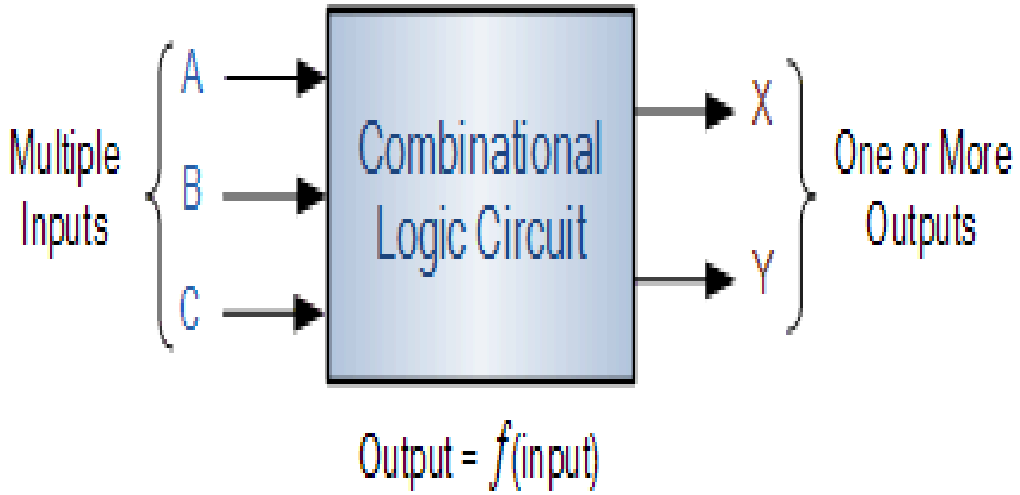
Jiangxi University of Science and Technology

DIGITAL DESIGN

Chapter 4 Combinational Logic

Review: Combinational Circuits

- A combinational circuit consists of logic gates whose outputs, at any time, are determined by combining the values of the inputs.
- For n input variables, there are 2^n possible binary input combinations.
- For each binary combination of the input variables, there is one possible output.



Hence, a combinational circuit can be described by:

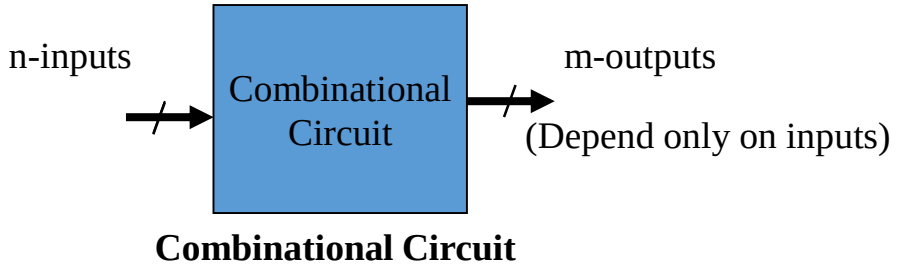
1. A truth table that lists the output values for each combination of the input variables, or
2. m Boolean functions, one for each output variable.

Other Combinational Circuits

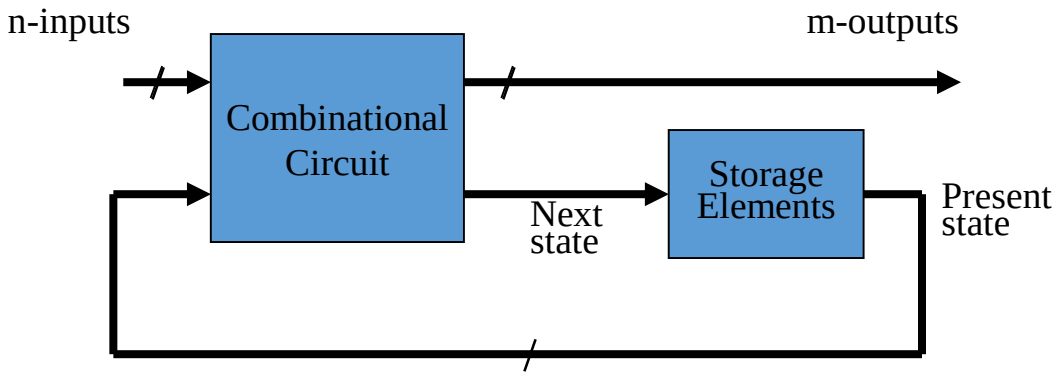
- Multiplexer
- Encoder
- Decoder
- Parity Checker
- Parity Generator
- etc

Combinational vs. Sequential Circuits

- Combinational circuits are memory-less. Thus, the output value depends ONLY on the current input values.
- Sequential circuits consist of combinational logic as well as memory elements (used to store certain circuit states).
- Outputs depend on BOTH current input values and previous input values (kept in the storage elements).



Combinational Circuit



Sequential Circuit

Classification of Combinational Logic

- One of the most common uses of combinational logic is in Multiplexer and De-multiplexer type circuits. Here, multiple inputs or outputs are connected to a common signal line and logic gates are used to decode an address to select a single data input or output switch.
- A multiplexer consist of two separate components, a logic decoder and some solid state switches, but before we can discuss multiplexers, decoders and de-multiplexers in more detail we first need to understand how these devices use these “solid state switches” in their design.

Multiplexer

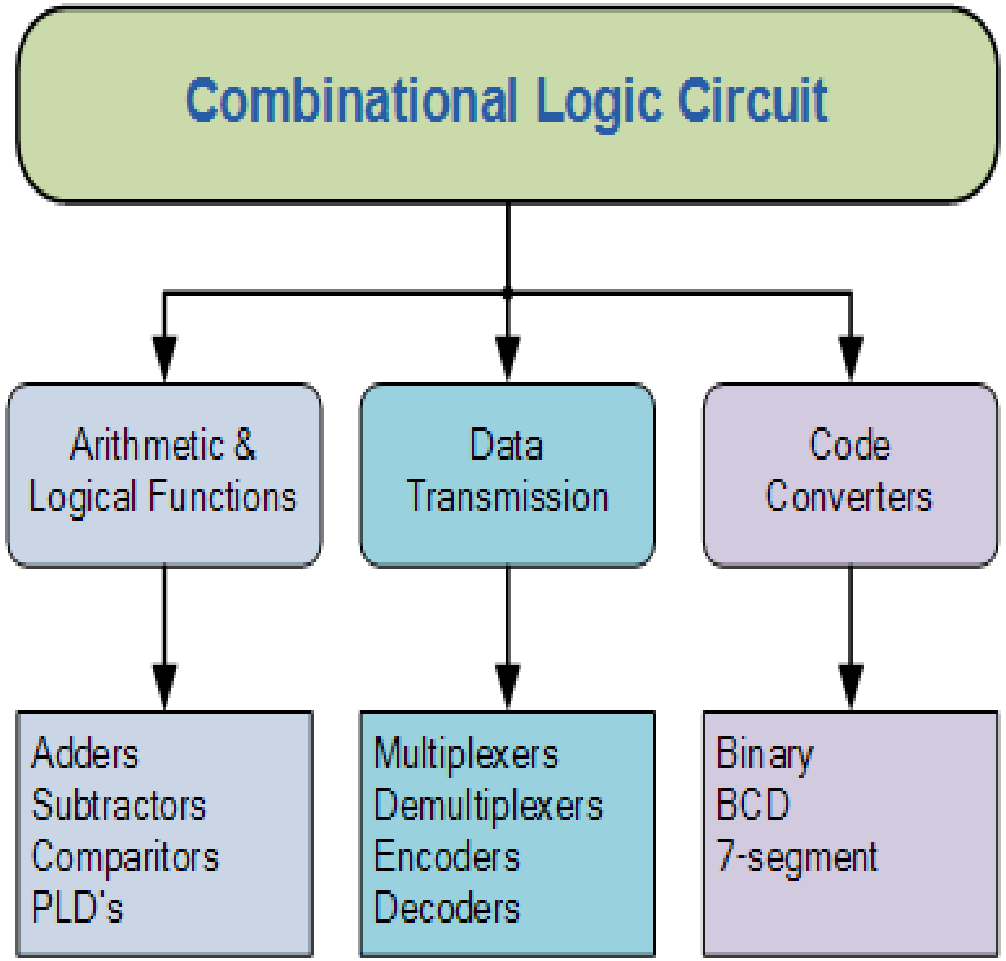
Encoder

Decoder

Parity Checker

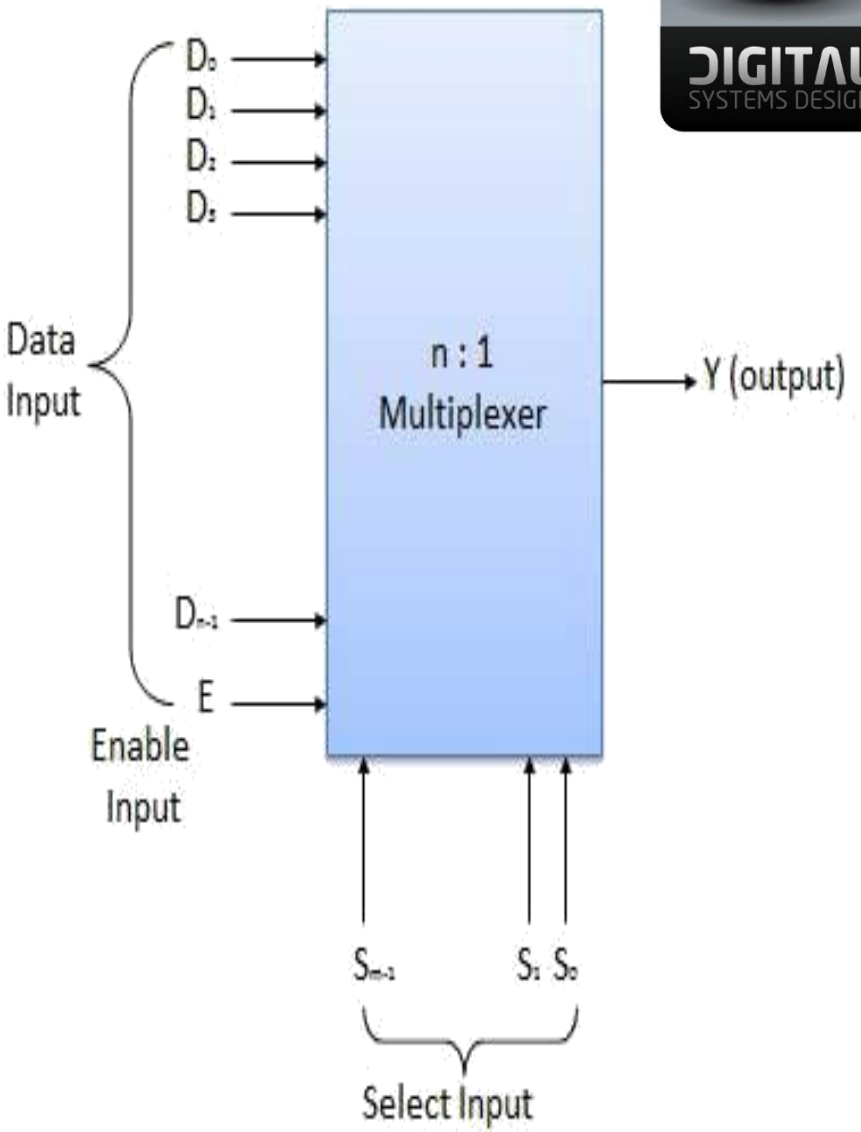
Parity Generator

etc



Multiplexers

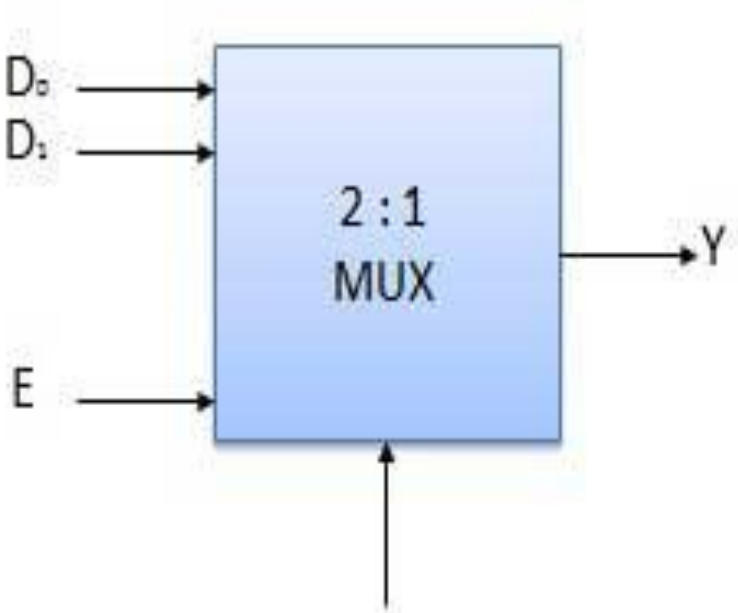
- Multiplexer is a special type of combinational circuit. There are n-data inputs, one output and m select inputs with 2m = n.
- It is a digital circuit which selects one of the n data inputs and routes it to the output. The selection of one of the n inputs is done by the selected inputs. Depending on the digital code applied at the selected inputs, one out of n data sources is selected and transmitted to the single output Y. E is called the strobe or enable input which is useful for the cascading. It is generally an active low terminal that means it will perform the required operation when it is low



Multiplexers

Multiplexers come in multiple variations

- 2 : 1 multiplexer
- 4 : 1 multiplexer
- 16 : 1 multiplexer
- 32 : 1 multiplexer



Enable	Select	Output
E	S	Y
0	x	0
1	0	D_0
1	1	D_1

x = Don't care

Multiplexers

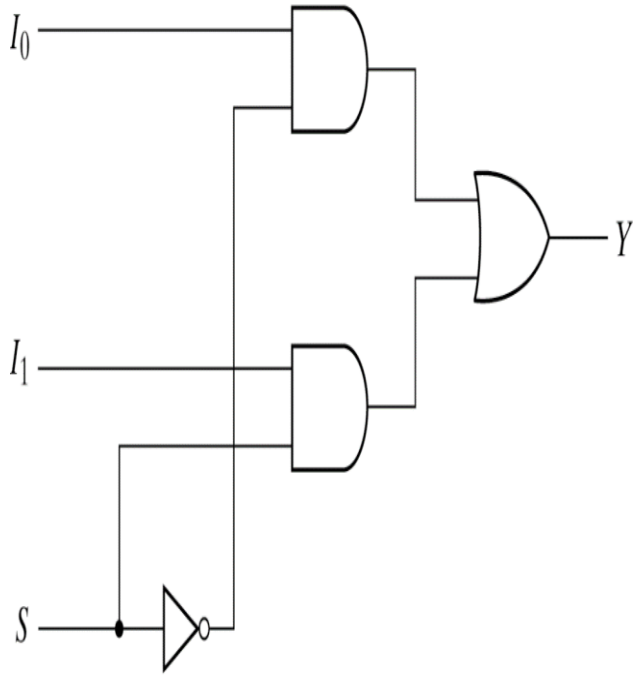
$S = 0, Y = I_0$

$S = 1, Y = I_1$

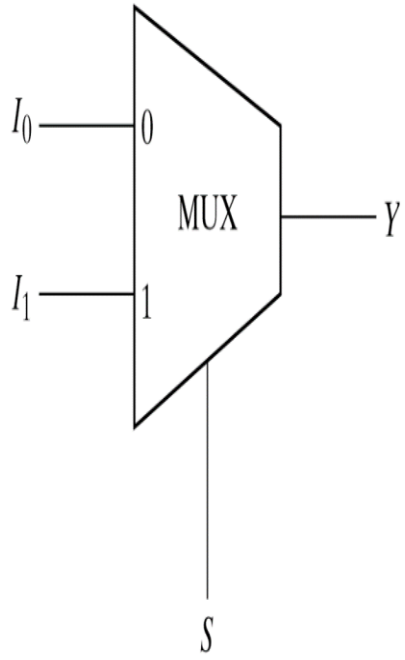
Truth Table→

S	Y
0	I_0
1	I_1

$$Y = S'I_0 + SI_1$$



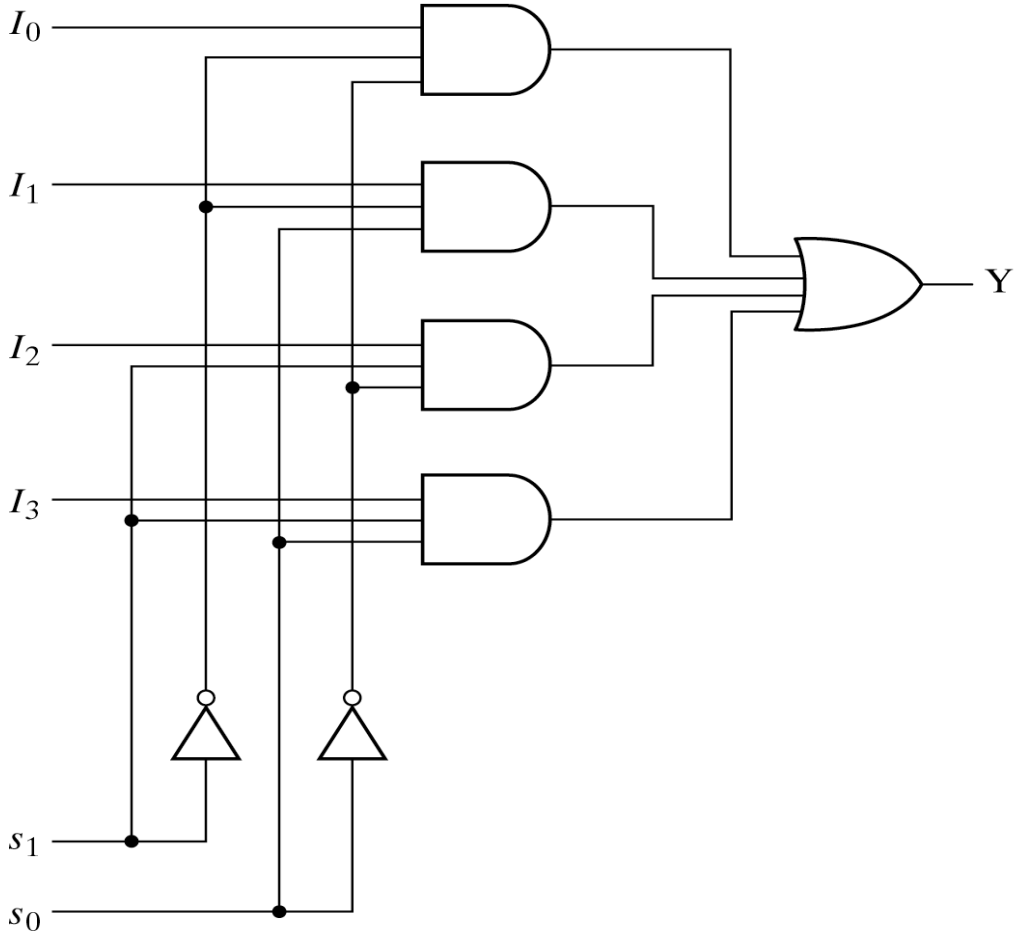
(a) Logic diagram



(b) Block diagram

Fig. 4-24 2-to-1-Line Multiplexer

4-to-1 Line Multiplexer



(a) Logic diagram

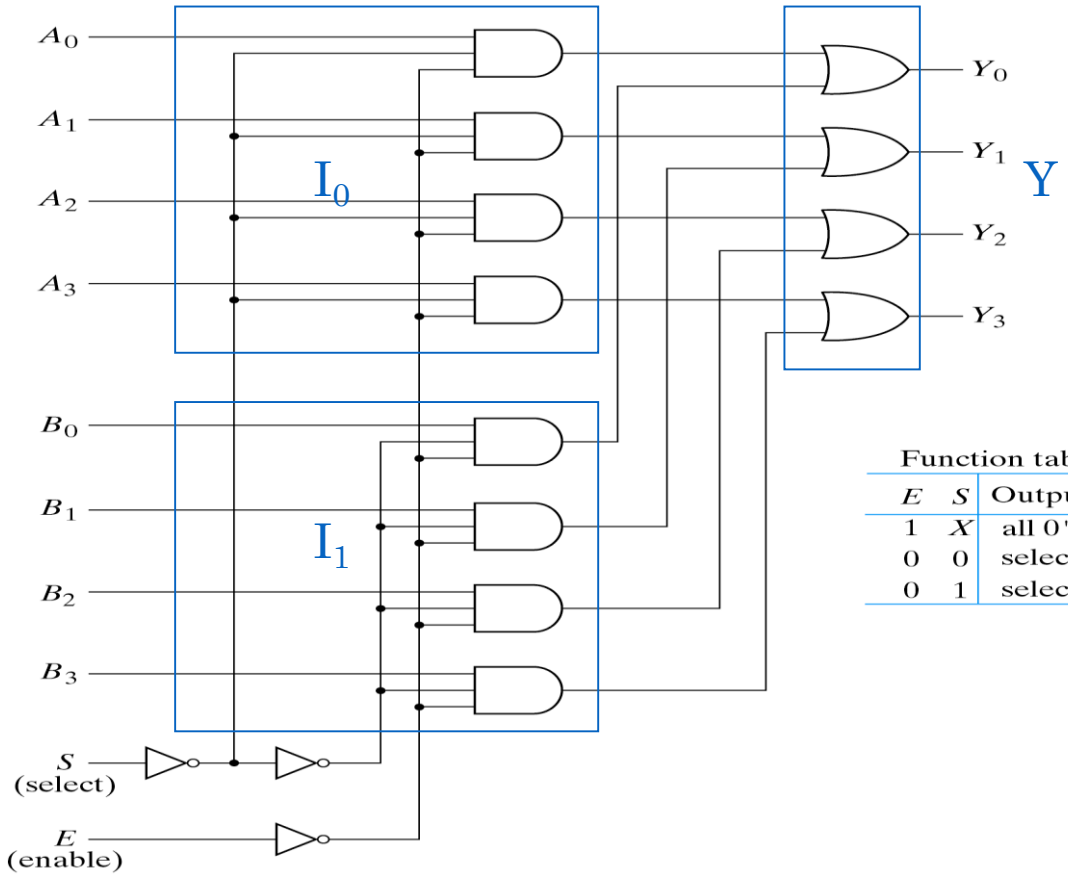
s_1	s_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

(b) Function table

Fig. 4-25 4-to-1-Line Multiplexer

Quadruple 2-to-1 Line Multiplexer

- Multiplexer circuits can be combined with common selection inputs to provide multiple-bit selection logic. Compare with Fig4-24.



Function table

E	S	Output Y
1	X	all 0's
0	0	select A
0	1	select B

Fig. 4-26 Quadruple 2-to-1-Line Multiplexer

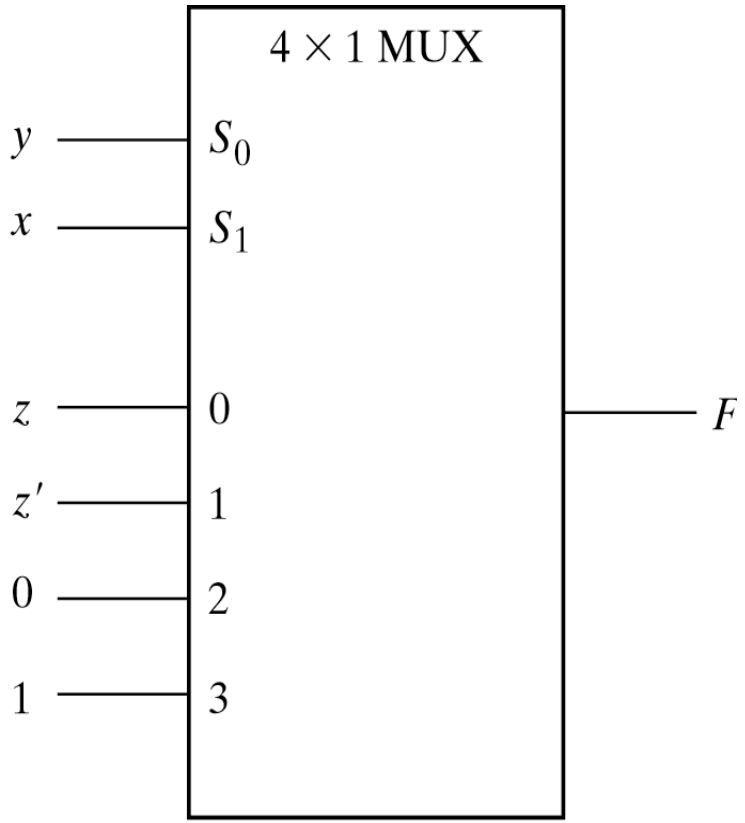
Boolean function implementation

- A more efficient method for implementing a Boolean function of n variables with a multiplexer that has n-1 selection inputs.

$$F(x, y, z) = \Sigma(1,2,6,7)$$

x	y	z	F	
0	0	0	0	$F = z$
0	0	1	1	
0	1	0	1	$F = z'$
0	1	1	0	
1	0	0	0	$F = 0$
1	0	1	0	
1	1	0	1	$F = 1$
1	1	1	1	

(a) Truth table



(b) Multiplexer implementation

Fig. 4-27 Implementing a Boolean Function with a Multiplexer

4-input function with a multiplexer

$F(A, B, C, D) = \Sigma(1, 3, 4, 11, 12, 13, 14, 15)$

A	B	C	D	F	
0	0	0	0	0	$F = D$
0	0	0	1	1	
0	0	1	0	0	$F = D$
0	0	1	1	1	
0	1	0	0	1	$F = D'$
0	1	0	1	0	
0	1	1	0	0	$F = 0$
0	1	1	1	0	
1	0	0	0	0	$F = 0$
1	0	0	1	0	
1	0	1	0	0	$F = D$
1	0	1	1	1	
1	1	0	0	1	$F = 1$
1	1	0	1	1	
1	1	1	0	1	$F = 1$
1	1	1	1	1	

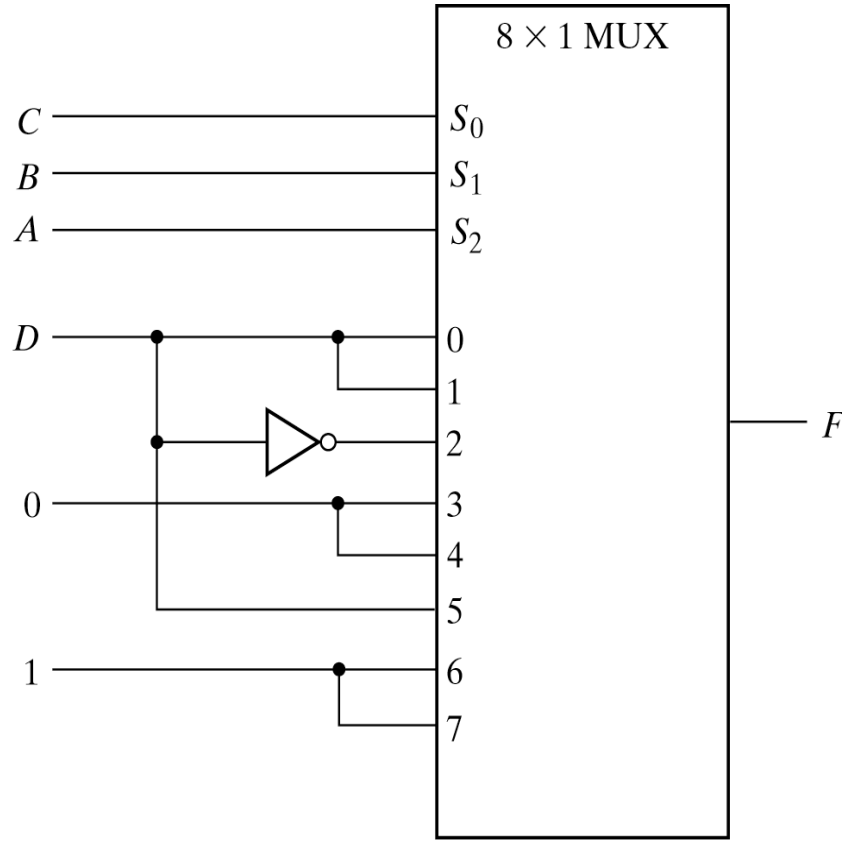
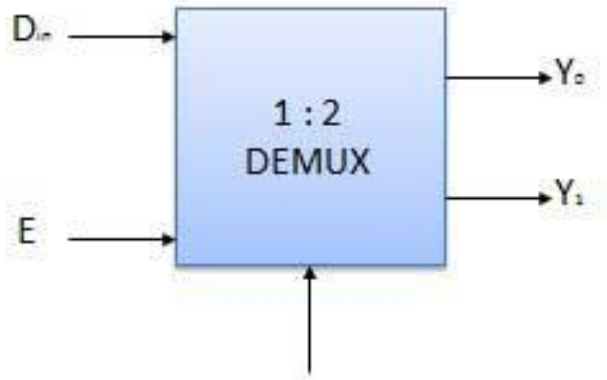


Fig. 4-28 Implementing a 4-Input Function with a Multiplexer

Demultiplexers

- A demultiplexer performs the reverse operation of a multiplexer i.e. it receives one input and distributes it over several outputs. It has only one input, n outputs, m select input. At a time only one output line is selected by the select lines and the input is transmitted to the selected output line. A de-multiplexer is equivalent to a single pole multiple way switch as shown in fig.
- Demultiplexers comes in multiple variations.
  - 1 : 2 demultiplexer
  - 1 : 4 demultiplexer
  - 1 : 16 demultiplexer
  - 1 : 32 demultiplexer

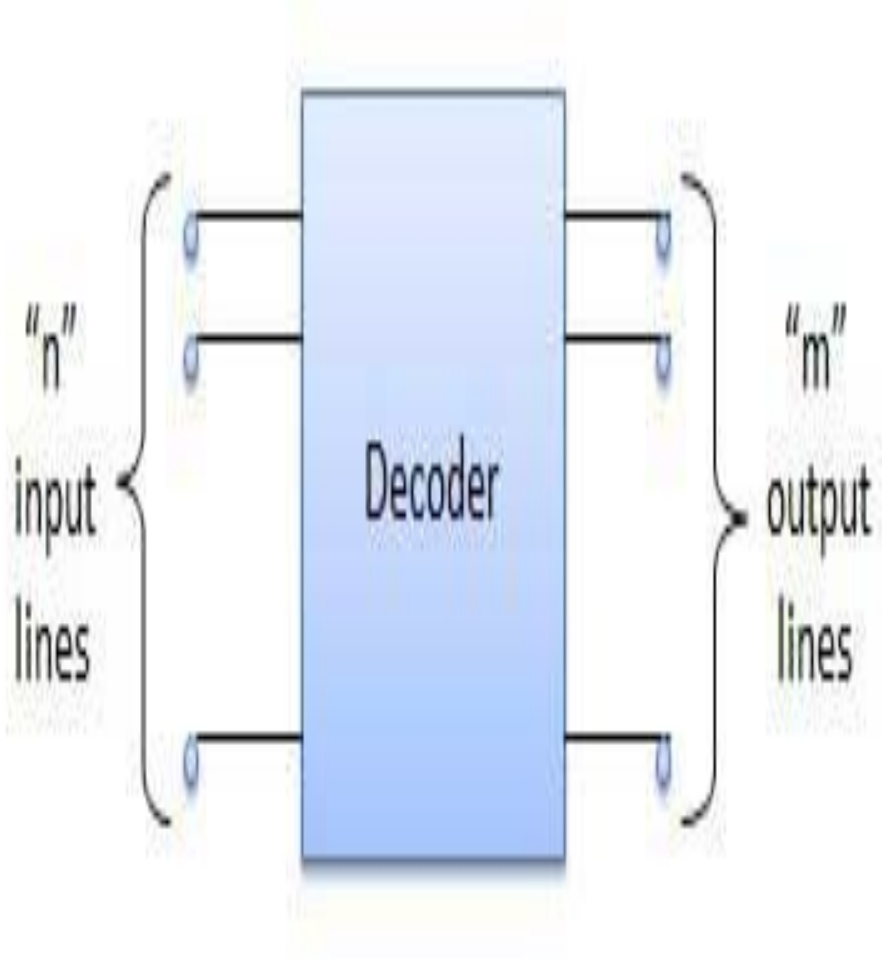


Enable	Select	Output	
E	S	Y0	Y1
0	x	0	0
1	0	0	Din
1	1	Din	0

x = Don't care

Decoder

A decoder is a combinational circuit. It has n input and to a maximum m = 2n outputs. Decoder is identical to a demultiplexer without any data input. It performs operations which are exactly opposite to those of an encoder.



Examples of Decoders are following.

- Code converters
- BCD to seven segment decoders
- Nixie tube decoders
- Relay actuator

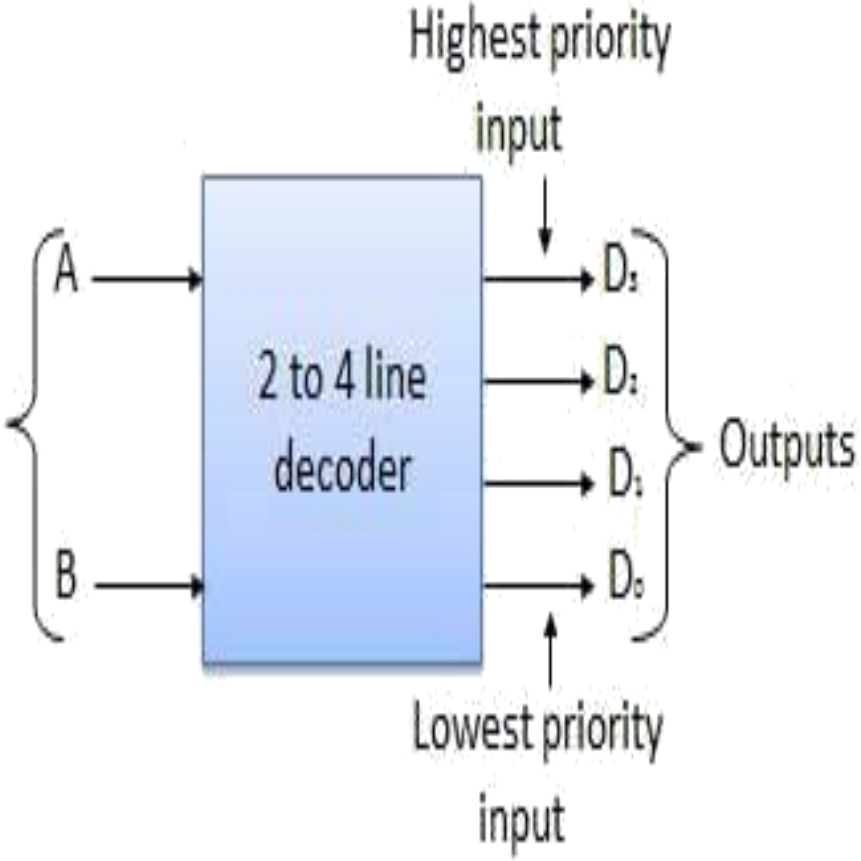
4-8. Decoders

- The decoder is called n-to-m-line decoder, where $m \leq 2^n$.
- the decoder is also used in conjunction with other code converters such as a BCD-to-seven_segment decoder.
- 3-to-8 line decoder: For each possible input combination, there are seven outputs that are equal to 0 and only one that is equal to 1.

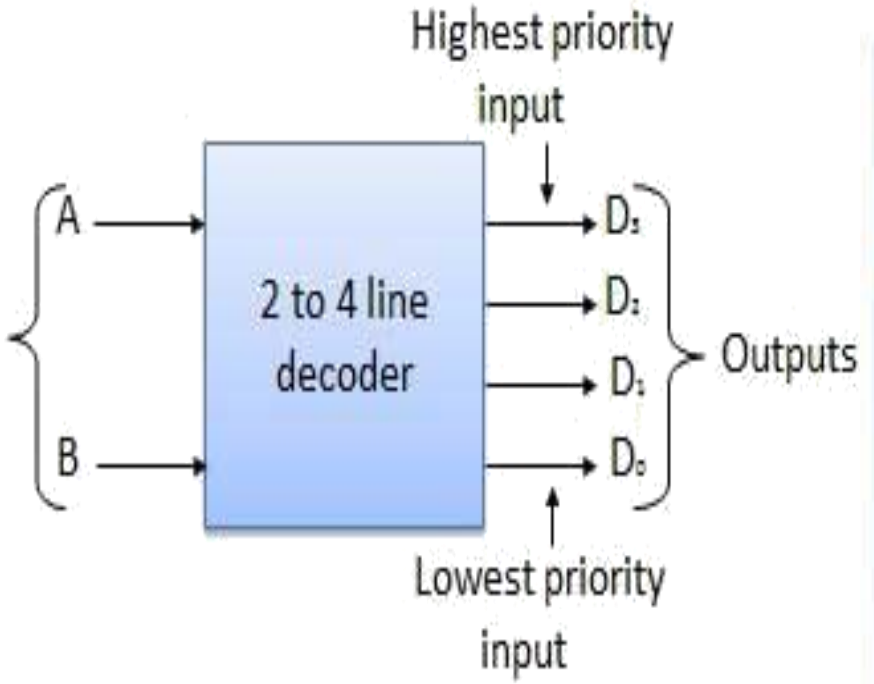
2 to 4 Line Decoder

The block diagram of 2 to 4 line decoder is shown in the fig. A and B are the two inputs where D through D are the four outputs. Truth table explains the operations of a decoder.

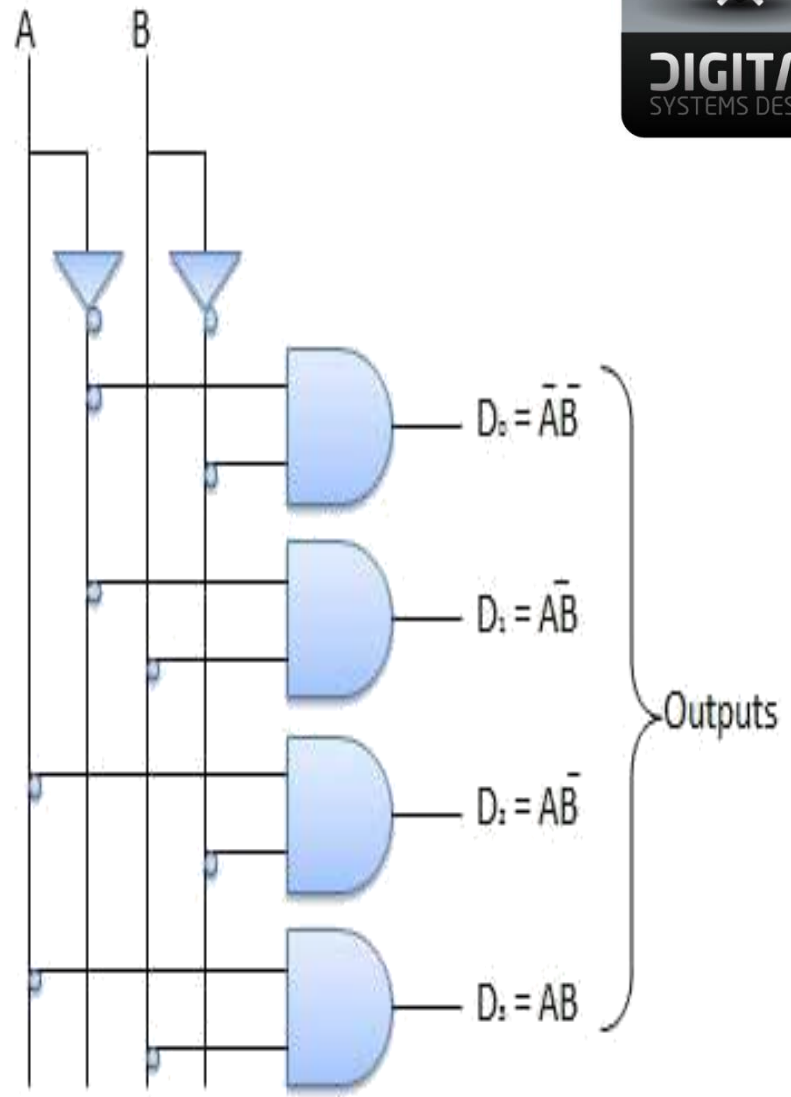
It shows that each output is 1 for only a specific combination of inputs.



2 to 4 Line Decoder



Inputs		Output			
A	B	D_0	D_1	D_2	D_3
0	0	1	0	0	0
0	1	0	1	0	0
0	1	0	0	1	0
1	1	0	0	0	1



Decoders

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Implementation and truth table

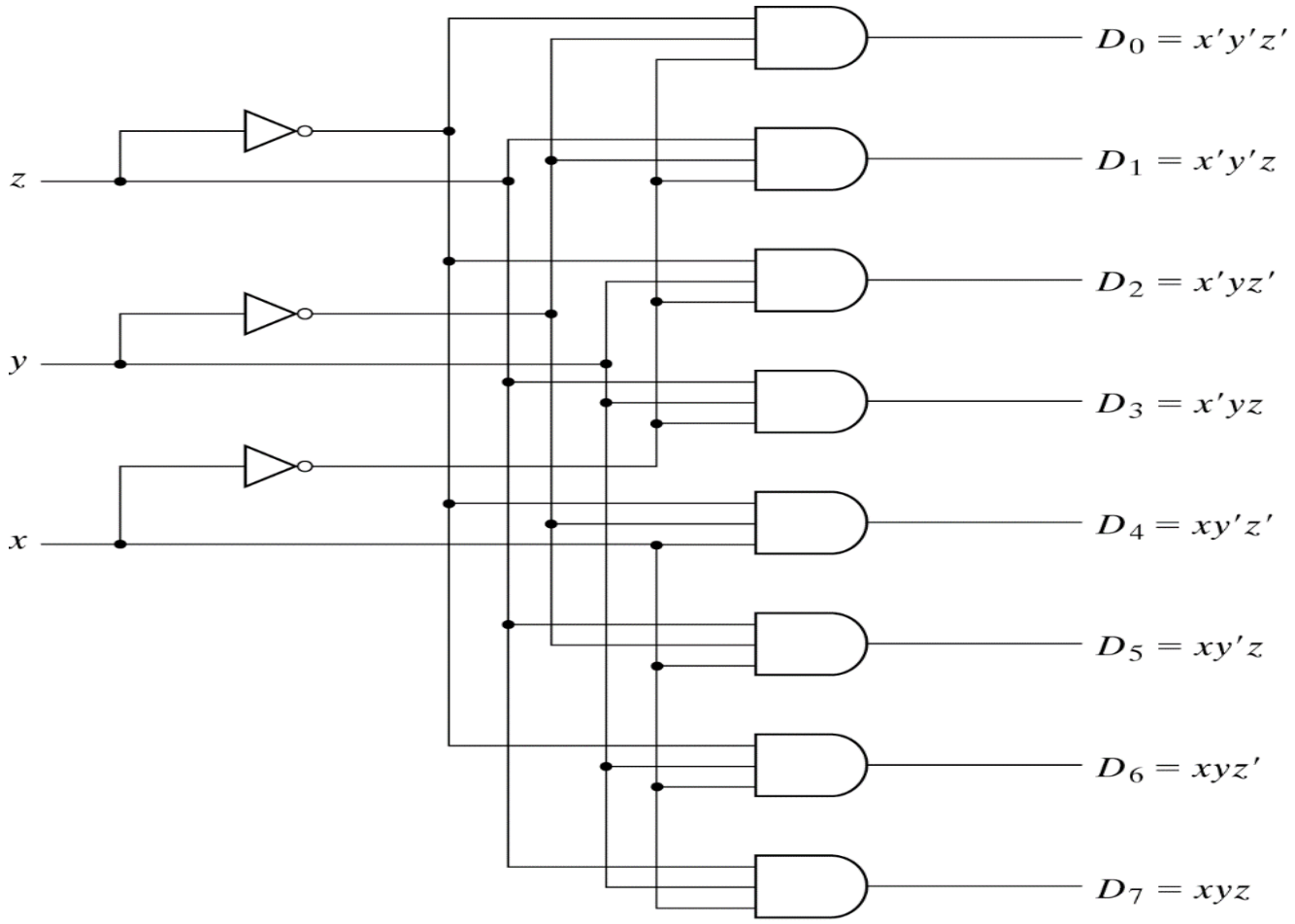


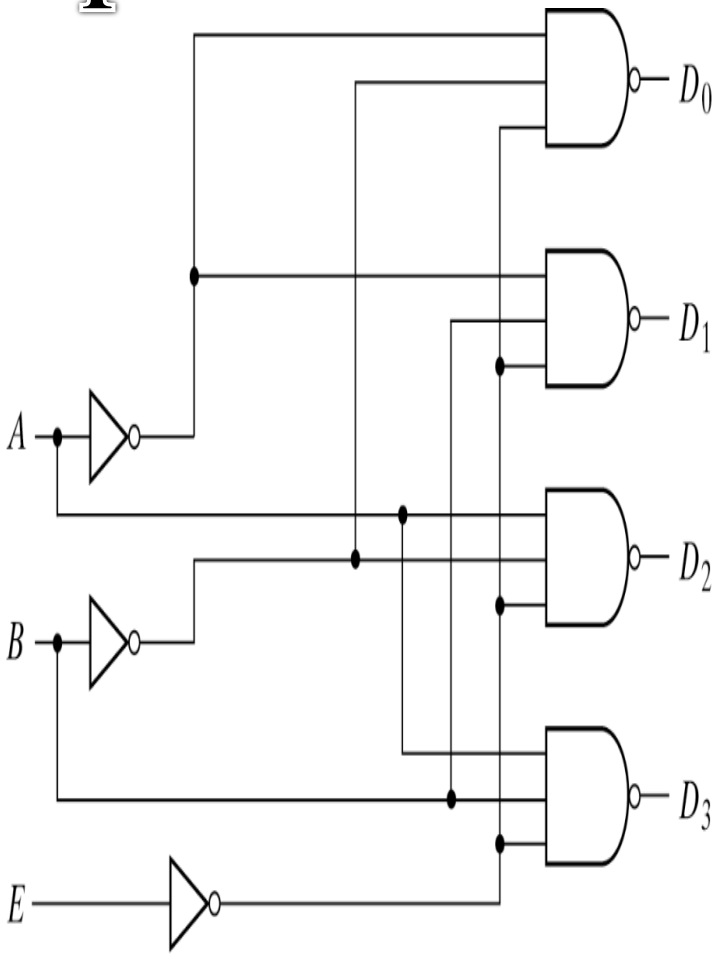
Fig. 4-18 3-to-8-Line Decoder

Table 4-6
Truth Table of a 3-to-8-Line Decoder

Inputs			Outputs							
x	y	z	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Decoder with enable input

- Some decoders are constructed with NAND gates, it becomes more economical to generate the decoder minterms in their complemented form.
- As indicated by the truth table , only one output can be equal to 0 at any given time, all other outputs are equal to 1.



(a) Logic diagram

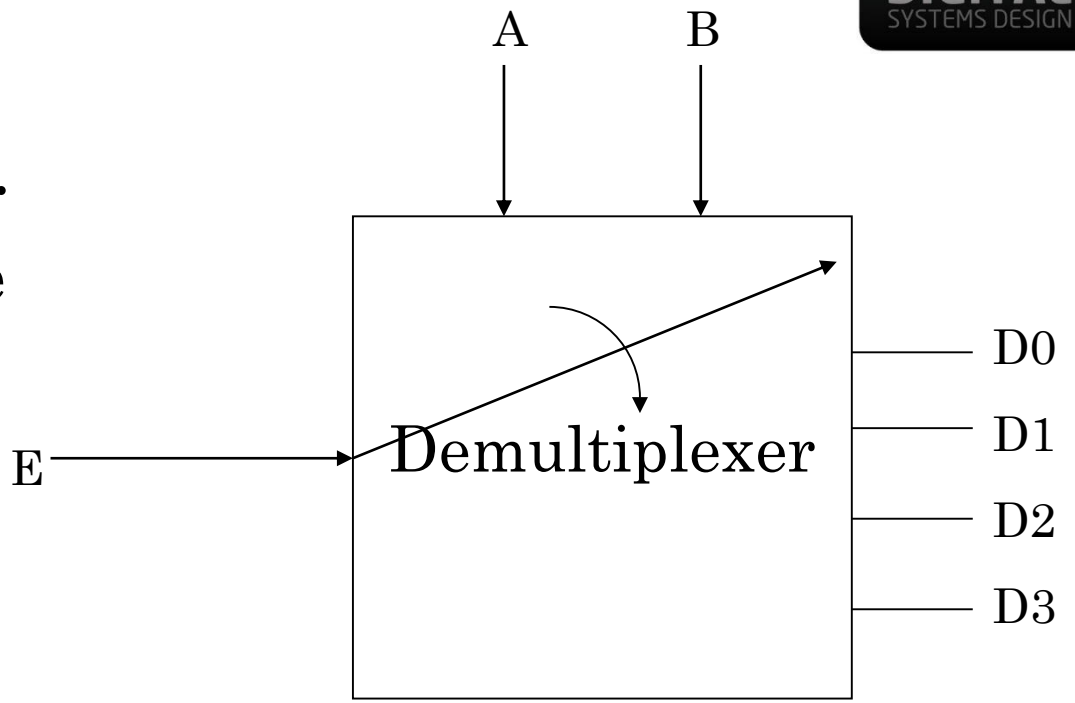
E	A	B	D_0	D_1	D_2	D_3
1	X	X	1	1	1	1
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0

(b) Truth table

Fig. 4-19 2-to-4-Line Decoder with Enable Input

Demultiplexer

- A decoder with an enable input is referred to as a decoder/demultiplexer.
- The truth table of demultiplexer is the same with decoder.



3-to-8 decoder with enable implement the 4-to-16 decoder

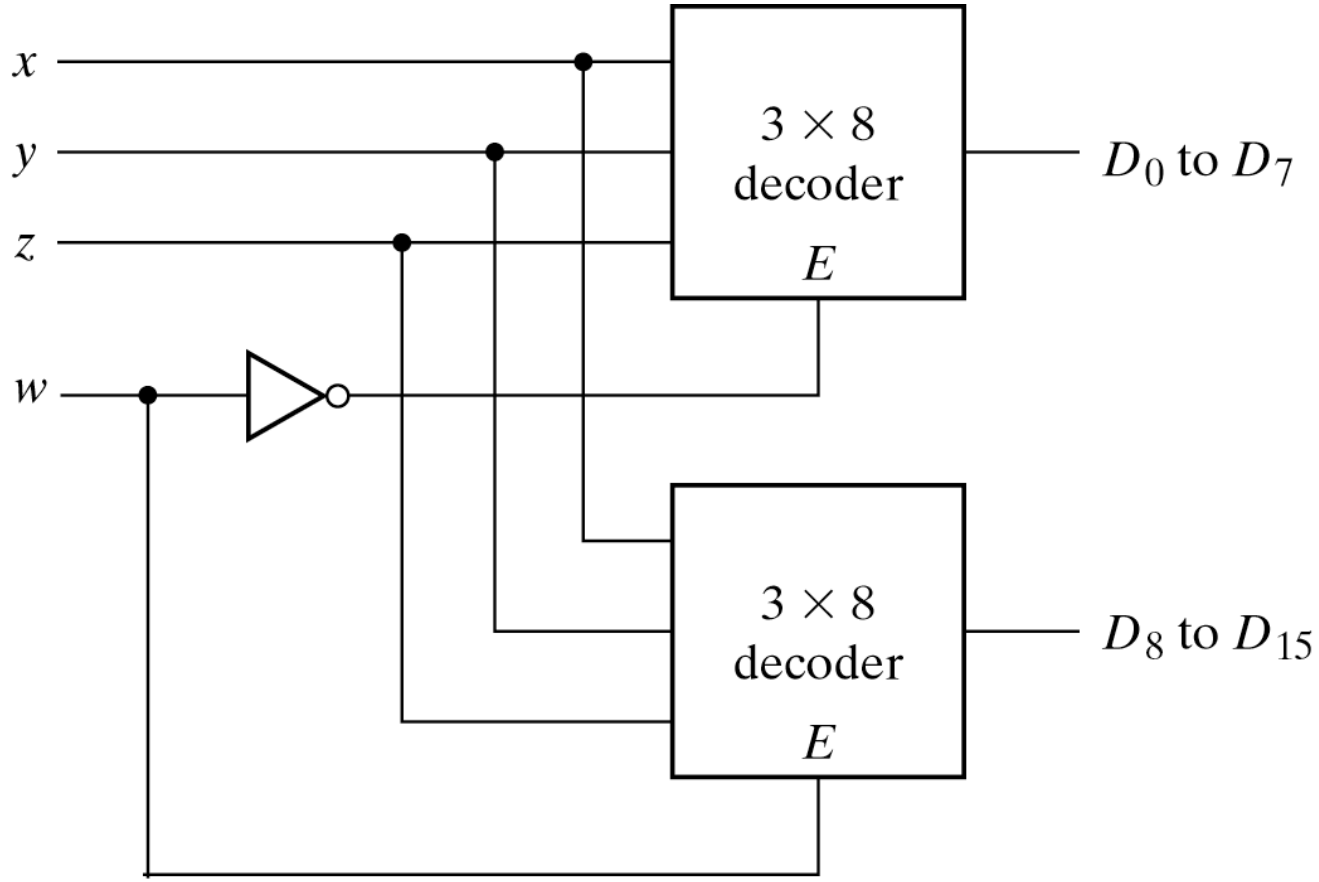
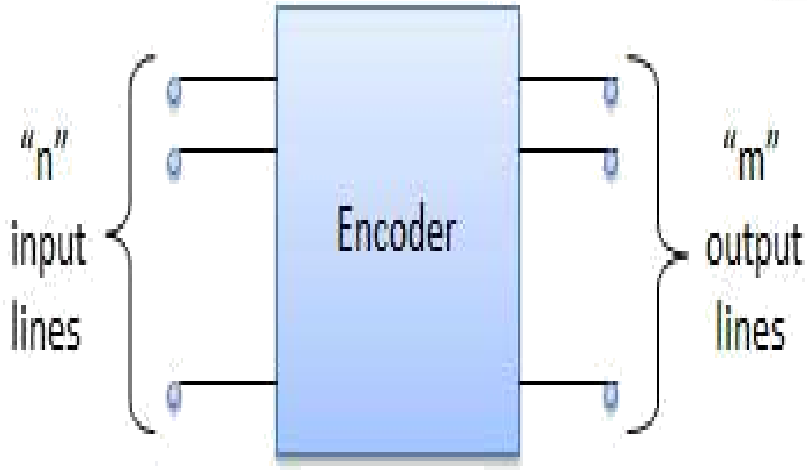


Fig. 4-20 4 × 16 Decoder Constructed with Two 3 × 8 Decoders

Encoder

- Encoder is a combinational circuit which is designed to perform the inverse operation of the decoder.
- An encoder has n number of input lines and m number of output lines.
- An encoder produces an m bit binary code corresponding to the digital input number.
- The encoder accepts an n input digital word and converts it into an m bit another digital word.



Examples of Encoders are following.

- **Priority encoders**
- **Decimal to BCD encoder**
- **Octal to binary encoder**
- **Hexadecimal to binary encoder**

Encoders

- An encoder is the inverse operation of a decoder.
- We can derive the Boolean functions by table 4-7

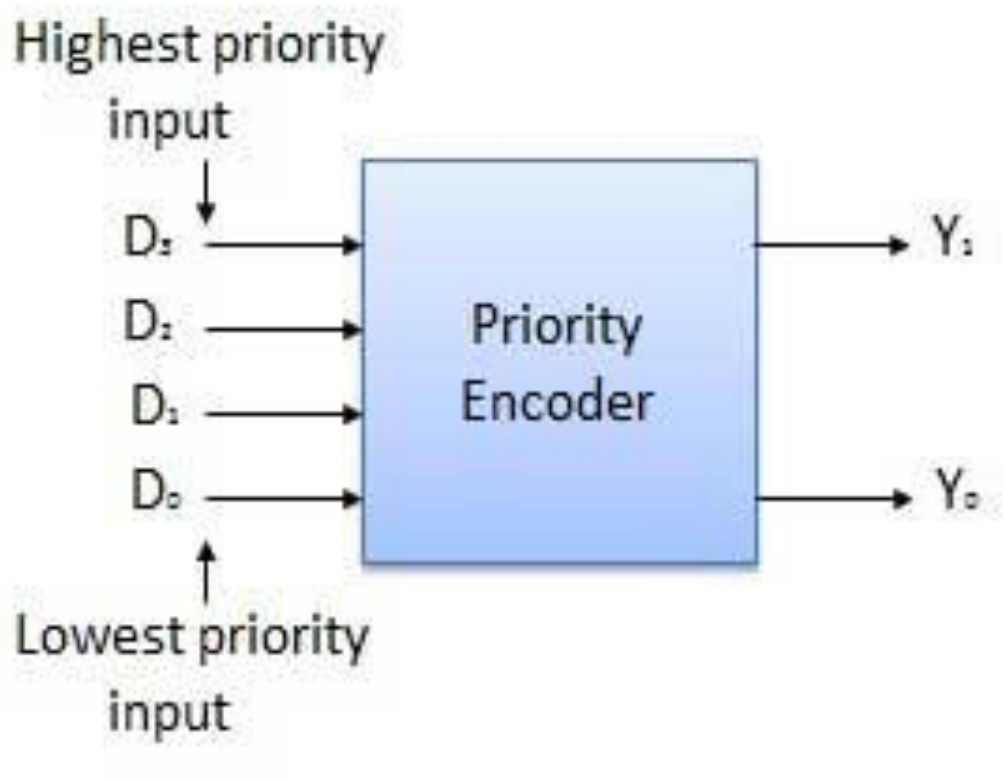
$$z = D_1 + D_3 + D_5 + D_7$$
$$y = D_2 + D_3 + D_6 + D_7$$
$$x = D_4 + D_5 + D_6 + D_7$$

Table 4-7
Truth Table of Octal-to-Binary Encoder

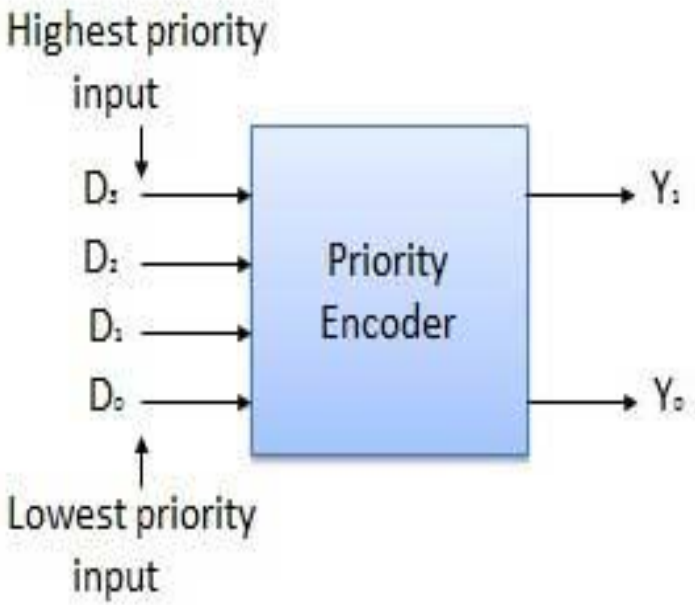
Inputs								Outputs		
D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	x	y	z
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

Priority Encoder

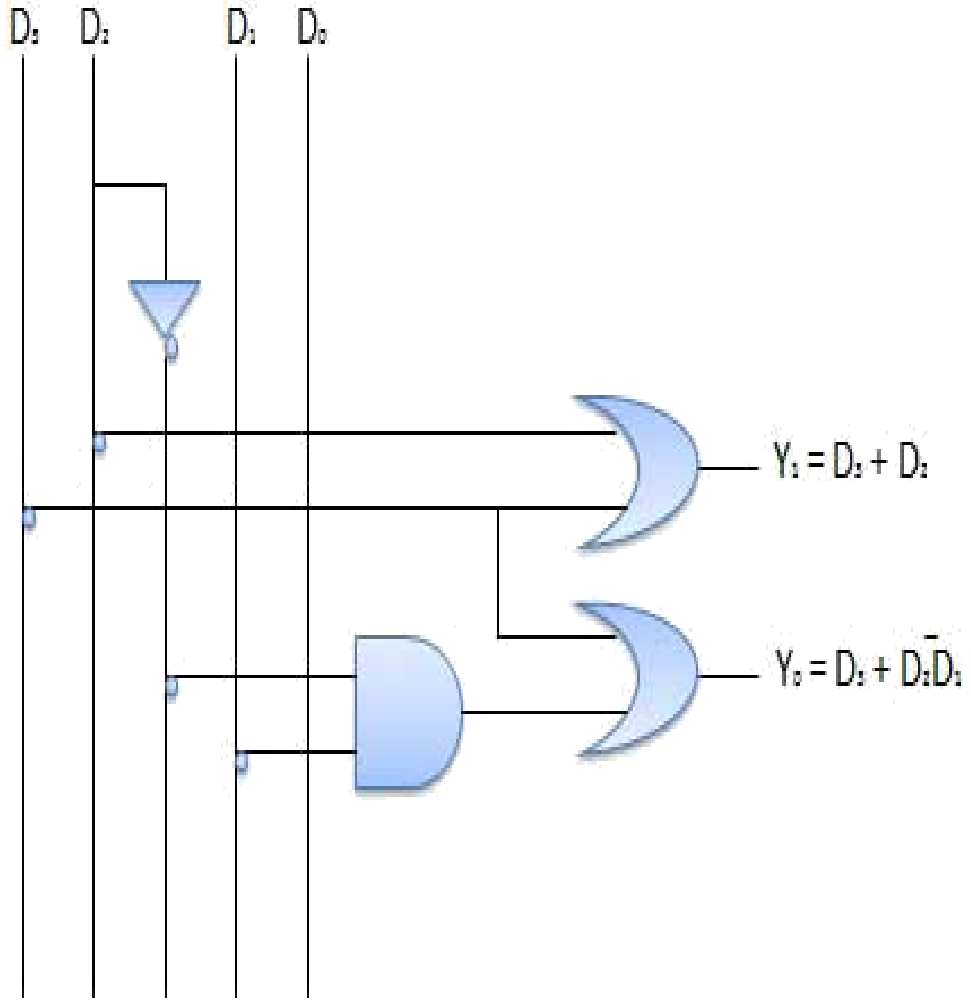
- This is a special type of encoder. Priority is given to the input lines. If two or more input line are 1 at the same time, then the input line with highest priority will be considered.
- There are four input D_0, D_1, D_2, D_3 and two output Y_0, Y_1. Out of the four input D_3 has the highest priority and D_0 has the lowest priority.
- That means if $D_3 = 1$ then Y_1 $Y_1 = 11$ irrespective of the other inputs. Similarly if $D_3 = 0$ and $D_2 = 1$ then Y_1 $Y_0 = 10$ irrespective of the other inputs.



Priority Encoder



Highest	Inputs		Lowest	Outputs	
D_3	D_2	D_1	D_0	Y_0	Y_1
0	0	0	0	x	x
0	0	0	1	0	0
0	0	1	x	0	1
0	1	x	x	1	0
1	x	x	x	1	1



Priority encoder

- If two inputs are active simultaneously, the output produces an undefined combination. We can establish an input priority to ensure that only one input is encoded.
- Another ambiguity in the octal-to-binary encoder is that an output with all 0's is generated when all the inputs are 0; the output is the same as when D_0 is equal to 1.
- The discrepancy tables on Table 4-7 and Table 4-8 can resolve aforesaid condition by providing one more output to indicate that at least one input is equal to 1.

Priority encoder

V=0→no valid inputs

V=1→valid inputs

X's in output columns represent don't-care conditions

X's in the input columns are useful for representing a truth table in condensed form.

Instead of listing all 16 minterms of four variables.

Table 4-8

Truth Table of a Priority Encoder

Inputs				Outputs		
D_0	D_1	D_2	D_3	x	y	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

4-input priority encoder

- Implementation of table 4-8

$x = D_2 + D_3$

$y = D_3 + D_1D'_2$

$V = D_0 + D_1 + D_2 + D_3$

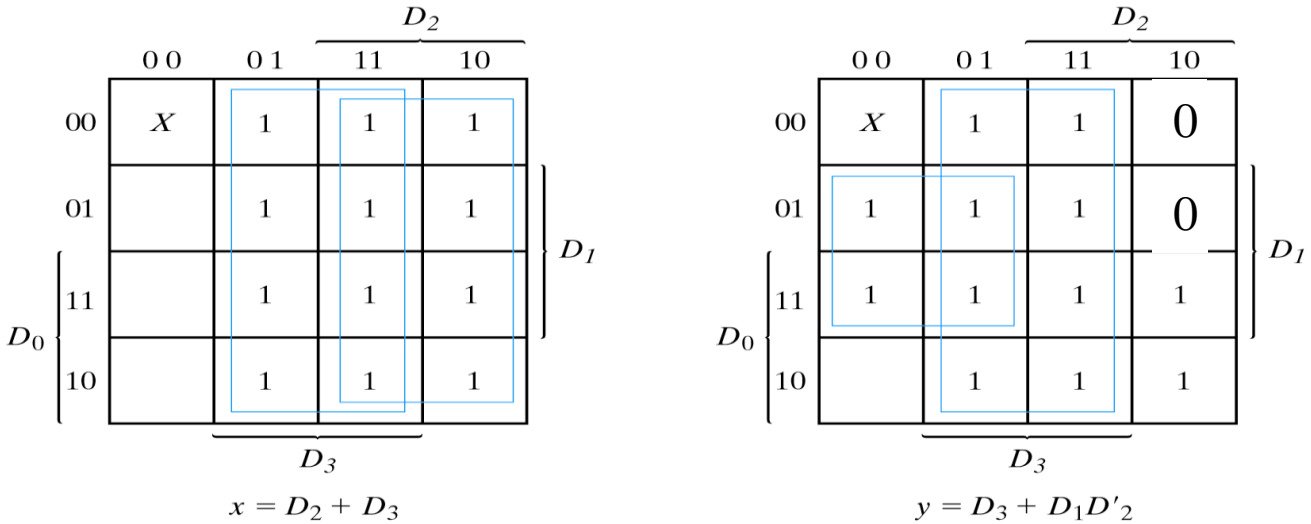


Fig. 4-22 Maps for a Priority Encoder

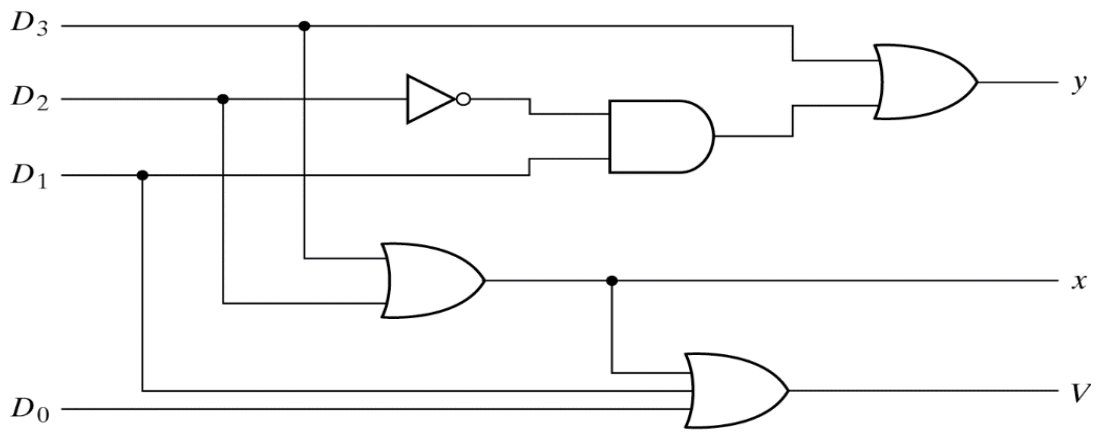


Fig. 4-23 4-Input Priority Encoder

Three-State Gates

- A multiplexer can be constructed with three-state gates.

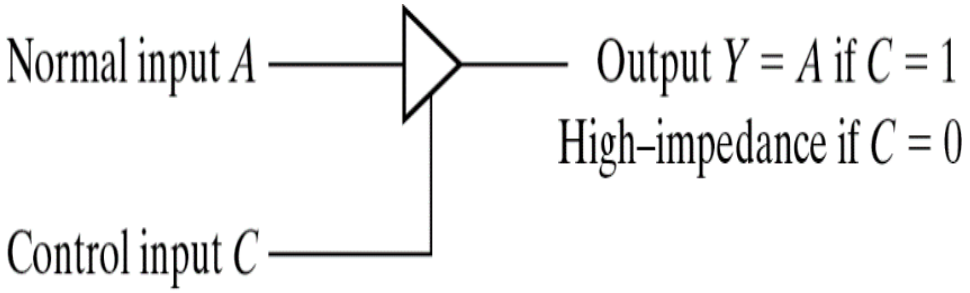
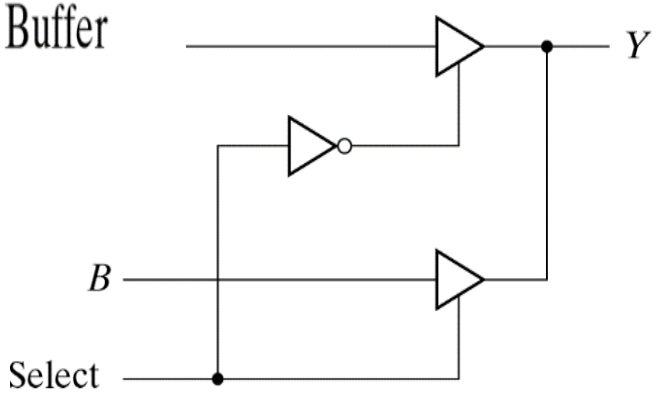
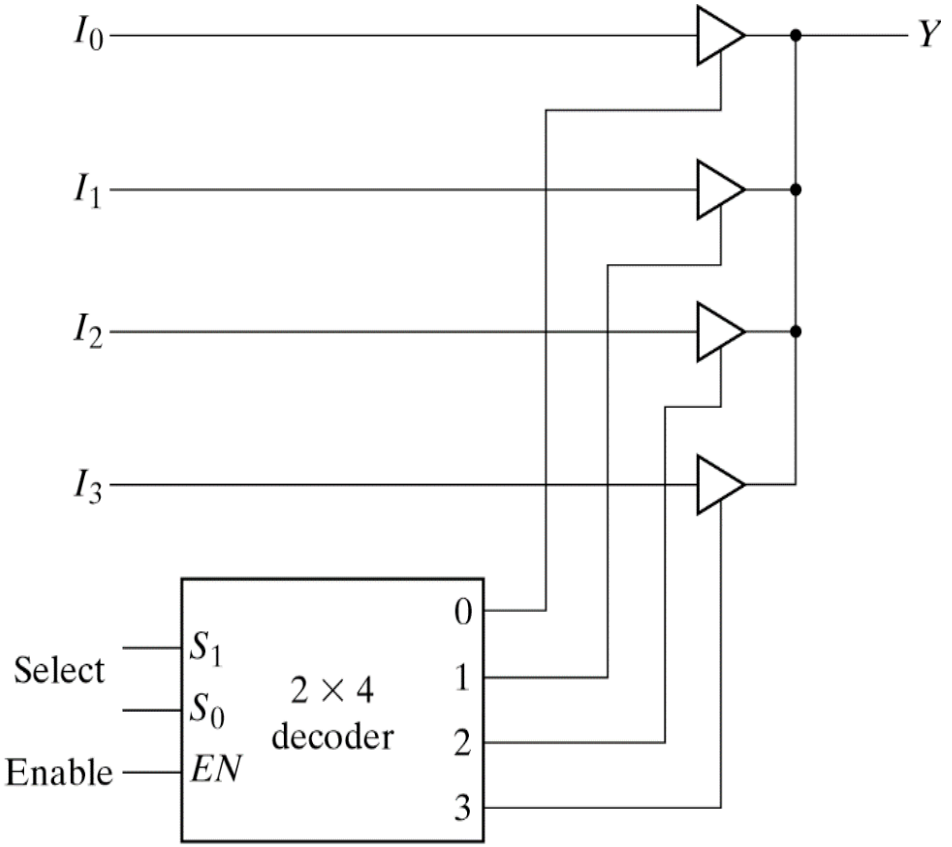


Fig. 4-29 Graphic Symbol for a Three-State Buffer



(a) 2-to-1- line mux



(b) 4 - to - 1 line mux

Fig. 4-30 Multiplexers with Three-State Gates

Gate-level Modeling

- A circuit is specified by its logic gates and their interconnection.
- Verilog recognizes 12 basic gates as predefined primitives.
- The logic values of each gate may be 1, 0, x(unknown), z(high-impedance).

Table 4-9
Truth Table for Predefined Primitive Gates

and	0	1	x	z
0	0	0	0	0
1	0	1	x	x
x	0	x	x	x
z	0	x	x	x

or	0	1	x	z
0	0	1	x	x
1	1	1	1	1
x	x	1	x	x
z	x	1	x	x

xor	0	1	x	z
0	0	1	x	x
1	1	0	x	x
x	x	x	x	x
z	x	x	x	x

not	input	output
	0	1
	1	0
	x	x
	z	x

Reference

1.Mano book
2.https://www.tutorialspoint.com/computer_logical_organization/combinational_circuits.htm