Project step 2: scenario-based stochastic optimisation

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1 Problem definition

The day-ahead power market-clearing problem with transport constraints from [1], that was solved in step 1 of the project, was extended to include stochastic real-time market clearing as in [2]. As before, the day-ahead bid and offer data for October 7, 2020 from the MISO Market Data site [3] were used as a base, with some clean-up and the following additions:

- generators were divided into three categories: flexible, inflexible, and wind; generators with an offer of zero were assumed to be wind, and those with an offer price above the 90th percentile were assumed to be flexible
- flexible generators were assumed flexible over their entire generation range, while wind generators could only spill wind from their day-ahead allocation

No attempt was made to price the flexibility: it was assumed that flexible generation costs were symmetric.

The notation follows a similar convention as before, extended from a single market to the day-ahead and real-time markets. For a set of generators $\{g\}$ with production costs $\{C_g\}$, demands $\{d\}$ with utility $\{U_d\}$ and common load shedding disutility U_{shed}, buses $\{n\}$, and connections $\{t_{n,m}\}$, denote:

- day-ahead generation and consumption as $p_g^{G,DA}$ and $p_d^{D,DA}$
- the set of real-time scenarios as Ω, the individual scenarios as ω, and the corresponding probability π_ω
- real-time change in scenario ω from day-ahead commitment as $p_{\omega,g}^{G,RT}$ and $p_{\omega,d}^{D,RT}$
- maximum generation for g as $\overline{P}_g^G$, maximum demand for d as $\overline{P}_d^D$, and maximum flow through connection $t_{n,m}$ as $F_{n,m}$
- actual wind generation in scenario ω as $\overline{P}_{\omega,g}^{G,RT}$
- the difference in voltage angle between buses n and m as $\theta_n - \theta_m$ (in scenario ω: $\theta_{\omega,n} - \theta_{\omega,m}$), the susceptance of their connection $t_{n,m}$ as $B_{n,m}$, and the (rough) approximation of flow between them as $f_{n,m} = B_{n,m}(\theta_n - \theta_m)$

- the set of generators and demands in bus n as Ψ_n, and the set of buses connected to bus n via transmission lines as Ω_n
- θ_{n_0} as the reference voltage angle of some arbitrary bus n_0

The problem is still the maximisation of social welfare (the sum of the producer and consumer surplus), extended to the real-time market:

$$\max_{p_g^{G,DA}, p_d^{D,DA}, \theta_n} \left[\sum_d U_d p_d^{D,DA} - \sum_g C_g p_g^{G,DA} \right] + \sum_\omega \pi_\omega \left(\sum_d U_{shed} p_{\omega,d}^{D,RT} - \sum_g C_g p_{\omega,g}^{G,RT} \right) \tag{1.1}$$

subject to the original day ahead constraints:

$$0 \leq p_d^{D,DA} \leq \overline{P}_d^D \quad \forall d \tag{1.2}$$
$$0 \leq p_g^{G,DA} \leq \overline{P}_g^G \quad \forall g \tag{1.3}$$
$$-F_{n,m} \leq f_{n,m} \leq F_{n,m} \quad \forall t_{n,m} \tag{1.4}$$
$$\theta_{n_0} = 0 \tag{1.5}$$
$$f_{n,m} = B_{n,m}(\theta_n - \theta_m) \quad \forall t_{n,m} \tag{1.6}$$
$$\sum_{d \in \Psi_n} p_d^{D,DA} + \sum_{m \in \Omega_n} f_{n,m} - \sum_{g \in \Psi_n} p_g^{G,DA} = 0 \quad \forall n \tag{1.7}$$

and to the real-time market constraints per scenario:

$$0 \leq p_d^{D,DA} + p_{\omega,d}^{D,RT} \leq \overline{P}_d^D \quad \forall \omega, d \tag{1.8}$$
$$p_{\omega,d}^{D,RT} \leq 0 \quad \forall \omega, d \tag{1.9}$$
$$0 \leq p_g^{G,DA} + p_{\omega,g}^{G,RT} \leq \overline{P}_g^G \quad \forall \omega, g \tag{1.10}$$
$$p_{\omega,g}^{G,RT} = 0 \quad \forall \omega, g \mid g \in \text{inflexible} \tag{1.11}$$
$$p_{\omega,g}^{G,RT} = \overline{P}_{\omega,g}^{G,RT} - p_g^{G,DA} - p_{\omega,g}^{spill} \quad \forall \omega, g \mid g \in \text{wind} \tag{1.12}$$
$$0 \leq p_{\omega,g}^{spill} \leq \overline{P}_{\omega,g}^{G,RT} \quad \forall \omega, g \mid g \in \text{wind} \tag{1.13}$$
$$-F_{n,m} \leq f_{\omega,n,m} \leq F_{n,m} \quad \forall \omega, t_{n,m} \tag{1.14}$$
$$\theta_{\omega,n_0} = 0 \quad \forall \omega \tag{1.15}$$
$$f_{\omega,n,m} = B_{n,m}(\theta_{\omega,n} - \theta_{\omega,m}) \quad \forall \omega, t_{n,m} \tag{1.16}$$
$$\sum_{d \in \Psi_n} \left(p_d^{D,DA} + p_{\omega,d}^{D,RT} \right) + \sum_{m \in \Omega_n} f_{\omega,n,m} - \sum_{g \in \Psi_n} \left(p_g^{G,DA} + p_{\omega,g}^{G,RT} \right) = 0 \quad \forall \omega, n \tag{1.17}$$

Once again, the flows are represented as additional variables with their own bounds (Eqs. 1.4 and 1.14) and explicitly linked to the voltage angles (Eqs. 1.6 and 1.16) for ease of retrieval in the implementation. Variable wind production is represented as a change in capacity which will either be used or spilled; the net change will still be represented by $p_{\omega,g}^{G,RT}$ (Eq. 1.10).

[6]

As anticipated, congestion of inter-region transmission causes prices to differ between nodes; in this particular constrained case, the marginal generating unit in the North region is zero-cost (such as wind or solar). Transmission constraints, unsurprisingly, also reduce

region	price ($/MW)	consumed (MW)	supplied (MW)
North	9.54	12854.2	31372.5
Central	9.54	28181.2	18817.1
South	9.54	16367.5	7213.3

(a) Unconstrained transmission (objective value: 1.13974e+08)

	price ($/MW)	consumed (MW)	supplied (MW)
North	0.00	12854.2	22854.2
Central	11.84	28178.1	23178.1
South	11.32	16367.5	11367.5

(b) Constrained transmission (objective value: 1.13911e+08)

Table 1: Market clearing summary

the objective value (the total social welfare). As this was a relatively straightforward linear optimisation problem, and MOSEK deals with high numbers of variables more quickly than it does high numbers of constraints, the solver completed in less than 20ms in both cases. Since problem setup with the Fusion API took longer than the optimisation itself, this suggests that when repeatedly solving large numbers of cases, it will be more effective to set up the problem once, and only change parameters, than set up the problem once per iteration.

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2 References

[1] J. Kazempour, "Lecture 1: Market clearing as an optimization problem," DTU 31792 – Advanced Optimization and Game Theory for Energy Systems, Jan. 2021. [Online]. Available: https://cn.inside.dtu.dk/cnnet/filesharing/download/dcb15e39-cb18-4656-a13b-fec8ef3201bd

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[3] "MISO Market Data." [Online]. Available: https://www.misoenergy.org/markets-and-operations/real-time--market-data/market-reports/#t=10&p=0&s=MarketReportPublished&sd=desc

[4] "MISO Energy and Operating Reserve Markets Business Practices Manual," MISO, Tech. Rep. BPM-002-r19, Oct. 2020.

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