Не инвариантность уравнений максвелла

I'm no expert on the historical development of the subject, however I will offer a derivation.

Consider two frames of reference S and S', and suppose that S' moves with speed $\mathbf{v}$ with respect to S. Coordinates in S and S' are related by a Galileian transformation:

$$\begin{cases} t' = t \\ \mathbf{x}' = \mathbf{x} - \mathbf{v}t \end{cases}$$

To find how the fields transform, we note that a Lorentz transformation reduces to a Galileian transformation in the limit $c \to \infty$. In fact, under a Lorentz transformation the fields transform like:

$$\begin{cases} \mathbf{E}' = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - (\gamma - 1)(\mathbf{E} \cdot \hat{\mathbf{v}})\hat{\mathbf{v}} \\ \mathbf{B}' = \gamma\left(\mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}\right) - (\gamma - 1)(\mathbf{B} \cdot \hat{\mathbf{v}})\hat{\mathbf{v}} \end{cases}$$

Taking the limit $c \to \infty$ so that $\gamma \to 1$, we obtain the Galileian transformations of the fields:

$$\begin{cases} \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} \\ \mathbf{B}' = \mathbf{B} \end{cases}$$

We can then invert the transformation by sending $\mathbf{v} \to -\mathbf{v}$:

$$\begin{cases} \mathbf{E} = \mathbf{E}' - \mathbf{v} \times \mathbf{B}' \\ \mathbf{B} = \mathbf{B}' \end{cases}$$

By the same reasoning, can obtain the Galileian transformation of the sources:

$$\begin{cases} \mathbf{J} = \mathbf{J}' + \rho'\mathbf{v} \\ \rho = \rho' \end{cases}$$

We know that the fields and sources satisfy Maxwell's equations in S:

$$\begin{cases} \nabla \cdot \mathbf{E} = \rho/\epsilon_0 \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \mu_0\left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \end{cases}$$

Replacing the fields and sources in S with those in S' we obtain:

$$\begin{cases} \nabla \cdot (\mathbf{E}' - \mathbf{v} \times \mathbf{B}') = \rho'/\epsilon_0 \\ \nabla \cdot \mathbf{B}' = 0 \\ \nabla \times (\mathbf{E}' - \mathbf{v} \times \mathbf{B}') = -\frac{\partial \mathbf{B}'}{\partial t} \\ \nabla \times \mathbf{B}' = \mu_0\left(\mathbf{J}' + \rho'\mathbf{v} + \epsilon_0 \frac{\partial(\mathbf{E}' - \mathbf{v} \times \mathbf{B}')}{\partial t}\right) \end{cases}$$

As a last step, we need to replace derivatives in S with derivatives in S'. We have:

$$\begin{cases} \nabla = \nabla' \\ \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \mathbf{v} \cdot \nabla \end{cases}$$

Substituting and removing the primes and using vector calculus, we obtain:

$$\begin{cases} \nabla \cdot \mathbf{E} + \mathbf{v} \cdot (\nabla \times \mathbf{B}) = \rho/\epsilon_0 \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \mu_0\left(\mathbf{J} + \rho\mathbf{v} + \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E} - \mathbf{v} \times \mathbf{B}) - \epsilon_0 \mathbf{v} \cdot \nabla(\mathbf{E} - \mathbf{v} \times \mathbf{B})\right) \end{cases}$$

In a vacuum, we can take the curl of the fourth equation to obtain:

$$c^2\nabla^2\mathbf{B} = \frac{\partial^2\mathbf{B}}{\partial t^2} + (\mathbf{v}\cdot\nabla)^2\mathbf{B} - 2\mathbf{v}\cdot\nabla\left(\frac{\partial\mathbf{B}}{\partial t}\right)$$

Substituting a wave solution of the form $\mathbf{B} \sim \exp i(\mathbf{k}\cdot\mathbf{x} - \omega t)$ We obtain an equation for ω, which we can solve to obtain:

$$\omega = -\mathbf{v}\cdot\mathbf{k} \pm c|\mathbf{k}|$$

Therefore the speed of propagation is the group velocity:

$$\frac{\partial\omega}{\partial\mathbf{k}} = -\mathbf{v} \pm c\ \hat{\mathbf{k}}$$

which gives you the expected $c \pm v$ with an appropriate choice of $\mathbf{v}$ and $\mathbf{k}$.