Drake Bridgewater
a2 MTH351

$\boxed{1}$ $\begin{bmatrix} 3 & -4\alpha \\ 3\alpha & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$x = 3, -3$

$\alpha = 4, -4$

$\alpha = 0$

$3(-1) - (-4\alpha)(3\alpha) = \det[A] = 0$

$-3 - 12\alpha = 0$

$\frac{-12\alpha}{-12} = 3/12 \quad \alpha = (1/4)^2 = 1/2$

if $\alpha = (1/4)^2$ the determinate is "0" implying that there are no solutions or there are infinitly many

in ~~the~~ All we can say that if $\alpha = 1/2$ we have infinity many solution if $\alpha = -1/2$ ther is no solutions and for all other values ther is a solution.

②

$$A = \begin{bmatrix} 2 & 2 & 2 \\ -1 & 0 & 1 \\ 3 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & -1 \\ -1 & 0 & 1 & 3 \\ 3 & 5 & 6 & 0 \end{array}\right] \xrightarrow[R_3 = R_3 - 3/2 R_1]{R_2 = R_2 + 1/2 R_1} \left[\begin{array}{ccc|c} 2 & 2 & 2 & -1 \\ 0 & 1 & 2 & 5/2 \\ 0 & 2 & 3 & 3/2 \end{array}\right]$$

$$\xrightarrow{R_3 = R_3 - 2R_2} \left[\begin{array}{ccc|c} 2 & 2 & 2 & -1 \\ 0 & 1 & 2 & 5/2 \\ 0 & 0 & -1 & 7/2 \end{array}\right]$$

$$\underbrace{LU}_{y} = \begin{bmatrix} 2 & 2 & 2 \\ -1/2 & 1 & 2 \\ +3/2 & 2 & -1 \end{bmatrix}$$

Solve

$$Ly = b \quad \begin{bmatrix} 1 & & \\ -1/2 & 1 & \\ +3/2 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$\boxed{y_1 = -1}$

$3 = -1/2\, y_1 + y_2 = -1/2 + 3 = y_2 = \boxed{y_2 = 5/4}$

$3/2\, y_1 + 2y_1 + 1y_3 = 0$

$-3/2 + 10/4 + y_3 = 0$

$y_3 = -4/4 = \boxed{-1 = y_3}$

$$y = \begin{bmatrix} -1 \\ 5/4 \\ -1 \end{bmatrix}$$

Solve $Ux = y$

$$\begin{bmatrix} 2 & 2 & 2 \\ & 1 & 2 \\ & & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 5/4 \\ -1 \end{bmatrix}$$

$2x_1 + 2x_2 + 2x_3 = -1$

$x_2 + 2x_3 = 5/4 \quad \boxed{x_2 = -3/4}$

$-1x_3 = -1 \Rightarrow \boxed{x_3 = 1}$

$2x_1 + \underset{+6/4}{-2(3/4)} + 2 = -1 \Rightarrow \dfrac{2x_1}{2} = \dfrac{-1/2}{2} \quad \boxed{-1/4 = x_1}$

$-1/2$

$\boxed{2b}$

$$A = \begin{bmatrix} 2 & 2 & 2 \\ -1 & 0 & 1 \\ 3 & 5 & 6 \end{bmatrix}$$

$$\det[A] = 2\underbrace{\begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix}}_{-5} - 2\underbrace{\begin{vmatrix} -1 & 1 \\ 3 & 6 \end{vmatrix}}_{-6-3=-9} + 2\underbrace{\begin{vmatrix} -1 & 0 \\ 3 & 5 \end{vmatrix}}_{-5} = -10 + 18 - 10 = -2$$

$\boxed{2c}$ Since we know that $A = PLU$ of a square matrix and ~~using~~ properties from linear Algebra we can infer that $\det[A] = \det[P] \cdot \det[L] \cdot \det[U]$

$$\det[L] = \det\begin{bmatrix} 1 & & \\ -1/2 & 1 & \\ 3/2 & 2 & 1 \end{bmatrix} = 1\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} - 0|X| + 0 = 1$$

$$\det[U] = \det\begin{bmatrix} 2 & 2 & 2 \\ & 1 & -1 \\ & & -1 \end{bmatrix} = 2\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix} - 2\begin{vmatrix} 0 & -1 \\ 0 & -1 \end{vmatrix} + 2\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = -2$$

$$\det[A] = \det[L] \cdot \det[U] = 1 \cdot -2 = -2$$

good news the problems have been verified and work

$\boxed{3a}$

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 8 & 6 \\ 1 & 6 & 19 \end{bmatrix} \xrightarrow[R_3 = R_3 - R_1]{R_2 = R_2 + 2R_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 4 & 8 \\ 0 & 8 & 18 \end{bmatrix}$$

$$\xrightarrow[R_3 = R_3 - 2R_2]{} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 4 & 8 \\ 0 & 0 & 2 \end{bmatrix} \qquad LU = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & 8 \\ 1 & 2 & 2 \end{bmatrix}$$

$\boxed{3b}$ $A = LDL^T$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \qquad L^T = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$A = LU$ $\qquad$ $U = DL^T$

$$U = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 4 & 8 \\ 0 & 0 & 2 \end{bmatrix} = DL^T$$

$\boxed{3c}$