Time Series Unit 1/HW 1 Solutions

Problem 1.1

$$\bar{X} = \frac{1}{n}\sum_{k=1}^{n} X_i = 71.25$$

$$\hat{\gamma}_0 = \frac{1}{n}\sum_{k=1}^{n}(X_k - \bar{X})^2 = \frac{1}{8}\sum_{k=1}^{8}(X_k - 71.25)^2 = \frac{279.5}{8} = 34.9375$$

$$\hat{\gamma}_1 = \frac{1}{8}\sum_{k=1}^{7}(X_k - 71.25)(X_{k+1} - 71.25) = \frac{117.9375}{8} = 14.74219$$

$$\hat{\rho}_0 = 1$$

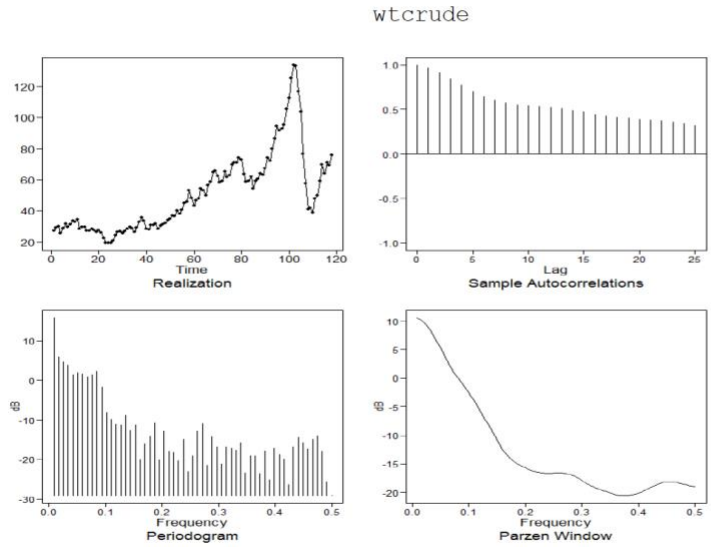
$$\hat{\rho}_1 = \frac{14.74219}{34.9375} = .422$$

Problem 1.2

Using the tswge R code

```
data(wtcrude)
plotts.wge(wtcrude)
```

we obtain the following plots.



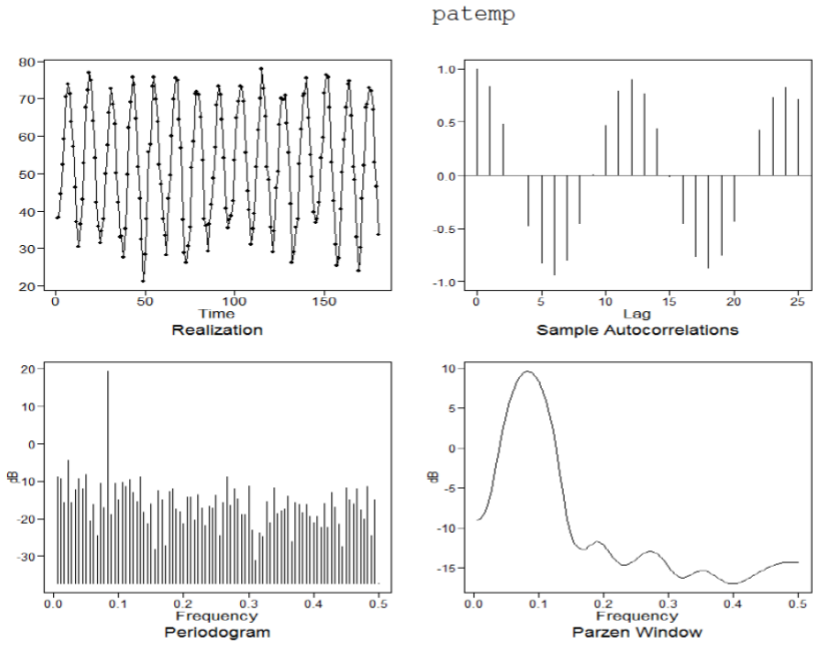
The data shows a non-cyclic wandering behavior with strong correlation between data values that are near each other in time. The sample autocorrelations show strong positive correlation (above 0.5 for $k \le 13$), and the periodogram and Parzen spectral estimator show peaks at zero with no indication of cyclic behavior.

However, none of the diagnostic plots provide an indication of the precipitous drop in oil prices around t=100.

Using the tswge R code

```
data(patemp)
plotts.wge(patemp)
```

we obtain the following plots.



The data show a strong cyclic behavior (with a period of about 12 which makes sense (because this is monthly data). The sample autocorrelations show a very slowly damping cyclic behavior with cycle length 12 while the periodogram and Parzen spectral estimator show a strong peak at about f=1/12.

Problem 1.4

Realization 1 has wandering behavior which is corresponds to fairly high positive autocorrelations for lags of modest length (d) and spectral density with a peak at zero (a): 1, d, a

Realization 2 has very little structure (random-line) which corresponds to small or zero autocorrelations (a) and flat spectral density (d): 2, a, d

Realization 3 is pseudo-cyclic with a period of about 10 which corresponds to the autocorrelations with damped sinusoidal behavior of about period 10 (b) and a peak in the spectral density at about f=0.1 (c): 3, b, c

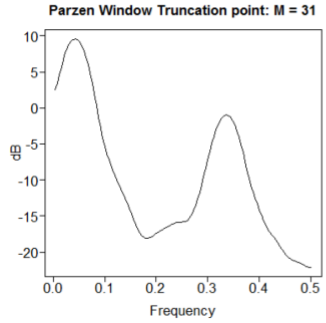
Realization 4 seems to have a pseudo-cyclic behavior with period a little less than 10 along with a higher frequency components. This corresponds to the autocorrelations in (c) that show a damped sinusoidal

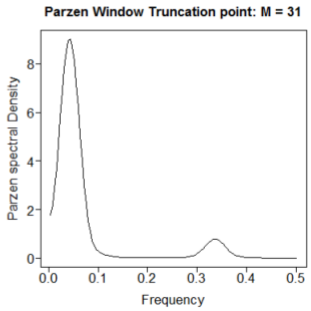
behavior of about 12 along with some high-frequency distortion. The spectral density in (b) shows a peak at about f=1/12 along with a higher frequency peak.: 4, c, b

Problem 1.5

data("fig1.21a")
parzen.wge(fig1.21a,dbcalc = FALSE)
parzen.wge(fig1.21a,dbcalc = TRUE)

We get the following plots (not showing the periodogram that is also plotted):





Both show strong indication of a peak at abouit *f*=0.05. However, the plot in dB shows the secondary peak at about *f*=0.33 much more clearly.

Problem 1.6

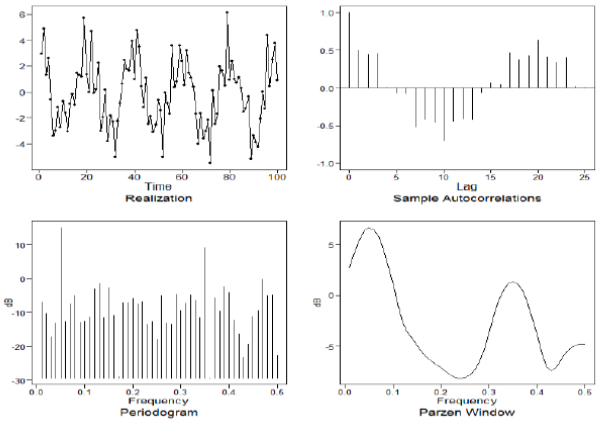
For UNIT 1 … only a – c are assigned. (d will be in Unit 2).

Problem 1.6

(a-d) The tswge R code

```
x=gen.sigplusnoise.wge(n=100,coef=c(3,1.5),freq=c(.05,.35),psi=c(0,2))
plotts.sample.wge(x)
```

produces the following plots.



The realizations shows a dominant frequency with period about 20 along with higher-frequency behavior. The autocorrelations show the periodic behavior of associated with the period 20 (f=0.05) with some indication of a higher frequency component. These two plots provide very little evidence regarding the nature of the higher frequency behavior. The two spectral plots clearly suggest frequency behavior at both *f*=0.05 and *f*=0.35.