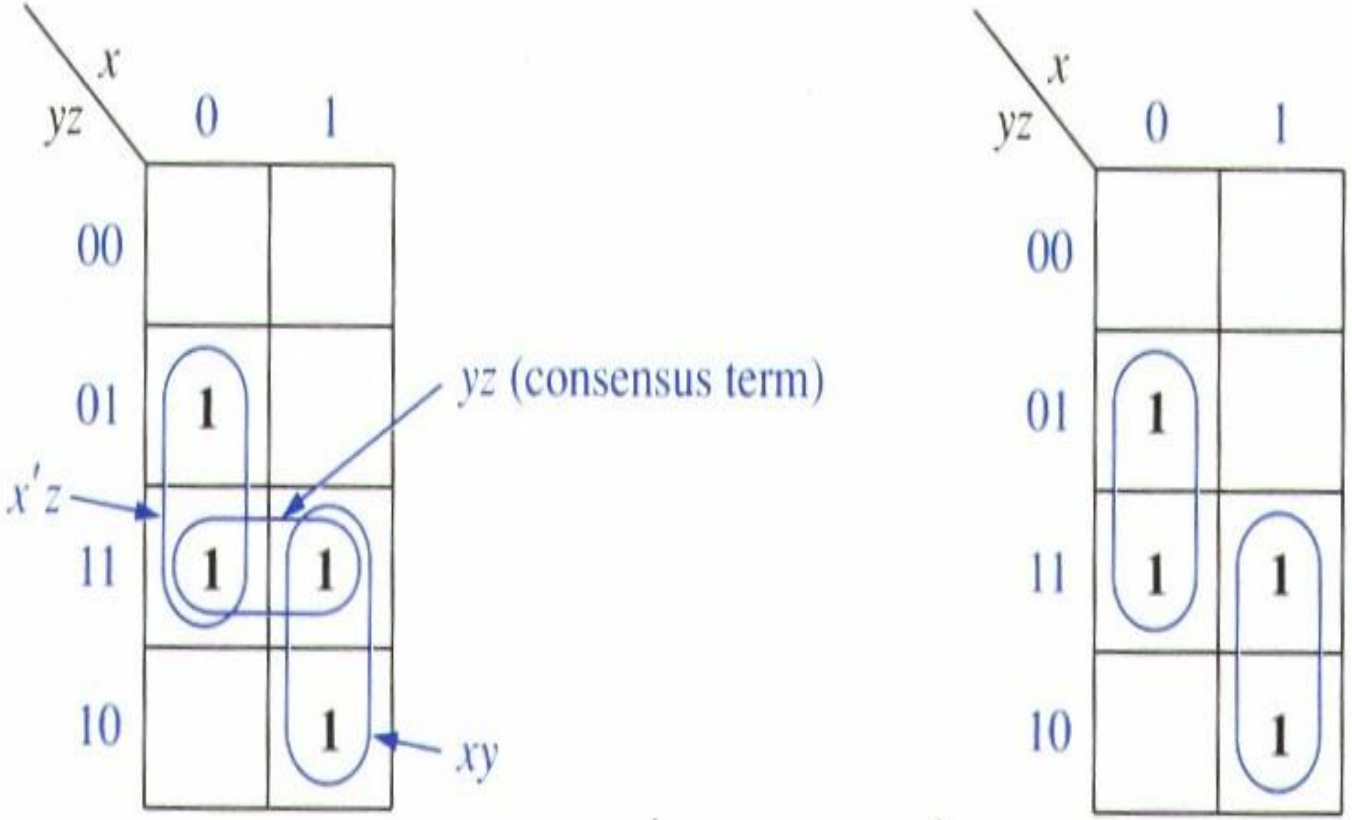
More Example

- The complement of F
  - Using four 1's to eliminate two variables.



Redundant Terms

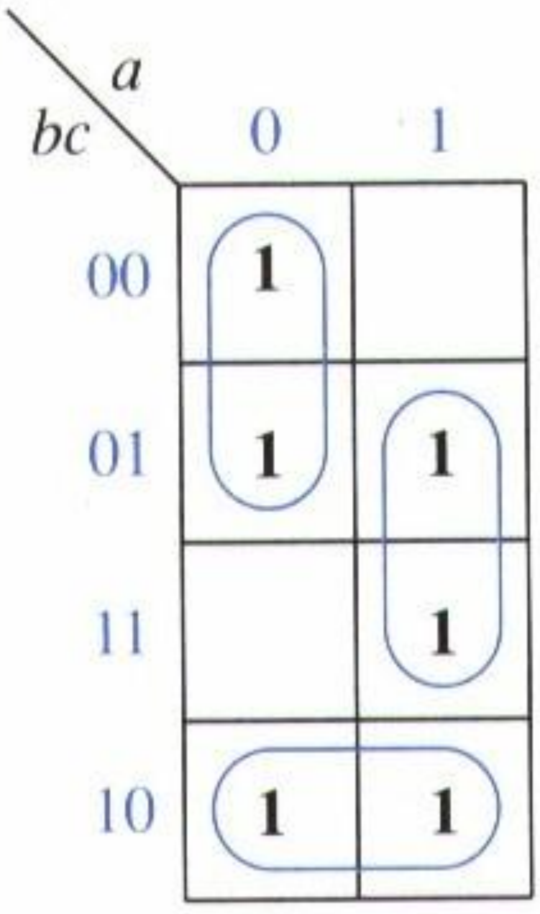
- If a term is covered by two other terms, that term is redundant. That is, it is a consensus term.
- Example: yz is the redundant.



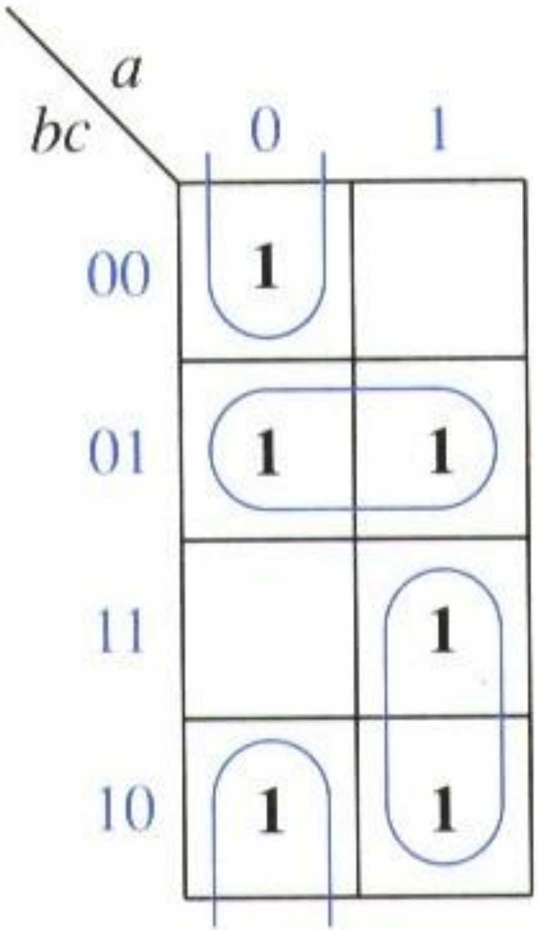
$$xy + x'z + yz = xy + x'z$$

More Than Two Minimum Solutions

- $F = \sum m(0,1,2,5,6,7)$



$F = a'b' + bc' + ac$



$F = a'c' + b'c + ab$

4-Variable K Map

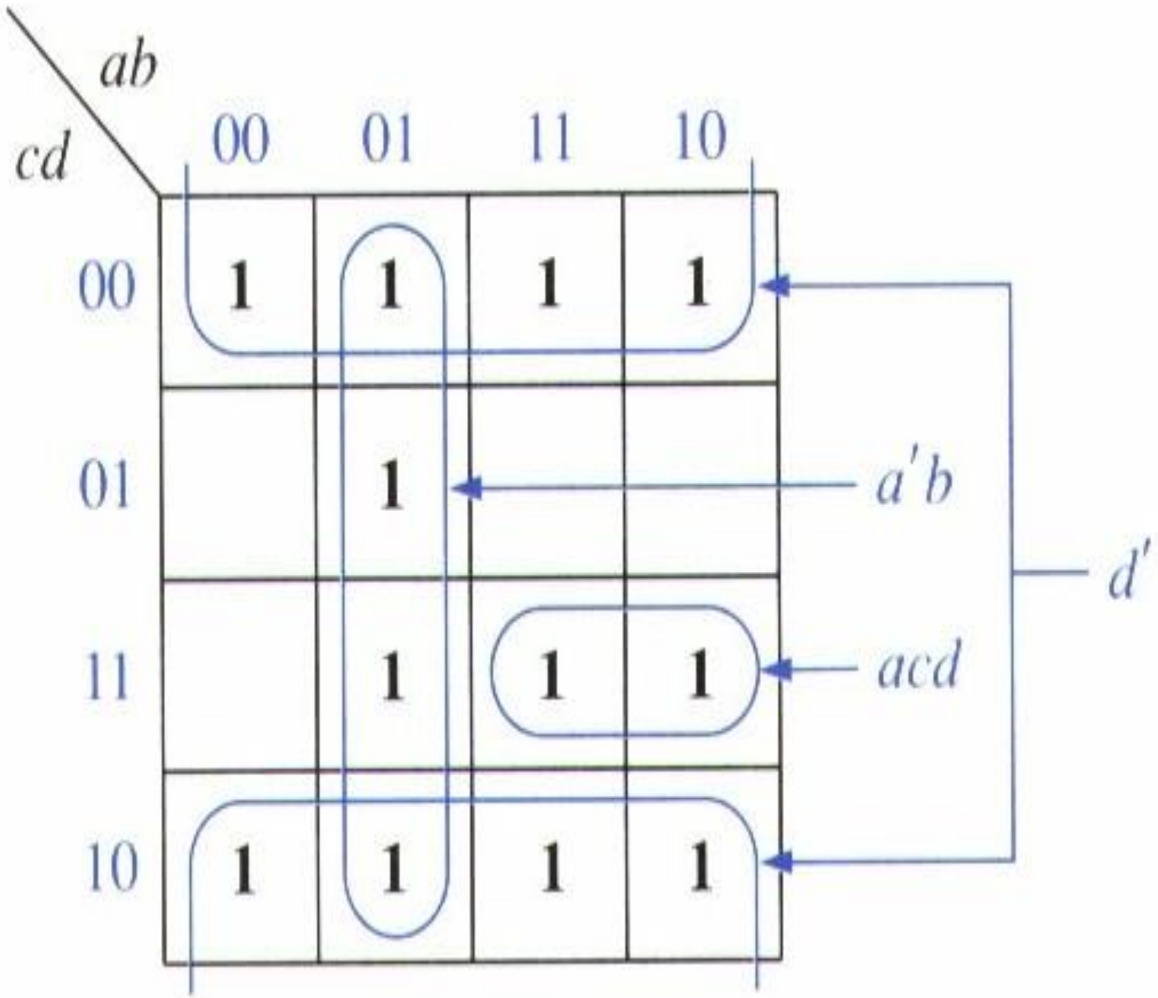
- Each minterm is adjacent to 4 terms with which it can combine.
  - 0, 8 are adjacent squares
  - 0, 2 are adjacent squares, etc.
  - 1, 4, 13, 7 are adjacent to 5.

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Plot a 4-variable Expression

- F(a,b,c,d) = acd + a'b + d'

  acd = 1 if a=1, c=1, d=1



Reference

http://www.ee.surrey.ac.uk/Projects/CAL/digital-logic/gatesfunc/