DS 6373: Time Series: Unit 4 HW Solution

Below are the homework (HW) problems for this Unit. You do not need to submit the solutions rather double check your solutions to the solutions posted. Solutions will be posted to the Wall a few days after the release of the HW. This is intended to let the student think about the problem and attempt it without the temptation to first look at the solution. Please write any questions to the Wall or in an email to myself and/or bring them up during office hours or even in the next Live Session. Remember that the concepts covered below are fundamental to the course and are fair game for the midterm and final.

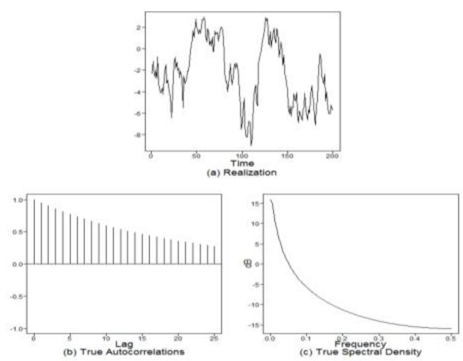
Have a blast!

Problems from Chapter 3 of the Textbook:
3.2

Problem 3.2

(a) Using the code
`x=plotts.true.wge(n=200,phi=.95)`

We obtained the plots



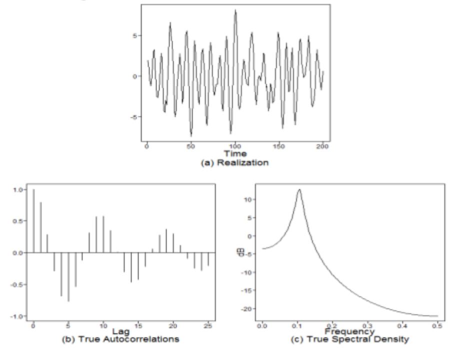
The data show a non-cyclic, wandering behavior around which can be explained by the fact that the autocorrelations for the lower lags are high-positive. That is, if an observation is above the mean, then the next few will likely also be above the mean, etc. The spectral density shows a spike at f=0 which further explains the non-cyclic behavior.

$\mu = 0$ and $\sigma_X^2 = \frac{1}{1-0.95^2} = 10.256$. The variance can also be found in `x$acv[1]` after running the above code.

`mean(x$data)= -2.375` and `var(x$data)=8.866.`

(b) Using the code
`x=plotts.true.wge(n=200,phi=c(1.5,-.9))`

We obtained the plots



Since the model has mean 10, I created and plotted the appropriate realization using

```
x10=x$data+10
plotts.wge(x10)
```

and obtained the following is the data with. The other two plots do not change with the mean change.

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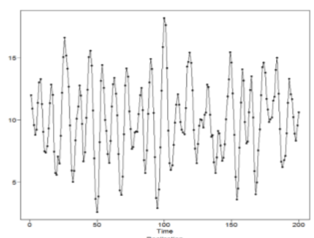

The data appear to be pseudo cyclic with a period of about 10 (there are about 20 cycles in the datab set of 200 points). The autocorrelations have a damped sinusoidal behavior with periods of about 10 and the spectral density has a opeak at about f=0.1 (=1/10).

$\mu = 10$ and $\sigma_X^2 = \dfrac{1}{1-\phi_1\rho_1-\phi_2\rho_2} = \dfrac{1}{1-1.5(.7894)+.9(.2842)} = 13.98$. The variance can also be found in `x$acv[1]` after running the above code (which gives 13.971). the autocorrelations ρ_1 and ρ_2 were obtained from x$aut[2] and x$aut[3], respectively, after running the above code.

`mean(x10)= 9.956` and `var(x10)=9.92` (which is quite a bit smaller than the actual variance in this case).

3.3 a,b, and d.

Problem 3.3

(a) Using the factor table below obtained using factor.wge (in this case the comand `factor.wge(phi=c(1.55,-1,.25))` we see that all the roots are outside the unit circle so this autoregressive process is stationary.

Factor	Roots	Abs Recip	System Freq
1-0.9877B+0.4446B^2	1.1107+-1.0076i	0.6668	0.1173
1-0.5623B	1.7785	0.5623	0.0000

(b) Using the factor table below obtained using factor.wge (in this case the comand `factor.wge(phi=c(2,-1.76,1.6,-.77))` we see that since the characteristic equation has a root inside the unit circle (0.8349) this autoregressive process is nonstationary.

Factor	Roots	Abs Recip	System Freq
1-1.1977B	0.8349	1.1977	0.0000
1+0.0012B+0.8001B^2	-8e-04+-1.1180i	0.8945	0.2501
1-0.8035B	1.2446	0.8035	0.0000

(d) Using the factor table below obtained using factor.wge (in this case the comand `factor.wge(phi=(c(1.9,-2.3,2,-1.2,.4))` we see that since all the roots are outside the unit circle, this autoregressive process is stationary.

Factor	Roots	Abs Recip	System Freq
1+0.0080B+0.7888B^2	-0.0050+-1.1259i	0.8882	0.2507
1-1.1206B+0.6440B^2	0.8700+-0.8921i	0.8025	0.1270
1-0.7873B	1.2701	0.7873	0.0000

Additional Question, with respect to question 3.3. What is the obvious difference between (3.3a,b and d) and (3.3c)?

33.c is a function of the white noise rather than lagged observations of X.