

Lab 5

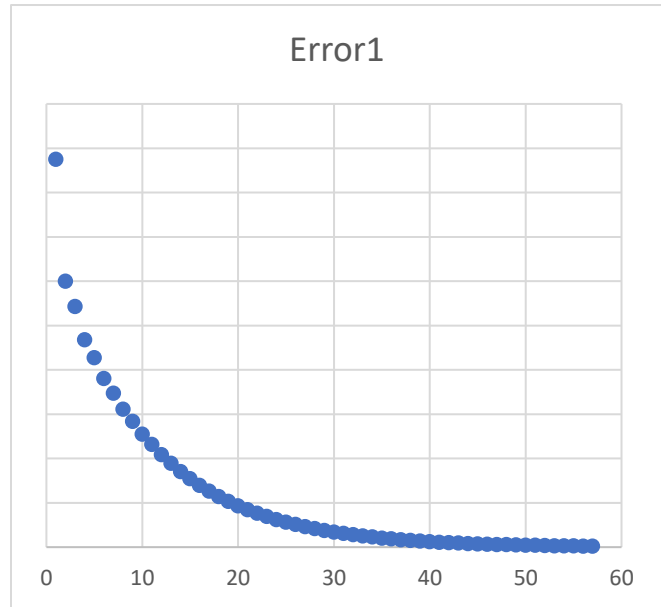
Drake Song

Problem 1

Tolerance 1 = 0.005

Tolerance 2 = 0.000000005

Iter	Error1	Error2
1	1.749779527558180	0.047302916609492
2	1.200184655800530	0.000034964048535
3	1.086679033695640	0.000000726867146
4	0.935498931902723	0.000000006204431
5	0.855689971871339	0.00000000043393
6	0.761113261754396	
7	0.695136924864191	
8	0.622654096186992	
9	0.567561616249839	
10	0.509604154601984	
11	0.463814987503336	
12	0.417029050595115	
13	0.379119635748372	
14	0.341238122506553	
15	0.309932272426954	
16	0.279206728175346	
17	0.253400748907834	
18	0.228443781515760	
19	0.207201101145229	
20	0.186904782271519	
21	0.169438183743130	
22	0.152914974971153	
23	0.138566522858425	
24	0.125103347567783	
25	0.113325377108180	
26	0.102347701100176	
27	0.092685784968479	
28	0.083729513347325	
29	0.075807563579564	
30	0.068496984516346	
31	0.062004407740830	
32	0.056034797264306	
33	0.050715535216471	
34	0.045839365221341	
35	0.041482619962926	
36	0.037498568516232	
37	0.033931004503634	
38	0.030675160567855	
39	0.027754382702944	
40	0.025093180655571	
41	0.022702302475767	
42	0.020526828293103	
43	0.018569963411309	
44	0.016791354144107	
45	0.015189884242581	
46	0.013735602735825	
47	0.012425095187852	
48	0.011235908080982	
49	0.010163574677811	
50	0.009191097261731	
51	0.008313702098451	

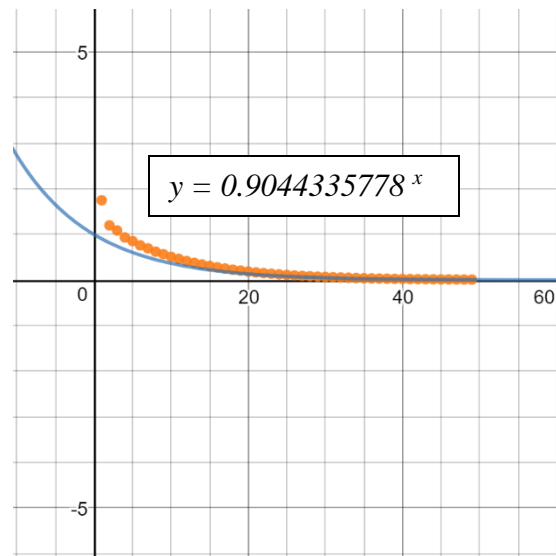


52	0.007518401251004
53	0.006800540417630
54	0.006150108508662
55	0.005562796868743
56	0.005030826625201
57	0.004550337772662

The convergence rate of Power Method is

$$|\lambda_2 / \lambda_1|$$

where λ_2 is the second largest eigenvalue in terms of magnitude and λ_1 is the largest eigenvalue in terms of magnitude. Graphing the errors and the rate of convergence line, it can be seen that the line is a fair representation of the convergence of the program. The second eigenvalue for this problem converges quite quickly so not too much analysis can be done with the second eigenvalue.



Eigenvalues:

[-18.14533333, 10.54134240 + 0.70809136j, 10.54134240 - 0.70809136j, 20.06264854]

Problem 2

Using numpy's linalg tool, the eigenvalues for this problem are:

[1.73205081, -1.73205081, -1.00000000, -3.29824209e-17, 1.00000000]

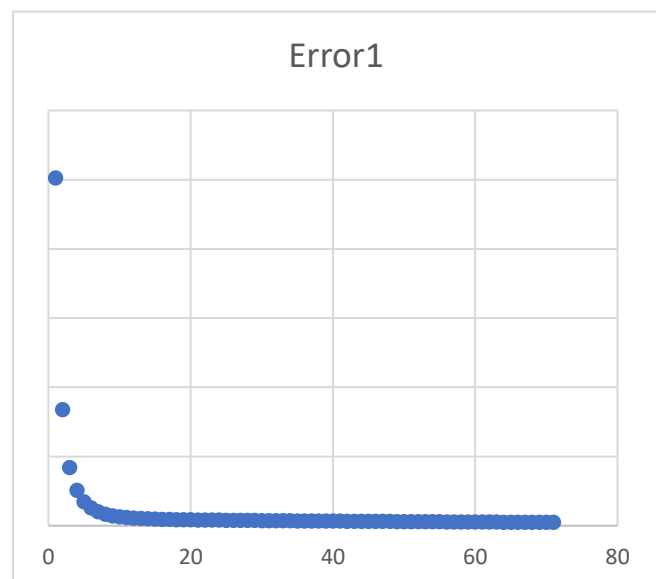
When the program is run, the iterations always exceed the maxCount set. This is due to the fact that there are two dominant eigenvalues: 1.73205081 and -1.73205081. Because there isn't one unique dominant eigenvalue, the Power Method fails for this problem.

Problem 3

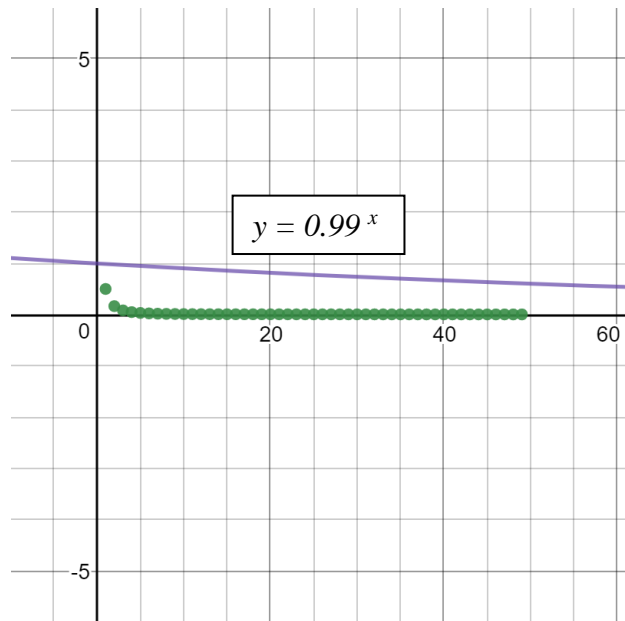
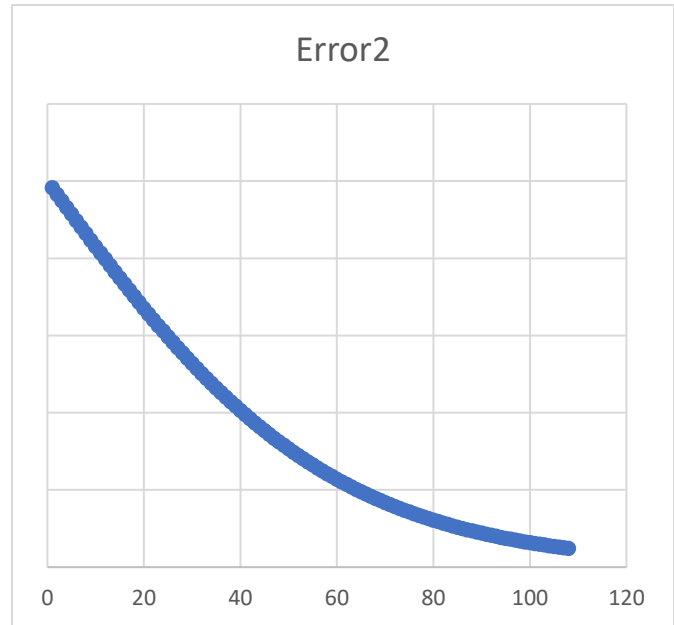
Tolerance 1 = 0.005

Tolerance 2 = 0.05

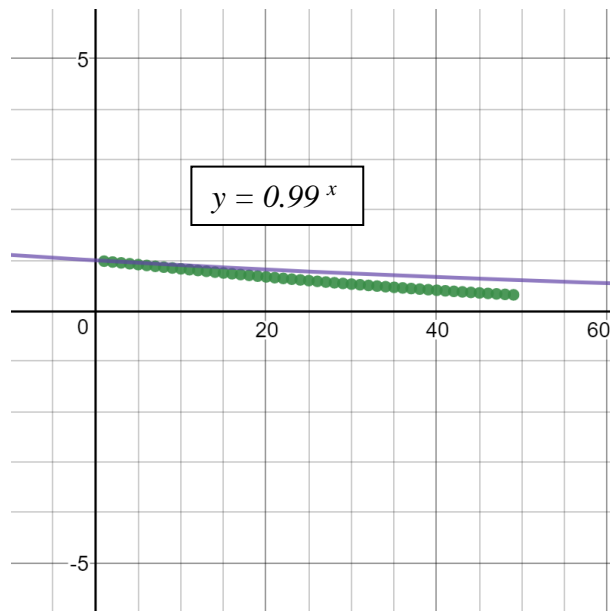
Iter	Error1	Error2
1	0.5026130552783910	0.9829072315559130
2	0.1677937652504600	0.9658244479344800
3	0.0843222016038899	0.9487616106412980
4	0.0511808731832370	0.9317286346634430
5	0.0348558624736815	0.9147353654979890
6	0.0257586421316224	0.8977915565238840
7	0.0202794745312183	0.8809068468263490
8	0.0168012727411883	0.8640907395797790
9	0.0145061195747115	0.8473525810909450
10	0.0129428090217375	0.8307015405976580



11	0.0118468540435076	0.8141465909136230
12	0.0110566614611843	0.7976964900004470
13	0.0104706233416570	0.7813597635445710
14	0.0100234566975424	0.7651446886045500
15	0.0096724531851887	0.7490592783888980
16	0.0093892073285611	0.7331112682151110
17	0.0091545244029503	0.7173081026912760
18	0.0089552314959249	0.7016569241534540
19	0.0087821509367530	0.6861645623832540
20	0.0086287944527710	0.6708375256200440
21	0.0084905101472876	0.6556819928752730
22	0.0083639176205043	0.6407038075469130
23	0.0082465287982187	0.6259084723238710
24	0.0081364899992816	0.6113011453633410
25	0.0080324041801076	0.5968866377167210
26	0.0079332068830530	0.5826694119713520
27	0.0078380786101038	0.5686535820730390
28	0.0077463822056052	0.5548429142830710
29	0.0076576176134176	0.5412408292245480
30	0.0075713888422168	0.5278504049636290
31	0.0074873796026150	0.5146743810701650
32	0.0074053351684914	0.5017151635983620
33	0.0073250487503013	0.4889748309254850
34	0.0072463511702116	0.4764551403845160
35	0.0071691029753569	0.4641575356264830
36	0.0070931883669366	0.4520831546453500
37	0.0070185104927892	0.4402328384007880
38	0.0069449877717719	0.4286071399715320
39	0.0068725510047439	0.4172063341752190
40	0.0068011410894906	0.4060304275902240
41	0.0067307072024584	0.3950791689172490
42	0.0066612053436414	0.3843520596197290
43	0.0065925971657132	0.3738483647850330
44	0.0065248490269516	0.3635671241499090
45	0.0064579312213475	0.3535071632371540
46	0.0063918173497549	0.3436671045528640
47	0.0063264838038834	0.3340453787966310
48	0.0062619093410219	0.3246402360393710
49	0.0061980747320598	0.3154497568291100
50	0.0061349624689852	0.3064718631841280
51	0.0060725565208619	0.2977043294404060
52	0.0060108421294839	0.2891447929197420
53	0.0059498056376303	0.2807907643913690
54	0.0058894343442186	0.2726396382994380
55	0.0058297163817252	0.2646887027353130
56	0.0057706406121103	0.2569351491334110
57	0.0057121965381745	0.2493760816741570
58	0.0056543742278242	0.2420085263795490
59	0.0055971642491710	0.2348294398886990
60	0.0055405576147565	0.2278357179045820
61	0.0054845457334787	0.2210242033039540
62	0.0054291203690449	0.2143916939054540
63	0.0053742736039663	0.2079349498927310
64	0.0053199978082716	0.2016507008904390
65	0.0052662856122533	0.1955356526941180
66	0.0052131298826619	0.1895864936545870
67	0.0051605237018636	0.1837999007206860
68	0.0051084603495445	0.1781725451441070
69	0.0050569332866139	0.1727010978519980
70	0.0050059361410044	0.1673822344929610
71	0.0049554626951194	0.1622126401648100
72		0.1571890138302900
73		0.1523080724314090
74		0.1475665547093420
75		0.1429612247407730
76		0.1384888752001520



77	0.1341463303579640
78	0.1299304488249070
79	0.1258381260535650
80	0.1218662966068100
81	0.1180119362039980
82	0.1142720635557310
83	0.1106437419969660
84	0.1071240809291530
85	0.1037102370812300
86	0.1003994155995820
87	0.0971888709764812
88	0.0940759078264932
89	0.0910578815199869
90	0.0881321986827714
91	0.0852963175700125
92	0.0825477483236056
93	0.0798840531198550
94	0.0773028462161846
95	0.0748017939035321
96	0.0723786143717006
97	0.0700310774946757
98	0.0677570045418605
99	0.0655542678215965
100	0.0634207902628247
101	0.0613545449402579
102	0.0593535545484655
103	0.0574158908296453
104	0.0555396739593305
105	0.0537230718960586
106	0.0519642996966780
107	0.0502616188033044
108	0.0486133363043421



The eigenvalues are:

[100, 98, 99]

$|\lambda_2 / \lambda_1|$ gives us the convergence rate of 0.99. This convergence rate only seems to be fitting the second eigenvalue from this program.

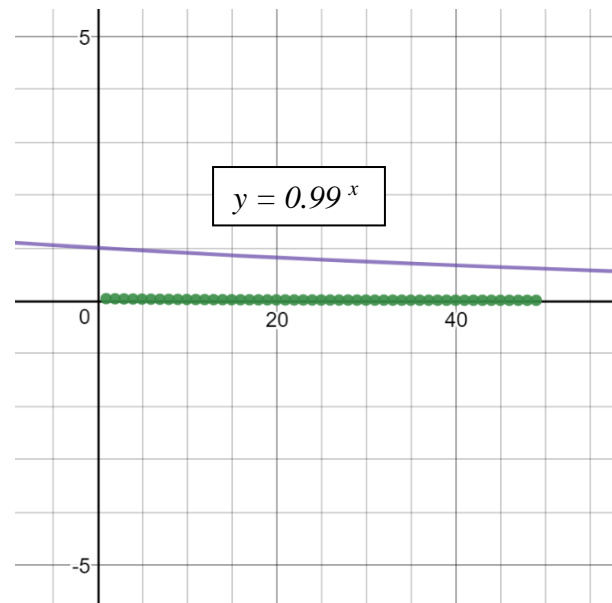
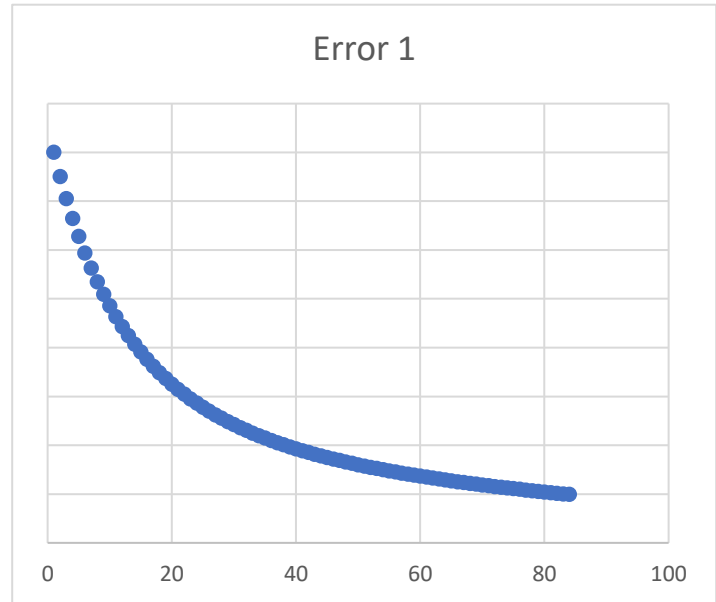
Problem 4

Tolerance 1 = 0.005

Tolerance 2 = 0.05

Iter	Error 1	Error 2
1	0.04003015170502590	0.01709276844408690
2	0.03753857747249620	
3	0.03528888644681630	
4	0.03325126018052920	
5	0.03140030402343750	
6	0.02971428152977470	
7	0.02817449930424790	
8	0.02676480942098260	
9	0.02547120441088940	
10	0.02428148563923210	
11	0.02318499025116610	
12	0.02217236514685900	

13	0.02123537894284420
14	0.02036676478600900
15	0.01956008835869450
16	0.01880963655563420
17	0.01811032320538630
18	0.01745760890959090
19	0.01684743262693090
20	0.01627615306839710
21	0.01574049832156030
22	0.01523752240327520
23	0.01476456766737630
24	0.01431923217786110
25	0.01389934130765290
26	0.01350292294521710
27	0.01312818579147010
28	0.01277350031185810
29	0.01243738197659490
30	0.01211847647847380
31	0.01181554666462710
32	0.01152746095777620
33	0.01125318307530310
34	0.01099176288200510
35	0.01074232823558270
36	0.01050407770348560
37	0.01027627404633230
38	0.01005823837720070
39	0.00984934491808359
40	0.00964901628504551
41	0.00945671924238565
42	0.00927196087363593
43	0.00909428512370445
44	0.00892326967204353
45	0.00875852310157578
46	0.00859968233226681
47	0.00844641029188943
48	0.00829839379967847
49	0.00815534164133331
50	0.00801698281624582
51	0.00788306493993827
52	0.00775335278655365
53	0.00762762695787548
54	0.00750568266678886
55	0.00738732862436429
56	0.00727238602087106
57	0.00716068759202179
58	0.00705207676262632
59	0.00694640686062751
60	0.00684354039517980
61	0.00674334839306065
62	0.00664570978825825
63	0.00655051086007261
64	0.00645764471551907
65	0.00636701081221235
66	0.00627851451827380
67	0.00619206670612400
68	0.00610758337730706
69	0.00602498531576323
70	0.00594419776718522
71	0.00586515014232366
72	0.00578777574228117
73	0.00571201150401808
74	0.00563779776444322
75	0.00556507804160890
76	0.00549379883165085
77	0.00542390942023915
78	0.00535536170740202



79	0.00528811004468839
80	0.00522211108371488
81	0.00515732363522937
82	0.00509370853788984
83	0.00503122853602531
84	0.00496984816570479

The eigenvalues are:

[100, 98, 99]

This problem was interesting in that the second eigenvalue this program found (99) after one iteration. Decreasing the tolerance even by slightest amount caused the program to exceed maximum number of iterations. The rate of convergence is also 0.99 here but the rate of convergence for the first eigenvalue (100) does not seem to follow the rate too closely.

Problem 5

Tolerance 1 = 0.005

Iteration	Error
1	9
2	0

The eigenvalues are:

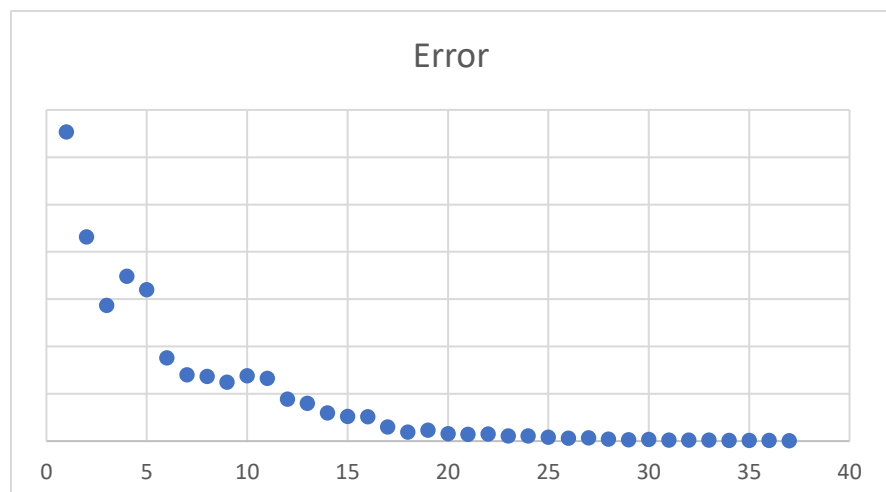
$[7.5 + 2.39791576j, 7.5 - 2.39791576j, 4.0]$

The eigenvalue 4 was found after 2 iterations so not much analysis can be done. Decreasing the tolerance caused the program to exceed maximum number of iterations. Also, the other eigenvalues could not be found because they are not real.

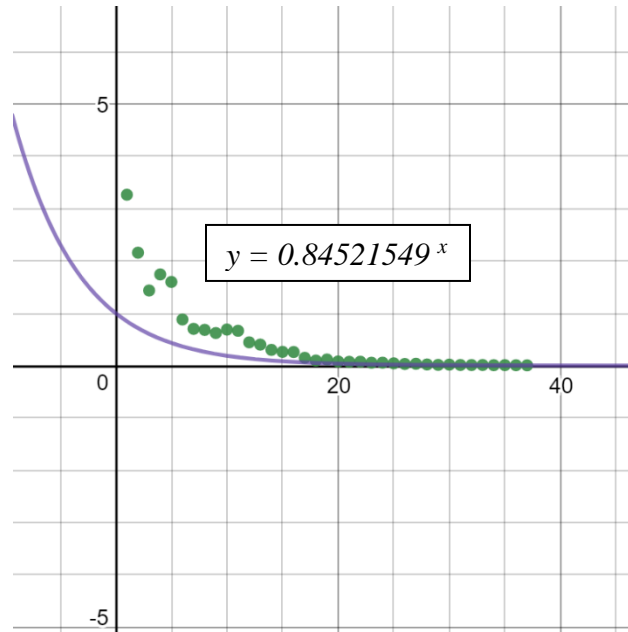
Problem 6

Tolerance 1 = 0.005

Iter	Error
1	3.267231961079590
2	2.158103149670110
3	1.434930792213300
4	1.742135529597690
5	1.598994841720940
6	0.880126617609123
7	0.700914496282795
8	0.681747516507147
9	0.621668792914787
10	0.688033871228638
11	0.663993004863340
12	0.443277839055898
13	0.399890202941338



14	0.298883539251917
15	0.260960888293451
16	0.258177270701831
17	0.147110186037756
18	0.095045837288158
19	0.114941284707525
20	0.077416784040026
21	0.070839346226724
22	0.072735426925268
23	0.053218516296898
24	0.054340149895504
25	0.041340785929741
26	0.029189385536548
27	0.033312022298274
28	0.019989611484919
29	0.012100320815564
30	0.017069619818900
31	0.010050550610871
32	0.008943417260320
33	0.009781209994151
34	0.006381489535914
35	0.007315217830840
36	0.005619875888317
37	0.003498936290925



The randomly selected matrix is:

$$\begin{bmatrix} -6 & -39 & 2 & 5 & -27 \\ -4 & 44 & 45 & 44 & -48 \\ -47 & -22 & -46 & -5 & 29 \\ -13 & 9 & 47 & 30 & 3 \\ -50 & 10 & -1 & -39 & -16 \end{bmatrix}$$

The eigenvalues are:

$$[65.46771615, 22.02333440 + 50.76278736j, 22.02333440 - 50.76278736j, \\ -51.75719248 + 14.69641382j, -51.75719248 - 14.69641382j]$$

The program could only find one eigenvalue because there is only one real eigenvalue for this matrix. The rate of convergence was found using the eigenvalue with the highest magnitude.