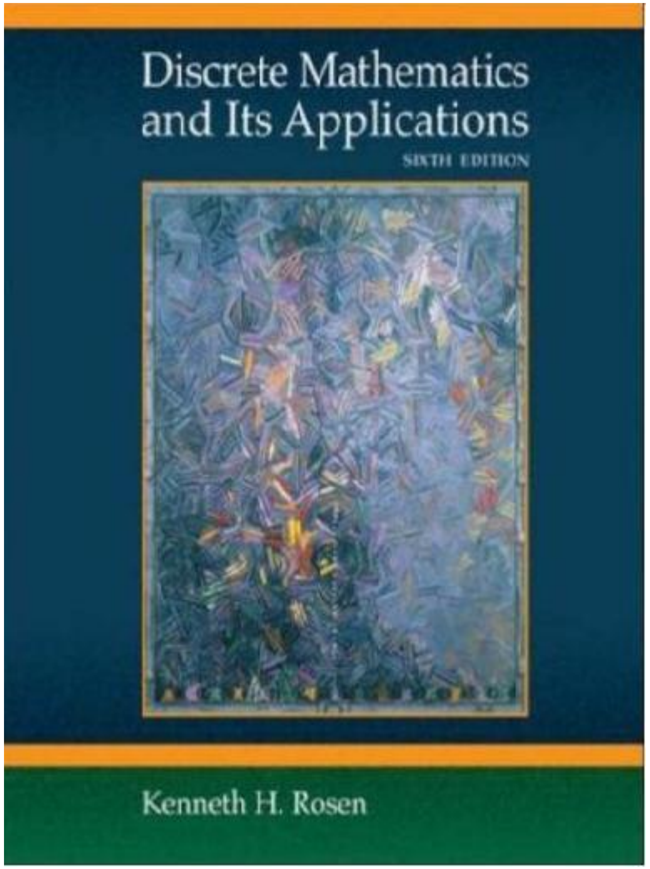
Jiangxi University of Science and Technology

Discrete Mathematics and Its Applications

Lecture 6:
Example



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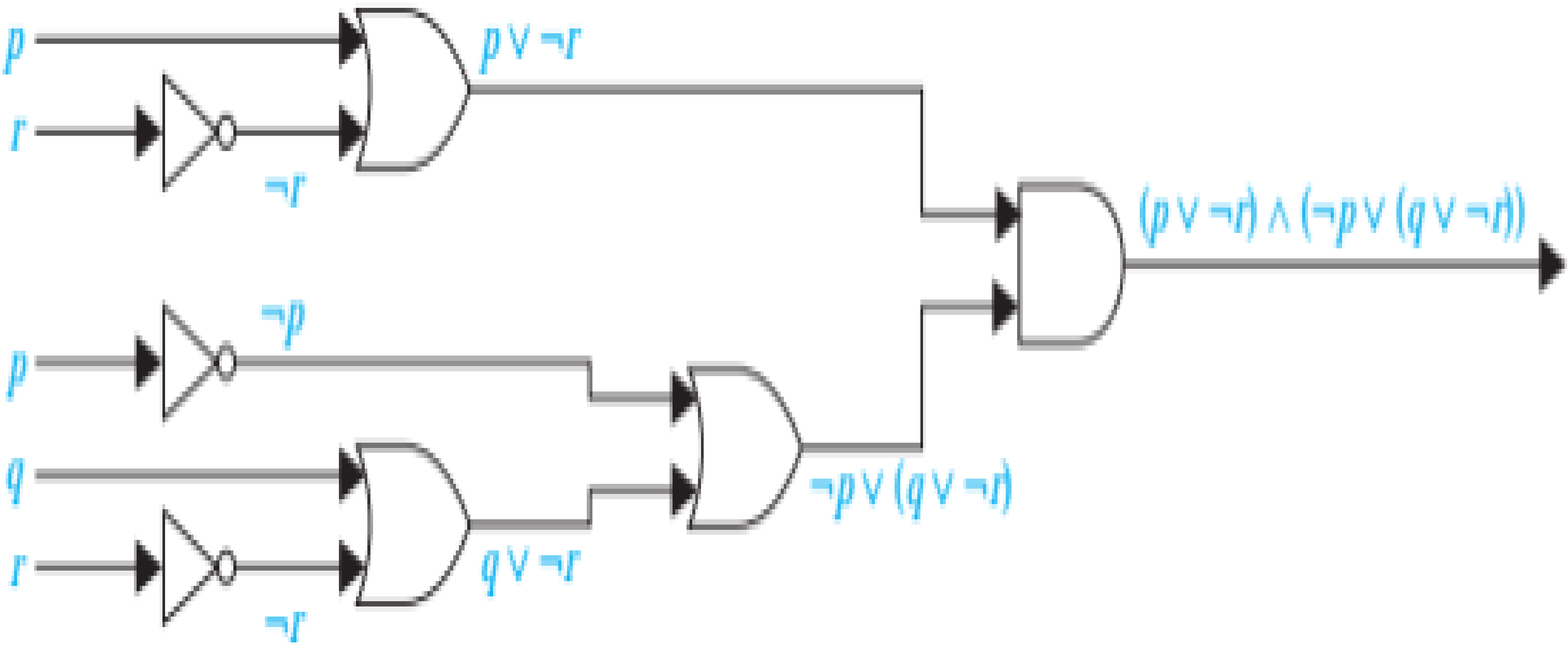


p
r
¬r
p ∨ ¬r
(p ∨ ¬r) ∧ (¬p ∨ (q ∨ ¬r))
¬p
p
q
r
¬r
q ∨ ¬r
¬p ∨ (q ∨ ¬r)

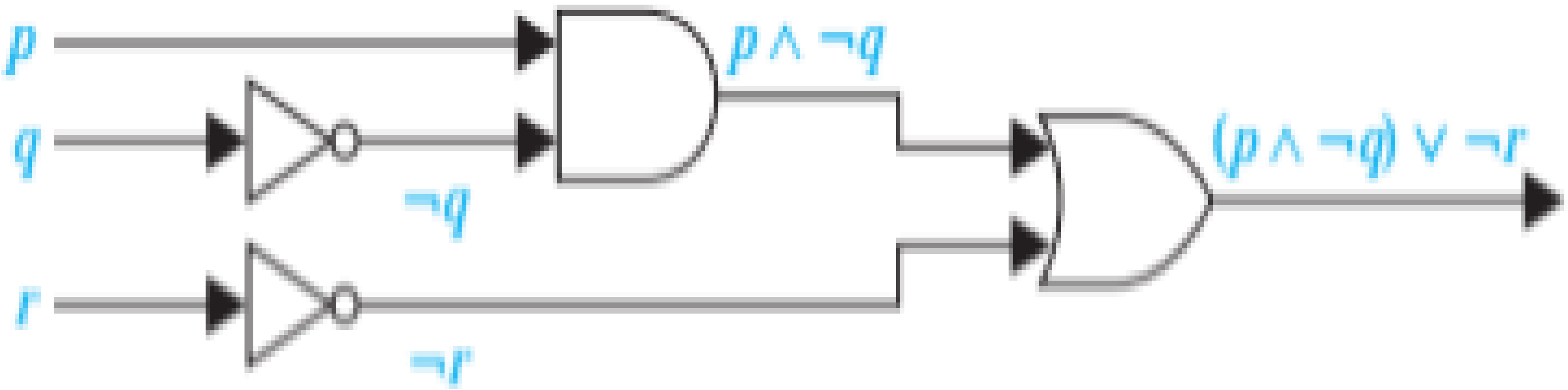
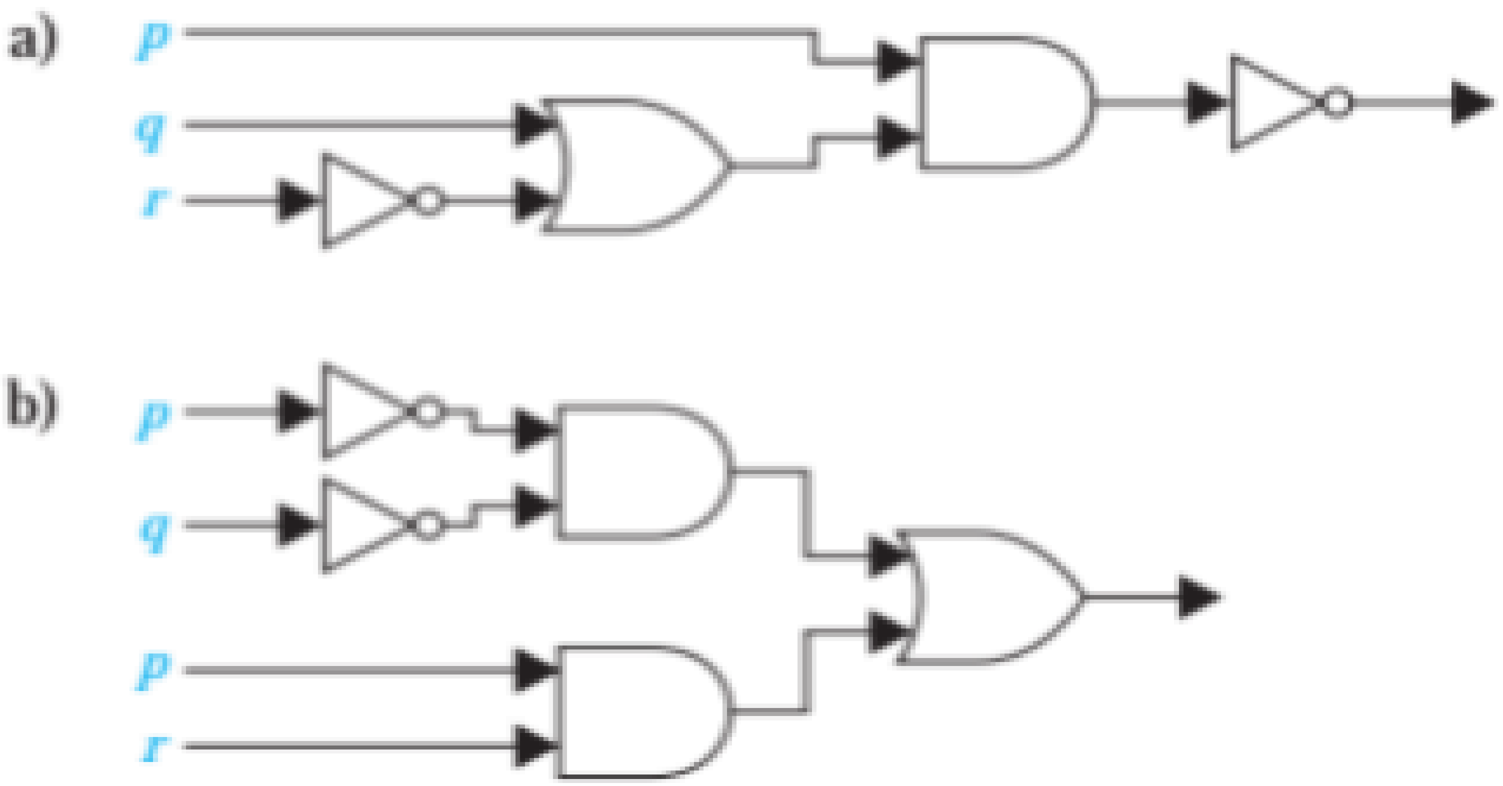


FIGURE 2 **A combinatorial circuit.**

41. Find the output of each of these combinatorial circuits.

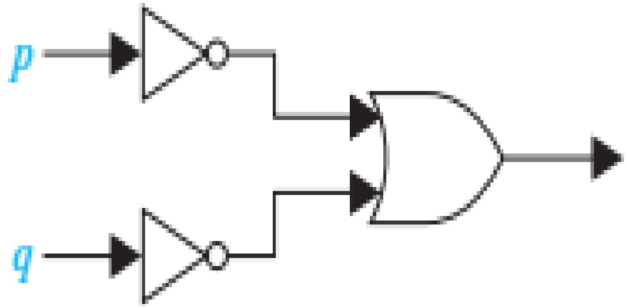


41. a) The output of the OR gate is $q \vee \neg r$. Therefore the output of the AND gate is $p \wedge (q \vee \neg r)$. Therefore the output of this circuit is $\neg(p \wedge (q \vee \neg r))$.

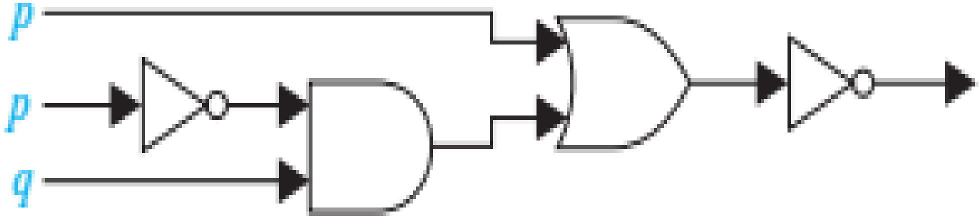
b) The output of the top AND gate is $(\neg p) \wedge (\neg q)$. The output of the bottom AND gate is $p \wedge r$. Therefore the output of this circuit is $((\neg p) \wedge (\neg q)) \vee (p \wedge r)$.

40. Find the output of each of these combinatorial circuits.

a)



b)



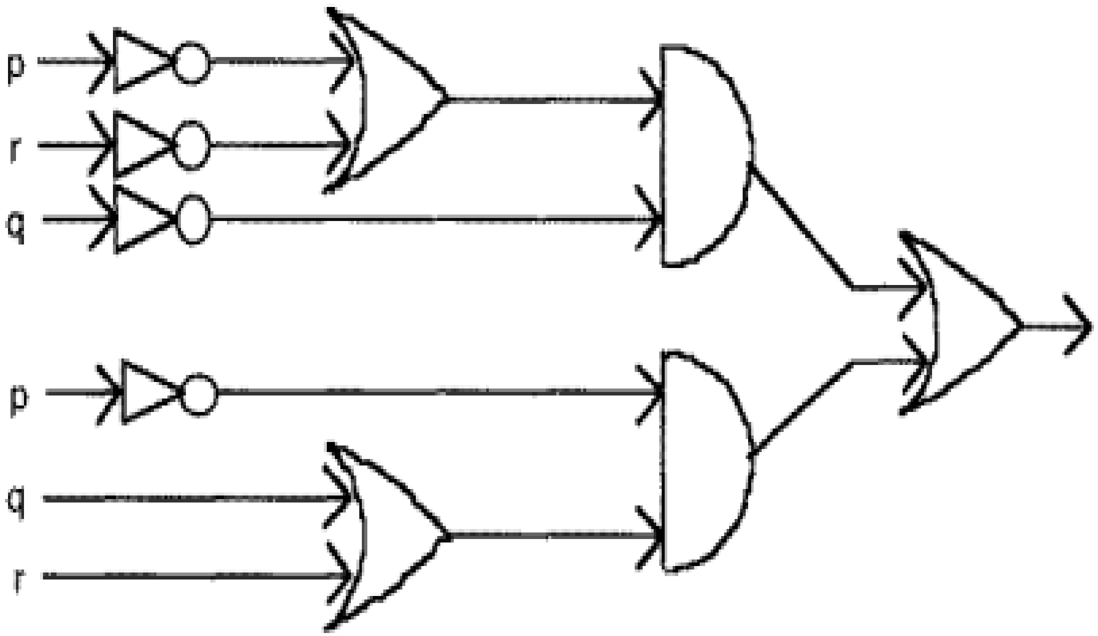
42. What is the value of x after each of these statements is encountered in a computer program, if $x = 1$ before the statement is reached?

a) **if** $x + 2 = 3$ **then** $x := x + 1$
b) **if** $(x + 1 = 3)$ *OR* $(2x + 2 = 3)$ **then** $x := x + 1$
c) **if** $(2x + 3 = 5)$ *AND* $(3x + 4 = 7)$ **then** $x := x + 1$
d) **if** $(x + 1 = 2)$ *XOR* $(x + 2 = 3)$ **then** $x := x + 1$
e) **if** $x < 2$ **then** $x := x + 1$

43. Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

a) 101 1110, 010 0001
b) 1111 0000, 1010 1010
c) 00 0111 0001, 10 0100 1000
d) 11 1111 1111, 00 0000 0000

43. We have the inputs come in from the left, in some cases passing through an inverter to form their negations. Certain pairs of them enter OR gates, and the outputs of these and other negated inputs enter AND gates. The outputs of these AND gates enter the final OR gate.



44. Evaluate each of these expressions.

a) $1\,1000 \wedge (0\,1011 \vee 1\,1011)$

b) $(0\,1111 \wedge 1\,0101) \vee 0\,1000$

c) $(0\,1010 \oplus 1\,1011) \oplus 0\,1000$

d) $(1\,1011 \vee 0\,1010) \wedge (1\,0001 \vee 1\,1011)$

41. Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

41. **a)** The output of the OR gate is $q \vee \neg r$. Therefore the output of the AND gate is $p \wedge (q \vee \neg r)$. Therefore the output of this circuit is $\neg(p \wedge (q \vee \neg r))$.

b) The output of the top AND gate is $(\neg p) \wedge (\neg q)$. The output of the bottom AND gate is $p \wedge r$. Therefore the output of this circuit is $((\neg p) \wedge (\neg q)) \vee (p \wedge r)$.

33. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \rightarrow (p \oplus q)$

b) $(p \oplus q) \rightarrow (p \wedge q)$

c) $(p \vee q) \oplus (p \wedge q)$

d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

f) $(p \oplus q) \rightarrow (p \oplus \neg q)$

34. Construct a truth table for each of these compound propositions.

a) $p \oplus p$

b) $p \oplus \neg p$

c) $p \oplus \neg q$

d) $\neg p \oplus \neg q$

e) $(p \oplus q) \vee (p \oplus \neg q)$

f) $(p \oplus q) \wedge (p \oplus \neg q)$

35. Construct a truth table for each of these compound propositions.

a) $p \rightarrow \neg q$

b) $\neg p \leftrightarrow q$

c) $(p \rightarrow q) \vee (\neg p \rightarrow q)$

d) $(p \rightarrow q) \wedge (\neg p \rightarrow q)$

e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

36. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \vee r$

b) $(p \vee q) \wedge r$

c) $(p \wedge q) \vee r$

d) $(p \wedge q) \wedge r$

e) $(p \vee q) \wedge \neg r$

f) $(p \wedge q) \vee \neg r$

37. Construct a truth table for each of these compound propositions.

a) $p \to (\neg q \lor r)$

b) $\neg p \to (q \to r)$

c) $(p \to q) \lor (\neg p \to r)$

d) $(p \to q) \land (\neg p \to r)$

e) $(p \leftrightarrow q) \lor (\neg q \leftrightarrow r)$

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

8. Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

a) $\neg p$ **b)** $p \vee q$ **c)** $p \rightarrow q$

d) $p \wedge q$ **e)** $p \leftrightarrow q$ **f)** $\neg p \rightarrow \neg q$

g) $\neg p \wedge \neg q$ **h)** $\neg p \vee (p \wedge q)$

Student

Page :54,Example:9

9. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

a) $\neg q$ b) $p \wedge q$ c) $\neg p \vee q$
d) $p \rightarrow \neg q$ e) $\neg q \rightarrow p$ f) $\neg p \rightarrow \neg q$
g) $p \leftrightarrow \neg q$ h) $\neg p \wedge (p \vee \neg q)$

Page :54,Example:9

9. **a)** We assume that this sentence is asserting that the same person has both talents. Therefore we can write $\exists x(P(x) \land Q(x))$.

b) Since "but" really means the same thing as "and" logically, this is $\exists x(P(x) \land \neg Q(x))$

c) This time we are making a universal statement: $\forall x(P(x) \lor Q(x))$

d) This sentence is asserting the nonexistence of anyone with either talent, so we could write it as $\neg\exists x(P(x) \lor Q(x))$. Alternatively, we can think of this as asserting that everyone fails to have either of these talents, and we obtain the logically equivalent answer $\forall x \neg(P(x) \lor Q(x))$. Failing to have either talent is equivalent to having neither talent (by De Morgan's law), so we can also write this as $\forall x((\neg P(x)) \land (\neg Q(x))$. Note that it would *not* be correct to write $\forall x((\neg P(x)) \lor (\neg Q(x))$ nor to write $\forall x \neg(P(x) \land Q(x))$.

Page :54,Example:7

7. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

   a) Quixote Media had the largest annual revenue.
   b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
   c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.
   d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
   e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

PAGE :54 Example:7

7. **a)** This statement is that for every x, if x is a comedian, then x is funny. In English, this is most simply stated, "Every comedian is funny."

b) This statement is that for every x in the domain (universe of discourse), x is a comedian *and* x is funny. In English, this is most simply stated, "Every person is a funny comedian." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of boring comedians; it's surely false, because there exist lots of x for which $C(x)$ is false. This illustrates the fact that you rarely want to use conjunctions with universal quantifiers.

c) This statement is that there exists an x in the domain such that if x is a comedian then x is funny. In English, this might be rendered, "There exists a person such that if s/he is a comedian, then s/he is funny." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of funny comedians; it's surely true, because there exist lots of x for which $C(x)$ is false (recall the definition of the truth value of $p \to q$). This illustrates the fact that you rarely want to use conditional statements with existential quantifiers.

d) This statement is that there exists an x in the domain such that x is a comedian and x is funny. In English, this might be rendered, "There exists a funny comedian" or "Some comedians are funny" or "Some funny people are comedians."

PAGE :54 , Example:9

9. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.
   - a) There is a student at your school who can speak Russian and who knows C++.
   - b) There is a student at your school who can speak Russian but who doesn't know C++.
   - c) Every student at your school either can speak Russian or knows C++.
   - d) No student at your school can speak Russian or knows C++.

9. a) We assume that this sentence is asserting that the same person has both talents. Therefore we can write $\exists x(P(x) \wedge Q(x))$.

b) Since "but" really means the same thing as "and" logically, this is $\exists x(P(x) \wedge \neg Q(x))$

c) This time we are making a universal statement: $\forall x(P(x) \vee Q(x))$

d) This sentence is asserting the nonexistence of anyone with either talent, so we could write it as $\neg\exists x(P(x) \vee Q(x))$. Alternatively, we can think of this as asserting that everyone fails to have either of these talents, and we obtain the logically equivalent answer $\forall x \neg(P(x) \vee Q(x))$. Failing to have either talent is equivalent to having neither talent (by De Morgan's law), so we can also write this as $\forall x((\neg P(x)) \wedge (\neg Q(x))$. Note that it would *not* be correct to write $\forall x((\neg P(x)) \vee (\neg Q(x))$ nor to write $\forall x \neg(P(x) \wedge Q(x))$.

Page :54

10. Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog, and a ferret.

b) All students in your class have a cat, a dog, or a ferret.

c) Some student in your class has a cat and a ferret, but not a dog.

d) No student in your class has a cat, a dog, and a ferret.

e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

Student

7. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$

b) $\forall x(C(x) \wedge F(x))$

c) $\exists x(C(x) \rightarrow F(x))$

d) $\exists x(C(x) \wedge F(x))$

Page :54

25. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) No one is perfect.

b) Not everyone is perfect.

c) All your friends are perfect.

d) At least one of your friends is perfect.

e) Everyone is your friend and is perfect.

f) Not everybody is your friend or someone is not perfect.

35. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x(x^2 \geq x)$

b) $\forall x(x > 0 \vee x < 0)$

c) $\forall x(x = 1)$