DS 6373: Time Series: Unit 3 HW Solutions

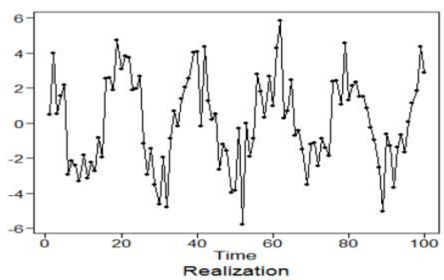
Below are the homework (HW) problems for this Unit. You do not need to submit the solutions rather double check your solutions to the solutions posted. Solutions will be posted to the Wall a few days after the release of the HW. This is intended to let the student think about the problem and attempt it without the temptation to first look at the solution. Please write any questions to the Wall or in an email to myself and/or bring them up during office hours or even in the next Live Session. Remember that the concepts covered below are fundamental to the course and are fair game for the midterm and final.

Have a blast!

Problems from Chapter 2 of the Textbook:
2.1

Problem 2.1 Below is the plot a data set generated using

```
x=gen.sigplusnoise.wge(100,b0=0,b1=0,coef=c(3,1.5),freq=c(.05,.35),psi=c(0,2))
```



(a) If x is the data set generated above, then the following command produces the low-pass filtered data.

```
xl2=butterworth.wge(x,order=3,type='low',cutoff=.2)
```

(b) If x is the data set generated above, then the following command produces the high-pass filtered data.

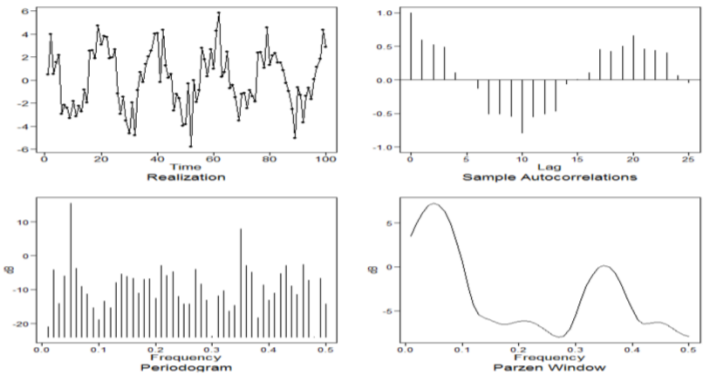
```
xh2=butterworth.wge(x,order=3,type='high',cutoff=.2)
```

(c) If xh2 is the high-pass filtered data in (b) then use the following command:

```
xhl2=butterworth.wge(xh2$x.filt,order=3,type='low',cutoff=.2)
```

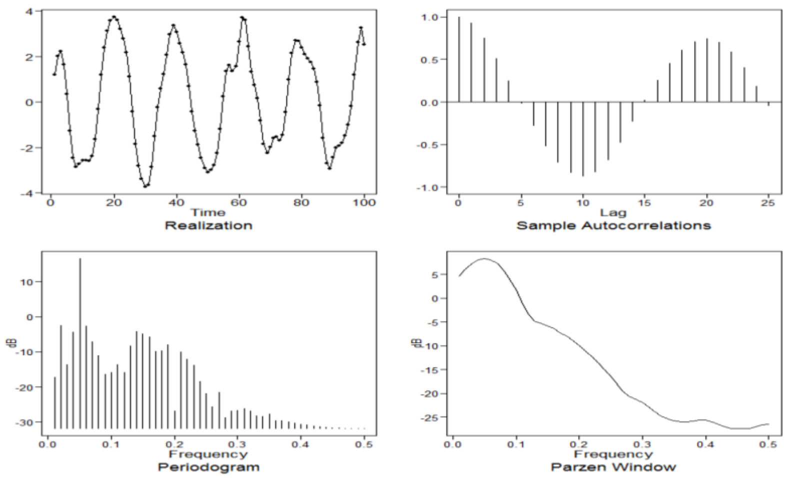
Original data set: command `plotts.sample.wge(x)`

This signal is chacterized by cyclic behavior (with period about 20) along with a high-frequency component. The sample autocorrelations primarily show the cyclic behavior woth period about 20 but may be affected slightly by the high-frequency behavior. The Parzen window (and periodogram) show two peaks at about .05 and .35.



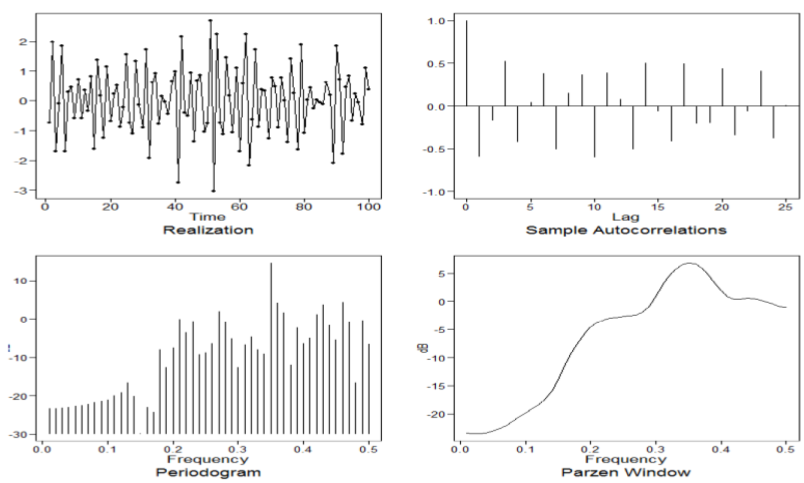
Low-pass filtered data set (`xl2`) : command `plotts.sample.wge(xl2)`

This signal is chacterized by a smooth cyclic behavior (with period about 20) with the high-frequency component removed. a high-frequency component. The sample autocorrelations clearly show the cyclic behavior with period about 20. The Parzen window (and periodogram) show a peak at about .05.



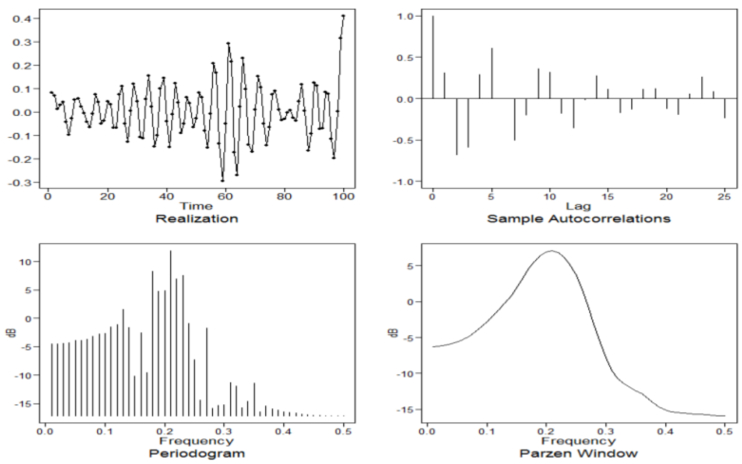
High-pass filtered data set (`xh2$x.filt`) : command `xh2=plotts.sample.wge(x)`

This signal is chacterized by a high-frequency (nearly up-and-down) behavior. The sample autocorrelations clearly show the cyclic behavior with period about 3. The Parzen window (and periodogram) show a peak at about .35.



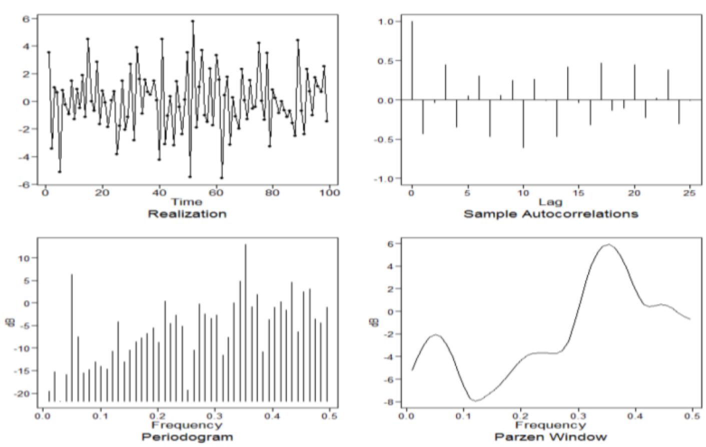
Low-pass filtering the high-pass filtered data: command `xhl2-plotts.sample.wge(xh2$x.filt)`

This signal is very weak. Whereas the original signal went from -6 to 6, the low-pass data went from -4 to 4 and the high-pass data had range -2 to 2, the double filtered data set goes from about -0.4 to 0.4. It would be clearer if all plots were plotted on the same scale. While we might have thought the double-filtered data would be essentially white noise, there does seem to be soime periodic behavior with period about 4-5 as characterized by the data, sample autocorrelations sand spectrum.



2.3

Problem 2.3 command `xdif=artrans,wge(x,phi.dif=1)`
`plotts.sample.wge(xdif)`



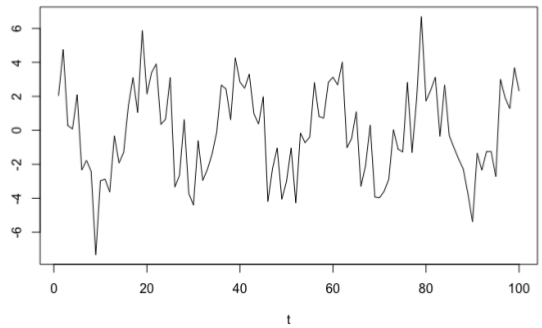
The fitering weakend the frequency component at f=0.05 and served as a high-pass filter. As Figure 2.2b shows, the filter allows some frequency behavior as low as 0.05 to leak into the filtered data. It is not as good as the Butterworth filter for filtering out one of the two signals.

Additional Problem

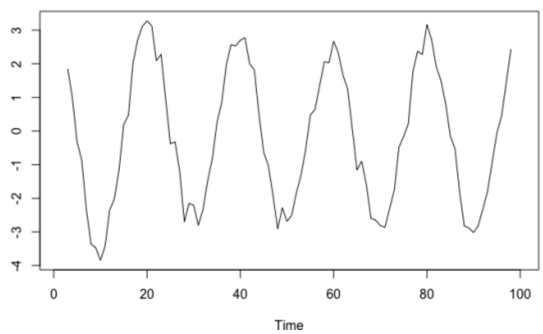
Apply a 5-point moving average to the series you created in 2.1. How does it compare to the difference and Butterworth filters? Specifically, is it a low pass or high pass filter?

I was not able to reproduce the realization in 2.1 above since the seed (sn) was not specified. However, below is another realization produced from the same parameters.

x=gen.sigplusnoise.wge(100,b0=0,b1=0,coef=c(3,1.5),freq=c(.05,.35),psi=c(0,2), sn = 8)



ma = filter(x,rep(1/5,5))
plot(ma)

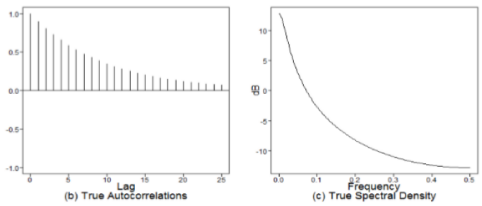


The 5 point moving average filter is a low pass filter the effects of which can be viewed above. While there is still possibly some remnants of the higher frequency, it the low frequency that dominates the filtered realization. It is most comparable to a Butterworth low pass filter with cutoff of .2 although this MA filter does not appear to filter out the higher frequency as well. This is partially because the order of the Butterworth filter was set to 3.

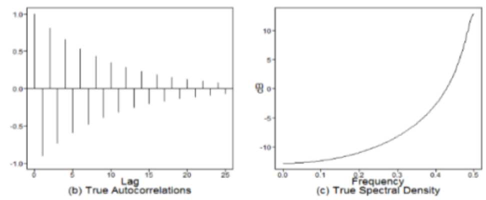
Problems from Chapter 3 of the Textbook:
3.4

Problem 3.4
(a,b) Using the command `plotts.true.wge(n=200,phi=.9)`, `plotts.true.wge(n=200,phi=-.9)`, `plptts.true.wge(n=200,phi=.5)`, and `plotts.true.wge(n=200,phi=-.5)`, we obtain the plots below. Note that this command also plots a realization, but it does not have the ability to specify the same seed.

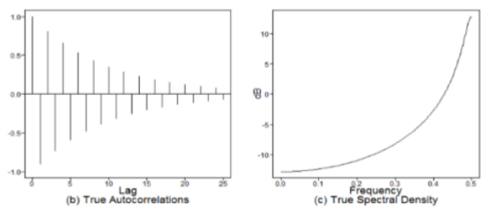
$\phi_1 = 0.9$



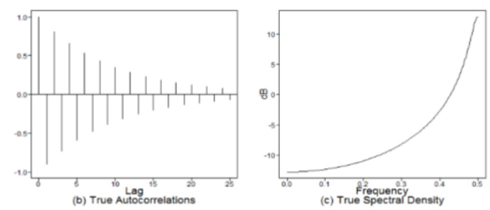
$\phi_1 = -0.9$



$\phi_1 = 0.5$



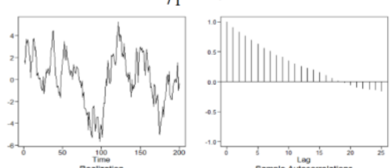
$\phi_1 = -0.5$



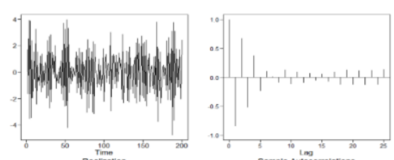
(c) For $\phi_1 = \pm 0.9$, $\sigma_X^2 = 1/(1-.81) = 5.263$ while for $\phi_1 = \pm 0.5$, $\sigma_X^2 = 1/(1-.25) = 1.333$. Note these can be obtained as $acv[1] after the above function calls.

(d) Using the commands `xp9=gen.arma.wge(n=200,phi=.9,sn=1)`, `xm9=gen.arma.wge(n=200,phi=-.9,sn=1)`, `xp5=gen.arma.wge(n=200,phi=.5,sn=1)`, and `xm5=gen.arma.wge(n=200,phi=-.5,sn=1)` to generate the realizations with the same seed (using `sn=1`) in each statement) and then issuing the commands `plotts.sample.wge(xxx)` where `xxx=xp9`, `xm9`, `xp5`, and `xm5`, we obtain the following plots (after cropping out the spectral density plots.

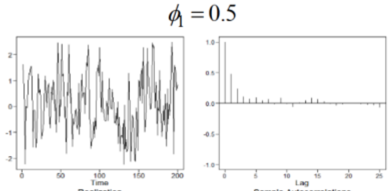
$\phi_1 = 0.9$



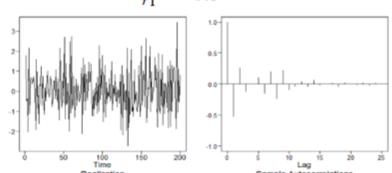
$\phi_1 = -0.9$



$\phi_1 = 0.5$

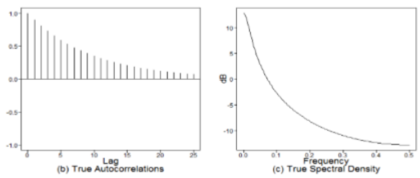


$\phi_1 = -0.5$

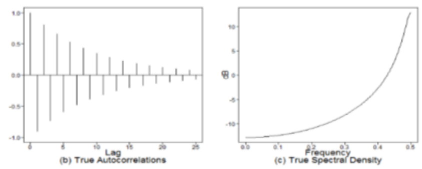


(a-c) with $\sigma_a^2 = 10$ (i.e. using `vara=10` in the function calls)

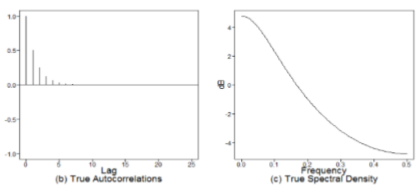
$\phi_1 = 0.9$



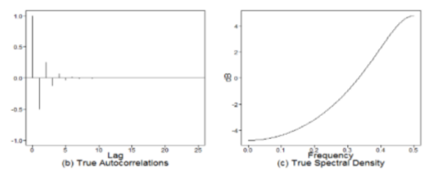
$\phi_1 = -0.9$



$\phi_1 = 0.5$



$\phi_1 = -0.5$



The autocorrelations and spectral densities are the same for $\sigma_a^2 = 1$ and $\sigma_a^2 = 10$

For $\phi_1 = \pm 0.9$ we have $\sigma_X^2 = 10/(1-.81) = 52.63$ and for $\phi_1 = \pm 0.5$ we have $\sigma_X^2 = 10/(1-.25) = 13.33$ (Again these can be obtained as \$acv[1] after the above function calls.) That is, the process variance for $\sigma_a^2 = 10$ is 10 time that for $\sigma_a^2 = 1$. Also, realizations (using the same seed) have the same appearance but those with $\sigma_a^2 = 10$ have larger range. (Each realization value for $\sigma_a^2 = 10$ is $\sqrt{10}$ times the corresponding value for $\sigma_a^2 = 1$.)