Oregon State University

MTH 351 - Numerical Analysis

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Assignment 1

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1 Cancellation Error

a $log(x+1) - log(x)$ can be rewritten to $log(1+x/1)$ using basic log properties to avoid cancellation error near x values of -.5 and you get overflow error around$> 100,000$

b $\sqrt{1+\frac{1}{x}} - 1$

c $\frac{x - sin(x)}{x^3}$ can be separated into to fractional parts leaving $\frac{1}{x^2} - \frac{sin(x)}{x^3}$ to eliminate the cancellation errors

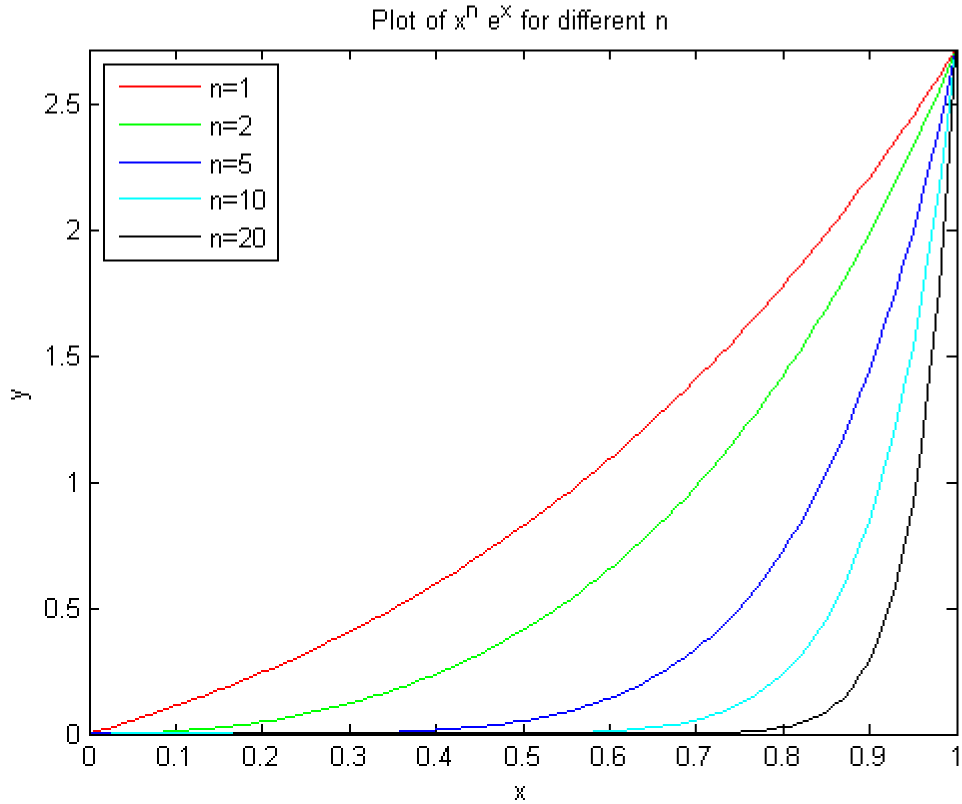
2 Eigenvalues

a Compute the eigenvalues of $\begin{bmatrix} 1 & 10^6 \\ 0 & 1 \end{bmatrix}$? The eigenvalues are 1 and 1
Compute the eigenvalues of $\begin{bmatrix} 1 & 10^6 \\ 10^{-6} & 1 \end{bmatrix}$? The eigenvalues are 2 and 0
How large is the change in the result compared to the change from matrix one to the other? They are 10^6 apart

b Compute the eigenvalues of $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$? The eigenvalues are 1 and 3
Compute the eigenvalues of $\begin{bmatrix} 1 & 2 \\ 10^{-6} & 3 \end{bmatrix}$? The eigenvalues are 1 and 3
What can you conclude about the conditioning of problem b verses problem a? I can conclude that the problem a has not been well conditioned therefor it is more sensitive to input changes

3 Unstable Recursion

a What can you say about the value of the integral I_n as n increases? What is the limit as $n \to \infty$?



b Prove the following recursive identity using integration by parts:

$$I_0 = e - 1$$

$$\begin{array}{ll} u = e & v' = -1 \\ u' = e & v = -x + c \end{array}$$

$$\int_0^1 x^n e^x dx = \left[x^n e^x - \int e^x x^{n-1} \right]_0^1$$

$$I_{n+1} = e - nI_n$$

c Implement the recursive formula from (b) in matlab. Do the values appear correct?
They appear incorrect as the values do not continuously decrease.

```
   I=exp(1)-1;
   for n = 1:25
        I_new = exp(1) - n*I(end);
        I = [I I_new];   % sticks new value on end of array using concatenation
   end
   fprintf('   n \t  I_n \n-------\t--------\n');
   fprintf('   %d\t%7.6f \n',[0:25; I]);
RESULTS
   n    I_n
---------------
   0 1.718282
   1 1.000000
   2 0.718282
   3 0.563436
   4 0.464536
   5 0.395600
   6 0.344685
   7 0.305490
   8 0.274362
   9 0.249028
   10 0.228002
   11 0.210265
   12 0.195100
   13 0.181982
   14 0.170533
   15 0.160282
   16 0.153766
   17 0.104254
   18 0.841710
   19 -13.274218
   20 268.202632
   21 -5629.536995
   22 123852.532175
   23 -2848605.521754
   24 68366535.240386
   25 -1709163378.291363
```

d Explain part c? Not sure but it will have to deal with machine epsilon

e

```
I = 0;%ENTER STARTING VALUE FOR I HERE

for n = 50:-1:1
    %ENTER BACKWARD RECURSIVE FORMULA HERE
    I_new =


    I = [I_new I];          % This sticks the new value on the
                            %  BEGINNING of the array using concatenation.
                            % access the first element of array with I(1).
end

fprintf('   n \t  I_n \n-------\t--------\n');
fprintf('   %d\t%7.6f \n',[0:50; I]);
```

f Explain the results of part (e) versus part (c), in terms of what happens to the numerical error at each step.