DS 6373: Time Series: Unit 6 HW Solutions

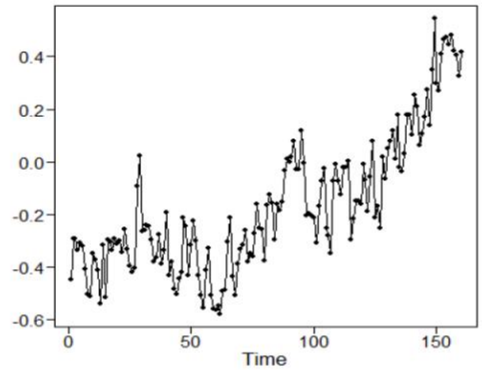
Below are the homework (HW) problems for this Unit. You do not need to submit the solutions rather double check your solutions to the solutions posted. Solutions will be posted to the Wall a few days after the release of the HW. This is intended to let the student think about the problem and attempt it without the temptation to first look at the solution. Please write any questions to the Wall or in an email to myself and/or bring them up during office hours or even in the next Live Session. Remember that the concepts covered below are fundamental to the course and are fair game for the midterm and final.

Have a blast!

Problems from Chapter 5 of the Textbook:
5.1

Problem 5.1

(a) Plot of `hadley` data



b) Using base R function lm the following code yields $\hat{a}$ and $\hat{b}$:

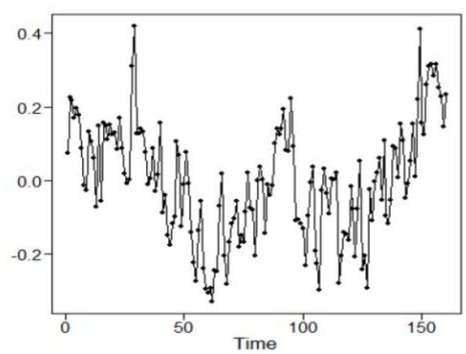
```
t=1:160
lm(hadley ~ t)
```

gives $\hat{a} = -.525737$ and $\hat{b} = 0.004438$

(c) We calculate the residuals using the code

```
t=1:160
hadley.res=hadley+.525737-.004438*t
```

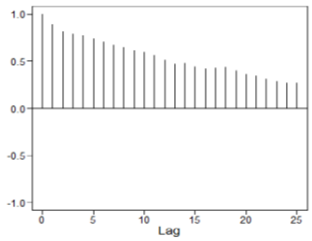
and a plot of the residuals is given below:



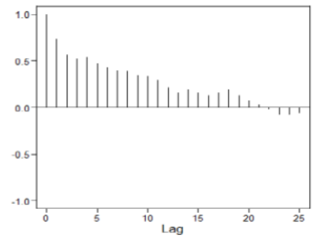
(d) The data appear stationary and have somewhat of a wandering behavior centered around a mean of about zero.

(e) Sample autocorrelations:

For Hadley data



For residuals from least squares line



Both sets of sample autocorrelations have a damped exponention appearance. The residuals have a faster damping (more stationary) appearance.

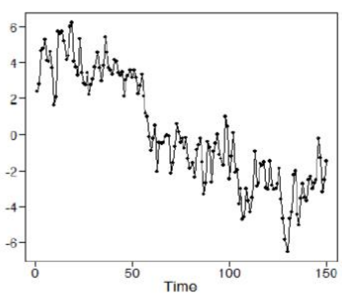
5.2

Problem 5.2

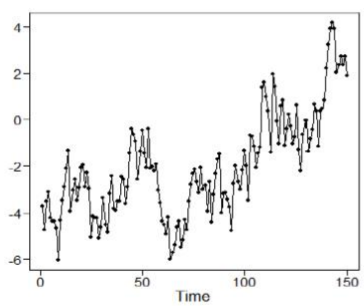
(a) The 4 realizations are generated as follows:

```
phi5.3=c(.66,-.02,.10,.24)
x1=gen.arma.wge(n=150,phi=phi5.3)
x2=gen.arma.wge(n=150,phi=phi5.3)
x3=gen.arma.wge(n=150,phi=phi5.3)
x4=gen.arma.wge(n=150,phi=phi5.3)
```

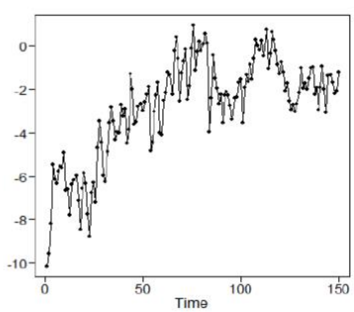
x1



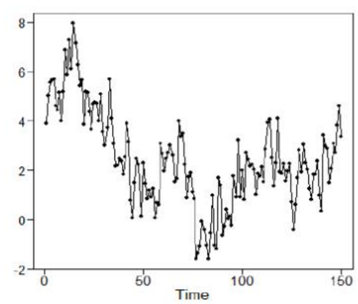
x2



x3



x4



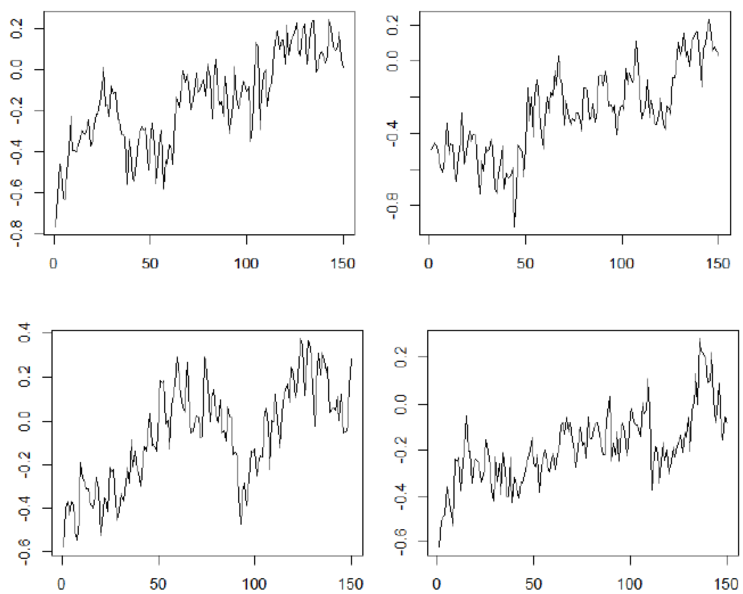
The first 4 realizations generated using the above code are shown above. All realizations show some ngndtrending behavior with `x1` showing strong (downward) trending behavior that persists throughout the

150 point time frame. x2 and x3 show some upward trending behavior similar to the temperature data while x4 shows wandering behavior with less trending. The lesson is that yes, this stationary model can produce realizations with strong trending behavior over 150 "years."

(b) The four realizations can be generated using the code:

```
phi1=c(.614,-.044,.077,.206)
x1=gen.sigplusnoise.wge(n=150,b0=-.526,b1=.0044,phi=phi1,vara=.01)
x2=gen.sigplusnoise.wge(n=150,b0=-.526,b1=.0044,phi=phi1,vara=.01)
x3=gen.sigplusnoise.wge(n=150,b0=-.526,b1=.0044,phi=phi1,vara=.01)
x4=gen.sigplusnoise.wge(n=150,b0=-.526,b1=.0044,phi=phi1,vara=.01)
```

we obtain the following realizatios:



These realizations are similar to those in Fig1.24a and part a of this problem. Note that because of the positive slope, all of these realizations have a tendency to climb whereas the trending behavior in part a could be either positive or negative. Using the strategy promoted throughout this text, we factor the AR(4) model using factor.wge(phi=phi1) and obtain the following factor table where the dominant factor is associated with a positive real root (not as close to the unit circle as in the factor table in Table 5.1) since a large part of the trending behavior in this model is accounted for by the linear trend component. It should be noted, however, that the wandering in the noise is enough to disrupt the linear trending behavior at some points in the above plots (specifically around t=90 in the bottom left plot.

Factor	Roots	Abs Recip	System Freq
|1-0.9208B	1.0860	0.9208	0.0000
1-0.2086B+0.4340B^2	0.2402+-1.4987i	0.6588	0.2247
1+0.5154B	-1.9402	0.5154	0.5000

5.3 (a,b and d)

(a) In the above, the factor table for ($(1-1.2B+.8B^2)$ follows which shows that this second order factor is associated with a pair of complex roots outside the unit circle associaed with a frequency of f=.133 (i.e. pseudo cyclic behavior with periods of about 7.5)

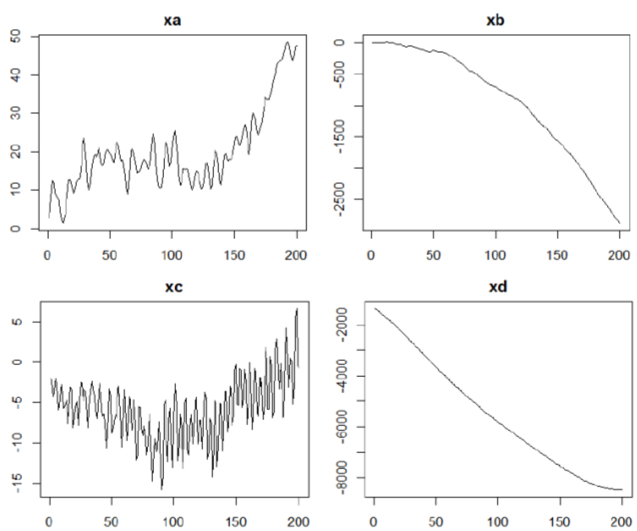
```
Factor              Roots              Abs Recip    System Freq
1-1.2000B+0.8000B^2  0.7500+-0.8292i    0.8944       0.1330
```

Also, the factor table for $(1+B+B^2)$ in model d hasis shown below wher itn can be seen that this is a nonstationary factor with a pair of complex roots on the unit circle associated with a frequency of f=1/3 (i.e. period length 3).

```
Factor              Roots              Abs Recip    System Freq
1+1.0000B+1.0000B^2  -0.5000+-0.8660i   1.0000       0.3333
```

The 4 realizations are generated as follows:

```
xa=gen.arima.wge(n=200,phi=c(1.2,-.8),d=1)
xb=gen.arima.wge(n=200,phi=c(1.2,-.8),d=2)
xc=gen.aruma.wge(n=200,phi=c(1.2,-.8),d=0,s=4)
xd=gen.aruma.wge(n=200,phi=c(1.2,-.8),d=2,s=4,lambda=c(-1,-1))
```



No models are stationary although they all share the stationary factor $(1-1.2B+.8B^2)$. Model a has a single unit root, Model b has two unit roots, Model 3 has a fourth order (nonstationary) seasonal factor, and Model

d has 2 unit roots, a 4[th] order nonstationary seasonal factor and the nonseason factor $(1+B+B^2)$ discussed above.

Realization xa shows the wandering behavyior associated with the unit root along with the cyclic behavior introcuded by the seasonal factor $(1-1.2B+.8B^2)$. Model b has two unit roots and this behavior dominates all other behavior in realization xb. The 4[th] order seasonal factor in Model c by the occurrence of about 50 seasonal-type cycles (which are more easily seen with a larger plot. Model d has 3 unit roots $(1-B)^2$ and one from $(1-B^4)$ (see Table 5.4). This behavior dominants all other components of the model in the realization.

5.5

Problem 5.5

(a) The calls to `factor.wge` below produces the factor table below for the AR and MA portions of the model, respectively:

```
factor.wge(phi=c(3,-4.5,5,-4,2,-.5))
factor.wge(phi=c(1.7,-.8))
```

AR portion

Factor	Roots	Abs Recip	System Freq
1-1.0000B	1.0000	1.0000	0.0000
1+0.0000B+1.0000B^2	0.0000+-1.0000i	1.0000	0.2500
1-1.0000B	1.0000	1.0000	0.0000
1-1.0000B+0.5000B^2	1.0000+-1.0000i	0.7071	0.1250

MA portion

Factor	Roots	Abs Recip	System Freq
1-1.7000B+0.8000B^2	1.0625+-0.3480i	0.8944	0.0504

There are no cancelling factors, and the factored form of the model is
$(1-B)^2(1+B^2)(1-B+.5B^2)X_t=(1-1.7B+.8B^2)a_t$, which is an ARUMA(2,4,2) model

(b) The call to `factor.wge` below produces the factor table below for the AR component:

```
factor.wge(phi=c(.5,-.3,.95,-.3,.35,-.2))
```

Factor	Roots	Abs Recip	System Freq
1-1.0000B	1.0000	1.0000	0.0000
1+1.0000B+0.8000B^2	-0.6250+-0.9270i	0.8944	0.3444
1+0.0000B+0.5000B^2	0.0000+-1.4142i	0.7071	0.2500
1-0.5000B	2.0000	0.5000	0.0000

There are no MA components to this model, and the AR component can be factored as
$(1-B)(1+B+.8B^2)(1+.5B^2)(1-.5B)X_t=a_t$, which is an ARUMA(5,1,0) model.

(c) The calls to `factor.wge` below produce the factor tables below for the AR and MA portions of the model, respectively:

```
factor.wge(phi=c(1.5,-1.3,-.35,1,-1.35,.7,-.4))
factor.wge(phi=c(0,-.95))
```

AR portion

Factor	Roots	Abs Recip	System Freq
1+1.0000B	-1.0000	1.0000	0.5000
1-1.5000B+1.0000B^2	0.7500+-0.6614i	1.0000	0.1150
1-1.0000B+0.8000B^2	0.6250+-0.9270i	0.8944	0.1556
1-0.0000B+0.5000B^2	0.0000+-1.4142i	0.7071	0.2500

MA portion

Factor	Roots	Abs Recip	System Freq
1+0.0000B+0.9500B^2	0.0000+-1.0260i	0.9747	0.2500

There are no cancelling factors, and the factored form of the model is
$(1+B)(1-1.5B+B^2)(1-B+.8B^2)(1+.5B^2)X_t=(1+.95B^2)a_t$, which is an ARUMA(4,3,2) model

(d) The calls to `factor.wge` below produce the factor tables below for the AR and MA portions of the model, respectively:

```
factor.wge(phi=c(.5,.5,0,1,-.5,-.5))
factor.wge(phi=c(0,.81))
```

AR portion

Factor	Roots	Abs Recip	System Freq
1-1.0000B	1.0000	1.0000	0.0000
1+1.0000B	-1.0000	1.0000	0.5000
1-0.0000B+1.0000B^2	0.0000+-1.0000i	1.0000	0.2500
1-1.0000B	1.0000	1.0000	0.0000
1+0.5000B	-2.0000	0.5000	0.5000

MA portion

Factor	Roots	Abs Recip	System Freq
1+0.9000B	-1.1111	0.9000	0.5000
1-0.9000B	1.1111	0.9000	0.0000

There are no cancelling factors, and the factored form of the model is
$(1-B)^2(1+B)(1+B^2)(1+.5B)X_t=(1+.9B)(1-.9B)a_t$, which is an ARUMA(1,5,2) model.

5.7

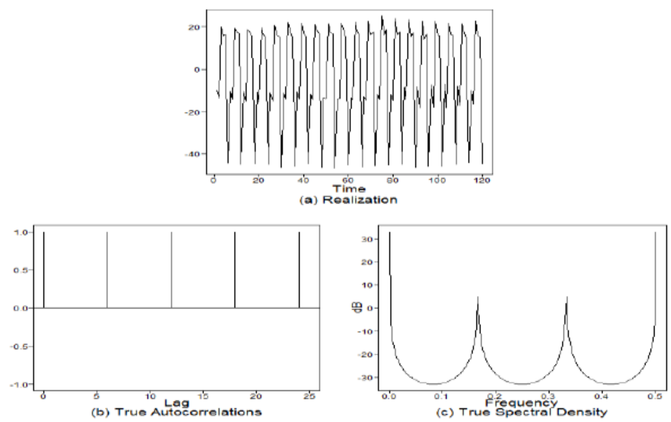
Problem 5.7

(a) Using the command `factor.wge(phi=c(0,0,0,0,0,1))` we obtain

Factor	Roots	Abs Recip	System Freq
1+1.0000B+1.0000B^2	-0.5000+-0.8660i	1.0000	0.3333
1-1.0000B+1.0000B^2	0.5000+-0.8660i	1.0000	0.1667
1-1.0000B	1.0000	1.0000	0.0000
1+1.0000B	-1.0000	1.0000	0.5000

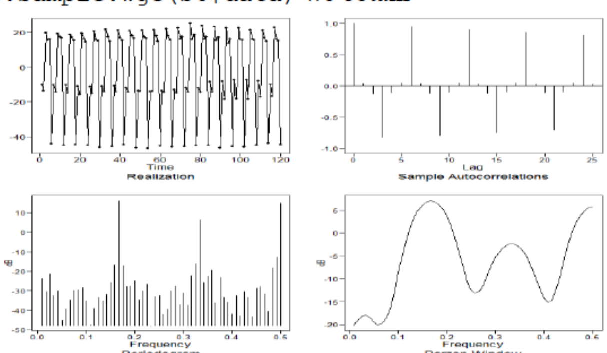
The 6 roots of unity are $+1, -1, -0.5000 \pm 0.8660i, 0.5000 \pm 0.8660\text{i}$.

(b-c) Although we could generate a realization using `s1=gen.aruma.wge(n=120,s=6)`, for this problem I will use `plotts.true.wge` using the nearly non-stationary counterpart. That is, we issue the command `s6=plotts.true.wge(n=120,phi=c(0,0,0,0,0,.999))` and produce the following:



The realization shows abouut 20 “period” that are similar to each other but are not sinusoidal. The true autocorrelations are zero except at muliples of 6 where the autocorrelations are =1. The sptral density shows peaks at the frequencies in the factor table.

(d) Using `plotts.sample.wge(s6$data)` we obtain



The periodogram and spectral density show peaks at the frequencies in the factor table, and the sample autocorrelations are near +1 at multiples of 6. However, the other sample autocorrelations are not zero and at k=3, 9, 15, 21 they are fairly large negative.