Hypercomplex Neural Networks

ft. Quaternions

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State of Deep Learning

Struggles

- Generalization
- Training Bias
- Over/Under fitting
- Model Size/Complexity

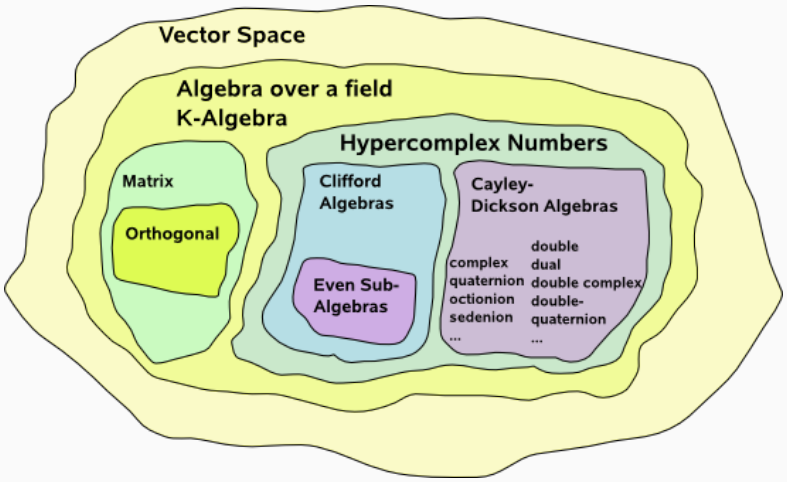
Solutions?

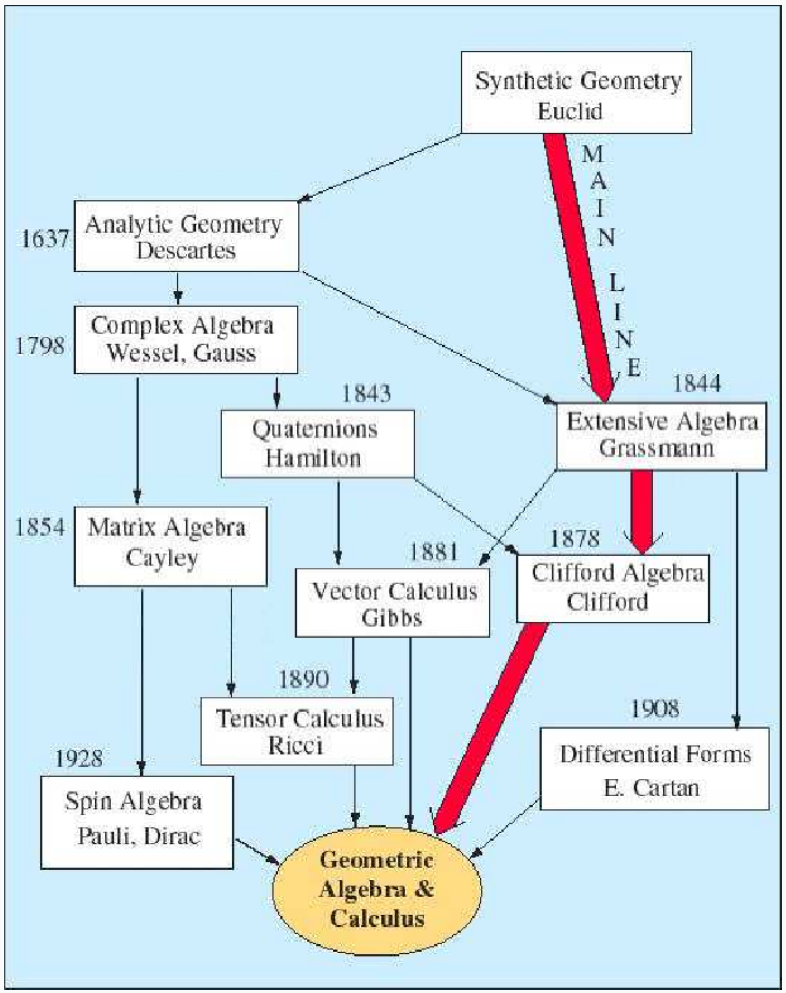
- More complex models
- Layer Stacking
- Aggressive Data Dollection
- Adding more humans into "machine" learning

image src: xkcd

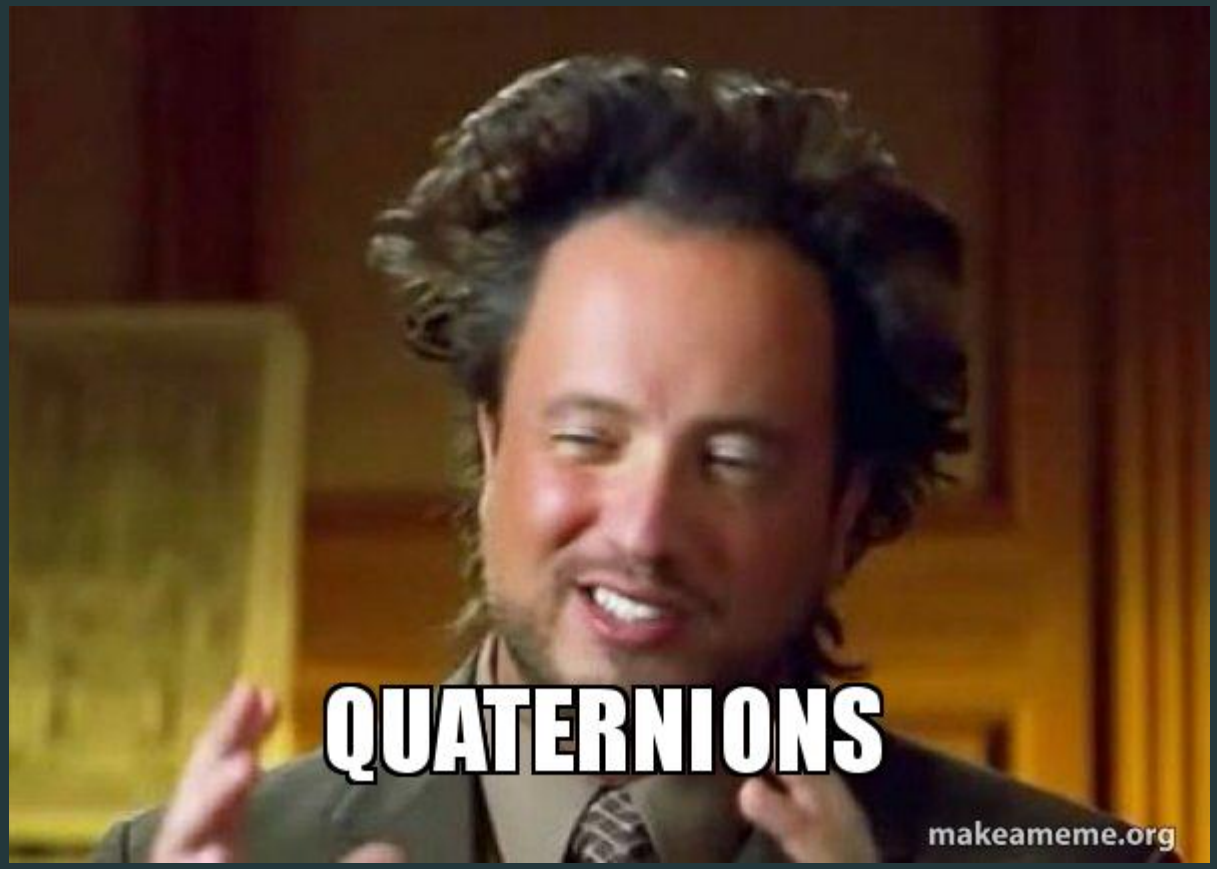
Other Options?

What about actual math?





left: www.euclideanspace.com/maths/algebra/multidimensional/vectorSpace.png
right: (Chappell, et al., 2016)



src: https://makeameme.org/meme/quaternions

Quaternion | Background

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

$$Q = r1 + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$q = (r, \vec{v}), q \in \mathbf{H}, r \in \mathbf{R}, \vec{v} \in \mathbf{R}^3$$

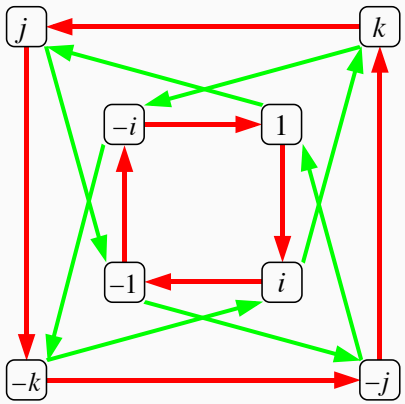
$$Q_{\text{mat}} = \begin{bmatrix} r & -x & -y & -z \\ x & r & -z & y \\ y & z & r & -x \\ z & -y & x & r \end{bmatrix}$$

History

- Hamilton in 1843
- "vector" and "scalar"
- Gibbs (and vector analysis)
- Resurgence
  - Quantum Mechanics
  - Number Theory

Quaternion | Properties

$\times$	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1



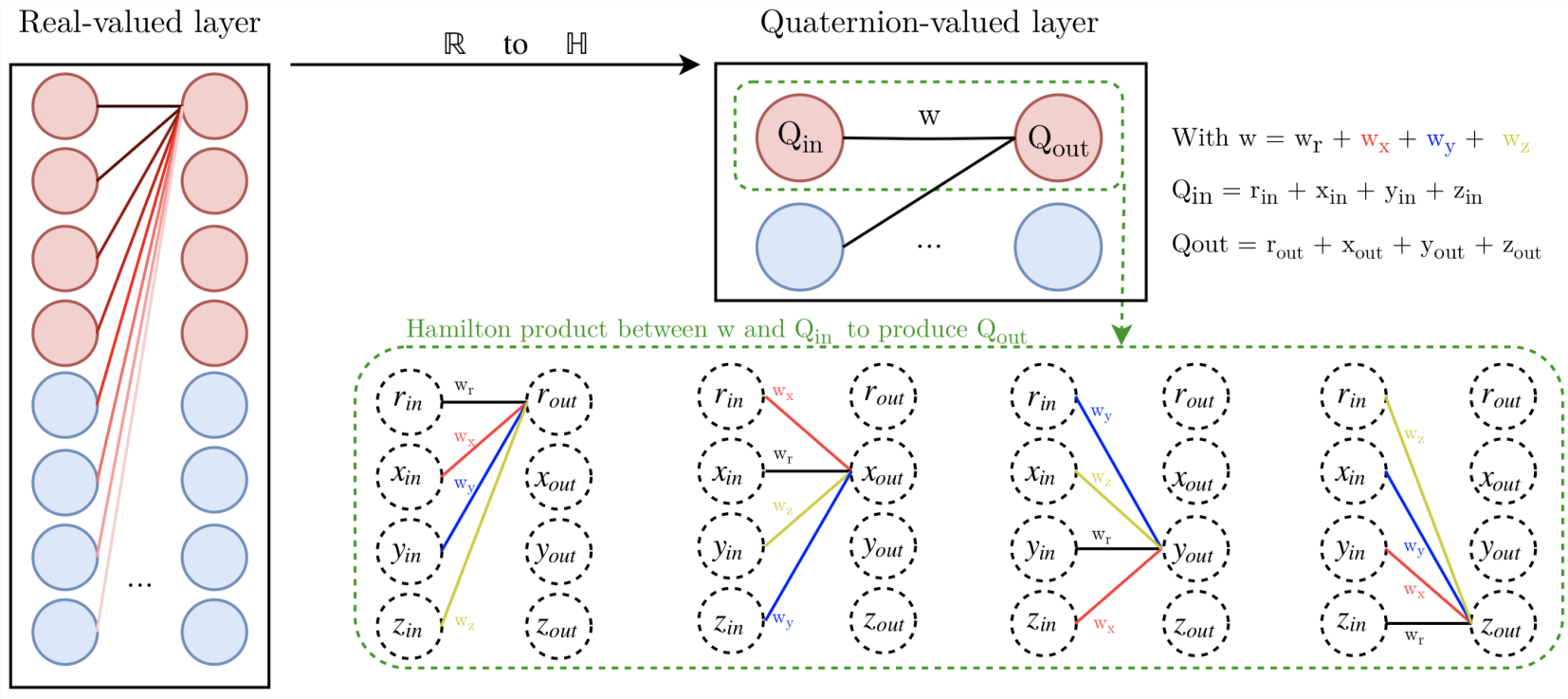
Properties

- Non-commuting
- Anti-commuting
- Double Cover

$$ij = -ji = k$$
$$jk = -kj = i$$
$$ki = -ik = j$$

Quaternion Valued Neural Networks

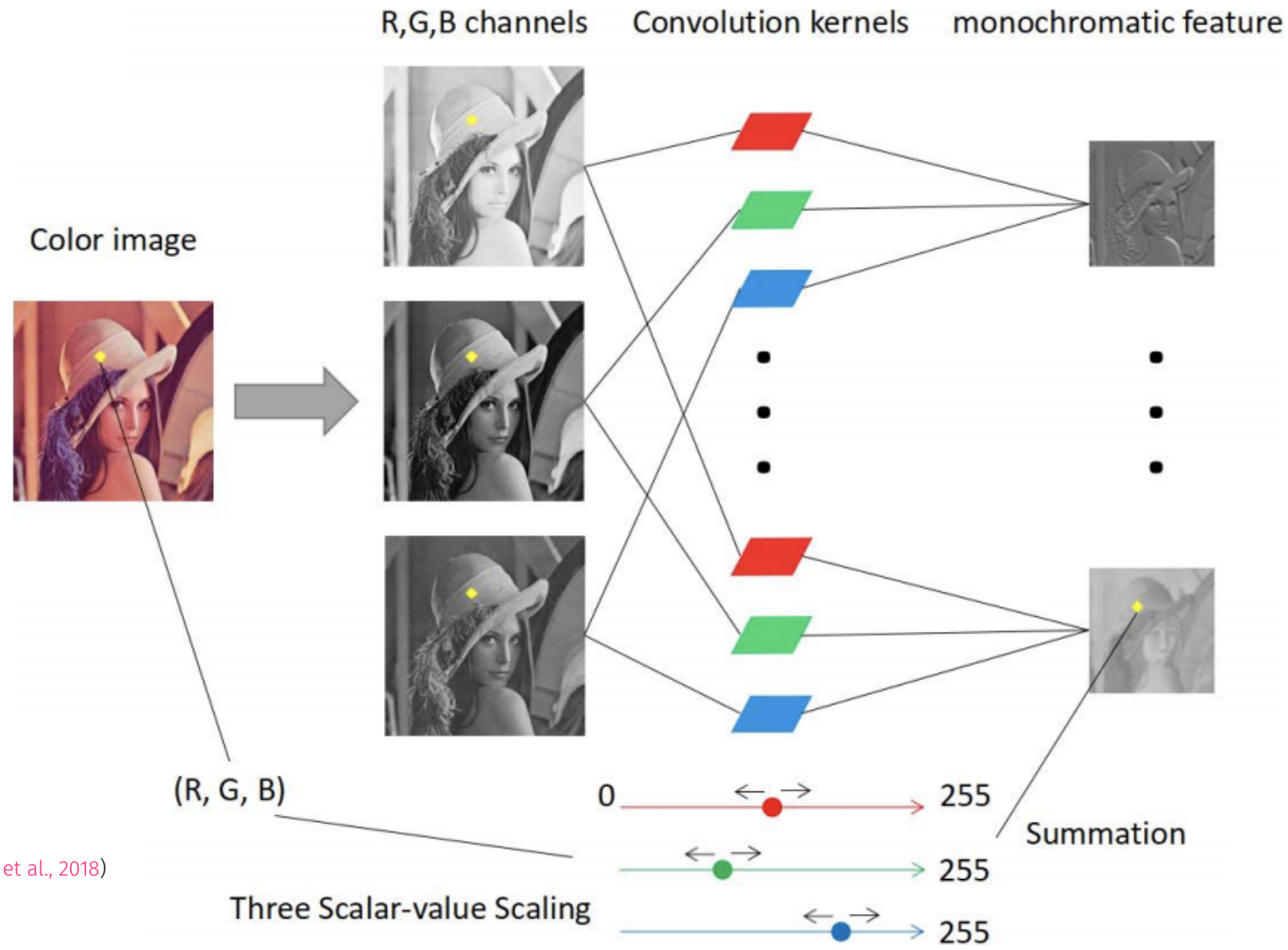
Hamilton Product



$$
\begin{aligned}
Q_1 \otimes Q_2 &\neq Q_2 \otimes Q_1 \\
Q_1 \otimes Q_2 &= (r_1 r_2 - x_1 x_2 - y_1 y_2 - z_1 z_2) \\
&\quad + (r_1 x_2 + x_1 r_2 + y_1 z_2 - z_1 y_2)\, i \\
&\quad + (r_1 y_2 - x_1 z_2 + y_1 r_2 + z_1 x_2)\, j \\
&\quad + (r_1 z_2 + x_1 y_2 - y_1 x_2 + z_1 r_2)\, k
\end{aligned}
$$

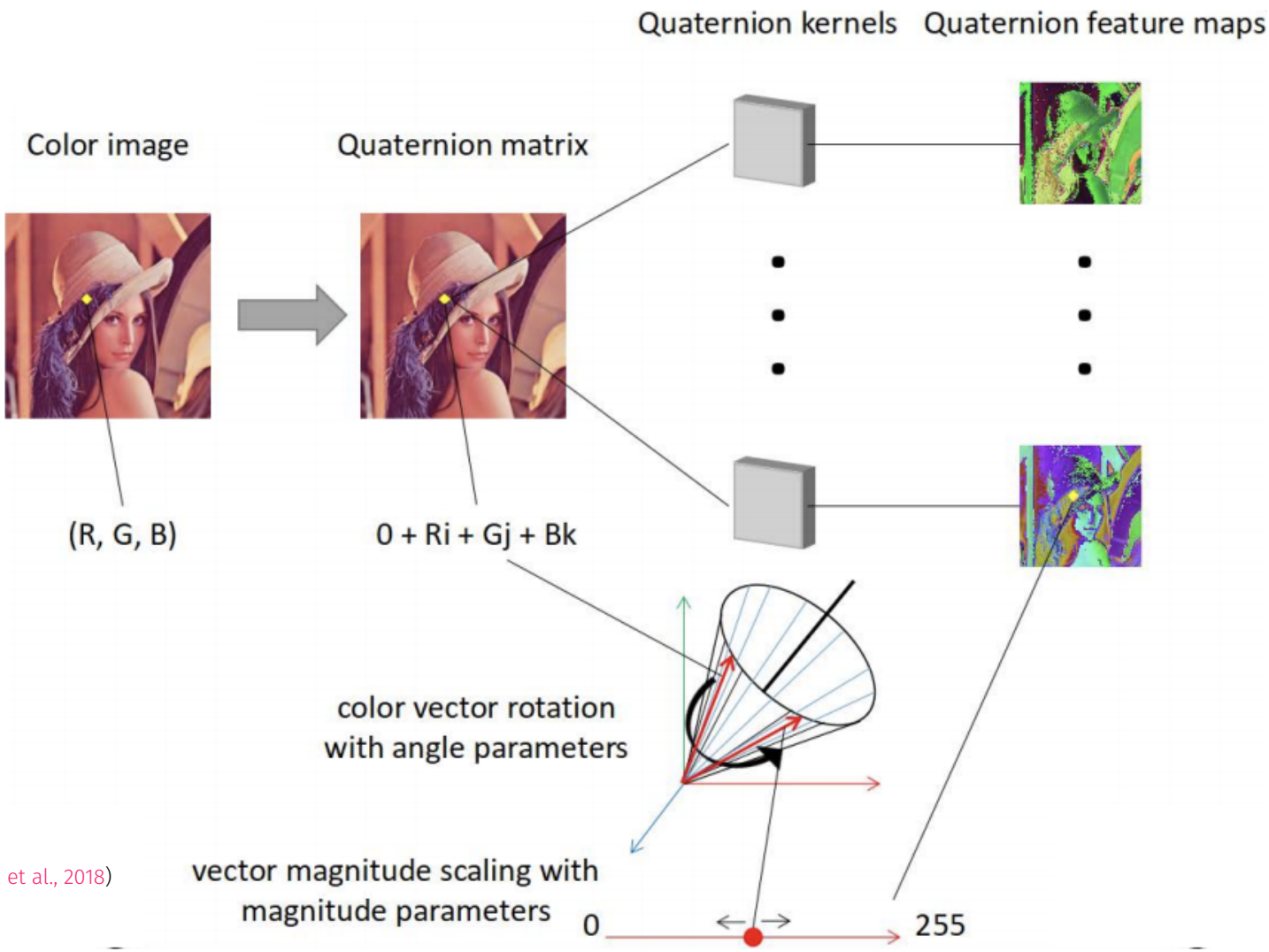
(Parcollet, et al., 2019)

CNN vs QCNN (Real)



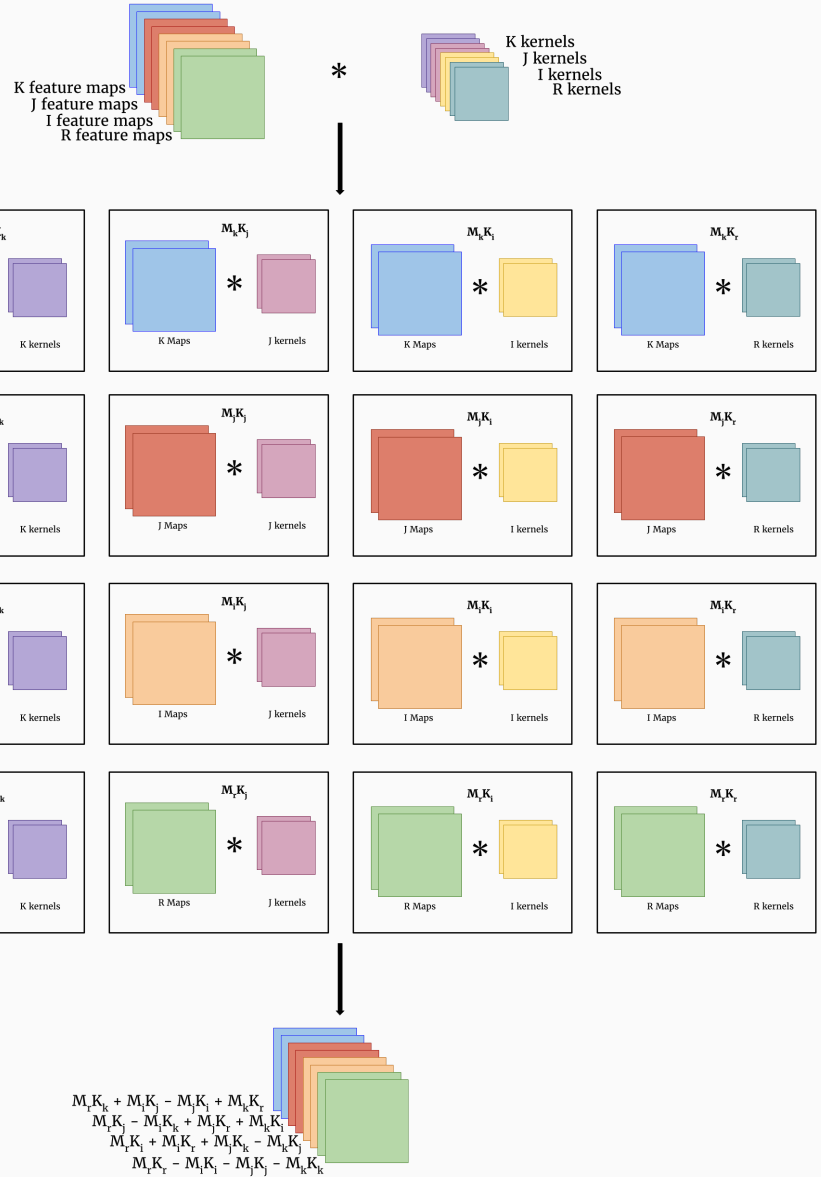
src: (Zhu, et al., 2018)

CNN vs QCNN (Quaternion)



src: (Zhu, et al., 2018)

CNN vs QCNN (Quaternion)



src: (Gaudet, et al., 2018)

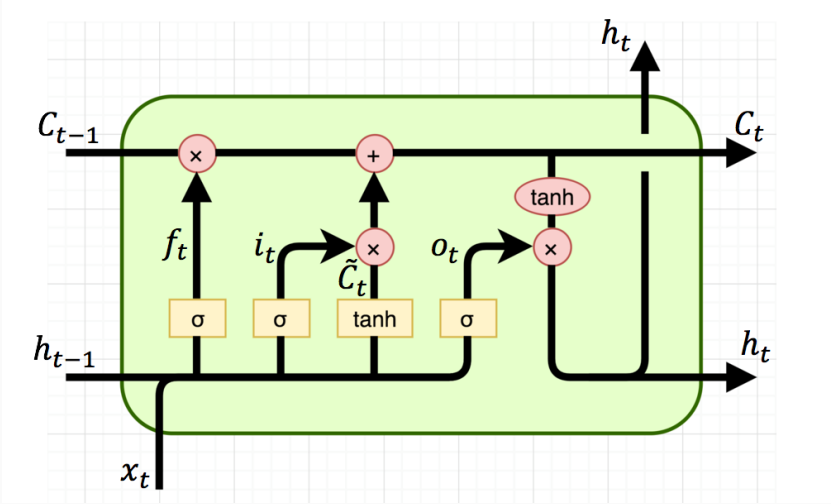
LSTM vs QLSTM

Quaternion-Valued

$$
\begin{aligned}
f_t &= \sigma\left(W_f \otimes x_t + R_f \otimes h_{t-1} + b_f\right) \\
i_t &= \sigma\left(W_i \otimes x_t + R_i \otimes h_{t-1} + b_i\right) \\
o_t &= \sigma\left(W_o \otimes x_t + R_o \otimes h_{t-1} + b_o\right) \\
c_t &= f_t \times c_{t-1} \\
&\quad + i_t \times \alpha\left(W_c \otimes x_t + R_c \otimes h_{t-1} + b_c\right) \\
h_t &= o_t \times \alpha\left(c_t\right)
\end{aligned}
$$

Real-Valued

$$
\begin{aligned}
f_t &= \sigma_g\left(W_f x_t + U_f h_{t-1} + b_f\right) \\
i_t &= \sigma_g\left(W_i x_t + U_i h_{t-1} + b_i\right) \\
o_t &= \sigma_g\left(W_o x_t + U_o h_{t-1} + b_o\right) \\
c_t &= f_t \circ c_{t-1} \\
&\quad + i_t \circ \sigma_c\left(W_c x_t + U_c h_{t-1} + b_c\right) \\
h_t &= o_t \circ \sigma_h\left(c_t\right)
\end{aligned}
$$



(Rathor, 2019)

QNN Research
Examples

Parcollet, Morchid, and Linarès et al.

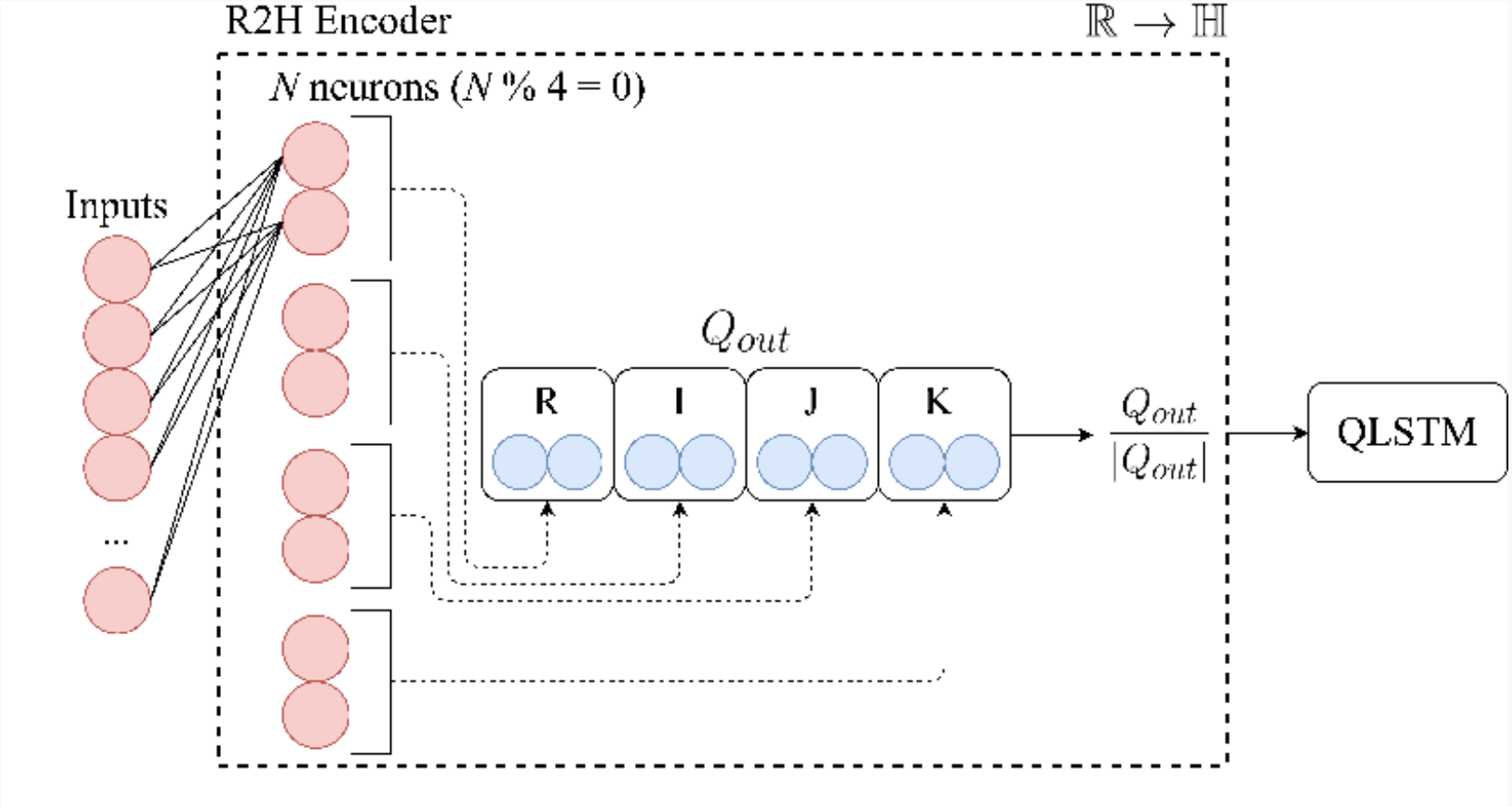
Recent Papers

- Real to H-Space Encoder for Speech Recognition https://arxiv.org/pdf/1906.08043.pdf
- Quaternion Convolutional Neural Networds for Heterogeneus Image Processing https://arxiv.org/pdf/1811.02656.pdf

Other

- Github Repo Orkis-Research
- A survey of quaternion neural networks Artificial Intelligence Review

R2H (Real to Quaternion) Layer



(Parcollet, et al., 2019)

Speech Recognition

TIMIT Phoneme Recognition Task

Quaternion acoustic features: $Q(f,t) = e(f,t) + \frac{\partial e(f,t)}{\partial t}\mathbf{i} + \frac{\partial^2 e(f,t)}{\partial^2 t}\mathbf{j} + \frac{\partial^3 e(f,t)}{\partial^3 t}\mathbf{k}.$

Models	Neurons	Dev.	Test	Params
RNN	256	22.4	23.4	1M
	512	19.6	20.4	2.8M
	1,024	**17.9**	**19.0**	**9.4M**
	2,048	20.0	20.7	33.4M
QRNN	64	23.6	23.9	0.6M
	128	19.2	20.1	1.4M
	256	**17.4**	**18.5**	**3.8M**
	512	17.5	18.7	11.2M

Models	Neurons	Dev.	Test	Params
LSTM	256	14.9	16.5	3.6M
	512	14.2	16.1	12.6M
	1,024	**14.4**	**15.3**	**46.2M**
	2,048	14.0	15.9	176.3M
QLSTM	64	15.5	17.0	1.6M
	128	14.1	16.0	4.6M
	256	**14.0**	**15.1**	**14.4M**
	512	14.2	15.1	49.9M

Phoneme Error Rate (PER%): Lower is better

Experiments conducted with Pytorch-Kaldi: https://github.com/mravanelli/pytorch-kaldi

Wall Street Journal Speech Recognition Task

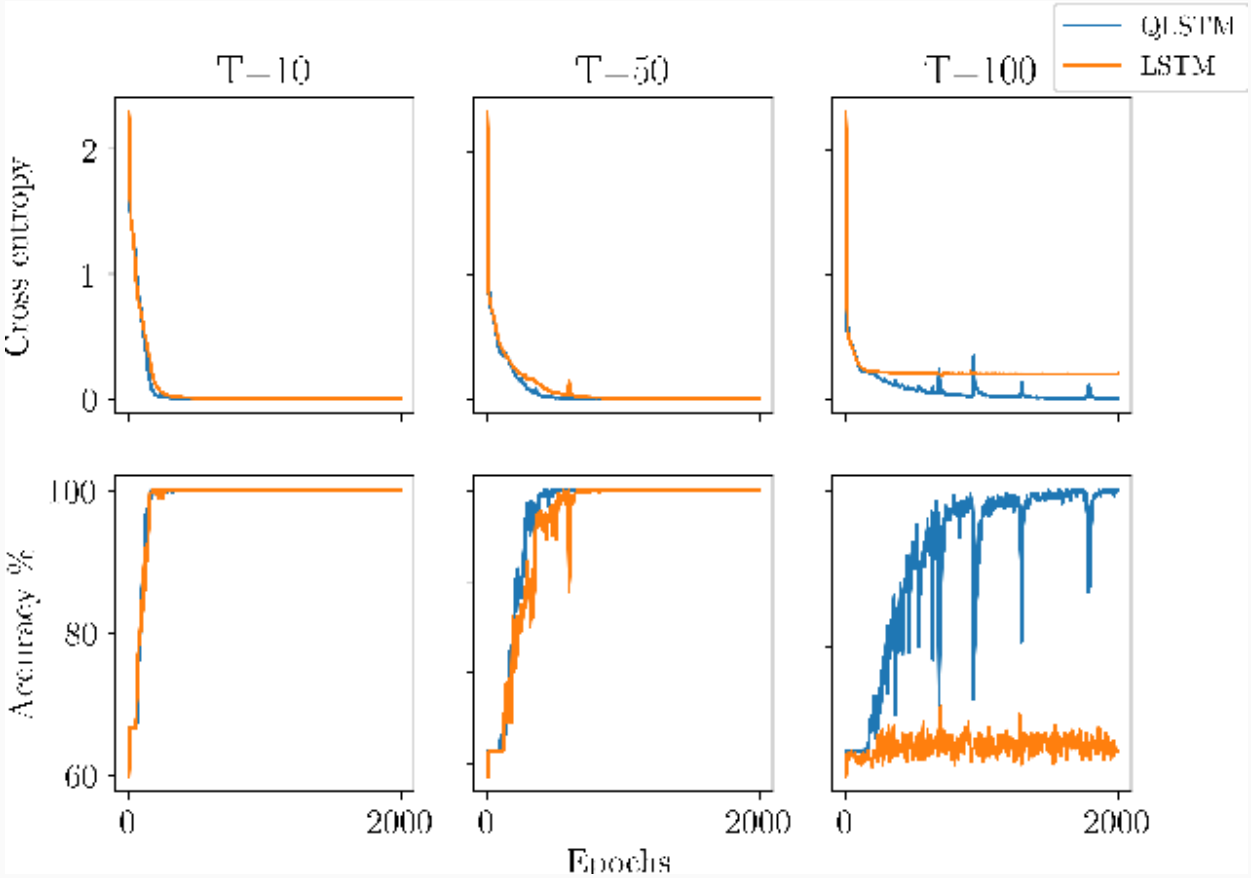
Models	WSJ14 Dev.	WSJ14 Test	WSJ81 Dev.	WSJ81 Test	Params
LSTM	11.2	7.2	7.4	4.5	53.7M
QLSTM	10.9	6.9	7.2	4.3	18.7M

Word Error Rate (WER%): Lower is better

Experiments conducted with Pytorch-Kaldi: https://github.com/mravanelli/pytorch-kaldi

(Parcollet, et al., 2019)

Speech Recognition Pt. 2



(Parcollet, et al., 2019)

Heterogeneous Image Processing

Oh my generalization!

(Parcollet, et al., 2019)

Going Forward...

References

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Reference II

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